Measurement of the combined rapidity and $p_T$ dependence of dijet azimuthal decorrelations in $p\bar{p}$ collisions at $\sqrt{s} = 1.96\text{ TeV}$

We present the first combined measurement of the rapidity and transverse momentum dependence of dijet azimuthal decorrelations, using the recently proposed quantity $R_{\Delta \phi}$. The variable $R_{\Delta \phi}$ measures the fraction of the inclusive dijet events in which the azimuthal separation of the two jets with the highest transverse momenta is less than a specified value for the parameter $\Delta \phi_{\text{max}}$. The quantity $R_{\Delta \phi}$ is measured in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV, as a function of the dijet rapidity interval, the total scalar transverse momentum, and $\Delta \phi_{\text{max}}$. The measurement uses an event sample corresponding to an integrated luminosity of 0.7 fb$^{-1}$ collected with the D0 detector at the Fermilab Tevatron Collider. The results are compared to predictions of a perturbative QCD calculation at next-to-leading order in the strong coupling with corrections for non-perturbative effects. The theory predictions describe the data, except in the kinematic region of large dijet rapidity intervals and large $\Delta \phi_{\text{max}}$.

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In high-energy collisions of hadrons, the production rates of particle jets with large transverse momentum with respect to the beam direction, $p_T$, are predicted by perturbative Quantum Chromodynamics (pQCD). At second order in the strong coupling constant, $\alpha_s$, pQCD predicts only the production of dijet final states. In the absence of higher-order radiative effects, the jet directions are correlated in the azimuthal plane and their relative azimuthal angle $\Delta \phi_{\text{dijet}} = |\phi_{\text{jet1}} - \phi_{\text{jet2}}|$ is equal to $\pi$. Deviations from $\pi$ (hereafter referred to as “azimuthal decorrelations”) are caused by radiative processes in which additional jets are produced. The amount of the decorrelation is directly related to the multiplicity and the $p_T$ carried by the additional jets. The transition from soft to hard higher-order pQCD processes can be studied by examining the corresponding range of azimuthal decorrelations from small to large values. This makes measurements of dijet azimuthal decorrelations an ideal testing ground for pQCD predictions of multijet production processes. In pQCD, dijet azimuthal decorrelations are predicted to depend not only on the transverse momentum of the jets, but also on the rapidity $y^* = |y_{\text{jet1}} - y_{\text{jet2}}|/2$, obtained from the rapidity difference of the two leading $p_T$ jets in an event [1]. In a previous analysis of dijet azimuthal decorrelations, we measured the dijet differential cross section as a function of $\Delta \phi_{\text{dijet}}$, integrated over a fixed jet rapidity range and normalized by the inclusive dijet cross section, for different requirements on the leading jet $p_T$ [2]. The same methodology was later used in analyses of $pp$ collision data at $\sqrt{s} = 7$ TeV from the CERN Large Hadron Collider [3, 4]. In all cases, dijet azimuthal decorrelations have been observed to decrease with increasing $p_T$; however, the combined rapidity and $p_T$ dependence has not yet been measured.

In this Letter, we perform a measurement of the rapidity and the $p_T$ dependence of dijet azimuthal decor-
relations. The analysis is based on a new quantity, $R_{\Delta \phi}$, which was recently proposed in Ref. [5] as

$$R_{\Delta \phi}(H_T, y^*, \Delta \phi_{\text{max}}) = \frac{d^2\sigma_{\text{dijet}}(\Delta \phi_{\text{dijet}} < \Delta \phi_{\text{max}})}{dH_T dy^*} \frac{d^2\sigma_{\text{inclusive}}}{dH_T dy^*}. \quad (1)$$

The quantity $R_{\Delta \phi}$ is defined as the fraction of the inclusive dijet cross section with a decorrelation of $\Delta \phi_{\text{dijet}} < \Delta \phi_{\text{max}}$, where $\Delta \phi_{\text{max}}$ is a parameter and $\sigma_{\text{dijet}}$ (inclusive) is the inclusive dijet cross section without a $\Delta \phi_{\text{dijet}}$ requirement. It is measured as a function of $\Delta \phi_{\text{max}}$, $y^*$, and of the total transverse momentum $H_T$ in the event, computed as the scalar $p_T$ sum from all jets $i$ with $p_{T1} > p_{T\text{min}}$ and $|y_i - y_{\text{boost}}| < y_{\text{max}}$ where $y_{\text{boost}} = (y_{\text{jet1}} + y_{\text{jet2}})/2$, $p_{T\text{min}} = 30 \text{ GeV}$, and $y_{\text{max}} = 2$, where jet1 and jet2 are the jets with the largest $p_T$ in the event. For $\Delta \phi_{\text{max}} \approx \pi$, $R_{\Delta \phi}$ is sensitive to soft QCD radiation, while it becomes sensitive to hard higher-order QCD processes for smaller values of $\Delta \phi_{\text{max}}$. The phase space region $\Delta \phi_{\text{max}} < 2\pi/3$ is dominated by final states with four or more jets. Since $R_{\Delta \phi}$ is defined as a ratio of cross sections, several experimental and theoretical uncertainties cancel. In pQCD, $R_{\Delta \phi}$ is computed as a ratio of three-jet and dijet cross sections which is (at leading order, LO) proportional to $\alpha_s$. While dependences on parton distribution functions (PDFs) largely cancel, $R_{\Delta \phi}$ is sensitive to the pQCD matrix elements and to $\alpha_s$.

The measurement is performed in $p\bar{p}$ collisions at $\sqrt{s} = 1.96 \text{ TeV}$, for an inclusive dijet event sample defined by the Run II midplane cone jet algorithm [6] with a cone of radius $R_{\text{cone}} = 0.7$ in $y$ and $\phi$. The dijet phase space is defined by the requirements $p_{T1} > H_T/3$, $p_{T2} > p_{T\text{min}}$, $y^* < y_{\text{max}}$, and $|y_{\text{boost}}| < 0.5$. Following the proposal in Ref. [5], $R_{\Delta \phi}$ is measured over the $H_T$ range of 180–900 GeV, in three rapidity regions of $0 < y^* < 0.5$, $0.5 < y^* < 1$, and $1 < y^* < 2$; and for $\Delta \phi_{\text{max}} = 7\pi/8, 5\pi/6, \text{ and } 3\pi/4$. The ranges in $y^*$ and $y_{\text{boost}}$, and the value of $p_{T\text{min}}$ ensure that all jets are always within $|y| < 2.5$ at $p_T$ values where the jet energy calibration and jet $p_T$ resolutions are known with high precision. The requirement $p_{T1} > H_T/3$ provides a lower boundary for the leading jet $p_T$ in each $H_T$ bin, which (together with $|y| < 2.5$) ensures that the jet triggers are efficient. The data are corrected for experimental effects and are presented at the “particle level,” which includes all stable particles as defined in Ref. [7].

A detailed description of the D0 detector is provided in Ref. [8]. The event triggering and selection, jet reconstruction, and jet energy and momentum correction are identical to those used in recent D0 multijet measurements [9–13]. Jets are reconstructed in the finely segmented liquid-argon sampling calorimeters that cover most of the solid angle. The central calorimeter covers polar angles in the range $37^\circ$–$143^\circ$ and the two endcap calorimeters extend this coverage to within $1.7^\circ$ of the nominal beamline [8]. The transition regions between the central and the endcap calorimeters contain scintillator-based detectors to improve the energy sampling. The jet transverse momenta are calculated using only calorimeter information and the location of the $p\bar{p}$ collision. The position of the $p\bar{p}$ interaction is determined from the tracks reconstructed based on data from the silicon detector and scintillating fiber tracker located inside a 2 T solenoidal magnet [8]. The position is required to be within 50 cm of the detector center in the coordinate along the beam axis, with at least three tracks pointing to it. These requirements discard (7–9)% of the events, depending on the trigger used. For this measurement, events are triggered by inclusive jet triggers. Trigger efficiencies are studied as a function of $H_T$ by comparing the inclusive dijet cross section in data sets obtained by triggers with different $p_T$ thresholds in regions where the trigger with lower threshold is fully efficient. The trigger with lowest $p_T$ threshold is shown to be fully efficient by studying an event sample obtained independently with a muon trigger. In each inclusive jet $H_T$ bin, events are used from a single trigger which has an efficiency higher than 98%. Requirements on the characteristics of the calorimeter clusters shower shapes are used to suppress the background due to electrons, photons, and detector noise that would otherwise mimic jets. The efficiency for the shower shape requirements is above 97.5%. Contributions from cosmic ray events are suppressed by requiring the missing transverse momentum in an event to be less than 70% (50%) of the leading jet $p_T$ (before the jet energy calibration is applied) if the latter is below (above) 100 GeV. The efficiency of this requirement for signal is found to be greater than 99.5% [14, 15]. After all selection requirements, the fraction of background events is below 0.1% for all $H_T$, as determined from distributions in signal and in background-enriched event samples.

The jet four-momenta reconstructed from calorimeter energy depositions are then corrected, on average, for the response of the calorimeter, the net energy flow through the jet cone, additional energy from previous beam crossings, and multiple $p\bar{p}$ interactions in the same event, but not for the presence of muons and neutrinos [14, 15]. These corrections adjust the reconstructed jet energy to the energy of the stable particles that enter the calorimeter except for muons and neutrinos. The absolute energy calibration is determined from $Z \rightarrow e^+e^-$ events and the $p_T$ imbalance in $\gamma +$ jet events in the region $|y| < 0.4$. The extension to larger rapidities is derived from dijet events using a similar data-driven method. In addition, corrections in the range (2–4)% are applied that take into account the difference in calorimeter response due to the difference in the fractional contributions of quark and gluon-initiated jets in the dijet and the $\gamma +$ jet event samples. These corrections are determined using jets simulated with the PYTHIA event generator [16] that have been passed through a GEANT-based detector simulation [17]. The total corrections of the jet four-momenta
FIG. 1: (Color online.) The results for $R_{\Delta \phi}$ as a function of $H_T$ in three different regions of $y^*$ and for three different $\Delta \phi_{\text{max}}$ requirements. The error bars indicate the statistical and systematic uncertainties summed in quadrature. The theoretical predictions are shown with their uncertainties.

vary between 50% and 20% for jet $p_T$ between 50 and 400 GeV. An additional correction is applied for systematic shifts in rapidity due to detector effects [14, 15].

The procedure that corrects the distributions $R_{\Delta \phi}(H_T, y^*, \Delta \phi_{\text{max}})$ for experimental effects uses particle-level events, generated with SHERPA 1.1.3 [18] with MSTW2008LO PDFs [19] and with PYTHIA 6.419 [16] with CTEQ6.6 PDFs [20] and tune QW [21]. The jets from these events are processed by a simulation of the detector response which is based on parametrizations of jet $p_T$ resolutions and jet reconstruction efficiencies determined from data and of resolutions of the polar and azimuthal angles of jets, obtained from a detailed simulation of the detector using GEANT.

The $p_T$ resolution for jets is about 15% at 40 GeV, decreasing to less than 10% at 400 GeV. To use the simulation to correct for experimental effects, the simulation must describe all relevant distributions, including the $p_T$ and $|y|$ distributions of the three leading $p_T$ jets, and the $\Delta \phi_{\text{dijet}}$ distribution. To achieve this, the generated events, which are used in the correction procedure, are weighted, based on the properties of the generated jets, to match these distributions in data. The bin sizes in the $H_T$ distributions are chosen to be approximately twice the $H_T$ resolution. The bin purity, defined as the fraction of all reconstructed events that were generated in the same bin, is above 50% for all bins, and only weakly dependent on $H_T$. We then use the simulation to determine correction factors for experimental effects for all bins. The correction factors are computed bin-by-bin as the ratio of $R_{\Delta \phi}$ without and with simulation of the detector response. These also include corrections for the energies of unreconstructed muons and neutrinos inside the jets. The total correction factors for $R_{\Delta \phi}$ using the weighted PYTHIA and SHERPA simulations agree typically within 1% for $\Delta \phi_{\text{max}} = 7\pi/8$ and $5\pi/6$ and between 1–4% for $\Delta \phi_{\text{max}} = 3\pi/4$. The total correction factors, defined as the average values from PYTHIA and SHERPA, are 0.98–1.0 for $\Delta \phi_{\text{max}} = 7\pi/8$, 0.95–0.99 for $\Delta \phi_{\text{max}} = 5\pi/6$, and 0.81–0.91 for $\Delta \phi_{\text{max}} = 3\pi/4$, with little $y^*$ dependence. The difference between the average and the individual corrections is taken into account as the uncertainty attributed to the model dependence.

In total, 69 independent sources of experimental systematic uncertainties are identified, mostly related to jet energy calibration and jet $p_T$ resolution. The effects of each source are taken as fully correlated between all data points. The dominant uncertainties for the $R_{\Delta \phi}$ distributions are due to the jet energy calibration ($2–5\%$), and the model dependence of the correction factors ($1–4\%$). Smaller contributions come from the jet $\phi$ resolution ($0.5–2\%$), from the uncertainties in systematic shifts in $y$ (< 2%), and the jet $p_T$ resolution (< 1%). All other sources are negligible. The systematic uncertainties are 2–3% for $\Delta \phi_{\text{max}} = 7\pi/8$ and $5\pi/6$ and 3–5% for $\Delta \phi_{\text{max}} = 3\pi/4$. A detailed documentation of the results, including the individual contributions to the uncertainties, is provided in the supplementary material [22].

The results for $R_{\Delta \phi}(H_T, y^*, \Delta \phi_{\text{max}})$ are listed in Tables I–III and displayed in Fig. 1 as a function of $H_T$, in different regions of $y^*$ and for different $\Delta \phi_{\text{max}}$. A subset of the data points from selected $H_T$ regions are
also shown in Fig. 2, where $R_{\Delta\phi}$ is displayed as a function of $y^*$ for different choices of $\Delta\phi_{\text{max}}$. The values of $H_T$ and $y^*$ at which the data points are presented correspond to the arithmetic centers of the bins. Figure 1 shows that for all choices of $\Delta\phi_{\text{max}}$ and in all $y^*$ regions, $R_{\Delta\phi}$ decreases with $H_T$. In all $y^*$ regions, the $H_T$ dependence increases towards lower $\Delta\phi_{\text{max}}$, and for all $\Delta\phi_{\text{max}}$ requirements the $H_T$ dependence becomes stronger for smaller $y^*$. This implies that the $y^*$ dependence of $R_{\Delta\phi}$ increases with increasing $H_T$, as shown in Fig. 2.

The theoretical predictions for $R_{\Delta\phi}$ are obtained from a pQCD calculation in next-to-leading order (NLO), in $\alpha_s$, with corrections for non-perturbative effects. The latter include contributions from hadronization and the underlying event. The non-perturbative corrections are determined using PYTHIA 6.426 with tunes AMBT1 [23] and DW [21] which use different parton shower and underlying event models. The hadronization correction is obtained from the ratio of $R_{\Delta\phi}$ on the parton level after the parton shower and the particle level including all stable particles, both without the underlying event. The underlying-event correction is computed from the ratio of $R_{\Delta\phi}$ computed at the particle level with and without underlying event. The total correction is given by the product of the two individual correction factors for hadronization and the underlying event. The total corrections vary between $+1\%$ and $-1\%$ for tune AMBT1 and between $+1\%$ and $-3\%$ for tune DW. The results obtained with the two tunes agree typically within $1\%$ and always within $3\%$ [5]. The central results are taken to be the average values, and the uncertainty is taken to be half of the difference. As a cross-check, the non-perturbative corrections are also derived with HERWIG 6.520 [24, 25], using default settings. The HERWIG and PYTHIA results agree typically within $0.5\%$, and always within $1\%$ (3\%) for $\Delta\phi_{\text{max}} = 7\pi/8$ and $5\pi/6$ (for $\Delta\phi_{\text{max}} = 3\pi/4$) [5].

The NLO (LO) pQCD prediction for $R_{\Delta\phi}$ is computed as the ratio of the NLO (LO) predictions for the numerator and the denominator. The NLO prediction for the numerator (denominator) is obtained from an $O(\alpha_s^3)$ ($O(\alpha_s^2))$ cross section calculation. These results are computed using FASTNLO [26, 27] based on NLOJET++ [28, 29], in the MS scheme [30] for five active quark flavors. The calculations use the next-to-leading logarithmic (two-loop) approximation of the renormalization group equation and $\alpha_s(M_Z) = 0.118$ in the matrix elements and the PDFs, which is close to the current world average value of $0.1184 \pm 0.0007$ [31]. The MSTW2008NLO PDFs [19] are used, and the central choice $\mu_0$ for the renormalization and factorization scales is $\mu_R = \mu_F = \mu_0 = H_T/2$, which is identical to $\mu_0 = p_T$ for inclusive jet and dijet production at LO. The theoretical predictions are overlaid on the data in Figs. 1 and 2, and some properties are displayed in Fig. 3. The PDF uncertainties are computed using the up and down variations of the 20 orthogonal PDF uncertainty eigenvectors, corresponding to the 68% C.L., as provided by MSTW2008NLO. The PDF uncertainties are typically 1\%, and never larger than 2\%. The $R_{\Delta\phi}$ results obtained with the CT10 [32] and NNPDFv2.1 [33] PDF parametrizations agree with those for MSTW2008NLO within 2\%. The theoretical uncertainties are dominated
by the uncertainties of the pQCD calculations due to the 
μ_R and μ_F dependencies. These are computed as the rela-
tive changes of the results due to independent variations 
of both scales between μ_0/2 and 2μ_0, with the restriction 
of 0.5 < μ_R/μ_F < 2.0. The uncertainties from the scale 
dependence are 4-6% for Δφ_{max} = 7π/8 and 5π/6, and 
6-20% for Δφ_{max} = 3π/4, decreasing with H_T. In addition 
to the scale dependence, the NLO k-factors provide addi-
tional information on the convergence of the pertur-
bative expansion, and therefore on the possible size of 
missing higher order contributions. The NLO k-factors 
are computed as the ratio of the NLO and the LO pre-
dictions for R_{Δφ} (k = R_{Δφ}^{NLO}/R_{Δφ}^{LO}). Figure 3 shows 
the inverse of the NLO k-factors and their dependence on y^* 
and Δφ_{max}.

Ratios of data and the theoretical predictions are dis-
played in Fig. 3 as a function of H_T in all regions of y^* 
and Δφ_{max}. To quantify the agreement, χ^2 values are 
determined that compare data and theory, taking into 
account the correlations between all uncertainties. The 
χ^2 definition is the same that was used in our recent α_s 
determinations [12, 34]. Table IV displays the χ^2 values 
for all H_T bins within each of the nine kinematic regions 
in y^* and Δφ_{max}. The results are shown for three dif-
ferent choices of μ_R and μ_F, including the central choice 
μ_R = μ_F = H_T/2 and the combined lower and upper 
variations, H_T/4 and H_T. The following discussion distinguishes 
between the three different kinematic regions, which are given by Δφ_{max} = 3π/4, by y^* > 1, and by y^* < 1 with Δφ_{max} = 7π/8 or 5π/6.

The region of large azimuthal decorrelations, Δφ_{max} = 
3π/4, is challenging for the theoretical predictions, since 
it receives large contributions from four-jet final states. 
These are only modeled at LO by the O(α_s^4) calculation 
for the numerator of R_{Δφ}, which causes the large NLO 
k-factors (up to 1.5) and the large scale dependence (up 
to 21%), seen in Fig. 3. In this kinematic region, the 
central theoretical predictions are consistently below the 
data (often by 15-25%). Within the large scale uncer-
tainty, however, they agree with the data, as the χ^2 val-
ues for the lower scale choice H_T/4 are all consistent with 
the expectations based on the number of degrees of free-
dom (N_{dof}, which corresponds here to the number of data 
points), of χ^2 = N_{dof} ± √2N_{dof}.

In the kinematic region y^* > 1, the theoretical predic-
tions exhibit a different H_T dependence as compared to 
lower y^*, as seen in Fig. 1. While at lower y^* the pre-
predicted H_T dependence of the R_{Δφ} distributions is mono-
tonically decreasing, the H_T distributions for y^* > 1 
have a local minimum around ≈ 0.5 TeV above which 
R_{Δφ} increases. For Δφ_{max} = 5π/6, the theoretical 
predictions give an adequate description of the data. For 
Δφ_{max} = 7π/8, however, the predicted H_T dependence 
differs from that of the measured R_{Δφ} distribution, as 
quantified by the large χ^2 regardless of the scale choice. 
This is the only kinematic region in (Δφ_{max}, y^*) for which 
the NLO k-factor is consistently below unity (0.89–0.81) 
over the entire H_T range. This may indicate a poor con-
vergence of the perturbative expansion.

The perturbative expansion works best in the kin-
while they decrease with φ intervals except in the kinematic region of large dijet rapidity in perturbative effects, give a good description of the data, total transverse momentum y at fixed recently proposed quantity relations is presented. The measurement is based on the scale dependence is small (< 6%) and the NLO k-factors are above unity but small (1.00 < k < 1.06). In all of those regions, the theoretical predictions give a good description of the data.

In summary, the first measurement of the combined rapidity and pT dependence of dijet azimuthal correlations is presented. The measurement is based on the recently proposed quantity RΔφ, which probes dijet azimuthal correlations in a novel way. It is measured in pp collisions at √s = 1.96 TeV as a function of the total transverse momentum HT, the rapidity y*, and the parameter Δφmax. For all values of Δφmax and at fixed HT, dijet azimuthal correlations increase with y*, while they decrease with HT over most of the HT range at fixed y*. Predictions of NLO pQCD, corrected for non-perturbative effects, give a good description of the data, except in the kinematic region of large dijet rapidity intervals y* > 1 and small correlations Δφmax = 7π/8.

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[1] Rapidity y is related to the polar scattering angle θ with respect to the proton beam direction by y = 1/2 ln[(1 + β cos θ)/(1 β cos θ)], where β is defined as the ratio of the magnitude of momentum and energy, β = |p|/E. In 2 → 2 processes, the variable y corre-
TABLE III: The results for $R_{\Delta \phi}$ with their relative uncertainties for $\Delta \phi_{\text{max}} = 3\pi/4$.

$$\begin{array}{ccc}
H_T & y^* & R_{\Delta \phi} \\
\text{(GeV)} & & \text{(percent)} \\
180-205 & 0.0-0.5 & 4.659 \times 10^{-2} \\
205-235 & 0.0-0.5 & 4.439 \times 10^{-2} \\
235-270 & 0.0-0.5 & 4.055 \times 10^{-2} \\
270-310 & 0.0-0.5 & 3.405 \times 10^{-2} \\
310-360 & 0.0-0.5 & 2.913 \times 10^{-2} \\
360-415 & 0.0-0.5 & 2.733 \times 10^{-2} \\
415-470 & 0.0-0.5 & 2.419 \times 10^{-2} \\
470-530 & 0.0-0.5 & 2.008 \times 10^{-2} \\
530-600 & 0.0-0.5 & 1.780 \times 10^{-2} \\
600-680 & 0.0-0.5 & 1.953 \times 10^{-2} \\
680-770 & 0.0-0.5 & 2.241 \times 10^{-2} \\
770-890 & 0.5-1.0 & 4.620 \times 10^{-2} \\
890-1010 & 0.5-1.0 & 4.261 \times 10^{-2} \\
1010-1220 & 0.5-1.0 & 4.152 \times 10^{-2} \\
1220-1530 & 0.5-1.0 & 3.510 \times 10^{-2} \\
1530-1800 & 0.5-1.0 & 3.578 \times 10^{-2} \\
1800-2100 & 0.5-1.0 & 2.962 \times 10^{-2} \\
2100-2410 & 0.5-1.0 & 3.107 \times 10^{-2} \\
2410-2700 & 0.5-1.0 & 2.984 \times 10^{-2} \\
2700-3000 & 0.5-1.0 & 2.532 \times 10^{-2} \\
3000-3300 & 0.5-1.0 & 2.587 \times 10^{-2} \\
3300-3600 & 1.0-2.0 & 6.873 \times 10^{-2} \\
3600-3990 & 1.0-2.0 & 6.402 \times 10^{-2} \\
3990-4700 & 1.0-2.0 & 6.169 \times 10^{-2} \\
4700-5300 & 1.0-2.0 & 6.741 \times 10^{-2} \\
5300-6000 & 1.0-2.0 & 5.218 \times 10^{-2} \\
6000-6800 & 1.0-2.0 & 5.049 \times 10^{-2} \\
6800-7700 & 1.0-2.0 & 4.505 \times 10^{-2} \\
7700-8900 & 1.0-2.0 & 4.899 \times 10^{-2} \\
8900-10100 & 1.0-2.0 & 3.504 \times 10^{-2} \\
\end{array}$$

Table shows the results for $R_{\Delta \phi}$ with their relative uncertainties for $\Delta \phi_{\text{max}} = 3\pi/4$.

TABLE IV: The $\chi^2$ values between data and theory for MSTW2008PDFs and $\Delta R = 0.118$ and for different choices of $\mu_R$ and $\mu_F$. The results are shown for each of the nine kinematic regions, defined by the $y^*$ and $\Delta \phi_{\text{max}}$ requirements, combining all $H_T$ bins inside those regions.

$$\begin{array}{cccc}
y^* & \Delta \phi_{\text{max}} & N_{\text{dof}} & \chi^2 \\
\text{range} & & & \text{for } \mu_R = \mu_F = H_T/2 \\
0.0-0.5 & 7\pi/8 & 12 & 15.1, 7.1, 12.7 \\
0.0-0.5 & 5\pi/6 & 12 & 15.7, 10.9, 20.9 \\
0.0-0.5 & 3\pi/4 & 11 & 13.1, 44.2, 104.5 \\
0.5-1.0 & 7\pi/8 & 11 & 11.8, 6.9, 8.6 \\
0.5-1.0 & 5\pi/6 & 11 & 5.6, 4.0, 12.6 \\
0.5-1.0 & 3\pi/4 & 10 & 15.4, 26.9, 60.2 \\
1.0-2.0 & 7\pi/8 & 10 & 29.7, 24.1, 19.8 \\
1.0-2.0 & 5\pi/6 & 10 & 9.3, 10.8, 10.7 \\
1.0-2.0 & 3\pi/4 & 9 & 10.3, 23.1, 45.5 \\
\end{array}$$