THE RESPONSE OF CATHETER MANOMETER SYSTEMS USED FOR DIRECT BLOOD PRESSURE MEASUREMENTS

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PROEFSCHRIFT
TER VERKRIJGING VAN DE GRAAD VAN DOCTOR IN DE WISKUNDE EN NATUURWETENSCHAPPEN AAN DE KATHOLIEKE UNIVERSITEIT TE NIJMEGEN, OP GEZAG VAN DE RECTOR MAGNIFICUS DR. S.J. GEERTS, HOOGLERAAR IN DE FACULTEITEN DER GENEESKUNDE EN DER WISKUNDE EN NATUURWETENSCHAPPEN, VOLGENS BESLUIT VAN DE SENAAT IN HET OPENBAAR TE VERDEDIGEN OP VRIJDAG 20 MEI 1966 DES NAMIDDAGS TE 2 UUR

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CHAPTER I

INTRODUCTION

Use
Catheter-manometer systems are widely used in clinical and physiological research for measuring blood pressure and, with differential manometers or two ordinary manometers, for measuring blood flow.

Static response
Faithful recording requires linearity of static response and absence of both hysteresis and drift of the zero level. In practice non linearity, hysteresis and drift are always present, but they should give an error of less than 1 percent of the full scale. These requirements are usually easily fulfilled by the commercially available catheter-manometer systems together with the associated electronic instruments.

Dynamic response
Another requirement not easily fulfilled is that the dynamic response should show a flat amplitude frequency characteristic and a linear phase-frequency characteristic in the frequency range of interest. Catheter-manometer systems are usually filled with water or a saline solution (Ringer's solution), the catheter fluid transferring the pressure at the tip of the catheter to the manometer. From a hydraulic point of view the catheter with its distributed hydraulic resistance, inductance and compliance can be considered as a hydraulic transmission line and the manometer as a hydraulic capacitance, being the load at the end of the catheter. This kind of system will generally not have a flat amplitude response curve.
as at least one resonance peak will occur. However, the amplitude response curve can still be sufficiently flat in the range of important frequencies, if the resonance frequency is high enough.

In this case the catheter-manometer system can be used if large catheter artefacts do not spoil the record. The latter are movements of the catheter, chiefly bending movements, which induce pressure fluctuations ringing out in the resonance frequency of the system. If the amplitude characteristic is flat throughout, then these resonance phenomena will not occur and the catheter artefacts are considerably smaller. Therefore, even if the flat frequency range below the resonance frequency is large enough, it is always better to flatten the amplitude response characteristic, while maintaining a linear phase response.

The narrower the catheter or the weaker the manometer, the more the resonance frequency shifts to lower values thus diminishing the flat part of the amplitude response curve. For this reason the record of blood pressure of children and small animals are often severely distorted. In these cases narrow catheters must be used, while at the same time the range of important frequencies is extended since the heart frequency is higher.

Filling the catheter and manometer air free is another difficulty. Air increases the hydraulic capacity of the manometer, thus shifting the resonance peak to lower values and decreasing the flat portion of the amplitude response characteristic. The slightest amount of air, not readily seen even with a transparent plastic manometer chamber can cause considerable changes in the amplitude response curve. Removing this air in the ordinary way by flushing and
flushing the system will not yield a reproducible amplitude response characteristic.

There are two different ways to flatten the amplitude response curve: hydraulic damping or more commonly electrical damping. The former is better than the latter as will be shown in chapter IX. However, only if the amplitude response curve is known can both methods be properly applied. It is possible to calculate theoretically the amplitude response characteristic - at least to estimate position and height of the resonance peak roughly if the manometer or catheter is air free. This is usually never the case unless the manometer is boiled out (Hansen 1949), as described in this thesis in chapter V. In other cases it is necessary to use a pressure generator, which has been adapted to the catheter-manometer system in use.

The work described in this thesis originated in an attempt to meet the practical requirements for faithful direct blood pressure measurements. The first problem to be solved was finding a method of filling catheter manometer systems such that reproducible amplitude response characteristics are obtained. For testing catheter-manometer systems it was necessary to have a good pressure generator. The design of such a generator requires theoretical and experimental knowledge of the behaviour of pressure generators loaded with catheter-manometer systems. In order to give physicians easily usable information about the relevant properties of catheter-manometer combinations, including the appropriate values of electrical or hydraulic damping, a theoretical description of the behaviour of catheter-manometer systems was necessary. Experimental data were needed to check the
theory.

The output-input relation of catheter-manometer systems was calculated by van Brummelen (1961) assuming that the catheter behaved like a hydraulic transmission line with a resistance and inductance calculated by Womersly (1955) for a rigid tube and a distributed hydraulic capacity originating from the radial compliance of the catheter. This catheter is loaded with the hydraulic capacitance of the manometer. That the system behaves like such a hydraulic transmission line must be proved experimentally, and as this thesis shows this is not the case. Moreover, van Brummelen used the value of the catheter compliance from the work of Hansen (1949), who measured the compliance of his catheter Charriere number 8 incorrectly, resulting in too large a value.

Experimental measurements concerning the output-input relation were done by Noble (1959) and Noble, et al (1963) using a pressure generator, whose properties had not been previously determined; hence, their results are open to question. Hansen (1949) also measured the output-input relation and concluded that the catheter-manometer system behaves as a hydraulic transmission line. However, his resistance and inductance did not have the correction factors of Womersley (1955). Moreover, the parameters of the catheters were measured too roughly and the fit of the experimental points with the calculated curves was very bad. Frank (1903), on the contrary, considered the catheter-manometer system as a system with one degree of freedom, a so-called lumped circuit, shown in Fig. 10. However, he also did not use the Womersley correction factors and the determination of the catheter parameters was also rough.
In chapter II the construction of a pressure generator is described and its properties are compared with those developed by others.

In chapter III a theoretical discussion of the hydraulic properties of the pressure generator and the manometer is given.

In chapter IV the hydraulic properties of the pressure generator are studied experimentally.

In chapter V the difficulties of testing catheter-manometer systems are described. Methods of getting reproducible experiments are given.

In chapter VI a theoretical formulation of catheter-manometer systems, considered both as a transmission line and as a lumped π-circuit, is given. In order to compare the results of both calculations knowledge of the hydraulic parameters of the catheters and manometer actually used was needed. The experimental determination of these parameters is discussed in chapter VII.

Chapter VIII deals with the experimental tests of the catheters in combination with the manometer. The experimental results were compared with the theoretically predicted values obtained by using measured hydraulic parameters from the equations of chapter VI. The predictions of the lumped π-circuit model agree reasonably well with the experimental data, while those of the transmission-line-model show large deviations from the data. The cause for this discrepancy is discussed and not found.

In chapter IX the methods for damping catheter-manometer systems are discussed. Both theoretical and experimental considerations indicate that parallel damping is superior to series damping and more flexible than electrical damping.
CHAPTER II

THE PRESSURE GENERATOR

SUMMARY
To test catheter-manometer systems a pressure generator is developed with an internal hydraulic capacity of 30 mm$^3$/100 mm Hg and a maximum output of 100 mm Hg$_{pp}$ in the frequency range of 0 - 100 c/s. Calculations show that the internal hydraulic impedance of the generator is sufficiently small in comparison with the impedance of the attached catheter-manometer system to guarantee an error in resonance frequency, damping constant and amplitude peak value of less than 1 percent. Other pressure generators constructed by Noble (1959), Linden (1959) and Yanof et al. (1963) do not fulfill these requirements because their internal impedance is too high.

REQUIREMENTS
An ideal pressure generator should generate sinusoidal pressures of variable frequencies with an amplitude constant in time and independent of the frequency and of the hydraulic impedance of the load connected to the generator. The latter requirement means that the internal hydraulic impedance of the generator should be zero. Even though in practice this is never the case, a pressure generator can be considered suitable if the pressure amplitude is constant in time and independent of frequency in a certain frequency range to within a specified accuracy, e.g., 1 percent, and if the hydraulic impedance is less than 1 percent of the hydraulic impedance of the systems to be tested.

CONSTRUCTION
The pressure generator consists of a fluid filled chamber with a thin flexible bottom, called the diaphragm (Fig.1). The diaphragm is a phos-
phorbronce membrane, 0.1 mm thick and with a diameter of 2 cm. The top of the chamber has two openings, one used to fill the chamber with water, and the other for inserting the catheter tip. A screw compresses an O-ring against the catheter so that a water-tight joint is formed. A pair of surgical Kocher scissors close a plastic insertion in the filling tube.

Fig. 1. The pressure generator for testing catheter-manometer systems.

To the diaphragm (at the lower end of the chamber) is attached a conical shaped cavity in which rests a pin connected to the center portion of a loudspeaker cone (Philips AD 3700). The loudspeaker solenoid is driven by a sine generator (0.15-15000 c/s) and a power amplifier. The sinusoidal voltage of constant amplitude at various frequencies produced by the sine generator induces a sinusoidal current in the loudspeaker solenoid. Thus a sinusoidal force of constant amplitude at various chosen frequencies is exerted on the flexible diaphragm. The bias current through the solenoid, produced by the power amplifier, provides for a positive minimum force and pressure in the fluid.
filled chamber. The sinusoidal force exerted on the diaphragm causes a sinusoidal pressure in the chamber. The amplitude dependence of this pressure and the internal hydraulic impedance of the generator will be analysed in the next chapter, together with the properties of the manometer.

COMPARISON WITH OTHER GENERATORS

Noble (1959) and Linden (1959) also used fluid filled chambers with a membrane fixed into the wall. The force $F_1$ on the membrane was exerted by means of a telephone system and a crystal respectively. If the chamber is closed and the fluid is considered to be incompressible, the fluid acts on the membrane with an opposing pressure $p$ equal to $3F_1/\pi r^2$ (see page 25), where $r$ is the radius of the membrane. If $F_1$ varies sinusoidally, so does $p$.

If a catheter-manometer system is connected to the chamber, a displacement of the membrane occurs, introducing restoring forces which determine the hydraulic impedance of the generator. The smaller these forces are the smaller the impedance. This impedance is a capacitance since the reaction forces are proportional to the displacement. The capacity (or compliance) of the membrane itself, being the ratio between the volume displacement and the pressure difference which causes this displacement, is

$$C_g = \frac{\pi r^6}{16Eh^3} \quad (E = E^*/(1 - \mu^2), \ E^* \text{ being the Young's modulus of the membrane material, } \mu \text{ the Poisson constant, } r \text{ and } h \text{ the radius and the thickness of the membrane respectively}).$$

Applying this formula, the calculated capacity of Linden's generator is 1.2 mm$^3$/100 mm Hg, and that of ours 25 mm$^3$/100 mm Hg. Experimentally 30 mm$^3$/100 mm Hg was found (see chapter IV). Noble's generator must have had a far lower hydraulic capacity because he used a thick stiff telephone membrane. The reaction forces of the loud-speaker system used in our case are negligible in comparison with the force $F_1$. Hence the internal hydraulic impedance of this generator is almost entirely determined by $C_g$. Linden (1959), who used a barium titanate crystal, does not give in his paper the magnitude of the rigidity
of the crystal. If the restoring force in this case were not negligible, a capacity $C_c$ would have to be put in series with $C_g$, resulting in a smaller hydraulic capacity and hence a greater internal hydraulic impedance.

An entirely different device was constructed by Yanof et al. (1963). They used a plunger pump, which delivered a sinusoidal volume displacement of water to the system to be tested. If the material of the pump chamber is assumed to be infinitely rigid, this system can be considered as a volume generator connected in parallel to the hydraulic capacity $C_w$ (due to the water compressibility) of the 65 cm$^3$ of water in the pump chamber. However, a volume generator $Q$ in parallel with a capacity $C_w$ is equivalent to a pressure generator $Q/C_w$ in series with a capacity $C_w$. For Yanof et al.'s case then $C_w = 65 \times 0.0065 = 0.4 \text{ mm}^3/100 \text{ mm Hg}$, a capacity even lower than that of the generator of Linden.

ERRORS CAUSED BY THE INTERNAL GENERATOR IMPEDANCE; THEORY

In chapter VI the error due to the internal impedance of the generator with the catheter-manometer systems used for this thesis is shown to be much less than 1 percent. However, if other manometers are used, which are not as stiff as the one used here, the error introduced by the generator impedance in the test of those catheter manometer systems is greater.

In chapter VIII it is shown that the catheter-manometer system can be approximately described by a lumped $\pi$-circuit shown in Fig. 10 (page 57). In Fig. 2, which shows the connection of this circuit to the pressure generator $R$, $L$ and $C$ are the hydraulic resistance, inductivity and capacity of the catheter respectively, while $C_m$ is the manometer capacity.

The values of $R$ and $L$ depend on $\omega$, the radial frequency, as is shown in chapter VI. For the following calculations the values of $R$ and $L$ at the resonance frequency are used.
The output pressure $P_o$ is:

$$P_o = P_g \frac{C_g}{C_g + C_m + C} \cdot \frac{1}{1/j\omega(C_m + C/2)} + 1/j\omega(C_g + C/2) \cdot \frac{1}{(C_m + C/2)(C_g + C/2)} + R + j\omega L.$$  \hspace{1cm} (1)

The undamped resonance frequency $f_o$ is:

$$f_o = \frac{1}{2\pi} \left[ \frac{1}{L(C_m + C/2)} + \frac{1}{C_g + C/2} \right] = \frac{1}{2\pi} \left[ \frac{1}{L(C_m + C/2)(C_g + C/2)} \right]$$

and the damping coefficient $\beta$ is:

$$\beta = \frac{1}{2} R \left[ \frac{1}{L} \frac{(C_m + C/2)(C_g + C/2)}{C_m + C_g + C} \right].$$  \hspace{1cm} (3)

From (2) and (3) it can be seen that with decreasing values of $C_g$ the undamped resonance frequency $f_o$ shifts to higher, and the damping coefficient $\beta$ to lower values. Relative to the values obtained with an ideal pressure generator ($C_g = \infty$) $f_o$ and $\beta$ are a factor

$$\left[ 1 + (C_m + C/2)(C_g + C/2) \right]^{\frac{1}{2}}$$

higher and lower respectively. For errors to be smaller than 1 percent, $C_g + C/2 > 50(C_m + C/2)$. 

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Fig. 2. The pressure generator $p_g$ with the internal capacity $C_g$ connected to a catheter with capacity $C$, resistance $R$, and inductivity $L$ and a manometer with capacity $C_m$. 
ERRORS CAUSED BY THE INTERNAL GENERATOR IMPEDANCE; PRACTICE

The commercially available manometers for arterial pressure recording have values for $C_m$ which are smaller than or equal to $0.1 \text{ mm}^3/100 \text{ mm Hg}$. The biggest value for $C$ is found for catheter Charriere number 8, being $C = 0.135 \text{ mm}^3/100 \text{ mm Hg}$ (see table IV of chapter VII). Then the error in $f_0$ and $\beta$ is smaller than 1 percent, if $C_g > 8.3 \text{ mm}^3/100 \text{ mm Hg}$. This condition is fulfilled by our pressure generator with the loudspeaker, but not by the other pressure generators mentioned above. Moreover, even a manometer for venous pressure recording with $C_m = 0.3 \text{ mm}^3/100 \text{ mm Hg}$ in combination with all current catheters of normal length (about 1 meter) can be tested with our loudspeaker pressure generator.

REMARKS ABOUT LEAKS

In a personal communication, Mr. Yanof told me that the frequency response curve of catheter manometer systems tested with his plunger pump generator had a low frequency cut off. This is probably due to the leaks around the plunger which introduces a hydraulic conductance in parallel to the output of the generator. Only devices which are closed by a membrane, and consequently leakproof guarantee a flat frequency response down to 0 c/s. All the connections between generator-catheter-manometer must also be watertight.
CHAPTER III

THE HYDRAULIC PROPERTIES
OF THE PRESSURE GENERATOR
AND OF THE MANOMETER

SUMMARY

In the previous chapter it was asserted that our pressure generator has a flat amplitude frequency characteristic in the range of 0 - 100 c/s and has an internal capacity of 30 mm³/100 mm Hg. Furthermore, it was asserted that the manometer used represents a hydraulic capacity of 0.057 mm³/100 mm Hg. Both assertions have to be demonstrated theoretically and experimentally. The theoretical calculations are given in this chapter while the experimental evaluation of the various parameters are described in the following one. The theoretical calculations are helpful because they provide an insight into the processes which play a role and show which parameters are relevant. This insight is necessary to interpret the experiments in the following chapter. These calculations result in values of the various parameters deviating slightly from the experimental ones. The reason for the discrepancy is that some physical quantities are known only to within 20 percent of their true value.

The theoretical results are that the pressure generator generates sinusoidal pressures with an amplitude of 100 percent at frequencies \( \nu < 10 \text{ c/s} \); 101 percent at \( \nu = 27 \text{ c/s} \); 104 percent at \( \nu = 55 \text{ c/s} \); 109 percent at \( \nu = 83 \text{ c/s} \). The capacity \( C_g \) of the generator is greater than 19 mm³/100 mm Hg for frequencies below 100 c/s. The manometer has a hydraulic capacity of \( C_m = 0.057 \text{ mm³/100 mm Hg} \), increasing 2.7 percent at 80 c/s.

THE BALANCE OF FORCES AT THE MEMBRANE

Both the pressure generator and the manometer consist of a fluid filled chamber closed at one end with a flexible membrane. At the open
air side of the membrane rests a pin, which in the case of the pressure generator is attached to a loudspeaker cone (Philips AD 3700), and in the case of the manometer is a part of a displacement transducer (Philips differential transformer PR 9310).

Fig. 3 gives a schematic diagram of the chamber, membrane and pin. The membrane bulges out to the right on account of the fluid pressure $\rho$ in the chamber and at the same time to the left on account of the force $F$, directed to the left, with which the pin presses against the membrane. The pressure $\rho$ acts on the whole surface of the membrane, the force $F$ only on the center. Therefore, the membrane takes the shape shown with some exaggeration in Fig. 3.

![Diagram of the chamber, membrane, and pin.](image)

Fig. 3. The model for the pressure generator and for the manometer.

The displacement $f$ of the center of the membrane is composed of the displacement $f_p$ caused by the pressure $\rho$ alone, $F$ being constant and the displacement $f_F$ caused by the force $F$ alone, $\rho$ being constant. For small displacements:

$$f = f_p + f_F. \quad (4)$$

The relations between $f_F$ and $F$, or $f_p$ and $\rho$ respectively are for a circular membrane, fixed at the rim:
\[
e_1 = -\frac{F}{f_F} = \frac{4\pi Eh^3}{3r^2} \quad \text{(Brömser, 1920)} \tag{5}
\]

and

\[
\frac{p}{f_p} = \frac{16Eh^3}{3r^4} \left[ 1 + 0.53 \left( \frac{fp}{h} \right)^2 \right] \quad \text{(Federhofer, 1918 and 1936).} \tag{6}
\]

\(E\) is equal to \(E^*/(1 - \mu^2)\), where \(E^*\) is the Young's modulus of the membrane material, \(\mu\) is the Poisson constant (about 1/3), and \(h\) is the thickness of the membrane and \(r\) its radius.

In the systems under consideration \(fp/h < 0.1\); hence the term between the brackets at the right side of (6) can be approximated by 1. Then

\[
\frac{p}{f_p} = \frac{16Eh^3}{3r^4} = \frac{4e_1}{\pi r^2}. \tag{6a}
\]

In considering the dynamical equation of forces the suspension of the loudspeaker pin is represented in Fig. 3 by a coiled spring with force constant \(e_2\). The pin has a mass \(m\) and a friction coefficient \(R\) symbolised by a dashpot. For a positive displacement \(f\), the inertial force \(m\frac{d^2f}{dt^2}\), the friction force \(R\frac{df}{dt}\), the suspension force \(e_2f\), the force \(F_1\) of the loudspeaker solenoid and the force \(e_1.f_F\) of the membrane on the pin are directed to the left side of Fig. 3.

Hence, the dynamical equation of forces is:

\[
m\frac{d^2f}{dt^2} + R\frac{df}{dt} + e_2.f + e_1.f_F + F_1(t) = 0. \tag{7}
\]

Here it is assumed that the mass of the fluid in the chamber, the mass of the membrane, and the viscosity index of the chamber fluid are zero. The influence of these masses and the viscosity will be regarded afterwards.

The pin of the manometer transducer exerts no driving force on the membrane center, so that \(F_1\) can be omitted if the manometer system is discussed.
The 6 variables $\rho$, $F$, $F_1$, $f$, $f_F$, $f_p$, in (4), (5), (6a) and (7) can be reduced to 3, which we take as $p$, $F_1$ and $f_p$.

The volume change of the fluid in the chamber is $V$. For our purposes the quantity $V$ is more important than the displacement $f_p$. To eliminate $f_p$ a relation between $V$ and the displacements $f_p$ and $f_F$ is needed. The displaced volume is equal to

$$V = \alpha f_p + \beta f_F.$$ (8)

For a membrane, secured at the rim, $\alpha = \pi r^2/3$ and $\beta = \pi r^2/4$.

If simultaneously a pressure $p$ and a force $F$ is applied to the membrane so that $f = f_p + f_F = 0$, then a volume displacement results equal to $(1/3 - 1/4) \pi r^2 + 0$.

This phenomenon is due to the fact that under the influence of the pressure $p$, the periphery of the membrane bulges more than under the influence of the force $F$. Hence the shape of the membrane is as is drawn in Fig. 3. A net volume displacement results.

**THE TRANSFERFUNCTION AND THE INTERNAL IMPEDANCE**

By the transferfunction of the pressure generator is meant the relation between $p$ and $F_1$ when $V = 0$. Applying a Laplace-transformation to the $s$-domain on equations (5), (6a), (7) and (8) gives:

$$(s^2m + sR + e_2 + e_1)f(s) = -F_1(s) + e_1 f_p(s);$$ (7a)

$$p(s) = \frac{4e_1}{\pi r^2} f_p(s);$$ (6b)

$$V(s) = \alpha f_p(s) + \beta f_F(s) = \beta f(s) + (\alpha - \beta) f_p(s);$$ (8a)

$$f(s) = f_F(s) + f_p(s).$$ (4a)

These equations give a relation between $p(s)$, $V(s)$ and $F_1(s)$:

$$p(s) = \frac{4e_1}{\pi r^2} \cdot \frac{\beta F_1(s)}{(\alpha - \beta)(s^2m + sR + e_2 + e_1) + \beta e_1} + \frac{4e_1}{\pi r^2} \cdot \frac{\beta e_1}{s^2m + sR + e_2 + e_1}. $$ (9)

Substituting $\alpha/\beta = 4/3$ we get:

24
The first term on the right side of (9a) is the transfer function multiplied by the force of the solenoid of the pressure generator. It is the pressure in the s-domain if the fluid chamber is watertight, i.e., if a catheter-manometer system is connected with an infinitely large impedance at the input. For very low frequencies (s → 0) and $e_2 << 4e_1$ the transfer function is equal to $3/4r^2$. The factor 3 is due to the leverage of the force at the membrane center on the more peripheral parts of the membrane (the rim is fixed).

The second term on the right side of (9a) is the internal capacity of the pressure generator, multiplied by the incoming fluid volume. If, instead of $V, V' = -V$, the outgoing volume is written in (9a), then the sign of this term becomes minus giving the more familiar equation. Under the conditions $s → 0$ and $e_2 << e_1$, the capacity for low frequencies is equal to $C_{go} = \pi r^6/16Eh^3$. For the generator membrane $r = 1$ cm, $h = 0.01$ cm, and $E = 10^{12}$ (dynes/cm²). Hence $C_{go} = \pi \times 10^{-6}$ cm³/dyne = 26 mm³/100 mm Hg. The measurement of $C_{go}$ is described in the following chapter.

For the manometer $F_1(s) = 0$, because there is no driving force on the transducer pin. Then only the second term on the right side of (9a) remains. For low frequencies and when $e_2 << e_1$, the manometer has a hydraulic capacity with a value of $C_{mo} = \pi r^6/16Eh^3 = \pi (0.01)^6/16 \times 10^{12} \times (0.01)^3 = 4.26 \times 10^{-10}$ cm³/dyne = 0.057 mm³/100 mm Hg. This will be verified experimentally afterwards.

EXPERIMENTAL EVALUATION OF $e_1, e_2, m$ AND $R$

In order to know the behaviour of the pressure generator and manometer for frequencies which are not zero or not very low, the values of
e₁, e₂, m and R of both the loudspeaker cone and pin and the manometer transducer pin must be evaluated.

THE MANOMETER

For the Philips differential transducer PR 9310 the catalogue gives m = 1 gram and e₂ = 150 gr/cm = 1.47 x 10⁵ dynes/cm. The latter value is confirmed by our own measurements. A weight of 10 grams on a little disk, screwed on the transducer pin, is applied and the displacement measured. A tiny metal cap is screwed on to the transducer pin to give the extremity a smooth surface. Thus the total mass is increased until m = 1.25 gram. The friction coefficient R is assumed to be zero, since the frictional force in the transducer is small in comparison with the frictional forces in the membrane and the fluid.

From equation (4), e₁ = \(4\pi Eh^3/3r^2\) = \(4\pi \times 10^{12} \times (0.01)^3/3 \times 0.36^2 = 3.2 \times 10^7\) dynes/cm.

THE PRESSURE GENERATOR

The mass, elastic force constant and friction coefficient of the loudspeaker cone are measured by connecting the transducer pin to the loudspeaker pin. The loudspeaker is driven by a sinusoidal solenoid current and the resonance frequency and maximum amplitude ratio are measured. These were found to be 81 c/s and 5 respectively. e₂ was measured separately by applying a weight of 10 grams on the loudspeaker cone and measuring the displacement. The total force constant was 18.5 x 10⁵ dynes/cm. This value, with the resonance frequency and the maximum amplitude ratio mentioned above, give a total mass of 7.1 grams and R = 728 dynes sec/cm. Thus the mass of the cone and the pin of the loudspeaker is 7.1 - 1.25 = 5.9 gram and e₂ = 18.5 x 10⁵ - 1.5 x 10⁵ = 17 x 10⁵ dynes/cm. The values for the various parameters are tabulated in Table I, together with C₀ and e₁ = \(4\pi Eh^3/3r^2\) = \(4\pi \times 10^{12} \times (0.01)^3/3 \times 1^2 = 4 \times 10^6\) dynes/cm.
Table I

<table>
<thead>
<tr>
<th></th>
<th>Pressure generator</th>
<th>Manometer</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>5.9 gram</td>
<td>1.25 gram</td>
</tr>
<tr>
<td>R</td>
<td>728 dynes/sec/cm</td>
<td></td>
</tr>
<tr>
<td>e₁</td>
<td>4 x 10^6 dynes/cm</td>
<td>32 x 10^6 dynes/cm</td>
</tr>
<tr>
<td>e₂</td>
<td>1.7 x 10^6 dynes/cm</td>
<td>0.15 x 10^6 dynes/cm</td>
</tr>
<tr>
<td>C₀</td>
<td>26 mm³/100 mm Hg</td>
<td>0.057 mm³/100 mm Hg</td>
</tr>
<tr>
<td>ν₁</td>
<td>227 c/s</td>
<td>1395 c/s</td>
</tr>
<tr>
<td>ν₂</td>
<td>156 c/s</td>
<td>806 c/s</td>
</tr>
<tr>
<td>ν₃ = \sqrt{\frac{ν₁^²}{1} + \frac{ν₂^²}{2}}</td>
<td>276 c/s</td>
<td></td>
</tr>
</tbody>
</table>

THE VALUES OF TABLE I SUBSTITUTED IN FORMULA 9a

Putting \( ν₁ = \frac{1}{2π} \sqrt{\frac{3e₁}{m}} \), \( ν₂ = \frac{1}{2π} \sqrt{\frac{e₂+e₁}{m}} \), \( ν₃ = \frac{2}{1} + \frac{ν₂}{2} \) and \( s = jω (ω = 2πν) \), than (9a) becomes:

\[
p(ν) = \frac{3F(ν)}{\pi r²} \cdot \frac{4ν₁²}{3ν₃²} \cdot \frac{1}{1 - (\frac{ν}{ν₃})² + j\frac{ν}{ν₃}} \cdot \frac{V(ν)}{C₀} \cdot \frac{4}{1 + (\frac{ν}{ν₂})²} \cdot \frac{R}{1 - (\frac{ν}{ν₂})² + j\frac{ν}{ν₂} \cdot \frac{R}{2πm}}
\]

Substituting the values shown in Table I for the pressure generator, (9b) becomes:

\[
p(ν) = \frac{3F(ν)}{\pi r²} \cdot \frac{0.91 x \frac{1}{2}}{1 - (\frac{ν}{276})² + j\frac{ν}{276} \cdot 7.1 \times 10^{-2}} - \frac{V'(ν)}{C₀} \cdot \frac{4}{1 + 2.1 x \frac{1}{2}} - \frac{1 - (\frac{ν}{156})² + j\frac{ν}{156} \cdot 0.126}{2πm}
\]
The imaginary terms on the right side of (9g), representing the frictional forces of the loudspeaker cone, may be neglected for frequencies below 100 c/s. In this case the pressure generator may be considered to have an internal capacity equal to \( \frac{C_{go}}{4}(1 + 2.1 \times \frac{1}{1 - (\nu/156)^2}) \) in series with it. This capacity has at zero frequency the value 0.78 \( C_{go} \). It increases for increasing frequencies. At 100 c/s the capacity is 1.14 \( C_{go} \).

It is possible to represent the internal impedance of the generator by a simple model with hydraulic resistances, inductivities and capacities, as shown in the second term on the right side in (9g). However, to within an error of 1 percent of the absolute value and an error of less than 0.01 radians the model can be simplified in the frequency range 0 - 100 c/s by a capacity ranging from 3/4 x 25 = 19 mm\(^3\)/100 mm Hg at 0 c/s to 1.14 x 25 = 28.5 mm\(^3\)/100 mm Hg at 100 c/s.

The solenoid force pressure conversion factor being the frequency independent portion of the transfer function is equal to 0.9 x 3/\( \pi r^2 \) for \( \nu << 276 \) c/s.

This can be proved experimentally and will be discussed later. The pressure amplitude increases for increasing frequencies, being 1 percent more at 27 c/s, 4 percent more at 55 c/s and 9 percent more at 83 c/s. This was also analysed experimentally and will be discussed after the discussion on the manometer.

Equation (9b), applied to the manometer gives (\( F_I(\nu) \) is non existent):

\[
p(\nu) = \frac{V(\nu)}{C_{mo}} \cdot \frac{4}{1 + 3.0 \times \frac{1}{1 - (\frac{\nu}{806})^2}}.
\]  

(9m)

The hydraulic impedance of the manometer appears to be a capacitance \( 1/j\omega C_{mo} \) at frequencies \( \nu << 806 \) c/s. The capacity increases with increasing frequencies, being 1 percent higher than \( C_{mo} \) at \( \nu = 80 \) c/s.

28
THE INFLUENCE OF THE CHAMBER FLUID ON THE PERFORMANCE OF THE PRESSURE GENERATOR AND THE MANOMETER

So far the masses of the membrane and of the chamber fluid have been considered to be zero. Also the fluid has been considered to be frictionless. Now, first it is assumed that the membrane has zero mass, but that the fluid is a normal one. In this case the pressure $p$ at the membrane surface differs from the pressure $p_0$ at the exit of the chamber (see Fig.1).

The fluid mass is $\rho \pi r^2 l$ if $l$ is the length of the chamber and $\rho$ is the specific mass of the fluid. The dynamical equation between the pressures $p$ and $p_0$ becomes then:

$$\pi r^2 (p - p_0) = \rho \pi r^2 \frac{\delta V}{\delta t} + W (\pi r^2)^2 v;$$

(10)

$v$ is the mean outgoing velocity of the fluid and $W$ a resistance coefficient. The resistance is proportional with the velocity because the fluid current is laminar. This will be proved in the following chapter.

Because $\pi r^2 v = \frac{\delta V'}{\delta t}$ (10) becomes:

$$p - p_0 = \frac{\rho l}{\pi r^2} \frac{\delta^2 V'}{\delta t^2} + W \frac{\delta V'}{\delta t} = L \frac{\delta^2 V'}{\delta t^2} + W \frac{\delta V'}{\delta t}.$$  

(10a)

$L$ is a hydraulic inductivity equal to $\rho l/\pi r^2$, $W$ the Poiseuille resistance equal to $8\eta l/\pi r^4$, and $\eta$ the viscosity index of the fluid. For the pressure generator chamber filled with water $L = 0.89 \text{ dynes} \cdot \text{sec}^2/\text{cm}^5$ and $W = 0.071 \text{ dynes} \cdot \text{sec} / \text{cm}^5$. These values are valid, because the catheter is inserted so that the tip is situated in the chamber.

The $L$ and $W$ of the insertion tube, which has a diameter of $2$ mm and a length of $2.5$ cm (see Fig.1) is: $L = 79.5 \text{ dynes} \cdot \text{sec}^2/\text{cm}^5$ and $W = 636 \text{ dynes} \cdot \text{sec} / \text{cm}^5$. If the manometer is connected directly to the insertion tube then these values of $L$ and $W$ must be used.

Setting $\delta V'/\delta t = I$ (Equation 10a) for catheter manometer systems becomes in the $\omega$-domain:

29
\[ p_0(\omega) = p(\omega) - j\omega LI - WI = \frac{3 \times 0.9}{\pi r^2} \cdot F_1(\omega) - I\left( \frac{1}{j\omega C_{go} \times 0.78} \right) + j\omega L + W = \frac{3 \times 0.9}{\pi r^2} \cdot F_1(\omega) - I\left( \frac{1}{j\omega \times 1.5 \times 10^{-7}} \right) + 0.89 \times j\omega + 0.0715). \] (10b)

For frequencies up to 100 c/s the last two terms on the right side of (10b) (the inductive and resistive term of the internal impedance) can be neglected in comparison with the capacitive term, because even at 100 c/s the inductive term is only 5 percent of the capacitive term. The resistive term is completely negligible as one may observe. Because the external impedance must be much higher than the internal one (at least 100 times more) to have an amplitude error less than one percent, the contribution of the inductive term to the total circuit impedance is less than half per thousand and negligible. Hence the influence of the fluid in the chamber may be neglected.

The fluid chamber of the manometer is conically shaped with the diameters 0.74 and 0.2 cm, to which a tube of a diameter of 0.2 cm and a length of 8 cm is attached (see Fig.4). If the chamber is filled with water,

\[ L = \frac{\rho l}{\pi r^2} = \frac{8/\pi \times 0.1^2}{1} = 254 \text{ dynes.sec.}^2/\text{cm}^5 \text{ and} \]
\[ W = \frac{8\eta l}{\pi r^4} = \frac{8 \times 0.01 \times 8/\pi \times 0.1^4}{1} = 2040 \text{ dynes.sec.}./\text{cm}^5. \]

Thus the hydraulic impedance of the manometer is:

\[ 1/j\omega C_m + j\omega x 254 + 2040 \text{ with } C_m = 4.26 \times 10^{-10} \text{ cm}^5/\text{dynes}. \] For \( \nu = 100 \text{ c/s} \) the impedance is equal to:

\[ - j \times 3.74 \times 10^6 + j \times 1.6 \times 10^5 + 2040. \]

Hence, the resistance, represented by the last term, can be neglected and the apparent capacity is increased with about 4 percent at 130 c/s. For \( \nu = 80 \text{ c/s} \) the increment in capacity is only 2.7 percent, which for practical purposes can be neglected. If the manometer is attached directly to the generator the \( L \) and \( W \) of the manometer must be considered, and will be dealt with in the following chapter. If the manometer is connected via a narrow catheter with a \( L \) and \( W \) much larger than those of the manometer, then the \( L + W \) of the latter can be ne-
glected. The L and W for the pressure generator, its insertion tube and the manometer are tabulated in Table II.

**Table II**

<table>
<thead>
<tr>
<th></th>
<th>Pressure generator</th>
<th>Insertion tube</th>
<th>Manometer</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L ) (dynes.s(^2)/cm(^5))</td>
<td>0.89</td>
<td>79.5</td>
<td>254</td>
</tr>
<tr>
<td>( W ) (dynes.s/cm(^5))</td>
<td>( 7.1 \times 10^{-2} )</td>
<td>636</td>
<td>2040</td>
</tr>
<tr>
<td>( C ) (cm(^5)/dynes)</td>
<td>( 1.5 \times 10^{-7} )</td>
<td></td>
<td>( 4.26 \times 10^{-10} )</td>
</tr>
</tbody>
</table>

**Fig. 4.** Constructional details of the manometer with cocks and showing a hydraulic needle resistance as a parallel damper.


In the preceding paragraphs it was assumed that the membrane had zero mass. If now the membrane material is considered to have a specific mass equal to that of the fluid, the contribution of the membrane to the equation (10b) is given by increasing \( L \) by the factor \( (1 + h) / 1 = 1 + 10^{-2}/2 \) for the pressure generator; this is valid because the membrane can be considered as a part of the fluid. If the membrane material has a density \( \rho_m \) it can be replaced with a membrane with the thickness...
\[ h \rho_m / \rho \text{ and a density } \rho. \text{ This contribution can be neglected.} \]

There is no contribution to \( W \) if there is no internal friction in the membrane. It is assumed throughout that the latter contribution is negligible. For the manometer the contribution to \( L \) of the mass of the membrane is much smaller relatively than for the pressure generator. Without the conically shaped chamber, the increment of \( L \) would be \((10^{-2/8}) \times 100\%\). By narrowing the chamber to the attached tube of 2 mm diameter this increment is much smaller. Hence, for both devices the mass of the membrane has no influence.

**THE DISPLACEMENT OF THE DIFFERENTIAL TRANSFORMER-CORE IN RELATION TO THE MANOMETER PRESSURE**

Equations (7a) and (6b), applied to the manometer \( (F_l(s) = 0) \) give:

\[
 f(s) = p(s) \cdot \frac{e_1 \cdot \pi r^2 / 4e_1}{s^2 m + sR + e_2 + e_1} \quad (10c)
\]

Substitution of \( \nu_2 = \frac{1}{2\pi} \sqrt{\frac{e_2 + e_1}{m}} \) in (10c) gives:

\[
 f(\nu) = p(\nu) \cdot \frac{\pi r^2}{4(e_2 + e_1)} \cdot \frac{1}{1 - (\frac{\nu}{\nu_2})^2 + j \cdot \frac{\nu}{\nu_2} \cdot \frac{R}{2\pi \nu^2 m}} \quad (10d)
\]

In this equation \( R \) is unknown; however, for membranes the damping factor \( R / 4\pi \nu^2 m \) is always smaller than 0.7, which is the critical damping factor. Hence, for \( \nu \leq 0.1 \nu_2 \) the imaginary term in (10d) can be omitted. Furthermore \( \nu_2 = 806 \text{ c/s} \) as in equation (9m) and Table I.

Then:

\[
 f(\nu) = p(\nu) \cdot \frac{\pi r^2}{4(e_2 + e_1)} \cdot \frac{1}{1 - (\frac{\nu}{806})^2} \quad (10m)
\]

For frequencies up to about \( \nu_2 / 10 = 80 \text{ c/s} \) the relative amplitude of \( f(\nu) \) is equal to that of \( p(\nu) \) within an error of 1 percent. The output pressure then gives the right relative amplitude.
CHAPTER IV

THE EXPERIMENTAL TEST
OF THE PRESSURE GENERATOR

SUMMARY

In the preceding chapter the transfer function of the pressure generator was derived. The internal impedance of the pressure generator and the manometer was calculated.

In this chapter the measurement of the frequency response curve is discussed.

This is done by connecting the manometer directly to the exit of the pressure generator from which the experimental values of the capacity of the generator and of the transfer function pressure force ratio can be obtained.

The measured capacity of the generator was 30 mm³/100 mm Hg and the transfer function amplitude is flat from 0 to 100 c/s.

THE STATIC CONVERSION FACTOR SOLENOID FORCE-PRESSURE

Theoretical calculations in the previous chapter showed that the transfer function pressure over solenoid force for zero frequency is equal to 0.9 \times \frac{3}{\pi r^2}. To check this experimentally, the solenoid current to pin force conversion factor is needed. This is measured by applying a weight of 10 grams to the loudspeaker cone and introducing a solenoid current of such a magnitude that the pin resumes the same position it had before the weight was applied. This yielded 250 gr/Ampere. Thus, the pressure over solenoid current should be 0.9 \times 250 \times \frac{3}{\pi} \times 1.0^2 \times 1.36 = 1.6 \times 10^2 \text{ mm Hg/Ampere}.

Experimentally a conversion factor of 163 mm Hg/Ampere is obtained. A discrepancy between the experimental and calculated value is not too important, because only the relative pressure amplitude variations with frequency are of interest.
With our pressure generator an amplitude of 50 mm Hg is readily obtainable.

THE CAPACITY OF THE PRESSURE GENERATOR

The capacity \( C_g \) is measured in the following way. The filling entrance of the chamber is connected (see Fig. 1) with a soft polyethylene tube. A pair of Kocher scissors close this tube at such a distance from the chamber that the amplitude recorded by the manometer is half of that obtained with the scissors placed near the chamber. The measurement was done at sinusoidal pressures of about 10 c/s. The capacity of the tube, which was in parallel with that of the generator, as measured by a volumetric method was 30 mm\(^3\)/100 mm Hg.

Thus \( C_g = 30 \cdot \text{mm}^3/100 \text{ mm Hg} \). This value is somewhat bigger than the calculated value of 26 mm\(^3\)/100 mm Hg. In chapter VII a method will be described to measure the capacity of the manometer, which is the same as the calculated value, namely 0.057 mm\(^3\)/100 mm Hg.

THE MANOMETER DIRECTLY CONNECTED TO THE PRESSURE GENERATOR: THEORETICAL RESULTS

The manometer was directly connected to the pressure generator and the amplitude of the pressure oscillations was measured at various frequencies with the manometer. The electrical analogue for this experiment is shown in Fig. 5.

According to this model:

\[
\frac{p_0}{p_g} = \frac{1}{j\omega C_m} = \frac{1}{j\omega C_m + \frac{1}{j\omega C_g} + W_m + W_g + j\omega(L_m + L_g)}
\]

\[
= \frac{1}{1 + \frac{C_m}{C_g} - \omega^2 C_m(L_m + L_g) + j\omega C_m(W_m + W_g)}
\]

(11)

Here \( p_g \) is the frequency dependent amplitude of the pressure, given by the first term at the right side of (9g). The indexes \( g \) and \( m \) refer to the generator and the manometer respectively.
According to (9g) and (9m):

\[
\frac{C_m}{C_g} = \frac{4.26 \times 10^{-10}}{1.97 \times 10^{-7}} \cdot \frac{1 + \frac{3}{1 - (\frac{v}{806})^2}}{1 + \frac{2.1}{\left[1 - (\frac{v}{156})^2 + j \cdot \frac{v}{156} \times 0.126\right]}}.
\]

Table III shows the value of \(C_m/C_g\) for various selected frequencies.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>(C_m/C_g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 c/s</td>
<td>0.28 x 10^{-2}</td>
</tr>
<tr>
<td>100 c/s</td>
<td>0.19 x 10^{-2} + j x 0.23 x 10^{-3}</td>
</tr>
<tr>
<td>276 c/s</td>
<td>0 + j x 8.3 x 10^{-2}</td>
</tr>
<tr>
<td>300 c/s</td>
<td>3.6 x 10^{-2} + j x 1.1 x 10^{-3}</td>
</tr>
<tr>
<td>400 c/s</td>
<td>1.7 x 10^{-2} + j x 0.61 x 10^{-3}</td>
</tr>
<tr>
<td>500 c/s</td>
<td>1.1 x 10^{-2} + j x 1.5 x 10^{-4}</td>
</tr>
<tr>
<td>630 c/s</td>
<td>2.2 x 10^{-2} + j x 1.1 x 10^{-4}</td>
</tr>
</tbody>
</table>
As can be seen from the table $C_m/C_g$ has the greatest absolute value in the neighbourhood of 276 c/s.

For frequencies below 100 c/s $C_m/C_g$ can be neglected in comparison with unity in Eq. (11).

$C_m$ is dependent on the frequency as Eq. (9m) shows.

$L_m + L_g$ and $W_m + W_g$ are also dependent on the frequency, because for oscillating flow in a tube the flow pattern is intermediate between Poiseuille flow and plug flow. Then the hydraulic resistance is higher than for stationary flow (Womersley 1955 and 1957).

The factor $p$ with which the hydraulic inductivity for plug flow and the factor $q$ with which the hydraulic resistance for Poiseuille flow must be multiplied to get the hydraulic inductivity and resistance respectively are only dependent on the quantity $\alpha = r_0 \sqrt{\frac{\omega \cdot \rho}{\eta}}$. This dependence is represented in Fig. 9, chapter VI.

For the manometer $r_0 = 0.1$ cm (see Fig. 4) and for the pressure generator: $r_0 = 0.1$ cm for the insertion tube, 0.275 cm for the filling tube on top of the chamber and 2 cm for the chamber itself (see Fig. 1). For the insertion tube and for the manometer $\alpha = 0.1 \sqrt{\frac{2\pi \nu}{0.01} \times 1.00} = 2.51 \sqrt{\nu}$ and for the filling tube $\alpha = 6.87 \sqrt{\nu}$.

With these values of $\alpha$, the correction factors $p(\nu)$ and $q(\nu)$ can be obtained from Fig. 9 (chapter VI). The hydraulic inductivities for plug flow of the generator output tube and manometer tube, $L_g$ and $L_m$ respectively, must be multiplied by their respective factor $p(\nu)$ to get the values of $L_g$ and $L_m$ in Eq. (11). The Poiseuille resistances for the generator output tube and manometer tube, $W_g$ and $W_m$ respectively, must be multiplied by their respective factor $q(\nu)$ to get the values of $W_g$ and $W_m$ in equation (11).

Equation (11) gives the transferfunction of $p_0/p_g$; equation (9g) the transferfunction of $p_g/F_1$; equation (10m) the transferfunction of $f/p_0$; and 250 grf/A is the conversion factor loudspeaker current to force. These four transferfunctions should be multiplied with each other to get the transferfunction $f$/current, represented in Fig. 6 by the signal flow diagram.
Fig. 6. The signal flow diagram, representing the center deflection of the membrane of the manometer to the transfer function of the solenoid current of the pressure generator.

THE MANOMETER DIRECTLY CONNECTED TO THE PRESSURE GENERATOR; EXPERIMENTS

The manometer is directly connected to the filling tube of the generator, because this tube has a smaller hydraulic inductivity than the insertion tube.

The full line in Fig. 7 represents the measurements of the relative amplitude of the displacement transducer of the manometer if the pressure generator solenoid is driven by a current with constant amplitude at various frequencies.

The ordinate should be the absolute value of the product of the transfer functions, represented in the signal flow diagram of Fig. 6.

From Eqs. (9g), (10m), and (11) peaks are expected to occur for the corresponding transfer functions, i.e., 276 c/s due to $\frac{p_g}{F_1}$ (9g), at 806 c/s due to $f/p_o$ (10m) and a peak at an unknown frequency due to the hydraulic system (11).

However, the experimental curve shows more than three peaks. In order to find out which peaks belong to the mechanical properties of the generator system, i.e., the transfer function of $\frac{p_g}{F_1}$, a mass of 20.85 gram was applied on the loudspeaker cone and the frequency response curve was again measured. The result is the broken line in Fig. 7. The peaks all shift to lower frequencies except for the peak at 630 c/s.

Hence, the resonance peak at 630 c/s must be due to the transfer function of the hydraulic system (Eq. 11) while the other peaks at 245, 380 and 445 c/s are due to the transfer function $\frac{p_g}{F_1}$. Eq. (9g) appears to be too simple for the description of the generator system. The loudspeaker cone can apparently oscillate in different modes in this frequency range.
Fig. 7. The amplitude frequency response curves of the manometer connected directly to the pressure generator.

The resonance frequency of the hydraulic system can be calculated from $1 - \omega^2 C_m (L_m + L_g) (1 - \nu^2 / 806^2) = 0$ (Eqs. 11 and 10m). $L_m + L_g$ can be calculated from the geometrical properties of the generator chamber, the filling tube, and the manometer tube. $C_m$ is given by Eq. (9m) and table II.
The calculation gives two solutions: $\nu = 490 \text{ c/s}$ and $\nu = 1930 \text{ c/s}$. The lower frequency deviates considerably from the measured frequency of 630 c/s. It is possible that the frequency of 806 c/s in Eqs. (9m) and (10m) is higher than assumed here, since it is calculated from $e_1$ (Eq. (5)) where the uncertainties of $h$ and $E$ are 10 and 50 percent, respectively. If instead of 806 c/s in Eq. (9m) the double value or an infinitely large value is substituted, then the resonance peak at the lower frequency would occur at 556 c/s and 582 c/s, respectively. The latter frequency deviates 8 percent and the former one 12 percent from the measured frequency of 630 c/s. This deviation can be easily explained by assuming that the frequency of 806 c/s is about two times higher and the value of $L_m + L_g$ is about ten percent lower than the calculated value. This would mean that the bore of the manometer tube is about 5 percent wider than assumed, which can easily be true.

The dotted line in Fig. 7 is the transfer function $p_g/\text{current } I$, calculated by dividing the ordinates of the full line by the absolute value of the right side of Eq. (11). There are more oscillating modes of the loudspeaker cone than are visible in the full line. Because these other modes disturb considerably the shape of the line no calculation of the damping factor in the transfer function of $p_g/ (\text{current } I)$ can be valid.

It is clear from the dotted line that the pressure generator generates a frequency independent amplitude to within an error of 1 percent in the frequency range of 0 - 100 c/s.

LAMINARITY OF THE FLOW THROUGH THE TUBES

It is easy to show using Reynolds criterion that the flow through the tubes is laminar

$$Re = \frac{\bar{\rho} \bar{v} \bar{d}}{\eta} = \frac{2\bar{\rho} \bar{i}}{\pi \eta r_0} < 2000.$$  

Here $\rho$ is the density of water, $\eta$ its viscosity index, $r_0$ the tube radius, $d = 2r_0$, $\bar{v}$ the mean velocity of flow and $\bar{i}$ the mean flow current. $\bar{i}$ is maximal in the resonance condition, being equal to
\[
\bar{I} = C_m \frac{d\rho_o}{dt} = \omega \rho_o \cdot C_m \cdot \rho o_{max} = \sqrt{\frac{C_m}{L_m + L_g \cdot \rho o_{max}}} = \frac{\pi \cdot C_m \rho o_{max}^2}{\rho \cdot l}
\]

Then: \( Re = \frac{2 \sqrt{\frac{\rho \cdot C_m \cdot \rho o_{max}}{\pi \cdot l}}}{} \).

In the experiments \( \rho o_{max} < 50 \text{ mm Hg} \) and \( C_m = 0.057 \text{ mm}^3/100 \text{ mm Hg} \), \( \rho = 1.00 \) and \( \eta = 0.0100 \) so that:

\[
Re = 200 \sqrt{\frac{0.606}{1}} = 155.6
\]

In the previous experiment \( 1 = 5.3 \text{ cm} \) so that \( Re < 2000 \) and the flow is laminar. In the experiments with a catheter between generator and manometer \( 1 \) is about \( 100 \text{ cm} \). Taking into account that the capacity of the catheter, being about equal to the manometer capacity \( C_m \), is in parallel with \( C_m \), the Reynolds number will be also much smaller than 2000.

**HIGH FREQUENCY BEHAVIOUR OF THE MANOMETER TRANSDUCER**

The question arises whether the pin of the transducer stays in contact with the manometer membrane at high frequencies. The pin of the transducer presses with a force of 28 grf against the membrane. The amplitude of the pin is 5 \( \mu \text{m} \) per 100 mm Hg. The manometer pressure amplitude in the experiments described in this chapter and in chapter VIII was smaller than 50 mm Hg. The membrane amplitude was 5/2 \( \mu \text{m} = 2.5 \times 10^{-4} \text{ cm} \). The frequency at which the pin should be lifted from the membrane is then \( \frac{1}{2\pi} \left[ 28 \times 981 \right. \left(2.5 \times 10^{-4}\right)^{1/2} \right] = 1670 \text{ c/s} \). Hence, there was no amplitude distortion by pin lift. This was also verified by tracing the same curve with a smaller signal. Generally in blood pressure measurements, amplitudes of 50 mm Hg (pulse pressures of 100 mm Hg) are seldom found. Even in that case the lift-frequency is very high, being equal to 1670 c/s.

At much lower frequencies the electronic equipment can also introduce another limiting frequency. The specifications for the transducer,
Philips PR 9310 and bridge Philips Pr 9300 state that the low frequency output of the bridge is flat from 0-200 c/s and increases with 10 percent at 500 c/s and 30 percent at 800 c/s. For the measurements, described in this chapter the slight increase beyond 200 c/s does not interfere with the results. In any case these two components are adequate up to 200 c/s, which is sufficient for the measurements to be done with catheter-manometer systems.

It should be remarked that the transducer and bridge have a phase shift of 1/4 degree per c/s, which means a delay time of 0.7 msec. in the frequency range from 0-100 c/s.
CHAPTER V

REPRODUCIBILITY OF THE RESPONSE OF THE CATHETER-MANOMETER SYSTEM

SUMMARY

Before the catheter-manometer system can be tested, preliminary experiments must be done to check the reproducibility of the measurement. This depends on the constancy of the hydraulic parameters, i.e., the capacity of the manometer and the inductivity and resistance of the catheter. The manometer capacity attains the same minimum value after each filling only if this is done by means of boiling (Hansen 1949). Therefore special manometers are constructed which can withstand high temperatures. Moreover, the catheters must be rinsed with pepsin before the experiment in order to obtain reproducible hydraulic impedances. Possible colonies of bacteria are removed by this procedure. In addition, it is necessary to fill the catheters with water, filtered by a micro pore filter to avoid increment of the impedance resulting from clogging.

THE TECHNIQUE FOR FILLING THE MANOMETER

The manometer must be filled with water free of air bubbles. An air bubble increases the capacity $C_m$. The hydraulic capacity of an air bubble is equal to $V/P$, $V$ and $P$ being the volume and the pressure respectively. A volume of 0.1 mm$^3$ of air has a capacity of about 0.01 mm$^3$/100 mm Hg, which is about 20 percent of $C_m$ ($C_m = 0.057$ mm$^3$/100 mm Hg). Hence, the resonance frequency of the catheter-manometer frequency response would shift to a value smaller by 10 percent if such an air bubble is captured in the manometer.

Measurements of the frequency response of commercially available manometers and catheters showed that the results were irreproducible if the manometers were filled or flushed between tests. The mano-
meters were transparent, fitted with a plastic dome, so that any re­tained air bubbles would be visible. However, one very serious dif­ficulty with a plastic dome manometer is that it can retain air at the plastic-metal interface. Such an air pocket is invisible, can be several centimeters in length, and unlikely to be removed even by flushing the manometer vigorously. Rough estimates indicate that the manometer capacity can be several times larger than the minimum, theoretical "air free" value.

Hansen (1949) introduced a filling method consisting of boiling the manometer and showed that this procedure improves the frequency response. The manometer which we have constructed can also be boiled out (see Fig. 4). We have made two types, one of brass and the other of a polycarbonate resin. This resin can withstand a temperature of 130 degrees centigrade. Both manometers were used in the experiments to test the reproducibility of the frequency response. The manometer is placed with all cocks open in the bottom of a pan, filled with boiling water. The temperature of the manometer is thus slightly higher than 100 degrees centigrade, because it is in contact with the bottom, which has a higher temperature than the water. The steam, produced in the manometer, drives the air out, new water comes in and evaporates, again driving out the remaining air. After a quarter of an hour the boiling is stopped, the water in the pan cooled and the cocks are closed. The manometer is now sterilised and ready for use.

THE INFLUENCE OF THE METHOD OF FILLING THE MANOMETER ON THE DYNAMIC RESPONSE OF THE CATHETER-MANOMETER SYSTEM

Fig. 8 shows the frequency amplitude response curves of the catheter manometer system (catheter No. 5 and our own manometer) if the manometer is filled in the usual way, i.e., by flushing with cold, air free water, till no air bubbles are visible. In this case the manometer of transparent polycarbonate is used. The dynamic response is now measured (curve 1). Then the manometer is relshed vigorously with water and the dynamic response measured again. Curve 2 results, shifted a
Fig. 8. The influence of the method of filling the manometer on the amplitude frequency response curves of the catheter-manometer system.

little to the right of curve 1. The manometer is flushed again, giving curve 3 shifted even more to the right. This indicates that every re-flushing removes a little bit of air. Since the peak frequency increases if $C_m$ diminishes. If the manometer is boiled, curve 4 is obtained. This curve is reproducible if the manometer is boiled before every test. This curve can be approximated by the old method only after many re-flushings.
THE PROPER USE OF DAMPING REQUIRES REPRODUCIBILITY OF RESPONSE

For measuring the blood pressure of an adult the frequency response curve must be flat from 0-6 c/s, and for children from 0-12 c/s, with an error of at most 10 percent in the upper frequency (Wood, 1956; Patel, 1963). Thus, the flat part of the response curve of the air-free filled manometer is not large enough for the blood pressure measurement of children as Fig. 8 shows. Therefore an additional hydraulic or electrical filter must be introduced to flatten the curve. In order to be able to choose the right filter position of the resonance peak must be known. Hence \( C_m \) must have a reproducible value, which is guaranteed only by boiling the manometer.

OTHER METHODS OF FILLING THE MANOMETER

Other procedures for filling bubble free have been used by us and by others (Hansen 1949), e.g., filling with water with a detergent or flushing the manometer first with CO\(_2\) and then reflushing with alkaline water. Our experience in agreement with Hansen's (1947), indicate that these methods do not yield results as satisfactory as boiling.

THE REPRODUCIBILITY OF THE CATHETER HYDRAULIC PARAMETERS

If the catheter is detached and boiled for a short time or if the catheter is flushed vigorously, the curve 4 of Fig. 8 shifts again. Our initial assumption was that the catheter contained air, which was difficult to remove; however, this turned out not to be true. The hydraulic resistance of the catheter was measured by means of Poiseuille experiments (chapter VII). This resistance increases steadily after each repeated experiment. Flushing the catheter with pepsin, an enzyme which dissolves proteins (bacteria), showed that the Poiseuille resistance obtained its lowest value, which was completely reproducible. The water for the Poiseuille measurements must then be boiled and filtered with a micro pore filter (10 μ) to prevent clogging of the catheter once more. The size of the required filter pores in relation to the inner dia-
meter of the catheter (0.5 - 1 mm) suggests that the clogging must be caused by bacteria or other such organisms.

Consequently, pepsin flushed catheters both with boiled manometers were used in the experiments in which the frequency response curves of catheter-manometer systems were studied. Every time the tests were repeated, the results were reproducible.

RECOMMENDATIONS FOR THE PHYSICIANS

It is recommended that physicians clean their catheters internally with pepsin. Reproducible frequency response curves result, so that the right damping filter can be chosen. In our experience the catheter resistance can become 10 times larger than its minimal value, with the result that the resonance frequency shifts to about half of the maximal value and the peak height diminishes by a factor 0.5. We do not know how long a catheter, rinsed with pepsin, retains its minimal resistance.

Furthermore, it is recommended that the physicians use only manometers which can be boiled. The dynamic response of the catheter manometer system is improved considerably as is shown in Fig. 8.
CHAPTER VI
THEORETICAL CALCULATIONS
ON THE CATHETER-MANOMETER SYSTEM

SUMMARY

On the basis of calculations by Womersley (1955) the equations for the impedance of a rigid catheter are derived. A distensible catheter is considered as a transmission line, which can be approximated by a model having the impedance of the rigid catheter in the axial direction and the distributed compliance of the distensible catheter in the radial direction. An equation for the output-input relation of this transmission line is derived (Eq. (19j)).

It is shown that the internal capacity of the pressure generator is sufficiently large to be neglected for the measurement of the output-input relation with an error of not more than 1 percent.

Furthermore, this transmission line model can be approximated by a lumped π-circuit, involving simpler calculations. An equation for the output-input relation of this circuit is given (Eq. (16i)).

A diagram is given (Fig. 11) which represents the relative amplitude frequency response curves of the lumped circuit. With this diagram it is possible to predict the output-input relation approximately for practical purposes (chapter VIII).

THE IMPEDANCE OF A RIGID CATHETER

Before we can compare the calculated output-input relation of a catheter manometer system with experimental data, it is necessary to calculate the hydraulic parameters of a catheter. If the catheter is rigid, these calculations are not difficult. If the catheter is distensible and has a thick wall in relation to its inner diameter, the calculations become cumbersome (Womersley 1955). The latter calculations of Womersley (1955), for the distensible catheter, however, are open to
question as the rheological equations for the wall are not correct.

First, the impedance of a rigid catheter will be calculated, following Womersley for the greater part. Fluid motion obeys the Navier-Stokes equation (Lamb 1953):

\[- \text{grad } p = \rho \frac{\partial v}{\partial t} + \rho (v \nabla) v - \eta \nabla^2 v - 1/3 \eta \text{grad div } v \]  

Here \( p \) is the pressure, \( v \) the fluid velocity, \( \rho \) the fluid density and \( \eta \), the fluid viscosity index. Cylindrical coordinates will be used, \( z, r \) and \( \phi \) being the axial, radial and azimuthal coordinate respectively.

Because of azimuthal symmetry: \( v_\phi = 0 \).

If the catheter is rigid and of uniform shape in the axis-direction, the diameter remains constant in time and over its entire length. If furthermore, the fluid is considered to be incompressible, then \( v_r = 0 \) and \( \text{div } v = 0 \). This combined with \( v_\phi = 0 \) gives \( \delta v_z/\delta z = 0 \) and thus \( (v \cdot \nabla)v_z = 0 \).

Hence Eq. (13) becomes:

\[ \frac{\delta p}{\delta z} = \rho \frac{\delta v_z}{\delta t} - \eta \left( \frac{\delta^2 v_z}{\delta r^2} + \frac{1}{r} \frac{\delta v_z}{\delta r} \right); \frac{\delta p}{\delta r} = 0; \frac{\delta p}{\delta \phi} = 0. \]  

Calling the catheter inner radius \( r_0 \), Womersley (1955) put

\[ \frac{r}{r_0} = y, \quad \frac{\delta p}{\delta z} = A e^{j \omega t}, v_z = U e^{j \omega t} \]  

transformations which apply for sinusoidal fluctuations and radial frequency \( \omega \).

Then Eq. (13a) becomes:

\[ \frac{\delta^2 U}{\delta y^2} + \frac{1}{y} \frac{\delta u}{\delta y} + \alpha^2 j^3 U = - \frac{A r_0^2}{\eta}. \]  

The solution of this Bessel equation for \( v_z(r = r_0) = 0 \) is:

\[ v_z = - \frac{A r_0^2}{\eta} \frac{1}{j^3 \alpha^2} \left( 1 - \frac{J_0(\alpha y^{3/2})}{J_0(\alpha^{3/2})} \right) e^{j \omega t}. \]  

To obtain the flow \( I_e^{j \omega t} \) through the catheter, Eq. (14) must be integrated as follows:

50
\[ I e^{i\omega t} = \int_0^r 2\pi r v z \, dr. \]  

Hence:
\[ I = -\frac{\pi r_0^4}{\eta} \cdot \frac{A}{j^3 \alpha^2} \left( 1 - \frac{2 J_1(a_j^{3/2})}{a_j^{3/2} J_0(a_j^{3/2})} \right). \]  

Womersley (1955) defined the complex quantity
\[ 2 J_1(a_j^{3/2}) / a_j^{3/2} J_0(a_j^{3/2}) = F_{10}(\alpha) \]
and published tabulated values from \( \alpha = 0 \) to \( \alpha = 10 \).

The hydraulic impedance \( Z' \) of the catheter per cm length is equal to:
\[ Z' = \frac{A}{\pi r_0^4} \cdot \frac{1}{1 - F_{10}(\alpha)} \cdot \frac{1}{\pi r_0^2 \cdot 1 - F_{10}(\omega)}. \]  

If \( \eta \to 0 \), \( \alpha = r_0 \sqrt{\frac{\omega \rho}{\eta}} \to \infty \), the value \( F_{10}(\alpha) \to 0 \) and
\[ Z' = \frac{j \omega \rho}{\pi r_0^2} = j \omega L'_0, \]  

where \( L'_0 \) is a hydraulic inductivity per cm length.

For \( \rho \to 0, \alpha \to 0 \) and \( 1 - F_{10} \to j^2/8 \) and
\[ Z' = \frac{8 \eta}{\pi r_0^4} = R'_0, \]  

where \( R'_0 \) is a hydraulic resistance per cm length.

For any other value of \( \alpha \) Eq. (16a) can be written as:
\[ Z' = R'_0 \cdot q(\alpha) + j \omega L'_0 \cdot p(\alpha) = R'(\alpha) + j \omega L'(\alpha) \]  
or:
\[ Z' = R'_0 \cdot q(\omega) + j \omega L'_0 \cdot p(\omega) = R'(\omega) + j \omega L'(\omega) \]

\( q(\alpha) \) increases and \( p(\alpha) \) decreases with \( \alpha \) as is shown in Fig.9.

The values for \( p \) and \( q \) are tabulated in Table III.
Fig. 9. The Womersley correction factors $p(\alpha)$ and $q(\alpha)$ for $\alpha = r_0 \sqrt{\frac{\omega \rho}{\eta}}$ for the hydraulic plug flow inductivity and Poiseuille resistance, respectively.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$p$</th>
<th>$q$</th>
<th>$\alpha$</th>
<th>$p$</th>
<th>$q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.330</td>
<td>1.000</td>
<td>6.00</td>
<td>1.220</td>
<td>1.490</td>
</tr>
<tr>
<td>1.00</td>
<td>1.330</td>
<td>1.010</td>
<td>8.00</td>
<td>1.168</td>
<td>1.835</td>
</tr>
<tr>
<td>2.00</td>
<td>1.328</td>
<td>1.015</td>
<td>10.00</td>
<td>1.138</td>
<td>2.175</td>
</tr>
<tr>
<td>2.50</td>
<td>1.325</td>
<td>1.030</td>
<td>12.00</td>
<td>1.118</td>
<td>2.525</td>
</tr>
<tr>
<td>3.00</td>
<td>1.318</td>
<td>1.065</td>
<td>14.00</td>
<td>1.110</td>
<td>2.870</td>
</tr>
<tr>
<td>3.50</td>
<td>1.304</td>
<td>1.120</td>
<td>16.00</td>
<td>1.098</td>
<td>3.215</td>
</tr>
<tr>
<td>4.00</td>
<td>1.288</td>
<td>1.170</td>
<td>18.00</td>
<td>1.075</td>
<td>3.570</td>
</tr>
<tr>
<td>4.50</td>
<td>1.271</td>
<td>1.240</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.00</td>
<td>1.254</td>
<td>1.320</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Eqs. (17b) and (17c) represent a hydraulic resistance \( R' \) in series with a hydraulic inductivity \( L' \) per cm of catheter length, both being dependent on the frequency \( \nu = \omega / 2\pi \).

For a distensible catheter Eq. (13a) is not valid, since \( v_r \) and \( \delta p / \delta z \) are not equal to zero. Moreover, there are additional boundary conditions which depend on the behaviour of the tube. Womersley (1955) tried to calculate the pressure flow relation of a distensible tube, but the basic equations for the elastic behaviour of the tube are not valid. The mathematical problems encountered are considerable; hence, in the following the transmission line will be approximated by a model having the impedance of the rigid catheter in the axial direction and the distributed compliance of the distensible catheter in the radial direction.

**THE MODEL FOR THE DISTENSIBLE CATHETER**

In this section the catheter will be considered as a transmission line with distributed hydraulic resistances, inductivities and capacities \( R'(\omega) \), \( L'(\omega) \) and \( C' \) per cm length, respectively. \( R'(\omega) \) and \( L'(\omega) \) are the parameters for a rigid catheter as defined in the preceding paragraph and \( C' = \delta V' / \delta p \), \( V' \) being the catheter inner volume per cm length and \( p \) the pressure difference across the catheter wall. For a visco-elastic catheter wall a hydraulic resistance must be put in series with the capacity \( C' \).

The error made is that the values of the resistance \( R'(\omega) \) and the inductivity \( L'(\omega) \) which are used, are derived with the assumption \( v_r = 0 \) and \( \delta v_z / \delta z = 0 \); this is inconsistent with the assumption of a distributed hydraulic capacity \( C' \). The error decreases with decreasing \( C' \), because the ratios \( v_r / v_z \) and \( R'(\omega) v_z / \delta z \) decrease. Thus, the weaker the catheter is, the greater the error. Using the same approximation van Brummelen (1961) calculated the output-input relations of the catheter-manometer system, taking for \( C' \) the value for catheter No. 8 as measured by Hansen (1949). Hansen's measurement was incorrect and yielded too high a value for \( C' \) (see chapter VII). Because \( C' \) of catheter No. 8 has a smaller value, our approximation will be better than van Brum-
melen's. He did not compare his theoretical results with experiments. Hansen (1949) used the same approximation, mentioned above, but without the Womersley corrections (called \( p(\omega) \) and \( q(\omega) \) in this thesis). His comparisons with experimental data were very rough, so that his statement that the catheter-manometer system can be described by such a transmission line is subject to doubt.

**THE OUTPUT-INPUT RELATION OF THE TRANSMISSION LINE**

The hydraulic transmission line is a four pole with the (impedance) matrix representation:

\[
\begin{pmatrix}
  p_i \\
  p_o
\end{pmatrix} =
\begin{pmatrix}
  r_{11} & r_{12} \\
  r_{21} & r_{22}
\end{pmatrix} \cdot 
\begin{pmatrix}
  i_1 \\
  i_0
\end{pmatrix},
\]

so that

\[
\begin{pmatrix}
  r_{11} & r_{12} \\
  r_{21} & r_{22}
\end{pmatrix} = 
\begin{pmatrix}
  Z_{\text{coth } \gamma} & Z/\sinh \gamma \\
  Z/\sinh \gamma & Z_{\text{coth } \gamma}
\end{pmatrix}.
\]

Here \( \gamma = \sqrt{(R(\omega) + j \omega L(\omega))j \omega C} \) and \( Z = \sqrt{\frac{R(\omega) + j \omega L(\omega)}{j \omega C}} \) where \( R \), \( L \) and \( C \) are the hydraulic resistance, inductivity and capacity, respectively, over the entire catheter length \( l \), i.e., \( R = R' l \); \( L = L' l \) and \( C = C' l \).

The determinant of the matrix is equal to \( \Delta = Z^2 \).

The transfer function \( p_o/p_i \) for the pressure at the end (\( p_o \)) and at the tip (\( p_i \)) is equal to:

\[
\frac{p_o}{p_i} = \frac{r_{21} \cdot Z_1}{\Delta + r_{21} \cdot Z_1},
\]

\( Z_1 \) being the load of the transmission line, in this case the impedance of the manometer. For frequencies below 100 c/s, \( Z_1 = 1/j \omega C_m \), as is shown in chapter III.

The input impedance is equal to:
\[ \frac{p_i}{i_i} = \frac{\Delta + r_{11} \cdot Z_1}{r_{22} + Z_1}. \] 

(19a)

Since the pressure generator has an internal impedance equal to \(1/j\omega C_g\) for frequencies below 100 c/s (chapter III), the ratio of the manometer pressure \(p_0\) to the generator pressure \(p_g\) is equal to:

\[
\frac{p_0}{p_g} = \left[ \frac{\Delta + r_{11} \cdot Z_1}{r_{22} + Z_1} \right] \cdot \frac{r_{21} \cdot Z_1}{\Delta + r_{11} \cdot Z_1} = \frac{r_{21} \cdot Z_1}{\Delta + r_{11} \cdot Z_1} \cdot \frac{1}{(r_{22} + Z_1) / j\omega C_g}.
\]

(19b)

From Eq. (18a), Eq. (19b) becomes:

\[
\frac{p_0}{p_g} = \frac{Z \cdot \frac{1}{\sinh \gamma} \cdot \frac{1}{j\omega C_m}}{Z^2 + Z \coth \gamma / j\omega C_m + (Z \coth \gamma + \frac{1}{j\omega C_m}) \cdot \frac{1}{j\omega C_g}} = \frac{1}{Z \cdot j\omega C_m \cdot \sinh \gamma + (1 + \frac{C_m}{C_g}) \cosh \gamma + \frac{1}{j\omega C_g} \cdot \frac{\sinh \gamma}{Z}}.
\]

(19c)

Since \(Z\) can be written \(Z = \gamma / j\omega C = (R + j\omega L)/\gamma\) then Eq. (19c) becomes:

\[
\frac{p_0}{p_g} = \frac{1}{\gamma \left[ (R + j\omega L) j\omega C_m + \frac{C}{C_g} \right] + (1 + \frac{C_m}{C_g}) \cosh \gamma}.
\]

(19d)

In the denominator on the right side of Eq. (19d), \(C_m/C_g\) and \(C/C_g\) can be neglected, since \(C_m/C_g = 0.057/30 = 1.9 \times 10^{-3}\) which is small in comparison with 1. Furthermore, catheter No.8, having the largest diameter, also has the largest capacity: \(C = 0.135 \text{ mm}^3/100 \text{ mm Hg}: C/C_g = 0.135/30 = 0.45 \times 10^{-2}\), which can be neglected in comparison with 1. The comparison of \(C/C_g\) with 1 is clear if Eq. (19d) is expanded in a powerseries in \(\gamma\) (see Eq. (19f)). Adding to every term \((R + j\omega L) j\omega C_m\) in (19f) the value \(C/C_g\) gives Eq. (19d) again.
If $C/C_g$ and $C_m/C_g$ are omitted in Eq. (19d) the shift in the resonance frequency and the error in the amplitude ratio is \( \frac{0.45 + 0.19}{2} = 0.3 \) percent. Both errors are not detectable and are much smaller than the observed discrepancies between the calculations of the transmission-line-model and the experiments (see chapter VIII).

Thus to within an error of 1 percent the pressure generator can be considered to have a negligible internal impedance. Then Eq. (19d) becomes, taking $C_g \to \infty$

\[
\frac{p_0}{p_g} = \frac{1}{\sinh \gamma (R + j\omega L)j\omega C_m + \cosh \gamma}.
\]  

(19e)

Expanding $\sinh \gamma / \gamma$ and $\cosh \gamma$ in a power series gives:

\[
\frac{p_0}{p_g} = \frac{1}{1 + (R + j\omega L)j\omega C_m + \frac{\gamma^2}{2!} \left[ 1 + (R + j\omega L)j\omega C_m \right] + \frac{\gamma^4}{4!} \left[ 1 + (R + j\omega L)j\omega C_m \right]^2 + \ldots}.
\]

(19f)

The last term of the denominator of Eq. (19f) arises from the substitution of $\gamma^2 = (R + j\omega L)j\omega C$.

Eq. (19f) can be used to calculate the frequency response curve of the transmission line. For the catheters we tested - 4, 5, 6 and 8 - the omission of higher terms in the power series in the denominator of Eq. (19f) causes an error smaller than 1 percent in the frequency range zero to about 1.5 times the resonance frequency (see chapter VIII).

THE CATHETER MANOMETER SYSTEM CONSIDERED AS A LUMPED CIRCUIT

Using only the first two terms of the denominator in Eq. (19f) results in a further approximation.

Then:
Eq. (19g) is the transfer function of the lumped circuit shown in Fig. 10. It would be convenient for theoretical calculations if this approximation would be good enough for practical purposes. In chapter VIII it will be shown that this is actually the case. Using this approximation a series of frequency response curves characterized by dimensionless parameters have been drawn (Fig. 11).

\[
\frac{P_O}{P_g} = \frac{1}{1 + (R + j\omega L)j\omega(C_m + \frac{C}{2})}.
\]

(19g)

The calculation of these curves require a modification of Eq. (19g) as follows. From Eqs. (16a) and (17) it can be shown that Eq. (19g) can be written as:

\[
\frac{P_O}{P_g} = \frac{1}{1 + \frac{j\omega L'_O.1}{1 - F_{10}(\alpha)} \cdot j\omega(C_m + \frac{C}{2})}
\]

(19h)

$L'_O.1 = L_O$ and $R'_O.1 = R_O \cdot L_O$. $C_m$ and $C$ can be expressed in terms of
the dimensionless quantities \( \alpha \) and \( \beta \) by the relation

\[
\omega^2 L_0 (C_m + \frac{C}{2}) = \left( \frac{\alpha_2 \beta}{4} \right)^2
\]

From the definition of \( \alpha \): \( \alpha^4 = r_0^4 \omega^2 \rho^2 / \eta^2 \) and from the definition of the damping coefficient \( \beta \) of the lumped circuit of Fig. 10:

\[
\beta^2 = \frac{R_0^2}{4} \frac{C_m + C/2}{L_0} = \frac{16 \times \eta^2 l^2}{\pi^2 r_0^8} \cdot \frac{C_m + C/2}{\rho l} \cdot \pi r_0^2 = \frac{16 \times \eta^2}{\pi r_0^6} \cdot \frac{(C_m + C/2)}{\rho}, 1
\]

it follows that:

\[
\left( \frac{\alpha_2 \beta}{4} \right)^2 = \frac{r_0^4 \omega^2 \rho^2}{\eta^2} \cdot \frac{\eta^2}{\pi r_0^6} \cdot \frac{(C_m + C/2)}{\rho} \cdot \frac{1}{\pi r_0^2} = \omega^2 L_0 (C_m + C/2)
\]

Then Eq. (19h) can be written as:

\[
\frac{p_0}{p_g} = \frac{1}{1 - \frac{(\alpha^2 \beta / 4)^2}{1 - F_10(\alpha)}}. \quad (19i)
\]

The absolute value \( \left| \frac{p_0}{p_g} \right| \) calculated from Eq. (19i) is shown in Fig. 11 as a function of \( \alpha_2 \beta / 4 \) with the value of parameter \( \beta \) written by each curve.

\( \beta \) can be calculated from the equation written in Fig. 11 if the inner diameter, the compliance of the catheter and the manometer compliance is known. Then the proper curve can be chosen. The horizontal axis can be changed to a frequency axis if \( \alpha_2 \beta / 4 \) is written in terms of the frequency \( \nu \). Thus, from the response curve for the lumped circuit it is possible to see at which frequency the resonance peak appears, and at which frequency the deviation from the flat portion becomes unacceptable.

In chapter VIII it will be shown that this lumped circuit is a sufficiently good approximation for the catheter-manometer system. This is of great practical advantage for the prediction of the behaviour of a particular combination of manometer and catheter.
Fig. 11. The calculated amplitude frequency response curves of catheter-manometer systems, considered as a lumped \( \pi \)-circuit model. The dimensionless parameters \( \alpha \) and \( \beta \) are expressed in the indicated units for the convenience of the physician as explained in chapter X.
COMPARISON OF THE TRANSMISSION LINE WITH THE LUMPED CIRCUIT

At the resonance frequency the absolute value of the second term \((R + j\omega)(C_m + C/2)\) in the denominator of Eq. (19f) approaches 1.

Hence, in case the lumped circuit of Fig. 10 is used as an approximation of the transmission line, the error \(e\) will be of the order of

\[
e = \frac{(R + j\omega L)^2 (j\omega)^2 C (C_m + C/4)}{6} = \frac{C(C_m + C/4)}{6(C_m + C/2)^2}.
\]

This must be compared with \(j\omega R (C_m + C/2)\) the value of the first two terms at the resonance frequency. Because \(j\omega R (C_m + C/2) = j\beta\) the relative error in the peak amplitude is then

\[
\left(\frac{1}{2\beta} - \frac{\sqrt{e^2 + 4\beta^2}}{2\beta}\right) = 1 - \frac{1}{\sqrt{1 + \frac{e^2}{4\beta^2}}}.
\]

The greater the peak amplitude is, the smaller \(\beta\) and the greater the relative error.

For catheter No. 5, \(\beta\) is about 0.05 (chapter VII) and \(C = 0.049 \text{ mm}^3/100 \text{ mm Hg}\), while \(C_m = 0.057 \text{ mm}^3/100 \text{ mm Hg}\) (chapter VII). Thus, \(e = 0.085\) and the error at the resonance frequency is about 25 percent. This value is large enough to be able to distinguish between the two models if an accuracy of better than 1 percent is obtained in the experiments (chapter VIII).

For damped catheters (chapter IX) \(\beta\) is about 0.6 to 0.7 and thus the error is \(e^2/8\beta^2 = 0.085^2/8 \times 0.6^2 = 2.5 \times 10^{-3}\) being less than 1 percent. Hence, if hydraulic damping is introduced, the approximation by a lumped circuit will be very good.

For practical use the frequency response characteristic of the catheter-manometer system must be known in the region below the resonance frequency, where the deviation of \(\frac{|P_o|}{|P_g|}\) from 1 is less than about 10 to 20 percent. In this case the second term in Eq. (19f) is of the order of 0.1 to 0.2 and the third term is the square of this value multiplied by \((1 + 4 C_m/C)/(1 + 2 C_m/C)^2\) so that the error is in most cases smaller than 1 percent. From theoretical point of view the curves
of Fig. 11 can thus be used from $\alpha^2\beta/4 = 0$ to 0.3 because at this last value the deviation of $|E_2|$ from 1 is about 12 percent. Another advantage is that all the curves in Fig. 11 almost coincide in this range. Hence, to evaluate the frequency at which the deviation from the flat portion of the curve becomes serious, it is sufficient to use the lumped circuit model for catheter-manometer systems. This is of great advantage for the physician because the elaborate calculations of a transmission-line model are thus avoided.

For this reason it is recommended to manufacturers of catheters that their catheters be provided with the values of the inner diameters and compliances, as the manufacturers of manometers do with their manometers.

Furthermore, the calculated error for frequencies below the resonance frequency (if the transmission line model is approximated by a lumped circuit model), is consistent with a generalised theoretical discussion published by Vierhout (1959).

SOME GENERAL REMARKS ABOUT THE DIFFERENCE BETWEEN THE LUMPED CIRCUIT AND THE TRANSMISSION LINE

The following equation is derived from Eq. (19f) for the transmission line model in the same manner as Eq. (19i) is derived from Eq. (19g) or (19h).

$$\frac{P_0}{P_g} = \frac{1}{\left(\frac{\alpha^2\beta}{4}\right)^2 + \frac{\left(\frac{\alpha^2\beta}{4}\right)^4}{1 - F_{10}(\alpha)} \cdot \frac{1 + 4 C_m/C}{\left[1 - F_{10}(\alpha)\right]^2} \cdot \frac{1}{1 + 2 C_m/C} \cdot \frac{1}{6} \ldots}$$

(19j)

This equation can be compared with Eq. (19i) of the lumped circuit model. The peak amplitude appears at values for the second term in the denominator of Eq. (19i) and Eq. (19j) of about 1. Then the third term in the denominator of Eq. (19j) is about equal to the factor with $C_m$ and $C$. This factor is zero for $C = 0$, which is consistent with the fact that in this case the catheter is rigid and the transmission line model approaches the lumped circuit model. For $C \rightarrow \infty$ this factor becomes 1/6, which is the maximum value. Therefore, in case the
second term in the denominator is about 1/2, the third term is smaller than 1/24 so that at values of $\frac{Po}{Pg} = 2$ the deviation between the two models is only a few percent. The deviation becomes greater at the peak values, so that in our experiments the behaviour of the catheter-manometer system at the peak amplitude was compared with the calculations using the transmission line and lumped circuit models (Fig. 15a and b of chapter VIII).

However, at the peak amplitude the error in the calculation of this amplitude is greater than at smaller amplitudes, because the parameters $L_0$, $R_0$ and $C$ needed for the calculation are known with an error of 1 percent. At the peak amplitude the denominator of Eq. (19i) or Eq. (19j) becomes small and therefore the relative error is large.

Catheter No.8 has the greatest peak amplitude, amounting to about 8 times above that at zero frequency. Therefore, it is necessary to evaluate the hydraulic parameters with the greatest accuracy possible. A procedure for obtaining an error of less than 1 percent is described in the following chapter. Since the pressure generator introduces an error of the same magnitude, it is not necessary to get more accurate values for the parameters.
CHAPTER VII
EXPERIMENTAL EVALUATION
OF THE HYDRAULIC PARAMETERS
OF CATHETERS AND MANOMETER

SUMMARY
In the previous chapter it was shown that before the frequency response curve of the catheter manometer system can be calculated, using either the transmission line model or the lumped circuit model of Fig. 10,

\[ L_0 = \frac{\rho}{\pi r_0^2}, \quad R_0 = 8\eta l/\pi r_0^4, \quad \alpha = r_0 \sqrt{\frac{\omega \rho}{\eta}}, \]

\( C \) and \( C_m \) must be determined and preferably with an error of less than 1 percent. \( R_0 \), the Poiseuille resistance, was measured directly and the catheter length \( l \) was measured with a rule. With \( R_0 \) and \( l \) known, \( L_0 \) and \( \alpha \) can be found by the above relations. The distensibility was measured by connecting in turn the catheter to a water filled capillary under pressure and to the atmosphere. This was done twenty \( (20) \) times for each experiment and the period was varied for repeated experiments. This procedure was also carried out for the evaluation of \( C_m \).

The values of the various hydraulic parameters of the catheters Charriere Nos. 4, 5, 6 and 8 are tabulated in Table IV. All these values are correct to within an error of less than 1 percent.

The value of \( r_0 \), the catheter radius, can be checked in two ways: firstly it can be measured directly with a glass blowers calipher, and secondly by filling the catheter with water and weighing it. To check the magnitude of \( C \), the Young's modulus in the longitudinal direction of the catheter is determined by measuring the bending of the catheter under an applied weight, while the catheter is rotated. The measurements show that the catheter is stiffer in the axial direction than in the radial direction.
THE DETERMINATION OF THE POISEUILLE RESISTANCE

The experimental determination of $R_o$ is as follows. The catheter is flushed with pepsin and then with distilled water which has been filtered by a micro-pore filter (10 μm-pores). One end of the catheter is then connected to a vertical calibrated tube; the other end is connected to a bottle placed so that it is higher than the tube filled with distilled, filtered water at room temperature. The height difference of the water levels diminishes as the water flows from the bottle into the calibrated tube via the catheter.

The volume flow into the tube through the catheter is an exponential function of the time. The time between the moments of passing two levels in the calibrated tube was measured. After having determined the relation between the height difference of the two levels and the displaced volume, the value $R_o$ can be calculated from the measured passage time. The error in $R_o$ in this measurement is less than 0.5 percent.

The correction in the pressure difference necessary for calculating the Poiseuille resistance (due to the kinetic energy of the waterflow at the entrance and the end of the tube) is of the order of $\rho \bar{v}^2$, if $\bar{v}$ is the mean velocity of the water in the tube. From Table IV, (page 65) $\bar{v}$ can be calculated for the maximum pressure difference used during the experiments (34 cm water). Hence, the correction in the pressure difference is at most $10^{-7}$ for catheter No.4 and $5 \times 10^{-5}$ for catheter No.8, and can be neglected.

REPRODUCIBILITY

It was observed that the resistance $R_o$ increased gradually with every experiment. This effect remained even when distilled water was sieved through filters with fine pores (bacterial filters with pores of 10 μm were used). Only after flushing the catheter before each experiment with pepsin, a protein solvent, could the value of $R_o$ be made reproducible within 0.5 percent. This effect can be explained if the catheter was being clogged by some material. Hill (1965) has shown that organisms
growing in water flowing through pipes will adhere to the wall of the pipe, even in tap water. Presumably, this is what happened in our experiments. The reason why the pepsin flushing gave reproducible results was that it desolved all substances with protein content.

THE CALCULATION OF THE CATHETER DIAMETER AND THE HYDRAULIC INDUCTIVITY

The radius \( r_0 \) of the catheter is calculated from the formula \( R_0 = \frac{8\eta l}{\pi r_0^4} \). \( l \) is the catheter length (from 100 to 105 cm) and can be measured with an error of a few parts per thousand. Since the viscosity index \( \eta \) is known to within an error of 0.2 percent (using the critical tables and reading the temperature with an error of 0.1°C) and since \( R_0 \) is known to within an error of 0.5 percent, the error in \( r_0 \) is 1.5 parts per thousand. The hydraulic inductivity of the catheter is \( L_0 = \frac{\rho l}{\pi r_0^2} \). This can be calculated with an error of 0.2 percent.

The values for \( r_0, R_0 \) and \( L_0 \) at 20°C are tabulated for catheters Nos. 4, 5, 6 and 8 in Table IV.

<table>
<thead>
<tr>
<th>Catheter No</th>
<th>( r_0 )(cm)</th>
<th>( l )(cm)</th>
<th>( L_0 )(g.sec(^{-1})cm(^{-4}))</th>
<th>( R_0 )(g.cm(^{-4}))</th>
<th>( C )(cm(^4)sec(^2)g(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.0260</td>
<td>104.5</td>
<td>5.87 x 10(^6)</td>
<td>4.96 x 10(^4)</td>
<td>2.21 x 10(^{-10})</td>
</tr>
<tr>
<td>5</td>
<td>0.0460</td>
<td>102.0</td>
<td>5.78 x 10(^5)</td>
<td>1.532 x 10(^4)</td>
<td>3.67 x 10(^{-10})</td>
</tr>
<tr>
<td>6</td>
<td>0.0573</td>
<td>102.0</td>
<td>2.425 x 10(^5)</td>
<td>0.996 x 10(^4)</td>
<td>6.51 x 10(^{-10})</td>
</tr>
<tr>
<td>8</td>
<td>0.0675</td>
<td>105.3</td>
<td>1.31 x 10(^5)</td>
<td>0.742 x 10(^4)</td>
<td>10.1 x 10(^{-10})</td>
</tr>
</tbody>
</table>

OTHER METHODS OF MEASURING \( r_0 \)

The values for radius \( r_0 \) were checked in two other ways. The diameter of the tip was measured several times with a glass blower's caliper. The calculated mean reading is in agreement with the measured values of the Poiseuille resistance method if the readings are
repeated so that the standard error is of the same order in the two cases. The second way was to weigh the amount of water in the filled catheter. This gave a value also in agreement with the other values. The agreement in the 3 ways of measuring $r_0$ indicates that the inside of the catheter is a cylinder.

![Diagram](image)

Fig. 12. The method of measuring the catheter and manometer capacity, respectively.

**THE HYDRAULIC CAPACITY OF THE CATHETER**

The distensibility of the catheter is the volume increment $C$ of the inside of the catheter per unit increment of the pressure difference across the catheter wall. This is given in Table IV. $C$ is measured in the following way. The catheter is connected to the manometer house, and its tip is closed with a tiny plug. A capillary, partly filled with water is connected to the stopcock A (see Fig. 12). A pressure of 100
mm Hg is applied on the top of the capillary. The stopcocks A and C remain in position I (Fig.12). Stopcock B is turned alternately to the positions I and II, thus connecting the catheter alternately to the capillary and to the atmosphere. A small volume of water disappears every time the stopcock B connects the capillary to the catheter. At the end of 20 cycles it is possible to read the distance the water level of the capillary is lowered with an error less than 1 percent. The cross section of the capillary is determined with Poiseuille experiments to within an error of less than 0.5 percent, so that the volume of water, disappearing from the capillary, is known. The capacity of the catheter is then calculated by dividing the disappeared volume through 20 times the pressure head on the catheter. The pressure reading on the manometer is compared with a mercury column and is corrected for the capillary hydrostatic column and the capillary rise due to the surface tension of the water. The error in the pressure value is less than 1 percent.

During this procedure there must be no leakage of water through the stopcocks. To check this, stopcock C is turned in position II (Fig. 12) and the cycle with stopcock B is repeated. If there is no lowering of the capillary water level, then the stopcocks are tight.

Furthermore the catheter must be air free. This is checked by flushing the catheter repeatedly with water, and connecting a vacuum pump to the tip. Hence, if any air bubbles are present, they are enlarged and easily swept away. Experimentally, it was found that the value of C converged to a limiting value; this value of C is tabulated in Table IV. Since the catheter wall exhibits plastic behaviour the period of revolution of stopcock B can have an effect on the value C. Hence, the procedure is repeated with cycle periods of one in ten seconds to one in half a second. No difference is observed in C, which means that the time constants of plastic deformation are greater than 10 seconds. Hence, C is constant for cycle times smaller than 10 seconds, or, C is constant for pressure fluctuations from 1 c/s to 100 c/s, as far as elasto-viscosity is concerned (see Chapter VIII). Mass effects of the catheter wall do not play a role (Chapter VIII).
The fluid must flow from the catheter to the capillary during the time the cock B is open. The RC-time of the catheters Nos. 4, 5, 6 and 8 are, according to Table IV, $12 \times 10^{-4}$, $2 \times 10^{-4}$, $1.5 \times 10^{-4}$ and $1 \times 10^{-4}$ seconds respectively. The resistance of the capillary and connections were equal to the catheter resistance. Hence, the rotation frequency of cock B was low enough.

VERIFICATION OF THE CATHETER CAPACITY BY OTHER EXPERIMENTS

The catheter capacity can be checked by measuring the Youngs modulus $E^*$ of the catheter and calculating the capacity from the equation (see Appendix):

$$C_{\text{cath.}} = \frac{2\pi d}{E^* r_0^2} \frac{1.5 - 3\mu + (1 + \mu)r_1^2/r_0^2}{r_1^2/r_0^2 - 1}$$

In Eq. (23) $r_0$, $r_1$, $l$, $E^*$ and $\mu$ are the inner radius, the outer radius, the length, the Youngs modulus, and the Poisson ratio of the catheter wall, respectively. It is assumed that the catheter can freely change its length under pressure influences.

$E^*$ is measured as follows: One end of the catheter is secured in a clamp and the other end was loaded with a weight. The catheter can now be treated as a hollow bar, sagging a distance $f$ at the tip, according to the equation:

$$f = \frac{P l^3}{3E^* J} \quad \text{or} \quad E^* = \frac{P l^3}{3fJ}.$$

$J$ is the moment of inertia of the cross-section of the bar with a diameter as rotation axis, at mass surface density 1. $P$ is the load at the end and $l$ the length from clamp to load. If the catheter does not rotate or if it rotates very slowly, the sag $f$ increases continually in time because of the plasticity of the catheter wall, and $f$ is not measurable. It was therefore necessary to rotate the clamp at various frequencies. With several weights $P$ and lengths $l$, $E^*$ is reproducible to within a few percents, using rotating frequencies from 0.1 to 3 cycles per second.

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If the measured value of $E^*$ is substituted in Eq. (23), the calculated capacity is about the half of the value obtained with the capillary method. This difference is probably due to the anisotropy of the catheter wall, which is stiffer in the axial direction than in the radial direction.

To check the reliability of this method for measuring $E^*$, the Young's modulus of an isotropic polythene tube was measured with the rotating clamp method. Then, from this value the capacity was calculated with Eq. (23) and compared with the capacity, obtained with the capillary method. The values were 0.240 and 0.284 mm$^3$/100 mm Hg, respectively. The difference of about 20 percent is due to the errors in measuring $r_0$, $r_1$, and the estimation of $μ$. This indicates that the method for measuring $E^*$ is reliable and that the catheter wall is anisotropic.

THE HYDRAULIC CAPACITY OF THE MANOMETER

The manometer capacity is measured by fixing cocks B and C in the position III and II, respectively and alternating the position of cock A from II to III and back, twenty (20) times (see Fig. 12). If B is turned to position II and the procedure is repeated, then a leak, if present, can be detected. A reproducible minimum value for the capacity of the manometer could be obtained if the manometer was boiled. This minimum value was 0.057 mm$^3$/100 mm Hg in agreement with the theoretical value (Chapter III). If the membrane is fixed at the rim, then this capacity must be equal to $\frac{π}{4} \times 7.4^2 \times \frac{h}{3}$, where the membrane diameter is 7.4 mm and $h$ (in mm) is the deflection of the center at an over pressure of 100 mm Hg. The deflection has been measured with a displacement meter (Philips PR 9300 and PR 9310). The calculated value of 0.052 mm$^3$/100 mm Hg added to the water compressibility of 0.002 mm$^3$/100 mm Hg for the manometer results in 0.054 mm$^3$/100 mm Hg, which agrees with the measured value to within 6 percent. This error is explicable from the inaccuracy of the displacement meter in the $μ$-range. The measured value with the capillary method is accepted, because a lower value could be never obtained and the error in this measurement is not more than 1 percent.
CHAPTER VIII

TESTING OF THE CATHETER-MANOMETER SYSTEM WITH THE PRESSURE GENERATOR

SUMMARY

The catheter-manometer system is tested with the pressure generator giving the input signal, and an oscilloscope and a vacuum tube-voltmeter, measuring the output signal. The amplitude frequency response curves of the U.S.C.I. - catheters Nos. 4, 5, 6 and 8 are depicted in Fig. 13. Also shown are those calculated from Eqs. (19h) or (19i) (the lumped circuit model). For practical purposes the agreement between the experimental data and the data obtained with the lumped circuit model is satisfactory. The deviation in resonance frequency and peak height is at most 10 percent.

In practice the calculated curves of Fig. 11 (page 59) can be used for the prediction of the amplitude response of catheter manometer systems. In Fig. 14a and 14b the response curves calculated from Eq. (19j) (the transmission line model) are compared with the experimental and the lumped-circuit model curves for catheter No. 8 and No. 5 respectively. The difference in resonance frequency and peak height between the experimental data and both models is discussed.

It is shown that the phase response curve measurement of catheter-manometer systems is not accurate enough to distinguish between the two models.

THE TESTING PROCEDURE

Catheter-manometer systems are tested by inserting the catheter tip in the pressure generator and connecting the other end to the manometer (Fig. 1, page 16). The transducer of the manometer (differential transformer Philips PR 9310) is connected to a displacement meter (Philips PR 9300). The recorder output of the displacement meter is
connected to an oscillograph (Tektronix 502 A) and also an a.c. voltmeter (Philips GM 6017). The voltmeter has a linear amplitude frequency response down to about 2 c/s, at which frequency the response is down 3db. The accuracy in the relative amplitude measurements was to within 1 percent f.s.d. Below 10 c/s the oscillograph is used which has an accuracy of 2 percent for relative measurements and amplitudes of more than 5 cm on the screen. It should be noted that the displacement meter has a built in galvanometer indicator with a response time of about 1 second. This indicator should be switched off, since the currents induced in the moving coil at a signal frequency of about 1 c/s distort the recorder output of the displacement meter.

The low frequency sine generator (Peekel 015A) connected to the pressure generator has been checked for amplitude constancy. The frequencies in the range of 0.5 to 100 c/s have been measured with a digital counter (Philips PW 4032).

Before the test the catheter is rinsed with pepsin and flushed with water while sucking at the tip with a vacuum pump. This has been continued until the catheter capacity has reached a constant minimum value. After the test the capacity was checked to see that it has the same value as before. The same procedure was followed with regard to the capacity of the boiled manometer.

The catheter is fixed to the table with tape to prevent mechanical oscillations of the catheter, which cause a shift in the resonance peak. These oscillations of the catheter itself have the same frequency as that of the inner pressure. These mechanical catheter oscillations have their own resonance frequency and introduce additional pressure oscillations within the catheter which disturb the measurement.

The measured data are depicted in Fig.13 for the catheters U.S.C.I. Nos. 4, 5, 6 and 8, respectively. The accuracy of the measurements is indicated by the size of the points.

The curves calculated for the lumped circuit from Eqs. (19g) or (19h) or (19i) of Fig.10 are also shown in Fig.13. The measured values of \( L_o \), \( R_o \), \( C \) and \( C_m \) (Table IV, chapter VII) are used taking into account the values of \( \rho \) and \( \eta \) at the water temperature. The errors are
Fig. 13. Comparison of the calculated amplitude frequency response curves of the lumped π-circuit model for the catheter-manometer system with the experimental data. The size of the points and the widths of the curves indicate the error.

The approximation of the lumped π-circuit is fairly good for practical purposes as Fig. 13 shows. This model can be used for relative amplitudes ranging from 1 to 1.5, and also for the estimation of the resonance frequency and peak height with an error of at most 10 percent. The latter estimation is sometimes necessary to choose the appropriate electrical filter to damp the curve (chapter IX).

COMPARISON OF THE EXPERIMENTAL DATA WITH THE TRANSMISSION LINE MODEL AND THE LUMPED π-CIRCUIT MODEL

Eq. (19j) represents the output-input relation for the transmission
line model as discussed in Chapter VI. Higher order terms in the denominator of the right side of Eq. (19j) can be neglected. The response curves calculated from Eq. (19j) for catheter No. 8 and 5 each connected to the manometer used in the experiments, are shown in Figs. 14a and 14b, respectively.

Fig. 14a. Comparison of the experimental amplitude frequency response curve of the catheter No. 8 manometer system with the predicted curves of the lumped π-circuit model and the transmission line model, respectively.
The transmission line model shows marked deviations from the experimental data. For catheter No.5 the lumped $\pi$-circuit model is a much better approximation than the transmission line model, while for catheter No.8 both models deviate from the experimental data. The discrepancy between the transmission line model and the experimental data can be due to various factors.

Firstly, it is possible that the compliance of the catheter is located for the most part at the end, where it is fixed to the manometer. In this case the transmission line model curve of Figs. 14a and 14b shifts to a position between the two curves in Figs. 14a and 14b, the resonance frequency of the transmission line model becoming slightly lower than shown in the figure. As is indicated in Fig. 14a this is not the case for the experimental data, which show a resonance frequency shifted to a value higher than that of the transmission line model. In Fig. 14b the experimental data lie completely on the curve of the lumped $\pi$-circuit model.
model; hence, it must be assumed in this case that the catheter is completely rigid and that half of its compliance is situated at each end. This is very unlikely, however.

Secondly, it is possible that the approximation of the transmission line model as discussed in chapter VI introduces a large error. We recall that for this model the axial impedance is used for a rigid catheter, which implies that there is no radial water movement, together with a distributed compliance $C' dx$, which implies that there rather is a radial water movement. In a catheter with radial water movement, however, there is more impulse-transport between the inner and outer water layers so that the inductivity approximates more that of plug flow, i.e. becoming smaller than in the case of the rigid catheter. Furthermore, the resistance of the compliable catheter increases relative to that of the rigid catheter. This would mean that a shift of the transmission line curve as represented in Figs. 14a and 14b to a higher resonance frequency and a lower peak value would occur. Hence, this accounts for the behaviour of catheter No.8 in Fig. 14a, but not for catheter No.5 in Fig. 14b.

Thirdly, it is possible that the mass of the catheter wall plays a big role. This mass can be considered as a hydraulic distributed inductivity in series with the hydraulic distributed capacity of the catheter. If the hydraulic distributed inductivity of the catheter wall per cm catheter length is called $L'_w$, which is in series with the distributed hydraulic capacity per cm catheter length $C'$, then the total impedance of this series connection is:

$$\frac{1}{j\omega C'} \left(1 - \omega^2 L'_w C'\right) = \frac{1}{j\omega C'} \left[1 - \left(\frac{v}{v_0}\right)^2\right] \text{ if } v_0^2 = 1/4\pi^2 L'_w C'.$$

The distributed capacity $C'$ then gets a frequency dependent correction so that: $C' \text{ (new)} = C' \text{ (old)} \left[1 - \left(\frac{v}{v_0}\right)^2\right]$. It can be shown that $L'_w = \frac{\rho_w}{4\pi} \left[\frac{r_1}{r_0}\right]^2 - 1$ if $r_1$ and $r_0$ are the outer and inner radius of the catheter respectively, and $\rho_w$ is the mass density of the wall material. $\rho_w$ is equal to about 1.5 and $r_1/r_0$ is about 1.8; hence, $L'_w = 0.27 \text{ g cm}^{-3}$. 76
For catheter No.8, \( C' = 10^{-11} \text{ cm}^3 \text{sec}^{-2} \text{g}^{-1} \) so that \( \nu_0 = 10^5 \text{ c/s} \). The correction factor for \( C' \) at 100 c/s is \( 10^{-6} \) and smaller for lower frequencies. For catheter No.5 the correction is even smaller. This means that the mass of the catheter need not to be considered in our calculations.

A fourth explanation is that the catheter wall is viscous. If the viscosity can be introduced in the model by means of a resistance, then this resistance can be put in series with \( C' \) or parallel to \( C' \). In the latter case the influence of the resistance at higher frequencies is lower. As can be shown from the experiments described in chapter VII, where cock \( B \) is turned with various frequencies without any difference in the measured capacities, the parallel resistance has no influence at frequencies above 1 c/s. The series resistance, on the contrary, has more influence at higher frequencies and gives a correction on \( C' \), which can be written as \( C'/(1 + j \omega \tau) \). \( \tau \) is the product of the capacity and the series resistance per unit catheter length. The resistance in series with \( C' \) screens the effect of \( C' \), resulting in a resonance frequency shift to higher values. This is consistent with the fact that the experimental data of catheter No.8 in Fig.14a show a higher resonance frequency than that of the transmission line model, but not consistent with the case of catheter No.5 in Fig.14b. For \( \omega_{\text{res}} \tau < 1 \), the maximal amplitude is indeed lower. Calculation shows that for large values of \( \tau \), i.e. \( \omega_{\text{res}} \tau >> 1 \) the maximum amplitude can be higher than for the case \( \tau = 0 \). This is not consistent with the experimental curves, shown in Figs.14a and 14b.

RESUMÉ

The discrepancies between the experimental data and the transmission line model cannot be explained satisfactorily by anyone of the above considerations. A viscous behaviour of the catheter wall in the sense of visco-elasticity (not elasto-viscosity) can explain the difference in height of the peaks and the shift in resonance frequency as found with catheter No.8. However, the resonance frequency shift occurring in catheter No.5,
is in the other direction. This could be explained by the simplification of the transmission line neglecting radial water movement. Probably both factors together should be taken into account. The inherent difficulties of formulating a more complete theory are in our opinion, probably not worth undertaking since the resistance of the catheter wall cannot be simply measured and as has been emphasized before, the lumped circuit model is for all practical purposes good enough.

It should be remarked that if the Womersley corrections are not applied, the resonance peaks of the calculated curves are much higher than for the transmission line with Womersley corrections depicted in Figs. 14a and 14b. Hence, the Womersley theory must certainly be used in calculating the impedance of tubes. Frank (1903) concluded that a catheter-manometer system behaves like a lumped circuit, while Hansen (1949) found that it behaved like a transmission line. Neither author, however, used the Womersley corrections. Hence, both their measurements are very probably incorrect.

Only the amplitude frequency response and not the phase response of the catheter-manometer system have been discussed in this chapter. The phase response does not distinguish between the two models (lumped circuit and transmission line model). At the resonance frequency, where the deviation between the two models is larger than at lower frequencies, the tangent of the phase angle for both models is very large. Even if there is a big difference between the values of the tangents, the difference between the phase angles is very small. Hence, phase response characteristics cannot be used to compare the experimental data with the models.
CHAPTER IX

DAMPING METHODS
FOR CATHETER MANOMETER SYSTEMS

SUMMARY

The high resonance peaks (Fig. 13) generally characteristic of catheter-manometer systems, have two disadvantages for the measurement of blood pressure. Firstly, the flat range of the amplitude frequency response curve may be too short, and secondly, by movement of the heart or body, pressure oscillations are often generated, which ring out in the resonance frequency - the so called catheter artefacts. Therefore, it is necessary to damp the amplitude frequency response, i.e., reducing the height of the resonance peak but retaining at the same time a flat response curve up to the original resonance frequency.

Two methods of damping, series damping and parallel damping (Fig. 15) are discussed. Parallel damping preserves the region of flat amplitude response up to the original resonance frequency, while series damping reduces not only the peak height but also the peak frequency.

Undistorted pressure recording requires that the delay time of the pressure waves through the catheter be independent of frequency in the frequency range of interest. This is the case for parallel damping and not for series damping (Fig. 17).

A theoretical discussion of parallel damping is given.

DAMPING METHODS

Generally the amplitude frequency response curve of a catheter-manometer system has a high resonance peak. This has two disadvantages. Firstly it may be that for faithful recording the flat part of this frequency response curve does not extend to a sufficiently high
frequency. Secondly, unavoidable movements of the catheter will give rise to pressure oscillations in the resonance frequency causing artefacts in the record. Therefore damping is always desirable and often necessary. Damping is particularly important when the resonance frequency of the system is relatively low as e.g. if a manometer is used which cannot be boiled.

Because the catheter-manometer system behaves as a transmission line, the deviation of the amplitude frequency response curve from the horizontal is due to reflections at the load, i.e., the manometer capacity. A flat response curve can be obtained by applying a load at the end equal to the characteristic impedance of the catheter. The characteristic impedance of the catheter is given by: \( \sqrt{\frac{R(\omega)+j\omega L(\omega)}{j\omega C}} \), where in contrast to an electrical transmission line, \( R(\omega) \) and \( L(\omega) \) are frequency dependent. It is difficult to develop a circuit which can be used as a load which has this characteristic impedance. Van der Tweel (1957) made plausible that the use of a parallel resistance with catheter No.8, having a small R and a very stiff manometer is a good approximation for the characteristic impedance.

As is shown in chapter VI, the catheter-manometer system can be considered as a lumped \( \pi \)-circuit if the damping coefficient is large, i.e., about 0.5 to 0.7. The error made by this approximation is less than one percent. Therefore, the damping method for a lumped circuit can be applied.

Two methods of damping a lumped circuit can be applied: firstly, a resistance can be put in series with the catheter inductivity and resistance (L and R in Fig.10), or secondly a resistance can be put in parallel to the manometer capacity (\( C_m \) in Fig.10). Theoretically, optimal damping can be achieved with both methods. In practice, however, the flat portion of the amplitude frequency response is always reduced by a series damper.

**PRACTICAL DAMPING DEVICES**

A schematic diagram of the two methods of damping is shown in Fig. 15. For parallel damping a needle resistance in series with a plastic
PARALLEL DAMPING

Fig. 15. Two methods of damping the response of the catheter-manometer system hydraulically.

tube closed by a syringe, is put in parallel to the manometer. The plastic tube with its big compliance is introduced because the fluid can flow through the needle resistance under the influence of the pressure oscillations around the mean level, so that the needle resistance actually can do its job. A steady flow of fluid through the needle resistance must be prevented because of the mean pressure, which exists during blood pressure measurements. Therefore the plastic tube must be closed at the other end. This is done here with a syringe, because this was advantageous during the filling and connecting procedure. It is possible to connect, instead of a simple syringe, the syringe of an automatically driven infusor to the plastic tube, so that the catheter tip is kept free from clotted blood by a slow stream of heparinized saline. An advantage
is then that the needle resistance has two functions, one to damp the response curve and the other to screen off the compliable tube connections of the infusor from the stiff manometer. Without such a resistance, the infusor tube would spoil the necessarily small manometer capacity by introducing the tube capacity in parallel.

For series damping a capillary is put in series with the catheter.

Fig. 16 shows the amplitude frequency response curve of the undamped, series damped, and parallel damped catheter-manometer system. In this particular case the manometer was flushed with cold water with the result that the resonance frequency was lower than can be obtained with air-free filling. This is done purposely in order to show that the damping is also effective in those cases where the damping is needed seriously.

With the proper parallel damping a curve can be obtained which is flat from zero frequency to the original resonance frequency. The needle resistance is adjusted so that the curve is as flat as possible. Furthermore, the position of the needle of the damper is reproducible for boiled manometers and air free catheters. The latter can be also easily obtained by boiling and connecting them under water to the manometer.

The series damper, on the contrary, causes the resonance peak to be shifted to a lower frequency. This is due to the fact that the capillary not only introduces a resistance but also an inductivity in series with the catheter.

This inductivity, in series with the catheter inductivity lowers the resonance frequency and tends to decrease the damping coefficient, counteracting the effect of the resistance of the capillary. The decrease of the resonance frequency decreases the flat part of the amplitude response curve. Experimentally, a short capillary of 3 mm was used. As Fig. 16 shows even with this sort capillary, a peak remains shifted to the left.

To prevent a large shift of the resonance frequency by series damping a capillary must be chosen with a large R/L ratio, which means that the capillary must be very narrow. For a fixed resistance this means
that the capillary must be very short. The shortest capillary is an orifice, but the R/L ratio of an orifice is not sufficiently large to prevent the shift in the resonance frequency. Furthermore, if the capillary length is of the order of the capillary diameter, the flow will no longer be linearly dependent on the pressure difference over the capillary. The capillary acts like an orifice, i.e., the second power of its flow is proportional to the pressure difference. Hence, this resistance being the greater part of the total resistance introduces non-linearity in the input output relation. Hansen (1949) suggested that an orifice with a needle be put in series with the catheter in order to in-
roduce the damping; however, it is doubtful whether the output input relation is linear.

The parallel damper also introduces an additional inductivity, but this does not cause the lowering of the resonance frequency; on the contrary, it can increase the resonance frequency somewhat. This will be shown in the last paragraph of this chapter.

TEMPERATURE INFLUENCE
ON AMPLITUDE FREQUENCY RESPONSE

The physician inserts the catheter in a vein for about half its length. Thus, the water temperature in the catheter will become 37°C and the viscosity index will be 0.7 centipoise, instead of 1.0 centipoise, the viscosity index of the water in the catheter at 20°C outside of the body. Then the resistance will decrease about 15 percent and the peak height at the resonance frequency will increase 15 percent. The resonance frequency will keep its original value. Even when the system is undamped the part of the amplitude response curve ranging from zero to more than three quarters of the resonance frequency will have the same shape. Hence, it is not necessary for the physician to test his catheter-manometer system with half the catheter dipped in a vessel with water of 37°C. If the physician has damped the system hydraulically, the hydraulic resistance remains at ambient temperature after insertion of the catheter in the vein. Because the damping resistance accounts for the greatest part of the damping, the amplitude resonance curve will not change its shape after insertion of the catheter in the vein. This is proved for the undamped and damped system by testing with temperatures of the water in the catheter of 20°C and 40°C.

THE PHASE RESPONSE CURVE OF DAMPED SYSTEMS

For an undistorted response of the catheter-manometer system, not only is a flat amplitude frequency response required, but also the phase angle should be proportional to the frequency. This means that the
time delay between input and output pressure is independent of the frequency.

Fig.17. The signal delay time in the catheter manometer system for parallel and series damped systems, respectively.

Fig.17 shows this delay time at various frequencies for a series damper and for a parallel damper. The parallel damper introduces a constant delay time, up to the resonance frequency. This combined with the flat amplitude response up to the resonance frequency (Fig.16) causes an undistorted response.

The delay time for the series damper shows a strong dependence on the frequency. Hence, in this respect the parallel damper is better
than the series damper, although a delay of 25 msec at 12 c/s (Fig.17) does not seriously affect the arterial blood pressure curve for practical purposes. For pressures with very steep upstrokes, e.g., the blood pressure of the left ventricle, the distortion may be noticeable.

The delay time is measured by comparing the waves of the recorded pressure with the sinusoidal current through the solenoid of the pressure generator on a dual beam oscilloscope (Tektronix 502A).

THEORY OF PARALLEL DAMPING

As is shown in chapter VI the lumped circuit model is a good approximation for transmission line models, which are damped with a damping coefficient of more than 0.5. Hence, in this theory the lumped circuit model will be used for the explanation of the operation of the parallel damper.

![Diagram of hydraulic circuit](image)

Fig.18. The hydraulic circuit of the pressure generator \( p_g \) connected to a catheter-manometer system with parallel damping \((R_d, L_d, C_d)\).

The needle resistance can be considered as a hydraulic resistance \( R_d \) in series with a hydraulic inductivity \( L_d \) as represented in Fig.18. This impedance is in series with the hydraulic capacity \( C_d \), the com-
plianc of the plastic tube connected to the needle resistance and the syringe (Fig.15 and 18). Hence, the impedance, in parallel with the manometer capacity $C_m$ in Fig.18 is equal to: $R_d = j\omega L_d + 1/j\omega C_d$. The output pressure over the input pressure becomes, instead of equation (19g):

$$\frac{P_0}{P_g} = \frac{1}{1 + (R + j\omega L)[j\omega(C_m + C/2) + 1/(R_d + j\omega L_d + 1/j\omega C_d)]]}.$$  \hspace{1cm} (20)

The damping needle has been constructed for catheter 4, 5 and 6. The length of the needle is about 0.5 cm, the conicity 1 : 20 and the crevice about 20 µm (Fig.4).

With these dimensions $\omega_1 = R_d/L_d \gg \omega_0 = 1/\sqrt{L(C_m + C/2)}$. Thus for $\omega < \omega_0$, $j\omega L_d$ can be neglected with respect to $R_d$. Hence for $0 < \omega < \omega_0$ Eq (20) becomes:

$$\frac{P_0}{P_g} = \frac{1}{1 - \omega^2 L(C_m + C/2) + j\omega R(C_m + C/2) + j\omega R C_d \frac{1 + j\omega L/R}{1 + j\omega R C_d C_d}}. \hspace{1cm} (20a)$$

If the capacity of the plastic tube $C_d$ and the resistance of the needle $R_d$ is chosen so that in the neighbourhood of $\omega_0$: $R_d \cdot C_d = L/R$, Eq. (20a) becomes:

$$\frac{P_0}{P_g} = \frac{1}{1 - \omega^2 L(C_m + C/2) + j\omega R(C_m + C/2) + j\omega R C_d} \hspace{1cm} (20b)$$

The system behaves as a second order system. The damping coefficient of this system is the sum of the original damping coefficient

$$\beta = -\frac{\omega_0 R(C_m + C/2)}{2} = -\frac{R}{2\omega_0 L}$$

and a newly introduced damping coefficient $\beta_d = \frac{1}{2}\omega_0 R C_d = \frac{1}{2}\omega_0 L/R_d$. Eq. (20b) can be written as

$$\frac{P_0}{P_g} = \frac{1}{1 - (\frac{\omega}{\omega_0})^2 + j\frac{\omega}{\omega_0}(2\beta + 2\beta_d)}. \hspace{1cm} (20c)$$
If for frequencies in the neighbourhood of $\omega_0$, $\beta + \beta_d$ is equal to about 0.5, the amplitude response curve is flat up to the resonance frequency. For $\omega = \omega_0$, the value of $\beta$ ranges from 0.2 to 0.05, while for $\omega = 0$, $\beta$ lies between 0.2 and 0.02 (Fig. 11). If, for example $\beta\omega = \omega_0 = 0.08$ and $\beta + \beta_d = 0.5$, and taking into account that $\beta\beta_d = \frac{1}{2} \frac{R}{\omega_0 L} \cdot \frac{1}{2} \frac{\omega_0 L}{R_d} = \frac{1}{4} \frac{R}{R_d}$, $R/R_d = 0.13$, or $R_d = 7.5 \times R$.

For optimal damping distinct values of $R_d$ and $C_d$ are required. However, the value of $C_d$ is not very critical as can be shown as follows. If we make $C_d$ very great i.e. $\omega R_d C_d >> 1$, Eq. (20) becomes:

$$
\frac{p_0}{p_g} = \frac{1}{1 + \frac{R}{R_d} - \omega^2 L(C_m + C/2) + j\omega R(C_m + C/2) + \frac{j\omega L}{R_d}} = 
\frac{1}{1 + \frac{R}{R_d} - \left(\frac{\omega}{\omega_0}\right)^2 + j\frac{\omega}{\omega_0}(2\beta + 2\beta_d)}. \tag{20d}
$$

The effect is that for very low frequencies the amplitude response is $1/(1 + R/R_d)$, which, in the example given above, is about 13% lower than unity. At zero frequency $p_0/p_g = 1$ according to formula (20a). Hence, the amplitude response curve will start with the value 1 at zero frequency and decrease till about the frequency $\omega = 1/R_d C_d$. For higher frequencies Eq. (20d) will be followed. A very large $C_d$ introduces a small negative hump in the low frequency-range. This can be flattened by increasing $R_d$ by a small amount. Experimentally, it was found that the curve of $p_0/p_g$, as calculated from Eq. (20b), did not depend critically on the value of $C_d$, since $R$ can be readjusted. For example, by taking $C_d$ as twice its optimal value and readjusting $R_d$ the amplitude response will remain flat up to the resonance frequency.

The greater part of the damping lies in $\beta_d = \omega_0 L/R_d$ which increases with decreasing frequency. This is because $L$ increases and $R_d$ decreases with decreasing frequency (Fig. 9). The result is that the response will be even flatter at lower frequencies than expected with a damping coefficient of 0.5 (Fig. 16).
For big catheters, like catheter No.8, $R_d$ must be made smaller than for thinner catheters. Then the ratio $R_d/L_d$ decreases. This, together with the fact that $\omega_0$ for the bigger catheter is larger than for the narrower one, prevents the condition $\omega_1 = R_d/L_d >> \omega_0$ from being fulfilled. If $\omega_1 << \omega_0$, then at frequencies $\omega << \omega_0$, $j\omega L_d$ may not be neglected in comparison with $R_d$. Neglecting $R$ now in comparison with $j\omega L_d$, Eq. (20) becomes:

$$\frac{p_0}{p_g} = \frac{1}{1 + (R + j\omega L) \left[ j\omega(C_m + C/2) + 1/j\omega L_d \right]} = \frac{L_d}{L + L_d} \cdot \frac{1}{1 - \frac{\omega^2 L(C_m + C/2)}{1 + L/L_d} + j\omega R \frac{C_m + C/2 - 1/\omega^2 L_d}{1 + L/L_d}} \cdot (20e)$$

This equation gives a response curve with a resonance frequency, which is $(1 + L/L_d)^{1/2}$ higher than the original one $(\omega_0/2\pi)$, and a damping coefficient smaller than the original one at very high frequencies by the same factor. At moderate frequencies the damping coefficient is even smaller than at very high frequencies because the term $1/\omega^2 L_d$ has a minus sign. The needle inductivity increases the resonance frequency, contrary to the inductivity of the series damper. Although in the range $0 < \omega << \omega_1$, the curve can be flattened by damping with the needle, it is not possible to do so for frequencies $\omega_1 << \omega$. With our needle, catheter artefacts, due to catheter movements and the undamped peak of the big catheter, remain.

The delay time of Fig.17 has been calculated with equation (20b), first calculating the original low damping coefficient and then $R/R_d$ with an assumed damping coefficient for the damped curve. The calculation shows that the delay time is independent of the frequency. The magnitude of this delay time cannot be calculated accurately because of the uncertainty in the value of the damping coefficient of the damped system. Taking into account this uncertainty the value of the delay time lies between 4 and 7 msec. Experimentally a value of 6 msec was found (Fig.17).
ELECTRICAL DAMPING

The amplitude response curve can also be electrically damped. If the damping filter contains one integrating section of resistance and capacitance, then Eq. (19g) (chapter VI) has to be multiplied by \( \frac{1}{1 + j\nu/\nu_f} \). For the resonance frequency \( \nu_0 \) Eq. (19g) gives

\[
\frac{\nu_0^2}{\nu_f} = \frac{P_o}{P_g} \max 
\]

If this peak is to be suppressed to the value 1 then

\[
\sqrt{1 + \frac{\nu_0^2}{\nu_f}} = \frac{P_o}{P_g} \max \quad \text{or} \quad \nu_f = \nu_0 / \left( \frac{P_o}{P_g} \max \right). 
\]

In this case the amplitude at \( \nu = \nu_0/2 \) will be in the neighbourhood of 1 for catheter No.4 but far below 1, namely \( \frac{8}{3} \frac{P_o}{P_g} \max \) for catheter No.8 with its high \( \frac{P_o}{P_g} \max = 7 \). Generally, electrical filters of the R-C-integrating type, even with several lumped sections, damp the amplitude response curve so that the curve first decreases below 1 and then rises to a hump, thereafter decreasing rapidly. An adequate filter for catheter No.6 or 8 will be very complex and the parameters of the filter will be very critical, since a slight deviation from the assumed undamped curve spoils the whole damped curve. Therefore, it is recommended that the continually adjustable hydraulic needle be used.
HOW TO OBTAIN UNDISTORTED BLOOD PRESSURE MEASUREMENTS

The amplitude frequency response of catheter manometer systems must be flat with an error of about 10\% at the upper frequency, this frequency being generally about equal to 5 times the normal heart beat frequency. For children this is usually 0 - 12 c/s, while for adults this is only 0 - 6 c/s. (Wood, 1956; Patel, 1963). Physiological experiments with small animals require quite a larger range; for example, for a rat under narcosis this is about 0 - 25 c/s.

To check whether these basic requirements are fulfilled, the amplitude frequency response curves of the catheter-manometer system must be investigated.

The behavior of a catheter-manometer system is qualitatively similar to that of a vibrating system consisting of a mass and damped spring, and like it will possess a resonance frequency whose peak height is determined by the damping coefficient. The amplitude frequency response characteristics of ordinary vibrating systems are well known and can be found in the common physics textbooks. Quantitatively, however, since there are major differences between the two, curves of catheter manometer systems for various damping coefficients, should be shown (Fig.11).

The so-called amplitude response, $p_o/p_g$, is plotted on the vertical axis. $p_o$ is the pressure amplitude of the catheter-manometer system, and $p_g$ is the pressure at the tip of the catheter, i.e., due to the pressure generator.

The dimensionless parameter $a \frac{2\beta}{42\beta}$ is plotted on the horizontal scale. As the equation in the figure shows, $a \frac{2\beta}{4}$ is proportional to the frequency...
Fig. 11. The calculated amplitude frequency response curves of catheter-manometer systems, considered as a lumped π-circuit model. The dimensionless parameters α and β are expressed in the indicated units for the convenience of the physician as explained in chapter X.
ν of the pressure oscillations, and the dimensionless horizontal scale can easily be converted to a frequency scale. Each catheter-manometer system will then have its own frequency scale.

The enhancement of the oscillations around the resonance frequency vary inversely with the damping coefficient β of the system. The narrower the catheter and the greater its length, the greater the damping coefficient and the lower the peak of the curve at the resonance frequency. On the top left of the figure the equation for β is given. The proper curve for the given catheter-manometer combination can be selected from Fig. 11 if β is known from the parameters of the catheter and manometer. The manometer parameter is given by the manufacturer and the catheter parameters can be determined by the physician himself as is described in the following paragraph.

For undistorted blood pressure measurements the amplitude response curve should be flat, for example, from 0 - 6 c/s for an adult. Although the curves in Fig. 11 are not really flat, a deviation of about 10% from the value 1 at the upper frequency for the amplitude ratio ρ₀/ρ₂ is permissible for faithful recordings. A deviation of 12% occurs for α₂β = 0.3. The value of ν which corresponds to this value of α₂β then represents an upper bound on the frequency of the blood pressure oscillations.

An example: if for catheter No. 61 = 1 m, d = 1 mm and its compliance C is 0.08 mm³/100 mm Hg and the manometer has a compliance of 0.05 mm³/100 mm Hg, then from the equation in Fig. 11:

\[ ν = 0.3 \times d / 0.0603 \sqrt{\frac{C}{2} + C_m} = 5 \times d / \sqrt{\frac{C}{2} + C_m} = 5 \times 1.00 / \sqrt{1.00(0.04 + 0.05)} = 17 \text{ c/s}. \]  

This upper bound frequency, at which the flat part of the curve begins to deviate more than 10 percent, is high enough to guarantee an undistorted response of the manometer. In the range α₂β = 0 to 0.5 all curves with different β coincide, so that it is not necessary to calculate
in advance the damping coefficient $\beta$ if only the upper bound frequency is needed. Furthermore, for a rough estimation of the resonance frequency it is also not necessary to calculate $\beta$. Fig. 11 shows that the peaks have an abscissa in the neighbourhood of $\alpha^2 = 0.9$, which is $3 \times 0.3$. This means that the resonance frequency is $3$ times the calculated upper bound frequency for $12$ percent deviation; in our example $3 \times 17 = 51$ c/s. Because $\alpha^2$ is proportional to frequency (see equation in Fig. 11) the horizontal axis can be transformed to a frequency-axis. In our example the point $\alpha = 0.3$ corresponds to $17$ c/s and the point $\alpha = 0.9$ to $51$ c/s. For this transformation of $\alpha$ to $\nu$ the actual compliances of the catheter and the manometer must be known. But it should be noted that if air is present in the catheter-manometer system the compliance can be considerably larger than the minimum values published in the folder of the manufacturer. In this case the upper bound frequency for $12$ percent deviation is lower than calculated with the minimum values. Therefore, if air free filling is not assured the compliances should be measured. However, as will be shown in the next paragraph, if one of the two compliances is known the value of the other can be determined from the first one:

THE DETERMINATION OF THE DIAMETER AND COMPLIANCE OF THE CATHETER

The diameter of the catheter can be measured at the tip with a glassblowers calliper, repeating the measurement several times and taking the mean of the readings. A second method is to weigh the empty catheter and the filled catheter. The diameter is easily calculated from the difference of the two weights. For this method a balance with an accuracy of only $10$ mg is sufficient. Thirdly, the Poiseuille method can be used. This consists essentially in measuring the rate of flow of a certain volume of water through the catheter per unit pressure difference. The catheter must first be rinsed with a solution of pepsin to dissolve any organisms in the catheter (Chapter VII). Their presence will increase the hydraulic resistance of the catheter. A syringe (without
a plunger), whose volume must be at least 15 cm$^3$ is attached to the top end of the catheter with the bottom end of the catheter closed. (This can be done with the finger). The stretched catheter and syringe are placed in a vertical position and filled with water. The water temperature should be between 15 - 25°C. The bottom end of the catheter is now opened and the time for the syringe to empty is measured with an ordinary watch. The diameter of the catheter is then obtained by use of the following relation.

$$d = 1.43 \sqrt[4]{\frac{l}{V} \cdot \frac{1}{h} \cdot \frac{t}{h}}$$  \hspace{1cm} (22)$$

where $d$ is the catheter diameter in mm and $l$ its length in cms, $V$ is the volume of water in the syringe, $t$ is the measured time in seconds and $h$ is the mean height in cms of the waterlevel in the syringe above the tip of the catheter. If an error of 10 percent is made in $l$, $h$, $V$ and $t$ each, then the error in $d$ is about 5 percent. Hence, if the syringe is shorter than 10 percent of the catheter length, $h$ can be put equal to 1 and (22) becomes:

$$d = 1.43 \sqrt[4]{\frac{V}{t}}$$  \hspace{1cm} (22a)$$

With an error of 5 percent in $d$ the frequency calculated from Eq.(21) will be accurate to within 5%.

In the previous example the error is 0.8 c/s, which can be neglected.

With this method the physician can detect the presence of any obstructing organisms in the catheter by measuring $t$ without rinsing the catheter with pepsin and another time after having rinsed the catheter with pepsin. It would be of interest to know if in practice such organisms develop in catheters frequently.

The length $l$ of the catheter can be measured with an ordinary rule.

The catheter compliance can be measured in the following way. First, the catheter is connected to the manometer via two three-way stopcocks; before closing the catheter at the tip the whole system is filled airfree with water. A plastic tube containing a little water is connected to the
third outlet of the stopcock nearest to the manometer. The stopcocks are turned so that the manometer is connected to the plastic tube, and the catheter is open to the atmosphere. Fig. 12 shows the various connections for the stopcocks (A and C). The stopcocks to begin with should be in position A II, C II. Stopcock B is superfluous for this experiment and should be in position BI. A pressure, \( p_0 \), is applied to the plastic tube with a handbulb and the pressure is read on the pressure recorder. The stopcocks are then turned so that the catheter is connected only to the manometer, and the plastic tube is closed. The cocks are then in the position C I, A III (Fig. 12). The pressure reading on the recorder diminishes to \( p_1 \). To check the presence of leaks in the stopcocks the pressure on the plastic tube is removed. If \( p_1 \) diminishes gradually, the stopcock has a leak. The catheter capacity is calculated from the following relation,

\[
C = C_m \left(\frac{p_0}{p_1} - 1\right),
\]

where \( C \) is the catheter capacity and \( C_m \) the manometer capacity, given by the manufacturer.

To check if there was any leak between manometer and catheter via the stopcocks, the reverse procedure is followed. First, the plastic tube is connected to a sphygmo-manometer and to the catheter, while the manometer is left open to the atmosphere. The pressure \( p'_0 \) is read on the sphygmo-manometer. The catheter is then connected to the manometer via the stopcocks, closing the plastic tube. If now the pressure-reading on the electronic manometer is \( p'_1 \) then the following relation holds:

\[
C = \frac{C_m}{\left(\frac{p'_0}{p'_1} - 1\right)}.
\]

If there were no leaks between catheter and manometer, the result of \( C/C_m \) should be the same in both experiments. If a leak exists \( C \) calculated from Eq. (23) is smaller than \( C \) in (23a). Because very small leaks are unavoidable, there usually will be a difference between the
two values of \( C \), but the difference should be not more than 15 percent. Taking the mean between the two \( C \)'s the error is at most 8 percent and the error in \( v \) in equation (21) at most 4 percent.

The catheter and manometer should be flushed again with boiled water and the experiments repeated in order to verify that the filling was air free. If any air is removed by flushing, the calculated value of \( C \) will increase, since the catheter seldom retains as much air as the manometer. Most of the air usually sticks to the plastic metal interface of the manometer.

INTRODUCING A DAMPING FILTER

Movements of the catheter introduce pressure oscillations - the so-called catheter artefacts - which ring out in the resonance frequency. These artefacts are larger, the higher the resonance peak. Therefore, damping is desirable in order to diminish the peak height.

Damping is necessary if the calculated upper bound frequency \( v \) (as calculated from Eq.21) is very low, because damping extends the flat part of the amplitude frequency response curve. Proper damping extends the flat part of the amplitude response curve up to the resonance frequency. Therefore, the resonance frequency, being 3 times the frequency \( v \), must be higher than the upper frequency present in the blood pressure oscillations.

The damping can be done with a parallel damper, a hydraulic resistance, connected in parallel to the manometer. This is described in chapter IX. The construction of the needle is shown in Fig.4. If there is no hydraulic parallel damper available, electrically damping must be tried.

The selection of the proper electrical filter can be done in the following way. A sine generator is connected to the electrical filter in the electronic manometer equipment. For manometer transducers with their own DC supply, the sine wave generator can be connected to the input of the electronic equipment, where normally the manometer is connected. The output is then recorded. With the filters off, the output
should be frequency independent. With the filters on, the amplitude response curves are then plotted on a graph. The values $p_g/p_0$ - the inverse of the values of Fig. 11 - from the appropriate curve of Fig. 11 are then plotted on the same graph. The filter, whose response agrees best with the $p_g/p_0$ curve, at least for the low frequencies, is selected. The deviation between the two curves should not be more than 10% in the frequency range of interest.

The above procedure needs to be done once for a given catheter-manometer combination. In the actual use of the catheter-manometer system, however, care must be taken to insure that the system is air free. If the manometer is one that can be boiled, boiling it and rinsing the catheter with pepsin will insure an air-free system and reproducible results. If the manometer has a perspex dome, the system will be air free only after hours of flushing. This can be checked by measuring the capacity of the catheter.

THE CHOICE OF MANOMETER OR PRESSURE GENERATOR

Physicians who are interested in buying a manometer are advised to buy one which can be boiled, and to boil it before use. If the manometer can be boiled, the physician should make certain either that it has a detachable electrical transducer, or if not, that no water comes in contact with the transducer during boiling. The manufacturer must guarantee this first. An air free catheter can easily be obtained by flushing vigorously with boiled water.

Manometers usually are sold with compliances of about 1, 0.05 and 0.01 mm$^3$/100 mm Hg, respectively. The first type is meant for low frequency, low pressure recording, such as venous blood pressure recording. The second type is used for catheter-manometer systems for arterial and heart pressures, and the third for needle-manometer systems. A needle has no compliance, except for the compliance of the water in it, which is $0.0067$ mm$^3$/100 mm Hg for every cm$^3$ of water in the needle. For a thin and short needle this can be neglected. A manometer with 0.01 mm$^3$/100 mm Hg can also be used for catheter-manometer systems.
meter systems, but it is not advisable to do so. Eq. (21) shows that half the catheter capacity is added to the manometer capacity. Since the catheter capacities are of the order of 0.05 mm$^3$/100 mm Hg, it does not really pay to select such a stiff manometer. The upper bound frequency $\nu$ of Eq. (21) is not improved, because the catheter capacity spoils the stiffness of the manometer. Hence, a value of about 0.05 mm$^3$/100 mm Hg is a good compromise between sensitivity and frequency-range. For needle-manometer systems the case is different. The needle is stiff and $C/2$ in Eq. (21) is about zero. Very high frequencies can be obtained, as can be seen from Eq. (21) by substituting $d$, and $1$ of the needle in the equation. The same graphs of Fig.11 can be used for needles.

The manufacturer publishes the compliance of the manometer; however, it should be ascertained whether this is an overall one, i.e., the compressibility of the water in the manometer must be included in the figure. For a chamber with 3 cm$^3$ of water a capacity of 0.02 mm$^3$/100 mm Hg, due to the water compressibility, should be added to the capacity of the manometer membrane. For the measurements of the catheter capacity described above the water compressibility is automatically taken into account.

For ease of operation the best thing to have is a pressure generator. In that case no knowledge of the parameters of the catheter or needle is necessary and one need not worry about whether the manometer or catheter are air free. The physician need only connect the catheter tip to the pressure generator, apply the proper damping device and take his measurements.

One requirement for the pressure generator is that its internal impedance be small in comparison with that of the particular catheter-manometer system. The manufacturer should supply the value of the internal impedance. In most cases the internal impedance of the pressure generator originates from a compliance, a hydraulic capacity. If the internal capacity of the generator is at least 50 times that of the manometer, then the error in the measurements will be less than 1 percent (chapter II). A pressure generator, which has a sterilisable
chamber is very convenient, because the amplitude response can be checked just before the catheterisation by inserting the sterile catheter in the sterile generator.
APPENDIX

The hydraulic capacity of an elastic tube is calculated as follows. If in the inside of the tube an over pressure $p_i$ exists let the radial and axial displacement of the tube material be $v$ and $w$, respectively, and $-p$ be the radial component of the stress on the cylindrical surfaces. Cylindrical coordinates $z, r$ and $a$ are used. Because of azimuthal symmetry all strains and stresses are independent of the azimuthal coordinate, $a$.

For an infinitesimal cylindrical segment with thickness $dr$ and length $dz$ the following equations hold, according to Hook's law:

\[ \frac{\delta v}{\delta r} = -\frac{p}{E^*} - \frac{\mu \sigma_t}{E^*} - \frac{\mu \sigma_z}{E^*}; \quad (24a) \]

\[ \frac{v}{r} = \frac{\sigma_t}{E^*} + \frac{\mu p}{E^*} - \frac{\mu \sigma_z}{E^*}; \quad (24b) \]

\[ \frac{\delta w}{\delta z} = \frac{\mu p}{E^*} - \frac{\mu \sigma_t}{E^*} + \frac{\sigma_z}{E^*}. \quad (24c) \]

Here $\mu$ is Poisson's contraction ratio, $E^*$ is Young's modulus, $\sigma_t$ is the tangential tension and $\sigma_z$ is the axial tension. On grounds of symmetry all shear stresses are zero. This is true if the tube forms a closed loop, in which case axial shear stresses are zero. However, since the tube is closed at one end with a plug and connected at the other end to a fluid reservoir, boundary conditions are introduced which cause deviations from the closed loop situation. Hence, axial shear stresses are present and extend over a length of the tube, equal to about its wall thickness. Because the tube is very long in comparison to its wall thickness, these axial shear stresses are neglected, and the tube be-
haves as a closed loop.

The condition of equilibrium is represented by:

\[ \frac{\delta(pr)}{\delta r} + \sigma_t = 0. \]  \hspace{1cm} (24d)

Since \( \sigma_Z \) is constant for the closed loop over the entire radius and along the axis it follows from Eq. (24a) and (24b) that:

\[ \frac{\delta(\sigma_t r)}{\delta r} + \mu \frac{\delta pr}{\delta r} = -p - \mu \sigma_t. \]  \hspace{1cm} (24e)

This combined with equation (24d) gives:

\[ \frac{\delta(\sigma_t r)}{\delta r} = -\frac{\delta}{\delta r} \left( r \frac{\delta(pr)}{\delta r} \right) = -p \text{ or } \frac{\delta^2(pr)}{\delta \ln r^2} = pr. \]  \hspace{1cm} (24f)

Hence: \( pr = Ae^{ln r} + Be^{-ln r} = Ar + \frac{B}{r} \) or \( p = A + \frac{B}{r^2} \) \hspace{1cm} (24g)

From Eqs. (24b), (24d) and (24g) one obtains:

\[ v = \frac{1}{E^*} (r\sigma_t + \mu pr - \mu \sigma_Z r) = \frac{1}{E^*} (-Ar + \frac{B}{r} + \mu Ar + \frac{B}{r} - \mu \sigma_Z r) = \frac{1}{E^*} \left[ -Ar(1 - \mu) + B(1 + \mu) - \mu \sigma_Z r \right]. \]  \hspace{1cm} (24h)

From Eqs. (24c), (24d) and (24g) we get

\[ \frac{\delta w}{\delta z} = \frac{\mu}{E^*} \left( \frac{A + \frac{B}{r^2} + A - \frac{B}{r^2}}{r^2} + \frac{\sigma_Z}{E^*} \right) + \frac{1}{E^*} = \frac{1}{E^*} (2\mu A + \sigma_Z). \]  \hspace{1cm} (24i)

Hence: \( w = \frac{1}{E^*} (2\mu A + \sigma_Z) z \). \hspace{1cm} (24j)

The plug at the end exerts a force \( \pi r_0^2 p_1 \) on the wall with area \( \pi (r_1^2 - r_0^2) \), so that \( \sigma_Z = p_1 / \left[ (\frac{r_1}{r_0})^2 - 1 \right] \). \hspace{1cm} (24k)

For \( r = r_0 \), \( p = p_1 \) and for \( r = r_1 \), \( p = 0 \).
Hence, with Eq. (24g):

\[
\frac{B}{r_0^2} = p_i \quad \text{and} \quad \frac{B}{r_1^2} = 0,
\]

so that:

\[
A = \frac{-p_i}{r_1^2 - 1} \quad \text{and} \quad B = \frac{p_i r_1^2}{r_1^2 - 1}.
\]  \hspace{1cm} (241)

The increase of the inner cross section of the tube is from Eqs. (24h), (24k) and (24l):

\[
2\pi r_0 v r_0 = \frac{2\pi r_0}{E^*} \left[ \frac{(1 - \mu) p_i r_0}{r_1^2/r_0^2 - 1} + \frac{p_i (1 + \mu) r_1^2/r_0}{r_1^2/r_0^2 - 1} - \frac{\mu r_0^2 p_i}{r_1^2/r_0^2 - 1} \right] =
\]

\[
= \frac{2\pi r_0^2 p_i}{E^* (r_1^2/r_0^2 - 1)} \left[ 1 - 2\mu + (1 + \mu) \frac{r_1^2}{r_0^2} \right].
\]  \hspace{1cm} (24m)

The length of the tube is now changed, according to (24j), (24k) and (24l) by:

\[
w = \frac{1}{E^*} \left[ -2\mu \frac{r_1^2/r_0^2 - 1}{r_1^2/r_0^2 - 1} + \frac{p_i r_1^2/r_0^2}{r_1^2/r_0^2 - 1} \right] = \frac{\frac{1}{E^*} \frac{1 - 2\mu}{r_1^2/r_0^2 - 1}}{r_1^2/r_0^2 - 1}.
\]  \hspace{1cm} (24n)

Hence, the total volume displacement can be calculated from Eqs. (24m) and (24n) and is given by

\[
2\pi r_0^2 v r_0 + w \pi r_0^2 = \frac{2\pi r_0^2 p_i}{E^* (r_1^2/r_0^2 - 1)} \left[ 1.5 - 3\mu + (1 + \mu) \frac{r_1^2}{r_0^2} \right].
\]

The distensibility C of the catheter is then:

\[
C = \frac{2\pi r_0^2 v r_0 + w \pi r_0^2}{p_i} = \frac{2\pi r_0^2}{E^*} \cdot \frac{1.5 - 3\mu + (1 + \mu) r_1^2/r_0^2}{r_1^2/r_0^2 - 1}.
\]  \hspace{1cm} (25)
SUMMARY

An investigation of the dynamic response of catheter-manometer systems has been carried out in an attempt to meet the practical requirements for faithful, direct blood pressure measurements.

For this purpose a pressure generator was designed and constructed and its characteristics were examined theoretically and experimentally. It was found that for our purposes this generator behaves as an ideal pressure generator. It has a negligible internal impedance and a flat amplitude frequency output in the frequency range of 0-100 c/s, with an accuracy of one percent.

The testing of catheter-manometer systems presents some difficulties with regard to the reproducibility of the measurements. To begin with, the manometer and catheter should be air free; this requirement is met only by boiling the manometer and by flushing the catheter vigorously with boiled water. A special manometer, which can be boiled, was constructed (Fig.4). It is shown theoretically that this manometer behaves dynamically as a hydraulic capacity in the frequency range of interest 0-100 c/s. This hydraulic capacity was measured experimentally by a special method. The experimental capacity value was equal to the theoretically calculated one if the manometer was boiled, showing that the manometer was air free. The hydraulic capacity (compliance) of the catheter can be measured in the same way as for the manometer. After repeated flushing the lowest possible value for the capacity is attained; then the catheter is assumed to be air free. It is also shown experimentally that the viscous component of the elasto-viscous behaviour of the catheter wall can be neglected for the frequency range of 1-100 c/s.

Boiled water filtered through micro-pore filters (10 μm-pores) was
used to fill the catheter and manometer. However, even when this precaution is taken, the measurement of the catheter resistance, using Poiseuille's law, will not yield reproducible results. This is presumably due to organisms sticking to the wall of the catheter, since rinsing with pepsin was an effective way of obtaining reproducible minimal values of the resistance. The Poiseuille resistance is used for the calculation of the plug flow inductivity of the catheter. The Poiseuille resistance, the plug flow inductivity and the capacity of the catheters U.S.C.I. Nos. 4, 5, 6 and 8 are tabulated in Table IV. (page 65).

It is shown theoretically, following the calculations of Womersley (1955 and 1957), that a rigid catheter behaves as a frequency dependent hydraulic resistance and hydraulic inductivity. Furthermore, equations are derived for two models of catheter-manometer systems. The first model is that of a lumped π-circuit with the hydraulic resistance and inductivity of the rigid catheter as series elements, and half of the catheter hydraulic capacity as a parallel element, and loaded at the output by the hydraulic manometer capacity. The second model is that of a transmission line, approximated by the hydraulic resistance and hydraulic inductivity of the rigid catheter as series elements and the distributed hydraulic capacity as a parallel element, loaded at the output by the hydraulic capacity of the manometer.

Using the parameters of the catheters from Table IV (page 65) the theoretical amplitude frequency response curves have been calculated for both models. The lumped circuit model curves for catheters Nos. 4, 5, 6 and 8 are shown in Fig. 13; the curves for both models are shown in Figs. 14a and 14b for catheters Nos. 8 and 5, respectively.

It can be seen from Fig. 13 where the experimentally determined amplitude frequency response curves of the catheter-manometer system, tested with the pressure generator, are also shown, that for practical purposes the catheter-manometer system can be considered as a lumped π-circuit. Therefore, the amplitude frequency response curves of catheter-manometer systems, based on this model, have been given as functions of dimensionless variables in Fig. 11. These curves are of practical use for the physician.
The experimental data of catheters Nos. 8 and 5 are shown in Figs. 14a and 14b, respectively. The discrepancy between the predictions of the transmission line model and the experimental points cannot be explained by the neglect of the mass of the catheter wall. It is shown that the visco-elastic behaviour of the catheter wall together with the error which is made by the approximation inherent to the used transmission line model can account for this discrepancy.

To avoid catheter artefacts the resonance peak of the catheter-manometer system should be damped. Furthermore, if the frequency range of interest extends beyond the flat part of the amplitude response characteristic, damping is necessary to increase the flat part in this range.

Two methods of hydraulic damping are discussed: the series damping and the parallel damping. It is shown that the parallel damping is preferable to the series damping because (1) the range of frequencies in which the amplitude response curve is flat is larger, and (2) the delay time of the oscillations is independent of the frequency. A theoretical discussion reveals the action of the hydraulic dampers.

Hydraulic parallel damping is superior to electrical damping. It is shown that electrical damping with R-C-circuits produces a negative hump, followed by a positive hump of too great a height to guarantee undistorted blood pressure measurements.

In the last chapter (X) recommendations for obtaining the best results with their catheter-manometer systems are given to physicians.
REFERENCES

3. Federhofer, K., Der Eisenbau 9, 152, (1918)
6. Frank, O., Zschr. f. Biol. 44, 445-613 (1903)
STELLINGEN

1. Linden, Noble en Yanof wijden bij de beschrijving van de door hen ontworpen drukgenerator geen beschouwing aan de inwendige impedantie en de frequentie-afhankelijkheid van de amplitude van de door hen ontworpen generators, hetgeen een ernstig verzuim is.
Linden, R.J., J.Sci. Instr. 36, 137 (1959)
Yanof, W.M., A.L. Rosen, W.M. MacDonald and D.A. MacDonald, Phys. in Med. and Biol. 8, 407, (1963)

2. De multipele resonanties bij zeer hoge frequenties van de Lilly manometer met catheter Courand Nr. 6L, zoals gemeten door Noble met zijn drukgenerator, kan mogelijk veroorzaakt worden door de frequentie-afhankelijkheid van de generator amplitude bij constante solenoide-stroom-amplitude, in plaats van door reflecties aan het catheter-manometer systeem, zoals hij suggereert.

3. De conclusie van Frank, dat een catheter-manometer systeem kan worden voorgesteld door een hydraulisch $\pi$-filter en de conclusie van Hansen, dat zo'n systeem kan worden beschouwd als een transmissielijn, zijn niet gerechtvaardigd op grond van hun experimenten.
Frank, O. Zschr. f. Biol. 44, 445-613, (1903)
Hansen, A.T., Thesis, Copenhagen (1949)

4. Het verdient aanbeveling catheters voor het gebruik door te spoelen met pepsine en daarna met gekookt en gefilterd water.
5. Het zou van groot nut zijn, indien fabrikanten van de door hen vervaardigde catheters de waarde van de hydraulische capaciteit en de inwendige diameter zouden opgeven, zowel in ongebruikte toestand als in de toestand na 20 maal uitkoken. Met de waarde van de hydraulische capaciteit als gegeven kan de arts de aanwezigheid van lucht in de manometer vaststellen. De waarde van de inwendige diameter is nodig voor het bepalen van de dynamische responsie in het frequentiegebied van de bloeddruk.

6. Een electronische regeling van bij voorkeur de lucht temperatuur in couveuzes, waarbij de regelgrootheid een inwendige temperatuur van de couveuze-baby is, schept de mogelijkheid door een geringe verstelling van de ingestelde waarde van deze regeling en meting van de deviatie van de regelgrootheid, de rondgaande versterkings-factor van de biologische thermische regeling van de baby te meten en deze van dag tot dag te vervolgen. Met deze gegevens kan een indruk verkregen worden van de ontwikkeling van de biologische thermische regeling in pasgeborenen en wel mede in afhankelijkheid van de waarde van de rondgaande versterking van de uitwendige electronische regeling, die waarschijnlijk de ontwikkeling beïnvloedt. Op deze gegevens kan een programma gebaseerd worden, waarbij de rondgaande versterking van de electronische regeling langzamerhand verminderd wordt, zo dat het ontwikkelingsproces van de biologische, thermische regeling voldoende voortgang vindt en de regelgrootheid binnen het veilige gebied blijft.

7. De indirecte meting van de systolische bloeddruk aan lichaamshanden met kleine, dikwandige arterien, volvoerd met dalende manchetdruk, zoals veelal gebruikelijk is, geeft te lage waarden. De systolische druk kan beter met oplopende manchetdruk gemeten worden.

8. Zuurstofpolarografie met de met een membraan bedekte electrode zal een uitkomst opleveren, onafhankelijk van de stroomsnelheid van
het omgevende medium, als de electrode intermitterend wordt verbonden met de uitwendige keten, waarbij de polarografische stroomsterkte op een vaste tijd na de verbinding (b.v. 50 msec) wordt afgelezen en daarna onmiddellijk de verbinding wordt verbroken.

Bij deze methode kunnen bij kortere intervallen van de ketensluiting, dunnere membranen gekozen worden, zodat de responsiesnelheid opgevoerd kan worden. De technische gevoeligheid kan opgevoerd worden omdat grotere elektroden genomen kunnen worden, t.o.v. het systeem waarbij zeer kleine elektroden gekozen worden bij continue polarografie, ter onderdrukking van de stroomsnelheidsinvloed van het omgevende medium.

9. Het bepalen van de diastolische bloeddruk volgens de indirecte methode van De Dobbeleer kan verklaard worden met een vermindering van de polsgolfsnelheid bij samengedrukte (flotterende) arteriën. De fysische aspecten van de door De Dobbeleer gegeven verklaring is onjuist.
G.de Dobbeleer, Acta Anaesthesiologica Belgica, 2, 7-18, (1962)

10. Met het gegeven, dat de maximale kracht per eenheid van spierdoorsnede voor de zoogdieren gelijk is, kan verklaard worden, dat gelijkvormige harten bij verschillende zoogdieren eenzelfde systolische bloeddruk veroorzaken.

11. De sterftekans van bejaarden kan worden verminderd door hen de nachtrust in twee of meer delen te doen ondergaan. Tussen deze gedeelten moet een periode van waken liggen, waarbij de fysiologische grootheden ongeveer op het niveau komen die voor de wakende toestand geldt.

12. Het overschot aan ongehuwde vrouwen wordt mede veroorzaakt door het gemiddelde positieve leeftijdsverschil tussen gehuwde mannen en vrouwen indien er geen bevolkingsafname is.
13. Met het gegeven, dat voor de zoogdieren het totale basaal metabo­lisme evenredig is met het lichaamsoppervlak kan verklaard wor­den, dat voor gelijkvormige harten de hartfrequentie ongeveer om­gekeerd evenredig is met de wortel uit het lichaamsoppervlak.