

PDF hosted at the Radboud Repository of the Radboud University Nijmegen

The following full text is a preprint version which may differ from the publisher's version.

For additional information about this publication click this link.

<http://hdl.handle.net/2066/103693>

Please be advised that this information was generated on 2019-06-18 and may be subject to change.

Renormalisation group invariants and sum rules: fast diagnostic tools for probing high-scale physics

Jamil Hetzel,^a Wim Beenakker^b

^a*Institut für Theoretische Physik, Universität Heidelberg,
Philosophenweg 16, D-69120 Heidelberg, Germany*

^b*Theoretical High Energy Physics, IMAPP, Faculty of Science, Mailbox 79,
Radboud University Nijmegen, P.O. Box 9010, NL-6500 GL Nijmegen, The Netherlands*

E-mail: hetzel@thphys.uni-heidelberg.de, W.Beenakker@science.ru.nl

ABSTRACT: A method is described to probe high-scale physics in lower-energy experiments by employing sum rules in terms of renormalisation group invariants. The method is worked out in detail for the study of supersymmetry-breaking mechanisms in the context of the Minimal Supersymmetric Standard Model. To this end sum rules are constructed that test either specific models of supersymmetry breaking or general properties of the physics that underlies supersymmetry breaking, such as unifications and flavour-universality.

KEYWORDS: Renormalization Group, Sum Rules, Beyond Standard Model, Supersymmetry Breaking

Contents

1	Introduction	2
2	Effective Lagrangians and renormalisation group equations	3
2.1	Effective field theories	3
2.2	Matching	5
2.3	Travelling along the EFT chain	6
2.4	How to probe the high scale	7
2.4.1	Top-down method	7
2.4.2	Bottom-up method	9
2.4.3	Renormalisation group invariants	10
3	Using RGIs and sum rules to study supersymmetry breaking	11
3.1	Supersymmetry	11
3.2	The Minimal Supersymmetric Standard Model	12
3.3	Constraints on broken supersymmetry	13
3.4	Breaking supersymmetry	15
3.5	Supergravity	16
3.6	Anomaly-mediated supersymmetry breaking	17
3.7	General gauge mediation	18
3.8	Mirage mediation	20
3.9	One-loop RGIs for the MSSM	21
3.10	RGIs in the literature	21
3.10.1	Using RGIs effectively	23
4	Results	24
4.1	Sum rules	25
4.1.1	Sum rules summary	30
4.2	Model-specific sum rules	30
4.2.1	General gauge mediation	30
4.2.2	Anomaly mediation.	33
4.3	Discussion	35
5	Conclusions and Outlook	38
A	One-loop RG equations for the MSSM	40
B	Deriving the one-loop RGIs for the MSSM	42

1 Introduction

Although the Standard Model of electroweak interactions has worked pretty well so far, it has a number of shortcomings. For example, it is difficult to explain why the Higgs mass is much smaller than the Planck scale (known as the “hierarchy problem”), a description of gravity is lacking and there is no good candidate for dark matter. Moreover, given the fact that the Higgs mass is either relatively light or rather heavy, i.e. $M_H < 127$ GeV or $M_H > 600$ GeV at 95% confidence level [1, 2], it is difficult to guarantee the stability of the Standard Model Higgs mechanism up to the Planck scale [3]. This might hint at the possibility that there is a scale of new physics between the electroweak scale and the Planck scale.

If signs of such new physics are observed at the upcoming runs of the Large Hadron Collider (LHC), the main theoretical challenge will be to unravel the underlying theory. Since the underlying physics can reside at energies that largely exceed the reach of the LHC, this is not going to be a simple task. The standard strategies for addressing this issue make use of renormalisation group techniques, linking the “low-energy” physics observed at present-day experiments to high-energy parameters at the energy scale where the underlying theory is formulated. The most widely studied approach is the top-down method, where one starts by choosing a specific new-physics model. Subsequently, the high-scale model parameters are evolved down to the collider scale and predictions can be made about the way the model will manifest itself phenomenologically in ongoing experiments, allowing a confrontation between theory and experiment. This approach is great for identifying where one should look for signs of new physics, but it is not really well-suited for deriving conclusive statements about the underlying model. Alternatively a bottom-up method can be used, where one is guided by experimental data. One starts by adopting a rather general phenomenological framework for describing the physics beyond the Standard Model, such as a supersymmetric extension of the Standard Model. Within this phenomenological context, the data are converted into running parameters at the collider scale. Subsequently, the running parameters are evolved up towards the scale where the underlying physics is presumably residing, allowing the high-scale parameters to be confronted with specific predictions from new-physics models. This method is better-suited for getting information on the underlying model. However, since the renormalisation group evolution is a numerical procedure, it has the tendency to enlarge uncertainties. Moreover, the fact that we do not know the scale of new physics can lead to misinterpretation of the data. A more detailed discussion of the advantages and drawbacks of the top-down and bottom-up methods will be given in section 2.

Recently, an elegant third approach to probing high-scale physics has gained some interest [4–7]. This approach is based on the same philosophy as the bottom-up method, but instead of using the full set of running parameters it makes clever use of so-called renormalisation group invariants. These are combinations of running parameters chosen in such a way that they do not evolve with energy. If we measure their values at the collider scale, we will immediately know their values at the threshold of new physics. This fact allows one to probe physics at high energy scales without having to evolve all parameters.

Subsequently, the renormalisation group invariants can be combined into sum rules that test the underlying physics. Up to now, sum rules have been constructed for testing specific models. We advocate to employ sum rules in a more model-independent way, by using them as fast diagnostic tools to test generic properties that are common to new-physics models, such as unification and universality properties. If a certain property is realised in Nature, all corresponding sum rules must be satisfied. So, the main strength of invariant sum rules is their falsifying power.

In order to give an idea of how these renormalisation-group-invariant sum rules work in practice, we work them out in detail for the study of supersymmetry-breaking mechanisms in the context of a phenomenological version of the Minimal Supersymmetric Standard Model. This model is one of the prime beyond-the-Standard-Model frameworks to be tested at the LHC, since it offers solutions to several of the problems that plague the Standard Model.

The structure of the paper is as follows. In section 2 we describe the concept of effective field theories and discuss renormalisation group techniques. In section 3 we give the salient details of the Minimal Supersymmetric Standard Model and a few popular supersymmetry-breaking mechanisms. Subsequently, the one-loop renormalisation group invariants are listed for a phenomenological version of the model. In section 4 we give a detailed discussion of the model-independent and model-specific sum rules that can be used for studying the supersymmetry-breaking mechanisms. We will conclude in section 5.

2 Effective Lagrangians and renormalisation group equations

When we perform calculations beyond tree level, we often encounter divergent integrals. In a renormalisable theory, these can be dealt with by a redefinition of the masses and couplings, which become scale dependent. Not all quantum field theories are renormalisable: if the Lagrangian contains operators with dimension greater than four, the theory is non-renormalisable. Such theories require an infinite number of counterterms and therefore have no predictive power, so one may wonder why we would ever consider non-renormalisable operators.

Suppose we regularise the divergences of a theory with a momentum cutoff Λ . The prevalent interpretation of renormalisation used to be that we should get rid of it by taking $\Lambda \rightarrow \infty$ at the end of our calculations [8]. However, by taking this limit we tacitly assume that the theory is valid up to arbitrarily large momenta. Since Wilson’s work on the renormalisation group (RG) [9, 10], this view has changed: now Λ is considered as a scale at which new physics becomes relevant. For processes at energies greater than Λ , the theory is not valid anymore and should be replaced by a more fundamental theory. This is the motivation for using effective field theories (EFTs).

2.1 Effective field theories

Suppose we had a ‘theory of everything’: a theory describing all fundamental dynamics of the basic constituents of Nature and unifying different kinds of interactions. Although this theory could in principle describe all physical phenomena, it would be unnecessarily

cumbersome to describe Nature at all physical scales. For example, the laws of chemistry arise from the electromagnetic interaction, yet it would be unwise to start a quantitative analysis from Quantum Electrodynamics. Instead, when we wish to analyse a particular physical system, we need to isolate its most relevant ingredients from the rest in order to obtain a simple description without having to understand every detail.

In order to do so, we have to make an appropriate choice of variables that captures the most important physics of the system. Physics problems usually involve widely separated energy scales, which allow us to study low-energy dynamics without needing to know the details of the high-energy interactions. The basic idea is to identify the parameters that are large (small) compared to the relevant energy scale and put them to infinity (zero). Eventually we can improve this approximation by taking into account the corrections of the high-energy physics in the form of small perturbations.

Effective field theories (see e.g. [11]) are the theoretical tool to describe low-energy physics, where ‘low’ means low with respect to some energy scale Λ . An EFT only takes into account states with mass $m \ll \Lambda$; heavier excitations with $m \gg \Lambda$ are integrated out from the action. The information about the heavy states is then contained in the couplings of the low-energy theory: we get non-renormalisable interactions among the light states, organised as an expansion in powers of energy/ Λ .

An effective field theory is characterised by some effective Lagrangian:

$$\mathcal{L} = \sum_i c_i O_i, \quad (2.1)$$

where the O_i are operators constructed from the light fields and the c_i are couplings containing information on any heavy degrees of freedom. Since the Lagrangian has dimension 4, dimensional analysis yields:

$$[O_i] \equiv d_i \quad \Rightarrow \quad c_i \sim \frac{1}{\Lambda^{d_i-4}}, \quad (2.2)$$

where Λ is some characteristic heavy scale of the system. At low energies, the behaviour of these operators is determined by their dimension:

- Operators with $d_i < 4$ are called *relevant*, since they give rise to effects that become large at low energies.
- Operators with $d_i > 4$ are called *irrelevant*: at energy scales E their effects are suppressed by powers of E/Λ , making them small at low energies. These are non-renormalisable operators that contain information about the underlying dynamics at higher scales.
- Operators with $d_i = 4$ are called *marginal*, because they are equally important at all energy scales.

This explains why we are able to include non-renormalisable operators in an EFT without spoiling its predictive power: at low energies E , their effects can be either neglected or incorporated as perturbations in powers of E/Λ . At high energies, it is more appropriate to use a different EFT. Thus at sufficiently low energies, an EFT automatically contains only renormalisable operators.

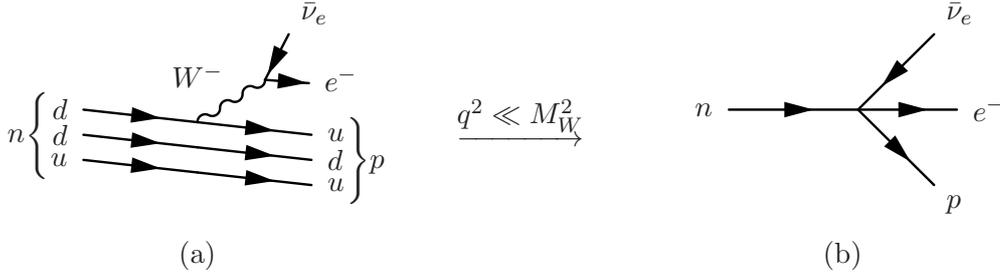


Figure 1. Tree-level diagram for beta decay of a neutron (a) in the Standard Model and (b) in the Fermi theory of weak interactions. In the Standard Model, this decay proceeds through the exchange of a W boson. If the momentum transfer q of the W boson is much smaller than its mass M_W , the W -boson propagator reduces to a contact interaction. In that case, the Fermi 4-vertex provides an effective description of this decay.

2.2 Matching

Suppose we have two EFTs: one that includes a heavy particle and one where its effects are included in the form of higher-dimensional operators, suppressed by inverse powers of the heavy particle mass M . Since physics around the mass scale M should not depend on our choice of theory, both EFTs should yield the same physical predictions. Hence they are related by the *matching condition*: at the threshold $\mu = M$, the two EFTs should give rise to the same S -matrix elements for light-particle scattering. This leads to relations between the parameters of the high-energy EFT (the one we use above threshold) and those of the low-energy EFT (the one we use below threshold). In other words, the matching conditions encode the effects of the heavy field into the low-energy EFT parameters.

As an example, consider the beta decay of a neutron. In the Standard Model, this decay is mediated by a W boson with mass M_W , which has the propagator

$$\rho \underset{q}{\sim} \nu = \frac{-ig_{\rho\nu}}{q^2 - M_W^2} \quad (2.3)$$

in the 't Hooft-Feynman gauge. If the momentum transfer q carried by the W boson is much smaller than its mass, this propagator reduces to a contact interaction (figure 1):

$$\frac{-ig_{\rho\nu}}{q^2 - M_W^2} \underset{q^2 \ll M_W^2}{\sim} \frac{ig_{\rho\nu}}{M_W^2} + \mathcal{O}\left(\frac{q^2}{M_W^4}\right). \quad (2.4)$$

At energies well below the W mass, there is not enough energy available to produce a physical W boson. Hence we might as well switch to an EFT that does not include the W field. Integrating out the W field from the action, we are left with the Fermi 4-vertex and higher-order interactions. Matching the two EFTs at $\mu = M_W$ yields the well-known formula for the Fermi coupling constant

$$G_F = \frac{\sqrt{2}}{8} \frac{g_2^2}{M_W^2}, \quad (2.5)$$

where g_2 is the weak coupling constant. Note that although the W field is not included in the low-energy EFT, its ‘fingerprints’ (namely its coupling constant g_2 and mass M_W)

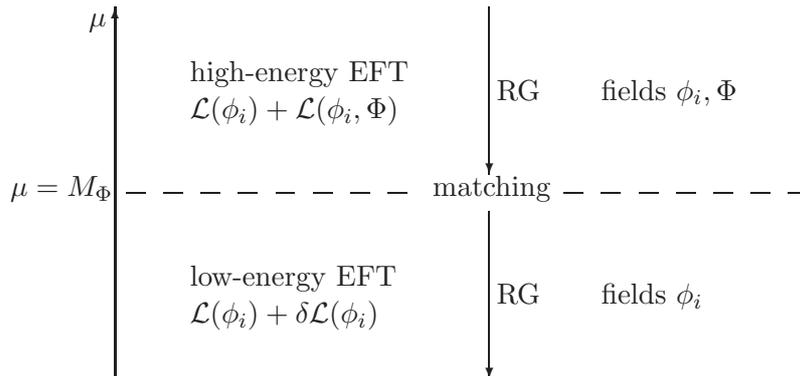


Figure 2. Schematic display of the procedure for evolving from high to low energies. A high-energy EFT, which contains light fields ϕ_i and a heavy field Φ with mass M_Φ , is evolved down using the renormalisation group equations. At the scale $\mu = M_\Phi$ we should switch to a low-energy EFT that includes only the light fields ϕ_i . The matching conditions yield the masses and couplings of the low-energy EFT at this scale. Then we continue to evolve down the theory, now using the RG equations of the low-energy EFT.

are still present in the low-energy coupling G_F . Also note that the irrelevant operator corresponding to the Fermi 4-vertex is indeed suppressed by powers of the W mass, as mentioned in section 2.1.

2.3 Travelling along the EFT chain

In the process of renormalising a theory, we redefine the masses and couplings by having them depend on a reference scale μ . This μ -dependence can be determined by noting that anything observable should be independent of μ . Consider for example any observable Γ , which is a function of some couplings $\{g_i(\mu)\}$ and masses $\{m_j(\mu)\}$. The above observation implies that

$$0 = \mu \frac{d}{d\mu} \Gamma = \left(\mu \frac{\partial}{\partial \mu} + \sum_i \mu \frac{dg_i(\mu)}{d\mu} \frac{\partial}{\partial g_i(\mu)} + \sum_j \mu \frac{dm_j^2(\mu)}{d\mu} \frac{\partial}{\partial m_j^2(\mu)} \right) \Gamma. \quad (2.6)$$

By explicitly calculating a set of observables that contains all necessary information, we obtain the renormalisation group equations: a set of coupled differential equations that govern the μ -dependence of the masses and couplings. These will depend on the loop order at which the observables are calculated and on the particle content of the theory, since this determines which particles can appear in loops.

Effective field theories, combined with the RG equations, allow us to evolve a theory from a high energy scale to a low one (figure 2). Suppose that we have an EFT describing physics at some (high) energy scale μ . The Lagrangian contains a field Φ with the largest mass M_Φ and a set of lighter fields ϕ_i :

$$\mathcal{L}_{\text{high}} = \mathcal{L}(\phi_i) + \mathcal{L}(\phi_i, \Phi), \quad (2.7)$$

where $\mathcal{L}(\phi_i)$ contains only the light fields and $\mathcal{L}(\phi_i, \Phi)$ contains the heavy field and its interactions with the light fields. If we want to describe physics at a lower energy scale, we have to evolve down the running parameters using the RG equations of this EFT. We can continue to do so until we reach the threshold $\mu = M_\Phi$. There we integrate out the heavy field Φ from the action, i.e. we switch to a different EFT containing only the light fields ϕ_i :

$$\mathcal{L}_{\text{low}} = \mathcal{L}(\phi_i) + \delta\mathcal{L}(\phi_i). \quad (2.8)$$

Note that $\mathcal{L}(\phi_i)$ contains the same operators in both theories, but with different couplings and masses due to the matching conditions. The second part $\delta\mathcal{L}(\phi_i)$ encodes the information on the heavy field Φ . It contains operators constructed with the light fields ϕ_i only, including new higher-order interactions that are suppressed by appropriate powers of $1/M_\Phi$. By matching the two EFTs at $\mu = M_\Phi$, we fix the values of the running parameters of the low-energy theory. From there, we can continue to evolve the theory down using the renormalisation group equations of the low-energy EFT.

Whenever we reach a new particle threshold, we should integrate out the corresponding field and match the two EFTs. Thus in the framework of effective field theories, physics is described by a chain of EFTs. Each one has a different particle content, and all theories match at the corresponding particle thresholds. Each theory below a threshold is considered as the low-energy EFT of the theory above that threshold, which is considered as a more fundamental theory. Then the ultimate goal of physics becomes to find the most fundamental theory of Nature, although strictly speaking we can never know whether we have found it, if there is a most fundamental one at all.

From this point of view, the Standard Model is only a low-energy effective field theory of Nature. The shortcomings of the Standard Model hint at the existence of a more fundamental theory. Even if that more fundamental EFT is appropriate only at energies beyond experimental access, the idea of a chain of EFTs certainly helps us study that more fundamental theory: we could measure the running parameters at a low scale μ and then evolve them upwards. At the threshold of the more fundamental theory, the matching conditions act as boundary conditions for the renormalisation group. Hence, by comparing our evolved masses and couplings with the predicted matching conditions, we can get information on the high-energy theory.

2.4 How to probe the high scale

2.4.1 Top-down method

The literature offers various approaches to using the renormalisation group to extract information about high-scale matching conditions. In the context of supersymmetry, the most widely studied one is the top-down method; see e.g. [12–14]. It is called this way because a top-down study is started from the high scale and the theory is evolved down to the collider scale (say $\mathcal{O}(1 \text{ TeV})$). One starts by choosing a new-physics model with few parameters and proceeds as follows:

- Pick a point in the parameter space of the model and translate this into values of the running parameters at the high scale (masses, gauge and Yukawa couplings, etc.).

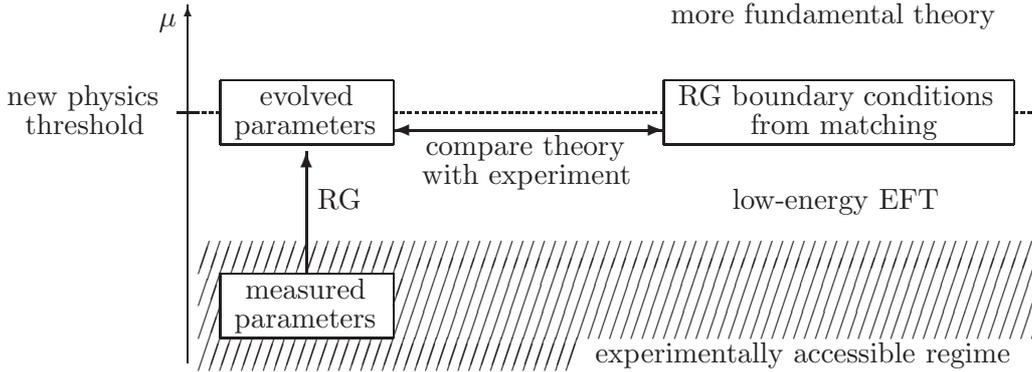


Figure 3. Scheme for studying physics at scales beyond experimental access. The running couplings are measured at a scale where the low-energy EFT is applicable. Using the RG equations, they are evolved towards the threshold where new fields presumably enter the theory. Then they can be compared with the matching conditions predicted by the more fundamental theory.

- Evolve the running parameters down to the collider scale using the renormalisation group equations.
- Using the resulting parameter values, perform a detector simulation to calculate relevant branching ratios and cross sections.
- Compare the results to experimental data and extract constraints on the parameter space of the model.

The top-down method is suitable for making general phenomenological predictions. For example, it is used to find collider signatures that are characteristic for supersymmetry. However, for the purpose of testing a new-physics model, this method has some serious limitations:

- With the top-down method one can only determine the regions in the parameter space of the model that are consistent with the data. If only small portions of the total parameter space seem phenomenologically viable, one might conclude that the model is neither likely to be correct nor natural. However, it seems unlikely that we can strictly exclude a model this way.
- Scanning the entire parameter space is very time-consuming. To scan it properly, one ought to use a reasonably fine grid and check each point separately. But the parameter space is usually too big to perform a full detector simulation for each point. For general predictions of supersymmetric phenomenology, for example, one usually resorts to using a limited set of benchmark points (see e.g. [15]), because many points in parameter space have a very similar phenomenology. However, for the purpose of excluding a certain model, this is no satisfying solution.
- Fitting the numerical predictions to the experimental data becomes much more difficult as the number of model parameters is increased. Therefore one always limits

oneself to a model with few parameters. But there is no reason to think that Nature would restrict itself to only a few parameters in the EFT beyond the next threshold.

2.4.2 Bottom-up method

The bottom-up method is an alternative to the top-down method; see e.g. [16–18] for bottom-up studies in the context of supersymmetry. It works by evolving the theory upwards from the collider scale to the new-physics threshold. A bottom-up analysis consists of the following steps:

- Convert experimental data into the running parameters at the collider scale.
- Using the renormalisation group equations pertaining to the low-energy EFT (e.g. the Standard Model above the electroweak scale, or the Minimal Supersymmetric Standard Model above the supersymmetric mass scale), evolve these running parameters towards the scale where new physics presumably comes into play.
- Analyse the structure of the high-scale parameters: do they fit the matching conditions predicted by any new-physics model?

This method seems more suitable for excluding new-physics models than the top-down method. Also, there is no practical need to only consider models with few parameters. Furthermore, no time-consuming scanning of the parameter space is involved. However, the bottom-up method presents challenges of its own:

- The running parameters at the collider scale will come with experimental errors. To determine the uncertainty in these parameters at a higher scale, we also have to evolve the error bars. These may become larger while numerically performing the renormalisation group evolution, which could make it difficult to tell for example whether certain parameters unify or not.
- We do not know the value of the high scale that should be taken as the new-physics threshold; this scale has to be guessed. In practice, one might evolve the running parameters until some of them unify and take the corresponding scale as the new-physics threshold. But a unification scale does not necessarily correspond to a threshold.¹ Also, there may be an intermediate new-physics threshold even though no unification occurs there. In both cases, we would extract incorrect boundary conditions for matching with the underlying high-energy EFT.
- Because the RG equations are coupled, all running parameters must be known. Hence if we fail to measure one mass or coupling, the bottom-up method cannot be used except for subsets of parameters whose RG equations contain only the parameters from that subset.

¹This occurs for example in a supersymmetry-breaking model called Mirage Mediation, see section 3.

2.4.3 Renormalisation group invariants

Recently, a third approach to probing the high scale has gained some interest [4–7]. This approach makes clever use of renormalisation group invariants (RGIs). These are combinations of running parameters chosen in such a way that they are independent of the renormalisation scale μ . A well-known example of an RGI is the following combination of the strong coupling g_3 , the weak coupling g_2 as well as the scaled hypercharge coupling $g_1 = g' \sqrt{5/3}$:

$$I_g \equiv (b_2 - b_3)g_1^{-2} + (b_3 - b_1)g_2^{-2} + (b_1 - b_2)g_3^{-2}, \quad (2.9)$$

where the gauge couplings satisfy the following renormalisation group equations at one loop:

$$16\pi^2 \frac{dg_a}{dt} = b_a g_a^3 \quad (a = 1, 2, 3). \quad (2.10)$$

Here $t \equiv \ln(\mu/\mu_0)$, where μ_0 is a reference scale that makes the argument of the logarithm dimensionless; its value is arbitrary since it drops out of the RG equations. The RG coefficients b_a are constants depending on the particle content of the model. For the Standard Model they are $b_a = (\frac{41}{10}, -\frac{19}{6}, -7)$ for $a = 1, 2, 3$, whereas for the Minimal Supersymmetric Standard Model (MSSM) they are $b_a = (\frac{33}{5}, 1, -3)$. It is easily checked that $dI_g/dt = 0$. Note that I_g is not exactly RG invariant, since we used the one-loop RG equations to construct it. We will come back to this issue in section 4.

A crucial property of RGIs is that if we measure their values at the collider scale, we will immediately know their values at the threshold of new physics. This fact allows us to probe physics at high energy scales without having to evolve all parameters. For example, I_g can be used to test whether the gauge couplings unify. Note that if the gauge couplings have a universal value g_U at some energy scale, then I_g will vanish. Since I_g is an RGI, it will consequently vanish at every scale where the renormalisation group equations (2.10) are valid. Hence, if we measure the gauge couplings at one scale, we can perform a quick diagnostic check to test whether gauge-coupling unification occurs within the context of a specific EFT. Also note that this consistency check is independent of the value of the scale where the gauge couplings unify.

To illustrate this, we perform this check explicitly for the Standard Model, using the measured couplings at $\mu = M_Z$ in the \overline{MS} scheme. The gauge couplings can be obtained from the measured quantities $\alpha_3^{-1}(M_Z) = 8.45 \pm 0.05$, $\alpha^{-1}(M_Z) = 127.916 \pm 0.015$ and $\sin^2 \theta_W(M_Z) = 0.23131 \pm 0.00007$ [19]. Here $\alpha_a^{-1} \equiv 4\pi g_a^{-2}$, α is the fine structure constant and θ_W is the weak mixing angle. The latter two are related to α_2 and α_1 by the relations

$$\begin{aligned} \alpha_2^{-1}(M_Z) &= \sin^2 \theta_W(M_Z) \alpha^{-1}(M_Z) = 29.588 \pm 0.010, \\ \alpha_1^{-1}(M_Z) &= \frac{3}{5} \cos^2 \theta_W(M_Z) \alpha^{-1}(M_Z) = 58.997 \pm 0.009. \end{aligned} \quad (2.11)$$

Using these values we find $I_g^{\text{SM}} = -3.252 \pm 0.030$, which lies many standard deviations from 0. Hence we find no compatibility with gauge-coupling unification within the Standard Model; this can be confirmed using the bottom-up method (figure 4).

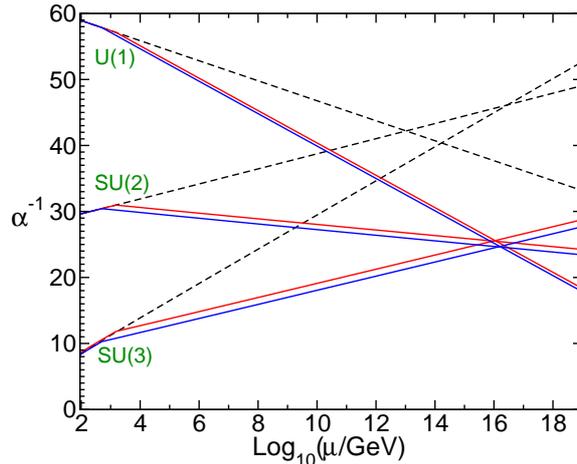


Figure 4. Two-loop renormalisation group evolution of the inverse gauge couplings squared $\alpha_a^{-1}(\mu) = 4\pi g_a^{-2}(\mu)$ for $a = 1, 2, 3$ in the Standard Model (dashed lines) and the Minimal Supersymmetric Standard Model (solid lines). For the latter, the supersymmetric particle thresholds are varied between 500 GeV and 1.5 TeV, and $\alpha_3(M_Z)$ is varied between 0.117 and 0.121. Figure taken from [20].

Similarly, we can make an estimate for I_g in the MSSM by taking the above values of $\alpha_a^{-1}(M_Z)$.² We find $I_g^{\text{MSSM}} = -0.059 \pm 0.024$, which is close to zero. Hence the MSSM might allow for gauge-coupling unification, depending on the actual values of the supersymmetric particle thresholds. This can be confirmed using the bottom-up method (figure 4).

To summarise, RGIs provide a fast diagnostic tool for probing matching conditions at high energy scales. They circumvent the need to evolve the running parameters numerically; we do not even need to know exactly at which energy scale new physics arises.

3 Using RGIs and sum rules to study supersymmetry breaking

In order to give an idea of how the RGI method works in practice, we work it out explicitly for models of supersymmetry breaking. To this end, we first give a short description of the salient details of the minimal supersymmetric extension of the Standard Model. This is followed by a discussion of the necessity to break supersymmetry and the ways to achieve this. For an extensive introduction to the subject the interested reader is referred to Refs. [20, 21].

3.1 Supersymmetry

Although the Standard Model has worked pretty well so far, it has a number of shortcomings. For example, it is difficult to explain why the Higgs mass is much smaller than the

²In order to determine the actual value of I_g^{MSSM} we ought to use the values of α_a^{-1} at the scale where the MSSM becomes valid, i.e. the highest supersymmetric particle threshold. This scale will be somewhat higher than M_Z . As can be seen using the bottom-up method (figure 4), this barely changes the prediction of gauge-coupling unification: only the value of the unification scale might change. Hence we should get a good estimate of I_g^{MSSM} using the values of $\alpha_a^{-1}(M_Z)$.

Planck scale (the hierarchy problem), a description of gravity is lacking and there is no good candidate for dark matter. Moreover, given the fact that the Higgs mass is either relatively light or rather heavy, i.e. $M_H < 127$ GeV or $M_H > 600$ GeV at 95% confidence level [1, 2], it is difficult to guarantee the stability of the Standard Model Higgs mechanism up to the Planck scale [3]. This might hint at the possibility that there is a scale of new physics between the electroweak scale and the Planck scale.

Supersymmetry offers possible solutions to these problems [20, 21]. It is a symmetry between fermions and bosons that would complete the list of possible spacetime symmetries [22, 23]. The supersymmetry generators Q, Q^\dagger are spinors that satisfy

$$\{Q, Q^\dagger\} \sim P_\mu, \quad (3.1)$$

where P_μ is the 4-momentum operator. The single-particle states of a supersymmetric theory fall into irreducible representations of the supersymmetry algebra, called supermultiplets. These contain an equal number of bosonic and fermionic degrees of freedom. The bosons and fermions in a supermultiplet are called *superpartners* of each other. A minimal supersymmetric extension of the Standard Model contains two types of supermultiplets: *chiral (matter) supermultiplets*, which consist of a two-component Weyl spinor and a complex scalar field, and *vector (gauge) supermultiplets*, which consist of a spin-1 gauge-boson field and a spin-1/2 Majorana spinor, called the *gaugino* field. The supersymmetry generators Q, Q^\dagger commute with the mass-squared operator P^2 and the generators of gauge transformations, so superpartners have the same mass and gauge quantum numbers. In view of the quantum-number structure of the Standard Model, this implies that a supersymmetric extension of the Standard Model introduces at least one new supersymmetric particle (or *sparticle* for short) for each Standard Model particle.

At this price, supersymmetry solves many of the problems of the Standard Model. The hierarchy problem is solved because divergences from diagrams with bosonic loops are compensated by those with fermionic loops and vice versa. Supersymmetry could also connect the Standard Model to gravity if we impose invariance of the theory under local supersymmetry transformations, as is apparent from the relation (3.1) between the supersymmetry generators and the generators P_μ of coordinate shifts. Furthermore, many of the phenomenologically viable supersymmetric theories provide an attractive candidate for dark matter (see below). As an added bonus, supersymmetry also encourages unification, as can be seen from figure 4.

3.2 The Minimal Supersymmetric Standard Model

The Minimal Supersymmetric Standard Model (MSSM) is defined to be a supersymmetric extension of the Standard Model with minimal particle content. This means that each Standard Model particle has a supersymmetric partner. Also, an additional Higgs doublet is needed because of the analytic structure of supersymmetric theories and in order to prevent gauge anomalies. The MSSM particle content and nomenclature is listed in tables 1–2.

The non-gauge interactions between these particles contain supersymmetric versions of the Yukawa interactions. Since we have two Higgs doublets, there is also a supersymmetry-preserving Higgs mixing term; this introduces one new parameter μ with respect to the

Name	Symbol	Spin 0	Spin 1/2	Gauge-group representation
squarks & quarks (3 generations)	Q	$\tilde{Q} = (\tilde{u}_L, \tilde{d}_L)$	(u_L, d_L)	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$
	\bar{u}	\tilde{u}_R^*	u_R^c	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$
	\bar{d}	\tilde{d}_R^*	d_R^c	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$
sleptons & leptons (3 generations)	L	$\tilde{L} = (\tilde{\nu}, \tilde{e}_L)$	(ν, e_L)	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
	\bar{e}	\tilde{e}_R^*	e_R^c	$(\mathbf{1}, \mathbf{1}, 1)$
Higgs & higgsinos	H_u	(H_u^+, H_u^0)	$(\tilde{H}_u^+, \tilde{H}_u^0)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$
	H_d	(H_d^0, H_d^-)	$(\tilde{H}_d^0, \tilde{H}_d^-)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

Table 1. Chiral supermultiplet content of the Minimal Supersymmetric Standard Model and the corresponding representations of the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, with superpartners indicated by a tilde. Note that we need an additional Higgs doublet compared to the Standard Model and that right-handed modes are charge conjugated in order to bring them into left-handed form.

Standard Model. Furthermore, the MSSM has the same gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ as the Standard Model and is defined to preserve a new quantum number called *R-parity*:

$$P_R \equiv (-1)^{3(B-L)+2s}, \quad (3.2)$$

where B , L and s stand for the baryon number, lepton number and spin of the particle respectively. This means that all Standard Model particles have $P_R = +1$, whereas their superpartners have $P_R = -1$ as a result of the half-unit shift in spin. Interactions that violate *R-parity* have the tendency to result in rapid proton decay, therefore such interactions are excluded in the MSSM. As a consequence, every interaction vertex contains an even number of supersymmetric particles. This implies that the lightest supersymmetric particle (LSP) is absolutely stable. If the LSP is electrically neutral and carries no colour charge, it would make an attractive dark-matter candidate.

3.3 Constraints on broken supersymmetry

If supersymmetry were an exact symmetry of Nature, each sparticle would have the same mass as its Standard Model partner and we would have discovered them already. Hence, if supersymmetry is a symmetry of Nature, it must be broken somehow. The requirement that broken supersymmetry should still solve the problems of the Standard Model puts constraints on the possible terms of a supersymmetry-breaking Lagrangian.

Firstly, in order to maintain the solution to the hierarchy problem, we must consider

Names	Spin 1/2	Spin 1	Gauge-group representation
gluino & gluon	\tilde{g}	g	$(\mathbf{8}, \mathbf{1}, 0)$
winos & W bosons	\tilde{W}^1 \tilde{W}^2 \tilde{W}^3	W^1 W^2 W^3	$(\mathbf{1}, \mathbf{3}, 0)$
bino & B boson	\tilde{B}^0	B^0	$(\mathbf{1}, \mathbf{1}, 0)$

Table 2. Gauge supermultiplet content of the Minimal Supersymmetric Standard Model and the corresponding representations of the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$.

soft supersymmetry breaking. This means that the Lagrangian can be written as

$$\mathcal{L} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{soft}}. \quad (3.3)$$

where $\mathcal{L}_{\text{SUSY}}$ is supersymmetry invariant and $\mathcal{L}_{\text{soft}}$ violates supersymmetry but contains only masses and couplings with *positive* dimension. By using only relevant operators (see section 2.1) to break supersymmetry, we guarantee that the high-scale physics responsible for supersymmetry breaking decouples at low energies. It also guarantees that non-supersymmetric corrections to the Higgs mass vanish in the limit $m_{\text{soft}} \rightarrow 0$, where m_{soft} is the largest mass scale associated with the soft parameters.

The most general soft-supersymmetry-breaking Lagrangian in the MSSM, compatible with gauge invariance and R -parity conservation, contains complex gaugino masses M_1, M_2, M_3 ; trilinear couplings $\mathbf{a}_u, \mathbf{a}_d$ and \mathbf{a}_e , which are complex 3×3 matrices in family space similar to the Yukawa couplings; sfermion mass terms $\mathbf{m}_Q^2, \mathbf{m}_u^2, \mathbf{m}_d^2, \mathbf{m}_L^2, \mathbf{m}_e^2$, which are Hermitian 3×3 mass matrices in family space; real Higgs masses $m_{H_u}^2$ and $m_{H_d}^2$ and a complex supersymmetry-breaking Higgs mixing parameter b .

Another constraint comes from experimental bounds on flavour-changing neutral current (FCNC) processes and new sources of CP-violation. Although supersymmetry itself introduces only one new parameter μ with respect to the Standard Model, supersymmetry breaking introduces 97 new masses, mixing angles and phases [24]. For arbitrary values of these parameters, the predictions for the FCNC and CP-violating processes would violate the experimental bounds. In order to suppress these processes, we additionally assume the following relations between the parameters:

- The soft sfermion masses are flavour diagonal and the first- and second-generation masses are degenerate.
- There are no sources of CP-violation in the soft-supersymmetry-breaking sector beyond those induced by the Yukawa couplings.

We also neglect the first- and second-generation Yukawa and trilinear couplings, because they give very small contributions to the evolution of the soft-supersymmetry-breaking

parameters. These contributions are smaller than the two-loop corrections associated with the gauge couplings and third-generation Yukawa couplings.

These assumptions resemble the ones that form the basis for the so-called phenomenological MSSM (pMSSM) [25]. Note, however, that we opt to work with the soft-supersymmetry-breaking parameters in our approach rather than the mass eigenstates that are used in the pMSSM. Under these assumptions, we are left with the following parameters:

- Twelve real soft scalar masses, which we denote as $m_{\tilde{Q}_1}^2, m_{\tilde{Q}_3}^2, m_{\tilde{u}_1}^2, m_{\tilde{u}_3}^2, m_{\tilde{d}_1}^2, m_{\tilde{d}_3}^2, m_{\tilde{L}_1}^2, m_{\tilde{L}_3}^2, m_{\tilde{e}_1}^2, m_{\tilde{e}_3}^2, m_{H_u}^2, m_{H_d}^2$, in accordance with the notation in table 1. The subscripts 1 and 3 refer to the first and the third generation respectively.
- Three real gauge couplings g_1, g_2, g_3 .
- Three real gaugino masses M_1, M_2, M_3 .
- Three real third-generation Yukawa couplings y_t, y_b, y_τ .
- Three real soft trilinear couplings A_t, A_b, A_τ defined by

$$a_i = A_i y_i \quad (i = t, b, \tau, \text{ no summation}). \quad (3.4)$$

- Two real Higgs mixing parameters μ, b .

One might even go one step further and assume full universality, i.e. take all mass matrices proportional to the unit matrix. Such apparently arbitrary relations between the soft parameters could make sense from the effective-field-theoretical point of view. If supersymmetry is exact in a more fundamental EFT than the MSSM, but is broken spontaneously at some high energy scale, then the terms in $\mathcal{L}_{\text{soft}}$ may arise as effective interactions. In that case, the universality relations could arise as matching conditions at the threshold where we switch from the more fundamental theory to the MSSM. Strictly speaking, flavour-universality is lost once the parameters are evolved down to the electroweak scale, but the numerical impact of this RG evolution is small [26]. Thus the desire for a theory that naturally explains supersymmetry breaking forces us to consider spontaneously broken supersymmetry.

3.4 Breaking supersymmetry

For a spontaneous breakdown of supersymmetry we need a Lagrangian that preserves supersymmetry but a vacuum state that breaks it. During the construction of a supersymmetric theory, one has to introduce *auxiliary fields* for each supermultiplet in order to make the supersymmetry algebra close off-shell. These are scalar fields that turn out to be suitable for breaking supersymmetry (see e.g. section 7 of [20]): if some of them acquire a non-zero vacuum expectation value (VEV), supersymmetry is broken.

It turns out to be difficult to make this happen using only renormalisable interactions at tree level. Therefore, the MSSM soft terms are expected to arise radiatively. In radiative supersymmetry-breaking models, supersymmetry is broken in a *hidden sector*, which

contains fields that have no direct couplings to the MSSM fields. The latter are said to be in the *visible sector*. The two sectors only interact indirectly; the interactions between them are responsible for mediating the supersymmetry breakdown from the hidden sector to the MSSM. If the mediating interactions are flavour blind, then the soft terms of the MSSM will automatically satisfy universality conditions. We will discuss several proposals for breaking mechanisms in the sections 3.5–3.8.

3.5 Supergravity

Supergravity (SUGRA) [27] is the theory that results from imposing local supersymmetry invariance. Recall that once we promote a global gauge symmetry (with bosonic generators, satisfying commutation relations) to a local one, we have to introduce a bosonic field with predetermined gauge-transformation properties. Similarly, by promoting supersymmetry (which has *fermionic* generators, satisfying *anticommutation* relations) to a local symmetry, we have to introduce a *fermionic* field Ψ_μ with spin-3/2. This is the gravitino, the superpartner of the spin-2 graviton. The resulting SUGRA Lagrangian is non-renormalisable; there is as yet no renormalisable quantum field theory of gravity. However, the non-renormalisable operators are suppressed by inverse powers of the Planck mass $M_{\text{Pl}} = \mathcal{O}(10^{19} \text{ GeV})$, so that their effects at low energies are small (see section 2.1).

The spontaneous breakdown of supersymmetry occurs in a hidden sector where the auxiliary component of some superfield gets a VEV. According to Goldstone’s theorem, spontaneously breaking a global symmetry yields a massless particle with the same quantum numbers as the broken symmetry generator. Since the broken generator Q is fermionic, the massless particle is a massless neutral Weyl fermion, called the *goldstino*. The goldstino then becomes the longitudinal component of the gravitino, which becomes massive.³ It turns out that when we consider the effects of the supersymmetry-breaking VEV, the gravitino mass $m_{3/2}$ sets the scale of all the soft terms. Moreover, the scalar masses are universal at the scale where supersymmetry becomes broken.

Minimal supergravity. The most widely used model of supersymmetry breaking is minimal supergravity (mSUGRA) [28, 29]. Despite the name, mSUGRA is not a supergravity model, but rather the low-energy EFT resulting from a minimal locally supersymmetric model. In the underlying model, one uses the simplest possible Ansatz for the scalar potentials. This leads to universal soft supersymmetry-breaking parameters in the scalar sector. Gauge-coupling unification in the MSSM suggests an additional simple Ansatz for the gauge kinetic function, which leads to universal gaugino masses. At the GUT scale $M_{\text{GUT}} = 2 \cdot 10^{16} \text{ GeV}$ the model is then described by four parameters and a sign: a universal scalar mass m_0 , a universal gaugino mass $M_{1/2}$, a universal proportionality factor A_0 between the trilinear couplings and the corresponding Yukawa couplings, the ratio $\tan \beta$ of the two non-zero Higgs VEVs, and the sign of the supersymmetric parameter μ . At the

³Because of the similarities with the Higgs mechanism, where the electroweak gauge bosons ‘eat’ the Goldstone bosons and become massive, this mechanism is called the *super-Higgs* mechanism.

GUT scale, the soft terms relevant to our study are therefore given by

$$m_i^2 = m_0^2, \tag{3.5a}$$

$$M_a = M_{1/2} \quad (a = 1, 2, 3), \tag{3.5b}$$

where m_i^2 are the scalar squared masses. From the supergravity point of view, the parameters $m_0, M_{1/2}, A_0$ depend on the hidden-sector fields and are all proportional to $m_{3/2}$ (for example, one has the relation $m_0 = m_{3/2}$). However, from the perspective of the low-energy EFT that we call mSUGRA, they are simply regarded as model parameters. The MSSM is assumed to be valid up to the GUT scale, where the relations (3.5) serve as RG boundary conditions. In addition, the soft Higgs mixing term $B = b/\mu$ has the GUT-scale value $B_0 = A_0 - m_{3/2}$.

As an aside, there is a model similar to mSUGRA: it is called constrained MSSM (CMSSM, see e.g. [30]). It has the same boundary conditions as mSUGRA and these two models are often confused in the literature. However, mSUGRA arises from a supergravity model whereas the CMSSM does not: the boundary conditions (3.5) are simply postulated. Also, in the CMSSM there is no relation between the model parameters and $m_{3/2}$, and the relation $B_0 = A_0 - m_{3/2}$ does not hold either.

Supergravity models are attractive since they provide a natural framework for supersymmetry breaking: a locally supersymmetric Lagrangian automatically contains terms that can mediate supersymmetry breaking. In addition, mSUGRA has great predictive power since it has only four free parameters. However, these models must necessarily appeal to Planck-scale physics, which is still poorly understood. Furthermore, though gravity is flavour blind, the supergravity invariance of the Lagrangian cannot prevent the occurrence of (Planck-scale suppressed) flavour-mixing operators that correspond to tree-level interactions between hidden-sector fields and visible-sector fields. In order to suppress sparticle-induced FCNC processes, one must resort to additional generation symmetries.

3.6 Anomaly-mediated supersymmetry breaking

In some models of supergravity, the visible and hidden sectors are physically separated by extra dimensions [31, 32]. In these ‘braneworld’ scenarios, often inspired by string theory, our four-dimensional world is embedded in a higher-dimensional bulk that has additional spatial dimensions, which are curled up.

The general idea is that the MSSM fields and the hidden-sector fields are confined to parallel, distinct three-branes (space-like hypersurfaces), separated by a distance r . Only the gravity supermultiplet (and possibly new heavy fields) resides in the bulk. In this scenario every flavour-violating term that plagues supergravity, caused by tree-level couplings with a bulk field of mass m , is suppressed by a factor e^{-mr} . Provided that r is large enough, the flavour-violating effects are exponentially suppressed without requiring any fine-tuning. This class of models is called Anomaly-Mediated Supersymmetry Breaking (AMSB), because the size of the soft supersymmetry-breaking terms is determined by the loop-induced superconformal (Weyl) anomaly [33]. Local superconformal invariance is a rescaling symmetry that is violated at the quantum level.

Anomaly-mediated terms are always present in supergravity, but they are loop-suppressed with respect to the gravitino mass and hence result in subleading-order contributions to the soft masses. AMSB is the scenario where there are no supergravity contributions at tree level, so that the anomaly-mediated terms become the dominant ones. At the scale M_{AMSB} where supersymmetry breaking occurs, the soft terms relevant to our study have the following values (using the usual pMSSM assumptions):

$$M_a = \frac{b_a}{16\pi^2} g_a^2 (M_{\text{AMSB}}) m_{3/2} \quad (a = 1, 2, 3), \quad (3.6a)$$

$$m_i^2 = \frac{1}{2} \dot{\gamma}_i m_{3/2}^2. \quad (3.6b)$$

Here, m_i^2 are again the scalar squared masses with γ_i being the corresponding anomalous dimensions and $b_a = (\frac{33}{5}, 1, -3)$ for $a = 1, 2, 3$. The derivatives $\dot{\gamma}_i \equiv d\gamma_i/dt$ are explicitly given by

$$(16\pi^2)^2 \dot{\gamma}_{H_u} = 6|y_t|^2 B_t - 3g_2^4 - \frac{99}{25} g_1^4, \quad (3.7a)$$

$$(16\pi^2)^2 \dot{\gamma}_{H_d} = 6|y_b|^2 B_b + 2|y_\tau|^2 B_\tau - 3g_2^4 - \frac{99}{25} g_1^4, \quad (3.7b)$$

$$(16\pi^2)^2 \dot{\gamma}_{\tilde{Q}_i} = \delta_{i3} (2|y_t|^2 B_t + 2|y_b|^2 B_b) + 16g_3^4 - 3g_2^4 - \frac{11}{25} g_1^4, \quad (3.7c)$$

$$(16\pi^2)^2 \dot{\gamma}_{\tilde{u}_i} = \delta_{i3} \cdot 4|y_t|^2 B_t + 16g_3^4 - \frac{176}{25} g_1^4, \quad (3.7d)$$

$$(16\pi^2)^2 \dot{\gamma}_{\tilde{d}_i} = \delta_{i3} \cdot 4|y_b|^2 B_b + 16g_3^4 - \frac{44}{25} g_1^4, \quad (3.7e)$$

$$(16\pi^2)^2 \dot{\gamma}_{\tilde{L}_i} = \delta_{i3} \cdot 2|y_\tau|^2 B_\tau - 3g_2^4 - \frac{99}{25} g_1^4, \quad (3.7f)$$

$$(16\pi^2)^2 \dot{\gamma}_{\tilde{e}_i} = \delta_{i3} \cdot 4|y_\tau|^2 B_\tau - \frac{396}{25} g_1^4, \quad (3.7g)$$

where we have defined the following quantities for convenience:

$$B_t \equiv 6|y_t|^2 + |y_b|^2 - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{13}{15} g_1^2, \quad (3.8a)$$

$$B_b \equiv 6|y_b|^2 + |y_t|^2 + |y_\tau|^2 - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{7}{15} g_1^2, \quad (3.8b)$$

$$B_\tau \equiv 4|y_\tau|^2 + 3|y_b|^2 - 3g_2^2 - \frac{9}{5} g_1^2. \quad (3.8c)$$

Minimal anomaly mediation. The advantage of AMSB over SUGRA is that it naturally conserves flavour. However, pure anomaly mediation leads to tachyonic sleptons, i.e. their squared soft masses become negative. This would cause them to acquire non-zero VEVs and break the electromagnetic gauge symmetry. The minimal AMSB (mAMSB) model uses a phenomenological approach to tackle this problem: a universal, non-anomaly-mediated contribution m_0^2 is added to the soft squared scalar masses (3.6b) at the scale M_{AMSB} . The origin of these terms may be for example additional fields in the bulk, but in the mAMSB model m_0 is simply considered as a parameter of the model.

3.7 General gauge mediation

Several models of Gauge-Mediated Supersymmetry Breaking (GMSB) have been proposed in the literature (see [34] for a review). Many of these models include a field X , called

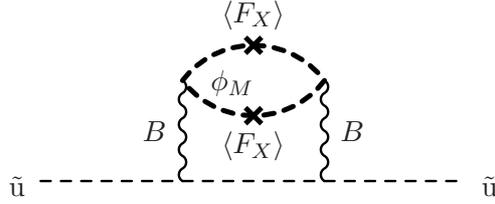


Figure 5. Contribution to the soft squared mass of the up squark in models of Gauge-Mediated Supersymmetry Breaking (GMSB). The auxiliary component of the spurion field X obtains a supersymmetry-breaking VEV $\langle F_X \rangle$. The up squark only couples indirectly to this VEV: the scalar component ϕ_M of a messenger supermultiplet couples at tree level to the spurion and through the MSSM gauge fields (in this diagram the B boson) with the MSSM. When the messengers are integrated out from the action, this diagram contributes to the soft mass of the up squark.

the *spurion*, that acquires a supersymmetry-breaking VEV, and a set of weakly coupled fields that are charged under the MSSM. The latter are called *messenger* fields since they communicate supersymmetry breaking to the MSSM fields: they interact at tree level with the spurion and through the MSSM gauge fields with the MSSM (see figure 5).

Recently, the framework of General Gauge Mediation (GGM) [35, 36] has been proposed to unify all earlier descriptions of GMSB. It describes the effects of an arbitrary hidden sector on the MSSM. It starts from the following definition of gauge mediation: *in the limit of vanishing gauge couplings, the theory decouples into the MSSM and a separate, supersymmetry-breaking hidden sector*. For example, the setup described above fits into this definition by taking the messenger and spurion fields as the hidden sector.

In the GGM framework, all MSSM soft terms can be described in terms of a small number of correlation functions involving hidden-sector currents. Essentially, the GGM framework parametrises the effects of the hidden sector on the MSSM. By constructing the effective Lagrangian, the following soft-mass formulae are found:⁴

$$M_a = g_a^2 B_a \quad (a = 1, 2, 3), \quad (3.9a)$$

$$m_i^2 = g_1^2 Y_i \zeta + \sum_{a=1}^3 g_a^4 C_a(i) A_a, \quad (3.9b)$$

with

$$C_1(i) = \frac{3}{5} Y_i^2, \quad (3.10a)$$

$$C_2(i) = \begin{cases} \frac{3}{4} & \text{for } \Phi_i = \tilde{Q}, \tilde{L}, H_u, H_d, \\ 0 & \text{for } \Phi_i = \tilde{u}_R, \tilde{d}_R, \tilde{e}_R, \end{cases} \quad (3.10b)$$

$$C_3(i) = \begin{cases} \frac{4}{3} & \text{for } \Phi_i = \tilde{Q}, \tilde{u}_R, \tilde{d}_R, \\ 0 & \text{for } \Phi_i = \tilde{L}, \tilde{e}_R, H_u, H_d. \end{cases} \quad (3.10c)$$

Here B_a , ζ and A_a are expressions involving the hidden-sector current correlation functions; Y_i is the hypercharge of the scalar field Φ_i and $C_a(i)$ is the quadratic Casimir of the

⁴For future convenience, a factor M (the messenger scale) has been absorbed into the definition of the B_a , cf. [6].

representation of Φ_i under the gauge group labeled by a . Usually a \mathbf{Z}_2 symmetry of the hidden sector is assumed in order to forbid the term containing ζ , since it would lead to tachyonic sleptons. The above conditions are the matching conditions at the messenger scale M where we integrate out the hidden sector. The seven numbers ζ, A_a, B_a contain information on the hidden sector, but are regarded as parameters of the low-energy EFT that we call the MSSM.

The GGM framework does not allow for additional interactions that could generate μ and b radiatively; that would require interactions between the MSSM and the hidden sector that remain in the limit of vanishing gauge couplings. The framework would have to be extended to allow for such couplings. To parametrise the effects of such an extension, additional contributions δ_u, δ_d to $m_{H_u}^2, m_{H_d}^2$ are often added.

Minimal gauge mediation. Minimal gauge mediation (MGM) is a GGM model that is restricted to a subset of the GGM parameter space, defined by the constraints $A_a = A$, $B_a = B$ and $A = 2B^2$. The term corresponding to ζ is taken to be zero. Additional non-gauge contributions δ_u, δ_d are added to the soft Higgs masses. Then the expressions for the soft masses become

$$M_a = g_a^2 B_a \quad (a = 1, 2, 3), \quad (3.11a)$$

$$m_i^2 = 2B^2 \sum_{a=1}^3 g_a^4 C_a(i), \quad (3.11b)$$

$$m_{H_u}^2 = 2B^2 \sum_{a=1}^3 g_a^4 C_a(H_u) + \delta_u, \quad (3.11c)$$

$$m_{H_d}^2 = 2B^2 \sum_{a=1}^3 g_a^4 C_a(H_d) + \delta_d, \quad (3.11d)$$

where this time m_i^2 denote only the squared masses of the squarks and sleptons.

3.8 Mirage mediation

Rather than restricting oneself to one of the three known mechanisms for radiative supersymmetry breaking (gravity, anomaly or gauge mediation), one could solve the problems of particular models by choosing two (or more) mechanisms and combining the best of both worlds. For example, one might tackle the tachyonic slepton problem of anomaly mediation by combining it with gauge mediation (see e.g. [37]).

Mirage mediation [38] is one such scenario in which gravity-mediated and anomaly-mediated soft terms have comparable contributions. In this scenario, the gravity-mediated terms are suppressed by a relative factor $\log(M_{\text{pl}}/m_{3/2})$, which is numerically of the order of a loop factor. This results in mirage unification: the gaugino and scalar masses unify at a scale far below the scale where the soft masses are generated. This *mirage messenger scale* does not correspond to any physical threshold, hence the name.

This class of phenomenological models are based on a class of string models with stabilised moduli, called the KKL T construction. It solves the tachyonic slepton problem that arises in pure anomaly mediation and has reduced low-energy fine-tuning [39].

3.9 One-loop RGIs for the MSSM

It should be noted that if we have a set of RGIs, then any function of those RGIs will also be RG invariant. Therefore, in order to find all RGIs, one should look for a maximal set of *independent* RGIs, i.e. invariants that cannot be expressed in terms of each other.⁵ Recently a complete list of independent one-loop RGIs for the MSSM was derived in [4, 5] (see table 3 for a listing). These one-loop RGIs have been derived under the pMSSM assumptions, using the relevant β -functions listed in appendix A. An alternative, systematic way of deriving this complete set can be found in appendix B.

Any other (one-loop) RGI we can think of can be written in terms of those in table 3. For example, the RGI in the example from section 2.4.3 can be written as

$$\begin{aligned} I_g &= 4g_1^{-2} - \frac{48}{5}g_2^{-2} + \frac{28}{5}g_3^{-3} \\ &= \frac{16}{11}I_{g_2} + \frac{28}{11}I_{g_3}. \end{aligned} \quad (3.12)$$

The last two RGIs in table 3 will not be relevant to our analysis. We will explain why in the next subsection.

3.10 RGIs in the literature

As we have seen, RGIs provide a new tool to test predictions about high-scale physics, such as gauge-coupling unification. The trick is to find sum rules for high-scale physics that can be written in terms of RGIs. In the literature, several such sum rules can be found.

Consider for example minimal gauge mediation, which has been studied in the context of RGIs in [6]. If one inserts the spectrum (3.11) at the messenger scale into the relevant RGI expressions from table 3, one immediately finds that $D_{B_{13}} = D_{L_{13}} = D_{\chi_1} = 0$. In terms of the model parameters of MGM, the non-vanishing RGIs are

$$D_{Y_{13H}} = -\frac{10}{13}(\delta_u - \delta_d), \quad (3.13a)$$

$$D_Z = -2\delta_d, \quad (3.13b)$$

$$I_{Y_\alpha} = g_1^{-2}(M)(\delta_u - \delta_d), \quad (3.13c)$$

$$I_{B_a} = B \quad (a = 1, 2, 3), \quad (3.13d)$$

$$I_{M_1} = \frac{38}{5}g_1^4(M)B^2, \quad (3.13e)$$

$$I_{M_2} = 2g_2^4(M)B^2, \quad (3.13f)$$

$$I_{M_3} = -2g_3^4(M)B^2, \quad (3.13g)$$

$$I_{g_2} = g_1^{-2}(M) - \frac{33}{5}g_2^{-2}(M), \quad (3.13h)$$

$$I_{g_3} = g_1^{-2}(M) + \frac{11}{5}g_3^{-2}(M), \quad (3.13i)$$

where M is the messenger scale. This gives us eleven equations in terms of six unknowns ($\delta_u, \delta_d, B, g_1(M), g_2(M), g_3(M)$). We can trade each unknown for an equation, i.e. for

⁵It is tempting to call this a ‘basis of RGIs’, as in [5]. Note however that it is not the same as a basis of a vector space. One should keep in mind that once we have found such a set, we are not restricted to making linear combinations of them, but can also take products, quotients and so on.

Invariant	Definition
$D_{B_{13}}$	$2 \left(m_{\tilde{Q}_1}^2 - m_{\tilde{Q}_3}^2 \right) - m_{\tilde{u}_1}^2 + m_{\tilde{u}_3}^2 - m_{\tilde{d}_1}^2 + m_{\tilde{d}_3}^2$
$D_{L_{13}}$	$2 \left(m_{\tilde{L}_1}^2 - m_{\tilde{L}_3}^2 \right) - m_{\tilde{e}_1}^2 + m_{\tilde{e}_3}^2$
D_{X_1}	$3 \left(3m_{\tilde{d}_1}^2 - 2 \left(m_{\tilde{Q}_1}^2 - m_{\tilde{L}_1}^2 \right) - m_{\tilde{u}_1}^2 \right) - m_{\tilde{e}_1}^2$
$D_{Y_{13H}}$	$m_{\tilde{Q}_1}^2 - 2m_{\tilde{u}_1}^2 + m_{\tilde{d}_1}^2 - m_{\tilde{L}_1}^2 + m_{\tilde{e}_1}^2$ $-\frac{10}{13} \left(m_{\tilde{Q}_3}^2 - 2m_{\tilde{u}_3}^2 + m_{\tilde{d}_3}^2 - m_{\tilde{L}_3}^2 + m_{\tilde{e}_3}^2 + m_{H_u}^2 - m_{H_d}^2 \right)$
D_Z	$3 \left(m_{\tilde{d}_3}^2 - m_{\tilde{d}_1}^2 \right) + 2 \left(m_{\tilde{L}_3}^2 - m_{H_d}^2 \right)$
I_{Y_α}	$\frac{1}{g_1^2} \left(m_{H_u}^2 - m_{H_d}^2 + \sum_{gen} \left(m_{\tilde{Q}}^2 - 2m_{\tilde{u}}^2 + m_{\tilde{d}}^2 - m_{\tilde{L}}^2 + m_{\tilde{e}}^2 \right) \right)$
I_{B_a}	$\frac{M_a}{g_a^2}$
I_{M_1}	$M_1^2 - \frac{33}{8} \left(m_{\tilde{d}_1}^2 - m_{\tilde{u}_1}^2 - m_{\tilde{e}_1}^2 \right)$
I_{M_2}	$M_2^2 + \frac{1}{24} \left(9 \left(m_{\tilde{d}_1}^2 - m_{\tilde{u}_1}^2 \right) + 16m_{\tilde{L}_1}^2 - m_{\tilde{e}_1}^2 \right)$
I_{M_3}	$M_3^2 - \frac{3}{16} \left(5m_{\tilde{d}_1}^2 + m_{\tilde{u}_1}^2 - m_{\tilde{e}_1}^2 \right)$
I_{g_2}	$\frac{1}{g_1^2} - \frac{33}{5g_2^2}$
I_{g_3}	$\frac{1}{g_1^2} + \frac{11}{5g_3^2}$
I_2	$\mu \left(\frac{g_2^9 g_3^{256/3}}{y_t^{27} y_b^{21} y_\tau^{10} g_1^{73/33}} \right)^{1/61}$
I_4	$\frac{b}{\mu} - \frac{27}{61} A_t - \frac{21}{61} A_b - \frac{10}{61} A_\tau - \frac{256}{183} M_3 - \frac{9}{61} M_2 + \frac{73}{2013} M_1$

Table 3. One-loop renormalisation-group invariants for the MSSM. The sum in I_{Y_α} runs over the three sfermion generations. Notation taken over from [5] (first 14 RGIs) and [4] (last two RGIs).

each parameter we use one of the above equations to express it in terms of RGIs only. Since we have more independent equations than unknowns, we can substitute the resulting six expressions into the remaining five equations to obtain five sum rules in terms of RGIs only. For example, using equation (3.13d) we can eliminate the model parameter B from the remaining equations. Then equations (3.13e)-(3.13g) can be used to eliminate the gauge couplings at the messenger scale. We can get the value of δ_d from equation (3.13b), and then (3.13a) gives the value of δ_u . After substituting the resulting six expressions into the five remaining equations, we are left with the following sum rules:

$$0 = I_{Y_\alpha} + \frac{13}{10} D_{Y_{13H}} I_{B_1} \sqrt{\frac{38}{5I_{M_1}}}, \quad (3.13c)$$

$$0 = I_{B_1} - I_{B_2}, \quad (3.13d)$$

$$0 = I_{B_1} - I_{B_3}, \quad (3.13d)$$

$$0 = I_{B_1} \sqrt{\frac{38}{5I_{M_1}}} - \frac{33}{5} I_{B_1} \sqrt{\frac{2}{I_{M_2}}} - I_{g_2}, \quad (3.13h)$$

$$0 = I_{B_1} \sqrt{\frac{38}{5I_{M_1}}} + \frac{11}{5} I_{B_1} \sqrt{\frac{-2}{I_{M_3}}} - I_{g_3}, \quad (3.13i)$$

To summarise, we have chosen a specific supersymmetry-breaking model and expressed the RGIs in terms of model parameters. Since we ended up with more equations than unknowns, we could construct eight sum rules in terms of RGIs only: three from vanishing RGIs and five by eliminating the model parameters. If any of these sum rules are violated, MGM is not consistent with experimental data. This test can be performed at any energy scale where the MSSM is valid, which implies that we don't have to know the messenger scale M in order to rule out the MGM model.

Now we can see why the last two RGIs in table 3 are not useful. Suppose we wish to test a specific breaking model. Let us denote the values of $B = b/\mu$ and μ at the new-physics threshold in this model as B_{thr} and μ_{thr} respectively. Now we apply the above procedure to this model: we express B_{thr} and μ_{thr} in terms of RGIs and the other couplings at the high scale; then we can insert these expressions into the remaining equations. But since B and μ both appear in only one independent RGI, there are no equations to insert these expressions into! In the above example, it was possible to combine all RGIs into sum rules because each running parameter appeared in more than one RGI. Since B and μ do not, their corresponding RGIs become useless to our analysis. Hence we will have to restrict ourselves to the RGIs that do not contain these running parameters. As will be shown in appendix B, it is not possible to construct RGIs that contain the Yukawa and soft trilinear couplings without using B and μ . That is why we will only use RGIs constructed out of soft masses and/or gauge couplings, i.e. the first 14 RGIs listed in table 3.

3.10.1 Using RGIs effectively

In studies of RGIs such as [5–7], a certain breaking mechanism is usually presupposed. Then one constructs sum rules that are tailor-made for that breaking mechanism. For

example, the sum rules constructed above all provide a test for consistency of MGM with experimental data. However, some of these sum rules will also hold for other breaking mechanisms. It is not always clear to what extent the validity of the sum rules depends on the unique features of the breaking mechanism under study. For example, in minimal gauge mediation the quantity M_a/g_a^2 unifies at the messenger scale; this follows directly from the matching condition (3.11a). However, in mSUGRA this quantity also unifies, but for a different reason: it is the consequence of the assumption of gauge-coupling unification and gaugino-mass unification at the same energy scale! Hence, the sum rules that test this unification property cannot be used to confirm that either of these specific models corresponds to reality. They can only provide consistency checks that should be satisfied if any of these models are realised in Nature.

Therefore, we will look for RG-invariant sum rules using a different, more model-independent approach. We will not presume any spectrum specific to a certain breaking mechanism. Instead, we will search for sum rules that test properties that are common in supersymmetry-breaking models (e.g. M_a/g_a^2 unification). Then any breaking model, be it an existing one such as those described in sections 3.5–3.8 or a new one contrived in the future, can be tested directly if it predicts any of these properties. For example, if the sum rules for M_a/g_a^2 unification are not satisfied by experimental data, then models that predict this property (mSUGRA and MGM, but not necessarily GGM) are falsified. But also anyone who would concoct a new model that has this property, would have to go back to the drawing board at once. In the next section, we will look for common properties to test and find sum rules for them.

4 Results

Supersymmetry-breaking models predict relations between the running parameters as a result of matching conditions at the new-physics threshold. These relations mostly involve the unification of certain parameters. Therefore we will construct sum rules for the following scenarios.

Scenario 1: gauge-coupling unification. As can be seen from figure 4, the MSSM may be consistent with gauge-coupling unification, depending on the values of the sparticle thresholds. The hypothesis that the gauge couplings unify is often made in supersymmetry-breaking models, for example in mSUGRA. Therefore it will be important to determine whether gauge-coupling unification occurs in Nature. We will call the special case where $g_1 = g_2 = g_3 = g$ scenario 1.

Scenario 2: gaugino-mass unification. In mSUGRA, the gaugino masses are assumed to unify. Since this model is widely used, it is useful to check whether gaugino-mass unification occurs in Nature. We will call the case where $M_1 = M_2 = M_3 = M_{1/2}$ scenario 2.

Scenario 3: Unification of M_a/g_a^2 . As we mentioned in section 3, the quantities M_1/g_1^2 , M_2/g_2^2 and M_3/g_3^2 may unify for different reasons. It could be the consequence of gaugino-mass and gauge-coupling unification at the same scale (as in mSUGRA) or it may

be the result of the gaugino-mass matching conditions (as in MGM). Therefore we will also test this property. We will call the case where $M_1/g_1^2 = M_2/g_2^2 = M_3/g_3^2 = C$ scenario 3.

Scenario 4: flavour-universality of high-scale sfermion masses. In many theories, the sfermion masses are assumed to be flavour-universal, i.e. the first (and therefore also the second) and third generation masses are equal: $m_{\tilde{Q}_1}^2 = m_{\tilde{Q}_3}^2 \equiv m_{\tilde{Q}}^2$, $m_{\tilde{u}_1}^2 = m_{\tilde{u}_3}^2 \equiv m_{\tilde{u}}^2$, $m_{\tilde{d}_1}^2 = m_{\tilde{d}_3}^2 \equiv m_{\tilde{d}}^2$, $m_{\tilde{L}_1}^2 = m_{\tilde{L}_3}^2 \equiv m_{\tilde{L}}^2$ and $m_{\tilde{e}_1}^2 = m_{\tilde{e}_3}^2 \equiv m_{\tilde{e}}^2$. We will call this scenario 4. This hypothesis is motivated by the need to suppress FCNC amplitudes. Flavour-universality may be a consequence of flavour symmetries (as postulated in mSUGRA) or of the flavour-blindness of the interactions that mediate supersymmetry breaking (as in GGM). Since this property occurs in many models, we will test flavour-universality of the soft masses.

Scenario 5: scalar-mass unification. Unification of the soft scalar masses, which occurs in mSUGRA, is very predictive: many matching conditions depend on a single parameter m_0 , which allows us to construct multiple sum rules. Sometimes non-universality of the soft Higgs masses is assumed, because suppression of FCNC amplitudes does not require them to be universal with the squark and slepton masses. Therefore we will distinguish between two cases. We will refer to the case where $m_{H_u}^2$ and $m_{H_d}^2$ have additional non-universal contributions δ_u and δ_d respectively as scenario 5a. The special case of universal scalar masses, i.e. $\delta_u = \delta_d = 0$, will be denoted as scenario 5b.

Mixed scenarios: multiple unifications at one scale. It is possible that several of these unification properties will turn out to be consistent with experimental data. In that case, we could test whether these unifications occur at the same energy scale. At first sight, it may seem strange to consider the possibility of two kinds of unifications at different energy scales. In supersymmetry-breaking models such unifications usually occur at a threshold where new physics enters the theory. Thus even if the MSSM were consistent with two kinds of unifications, the RG trajectories of the running parameters could be deflected from the MSSM trajectories after the first threshold, spoiling the second unification. However, recall that in mirage mediation (see section 3.8) the scale where the soft masses unify is lower than the scale at which the soft masses are generated. Thus it is possible that the unification scale does not correspond to any physical threshold. Therefore we will separately check whether multiple unifications occur at the same scale.

4.1 Sum rules

In this section we construct sum rules that test the scenarios described above. Recall that the RGIs that are useful to our analysis are defined as

$$D_{B_{13}} = 2 \left(m_{\tilde{Q}_1}^2 - m_{\tilde{Q}_3}^2 \right) - m_{\tilde{u}_1}^2 + m_{\tilde{u}_3}^2 - m_{\tilde{d}_1}^2 + m_{\tilde{d}_3}^2, \quad (4.1a)$$

$$D_{L_{13}} = 2 \left(m_{\tilde{L}_1}^2 - m_{\tilde{L}_3}^2 \right) - m_{\tilde{e}_1}^2 + m_{\tilde{e}_3}^2, \quad (4.1b)$$

$$D_{\chi_1} = 3 \left(3m_{\tilde{d}_1}^2 - 2 \left(m_{\tilde{Q}_1}^2 - m_{\tilde{L}_1}^2 \right) - m_{\tilde{u}_1}^2 \right) - m_{\tilde{e}_1}^2, \quad (4.1c)$$

$$D_{Y_{13H}} = m_{\tilde{Q}_1}^2 - 2m_{\tilde{u}_1}^2 + m_{\tilde{d}_1}^2 - m_{\tilde{L}_1}^2 + m_{\tilde{e}_1}^2 - \frac{10}{13} \left(m_{\tilde{Q}_3}^2 - 2m_{\tilde{u}_3}^2 + m_{\tilde{d}_3}^2 - m_{\tilde{L}_3}^2 + m_{\tilde{e}_3}^2 + m_{H_u}^2 - m_{H_d}^2 \right), \quad (4.1d)$$

$$D_Z = 3 \left(m_{\tilde{d}_3}^2 - m_{\tilde{d}_1}^2 \right) + 2 \left(m_{\tilde{L}_3}^2 - m_{H_d}^2 \right), \quad (4.1e)$$

$$I_{Y_\alpha} = \frac{1}{g_1^2} \left(m_{H_u}^2 - m_{H_d}^2 + \sum_{\text{gen}} \left(m_{\tilde{Q}}^2 - 2m_{\tilde{u}}^2 + m_{\tilde{d}}^2 - m_{\tilde{L}}^2 + m_{\tilde{e}}^2 \right) \right), \quad (4.1f)$$

$$I_{B_a} = \frac{M_a}{g_a^2} \quad (a = 1, 2, 3), \quad (4.1g)$$

$$I_{M_1} = M_1^2 - \frac{33}{8} \left(m_{\tilde{d}_1}^2 - m_{\tilde{u}_1}^2 - m_{\tilde{e}_1}^2 \right), \quad (4.1h)$$

$$I_{M_2} = M_2^2 + \frac{1}{24} \left(9 \left(m_{\tilde{d}_1}^2 - m_{\tilde{u}_1}^2 \right) + 16m_{\tilde{L}_1}^2 - m_{\tilde{e}_1}^2 \right), \quad (4.1i)$$

$$I_{M_3} = M_3^2 - \frac{3}{16} \left(5m_{\tilde{d}_1}^2 + m_{\tilde{u}_1}^2 - m_{\tilde{e}_1}^2 \right), \quad (4.1j)$$

$$I_{g_2} = g_1^{-2} - \frac{33}{5} g_2^{-2}, \quad (4.1k)$$

$$I_{g_3} = g_1^{-2} + \frac{11}{5} g_3^{-2}. \quad (4.1l)$$

Scenario 1. From equation (4.1) and table 4 we can see that we have 14 equations with 16 unknowns, so at first sight we expect to find no sum rules. However, equations (4.1k) and (4.1l) form a subset of two equations with only one unknown g . Hence we can make one sum rule:

$$I_{g_2} + \frac{7}{4} I_{g_3} = 0. \quad (4.2)$$

Scenario 2. In this scenario we also have 14 equations with 16 unknowns. However, equations (4.1g), (4.1k) and (4.1l) form a subset of five equations with four unknowns. This allows us to construct one sum rule:

$$\left(I_{B_1} - \frac{33}{5} I_{B_2} \right) I_{g_3} = \left(I_{B_1} + \frac{11}{5} I_{B_3} \right) I_{g_2}. \quad (4.3)$$

Scenario 3. Again we have 14 equations with 16 unknowns. This time equations (4.1g) form a subset of three equations with one unknown. This yields two sum rules:

$$I_{B_1} = I_{B_2}, \quad (4.4)$$

$$I_{B_1} = I_{B_3}. \quad (4.5)$$

Scenario 4. We can see directly from table 4 that this scenario yields two sum rules in the form of vanishing RGIs:

$$D_{B_{13}} = 0, \quad (4.6)$$

$$D_{L_{13}} = 0. \quad (4.7)$$

The remaining 12 equations contain 13 unknowns and no subset of them contains less unknowns than equations. Hence, we cannot construct any other sum rules.

Invariant	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5a	Scenario 5b
$D_{B_{13}}$	$D_{B_{13}}$	$D_{B_{13}}$	$D_{B_{13}}$	0	0	0
$D_{L_{13}}$	$D_{L_{13}}$	$D_{L_{13}}$	$D_{L_{13}}$	0	0	0
D_{X_1}	D_{X_1}	D_{X_1}	D_{X_1}	$D_{X_1} \left(m_{\tilde{f}_i}^2 \rightarrow m_{\tilde{f}}^2 \right)$	$5m_0^2$	$5m_0^2$
$D_{Y_{13H}}$	$D_{Y_{13H}}$	$D_{Y_{13H}}$	$D_{Y_{13H}}$	$D_{Y_{13H}} \left(m_{\tilde{f}_i}^2 \rightarrow m_{\tilde{f}}^2 \right)$	$-\frac{10}{13}(\delta_u - \delta_d)$	0
D_Z	D_Z	D_Z	D_Z	$D_Z \left(m_{\tilde{f}_i}^2 \rightarrow m_{\tilde{f}}^2 \right)$	$-2\delta_d$	0
I_{Y_α}	$I_{Y_\alpha} (g_1 \rightarrow g)$	I_{Y_α}	I_{Y_α}	$I_{Y_\alpha} \left(m_{\tilde{f}_i}^2 \rightarrow m_{\tilde{f}}^2 \right)$	$\frac{\delta_u - \delta_d}{g_1^2}$	0
$I_{B_a} \quad a = 1, 2, 3$	$I_{B_a} (g_a \rightarrow g)$	$\frac{M_{1/2}}{g_a^2}$	C	I_{B_a}	I_{B_a}	I_{B_a}
I_{M_1}	I_{M_1}	$I_{M_1} (M_1 \rightarrow M_{1/2})$	$I_{M_1} (M_1 \rightarrow Cg_1^2)$	$I_{M_1} \left(m_{\tilde{f}_i}^2 \rightarrow m_{\tilde{f}}^2 \right)$	$M_1^2 + \frac{33}{8}m_0^2$	$M_1^2 + \frac{33}{8}m_0^2$
I_{M_2}	I_{M_2}	$I_{M_2} (M_2 \rightarrow M_{1/2})$	$I_{M_2} (M_2 \rightarrow Cg_2^2)$	$I_{M_2} \left(m_{\tilde{f}_i}^2 \rightarrow m_{\tilde{f}}^2 \right)$	$M_2^2 + \frac{5}{8}m_0^2$	$M_2^2 + \frac{5}{8}m_0^2$
I_{M_3}	I_{M_3}	$I_{M_3} (M_3 \rightarrow M_{1/2})$	$I_{M_3} (M_3 \rightarrow Cg_3^2)$	$I_{M_3} \left(m_{\tilde{f}_i}^2 \rightarrow m_{\tilde{f}}^2 \right)$	$M_3^2 - \frac{15}{16}m_0^2$	$M_3^2 - \frac{15}{16}m_0^2$
I_{g_2}	$-\frac{28}{5}g^{-2}$	I_{g_2}	I_{g_2}	I_{g_2}	I_{g_2}	I_{g_2}
I_{g_3}	$\frac{16}{5}g^{-2}$	I_{g_3}	I_{g_3}	I_{g_3}	I_{g_3}	I_{g_3}

Table 4. Values of the MSSM RGIs in the unification scenarios 1–5. For each scenario, RGIs that contribute to sum rules that are specific to that scenario are listed in boldface. If an RGI is simplified with respect to its definition but is not used for a sum rule, its name with the appropriate substitutions is listed. For the substitution $(m_{\tilde{f}_i}^2 \rightarrow m_{\tilde{f}}^2)$ it is implied that $\tilde{f} = \tilde{Q}, \tilde{u}, \tilde{d}, \tilde{L}, \tilde{e}$ and $i = 1, 2, 3$. If an RGI does not simplify at all, only its name is listed.

Scenario 5a. In this scenario we also have two vanishing RGIs, because unified scalar masses imply flavour-universal scalar masses. The remaining RGIs yield twelve equations with nine unknowns. Hence, we can construct three sum rules that are specific to this scenario:

$$I_{g_2} = I_{B_1} \left(I_{M_1} - \frac{33}{40} D_{\chi_1} \right)^{-1/2} - \frac{33}{5} I_{B_2} \left(I_{M_2} - \frac{1}{8} D_{\chi_1} \right)^{-1/2}, \quad (4.8)$$

$$I_{g_3} = I_{B_1} \left(I_{M_1} - \frac{33}{40} D_{\chi_1} \right)^{-1/2} + \frac{11}{5} I_{B_3} \left(I_{M_3} + \frac{3}{16} D_{\chi_1} \right)^{-1/2}, \quad (4.9)$$

$$0 = I_{Y_\alpha} \sqrt{I_{M_1} - \frac{33}{40} D_{\chi_1}} + \frac{13}{10} I_{B_1} D_{Y_{13H}}. \quad (4.10)$$

Furthermore, non-universality of the Higgs masses can be tested directly because we can extract δ_u and δ_d from the RGIs:

$$\delta_d = -\frac{1}{2} D_Z \neq 0, \quad (4.11)$$

$$\delta_u = -\frac{13}{10} D_{Y_{13H}} - \frac{1}{2} D_Z \neq 0. \quad (4.12)$$

Scenario 5b. We can see directly from table 4 that this scenario yields three new sum rules in the form of vanishing RGIs:

$$D_{Y_{13H}} = 0, \quad (4.13)$$

$$D_Z = 0, \quad (4.14)$$

$$I_{Y_\alpha} = 0. \quad (4.15)$$

The remaining nine equations contain seven unknowns, which allows us to construct two more sum rules:

$$I_{g_2} = I_{B_1} \left(I_{M_1} - \frac{33}{40} D_{\chi_1} \right)^{-1/2} - \frac{33}{5} I_{B_2} \left(I_{M_2} - \frac{1}{8} D_{\chi_1} \right)^{-1/2}, \quad (4.16)$$

$$I_{g_3} = I_{B_1} \left(I_{M_1} - \frac{33}{40} D_{\chi_1} \right)^{-1/2} + \frac{11}{5} I_{B_3} \left(I_{M_3} + \frac{3}{16} D_{\chi_1} \right)^{-1/2}. \quad (4.17)$$

Note that these two sum rules also hold for scenario 5a. This makes sense, because scenario 5b is a special case of scenario 5a, so the sum rules (4.8)-(4.10) will also hold for scenario 5b. However, sum rule (4.10) has become redundant because it is automatically satisfied if (4.13) and (4.15) hold.

Mixed scenario 123. Note that if any two of the scenarios 1, 2 and 3 hold, the hypothesis that the corresponding unifications occur at the same scale is equivalent to the hypothesis that the third scenario also holds. For example, if the gauge couplings and the gaugino masses unify, these unifications occur at the same scale if and only if M_a/g_a^2 unification occurs. Hence, for simultaneous unification at least the sum rules (4.2)-(4.5) should hold. Note that (4.3) is automatically satisfied if the other three sum rules hold, so we have three independent sum rules for this mixed scenario.

From table 4 we conclude that if these three unifications occur simultaneously we have 14 equations with 14 unknowns, which gives no sum rules at first sight. However, equations (4.1g), (4.1k) and (4.11) constitute five equations with two unknowns, giving us three sum rules. We have already found three, so there are no new sum rules.

Mixed scenario 15. If both scenario 1 and scenario 5a hold, at least the sum rules (4.2) and (4.6)–(4.10) hold, as well as the inequalities (4.11)–(4.12). From table 4 we conclude that if these unifications occur simultaneously we have two sum rules from vanishing RGIs as well as twelve equations with seven unknowns. Thus we can construct five additional sum rules. We already found six of them, so there is one new sum rule:

$$I_{Y_\alpha} = \frac{13}{56} I_{g_2} D_{Y_{13H}}. \quad (4.18)$$

If both scenario 1 and scenario 5b hold, we have $D_Z = I_{Y_\alpha} = D_{Y_{13H}} = 0$. In that case this new sum rule becomes redundant.

Mixed scenario 25. If both scenario 2 and scenario 5a hold, at least the sum rules (4.3) and (4.6)–(4.10) hold, as well as the inequalities (4.11)–(4.12). From table 4 we conclude that if these unifications occur simultaneously we have two sum rules from vanishing RGIs as well as twelve equations with seven unknowns. Hence we can construct five additional sum rules. We already found six of them, so there is one new sum rule:

$$I_{M_1} - \frac{81}{25} I_{M_2} + \frac{56}{25} I_{M_3} = 0 \quad (4.19)$$

Note that we get the same additional sum rule if we choose scenario 5b instead of 5a.

Mixed scenario 125. If all unifications occur simultaneously, then we immediately know that the following sum rules should hold (we assume scenario 5a for the moment):

- Sum rule (4.2) for scenario 1.
- Sum rules (4.4)–(4.5) for scenario 3.
- Sum rule (4.3) for scenario 2. However, as we have mentioned before, this one is automatically satisfied if the sum rules for scenarios 1 and 3 are satisfied. Hence this one is redundant.
- Sum rules (4.6)–(4.7) for scenario 4, which is implied by scenario 5.
- Sum rules (4.8)–(4.10) for scenario 5, as well as inequalities (4.11)–(4.12).
- Sum rule (4.18) for the simultaneity of scenarios 1 and 5.
- Sum rule (4.19) for the simultaneity of scenarios 2 and 5. However, this one has become redundant: it can be retrieved by combining equations (4.4)–(4.5), (4.8)–(4.10) and (4.18). We could have expected this: if we have consistency with scenarios 1, 2 and 5, and we have established simultaneity of both scenarios 1 and 2 and scenarios 1 and 5, then it follows automatically that we have simultaneity of scenarios 2 and 5.

This adds up to nine independent sum rules. From table 4 we conclude that if all unifications occur simultaneously we have two sum rules from vanishing RGIs as well as twelve equations with five unknowns. Hence, we should get nine sum rules, so there are no new ones. Note that if we had taken scenario 5b instead of 5a, we would not get any additional new sum rules either. In that case the usual sum rules $D_Z = I_{Y_\alpha} = D_{Y_{13H}} = 0$ would hold and equation (4.18) would become redundant.

4.1.1 Sum rules summary

All scenarios discussed above and their corresponding sum rules have been summarised in figure 6. Related scenarios have been connected: if one starts at a given scenario, one should follow the arrows downwards to arrive at the underlying hypotheses. When we have determined the values of the RGIs from experimental data, we can test whether the listed scenarios are consistent with the data. One should proceed as follows: to test a hypothesis, check the validity of the sum rules in the corresponding box. Then check the validity of the sum rules in all boxes one encounters by following the arrows all the way down. If all these sum rules are satisfied, the hypothesis is consistent with the experimental data (as far as our sum rules are concerned).

4.2 Model-specific sum rules

Until now we have only considered hypotheses concerned with relations between the running parameters of the (p)MSSM. These hypotheses do not refer to any model-specific parameters. However, we can find additional sum rules for certain models because the soft masses are related by only a few parameters. For example, in MGM the gaugino and sfermion masses are determined by the gauge couplings and a single parameter B ; see equations (3.11a)–(3.11b). Furthermore, the question whether the messenger scale equals the gauge-coupling unification scale only makes sense if we consider gauge-mediation models. Therefore we consider model-specific sum rules separately in this section.

In the following, we will look for sum rules for GGM and AMSB that do not follow from the general hypotheses we have discussed above. Such sum rules will generically be referred to as “model-specific sum rules”. The sum rules for GGM can also be found in [6]. To our knowledge no sum rules for AMSB models have been presented in the literature. We will not discuss mSUGRA, because for our purposes the mSUGRA spectrum is completely characterised by simultaneous scalar-mass, gaugino-mass and gauge-coupling unification. So, there are no mSUGRA-specific sum rules.

4.2.1 General gauge mediation

In section 3.7 the RG boundary conditions for GGM are given in terms of 11 model parameters at the messenger scale M : $\delta_{u,d}$ and $B_a, A_a, g_a(M)$ for $a = 1, 2, 3$ (we take $\zeta = 0$). If we insert these RG boundary conditions into the RGIs, we immediately find $D_{B_{13}} = D_{L_{13}} = 0$, as expected from flavour-universality. In addition we get *one model-specific sum rule*:

$$D_{\chi_1} = 0. \tag{4.20}$$

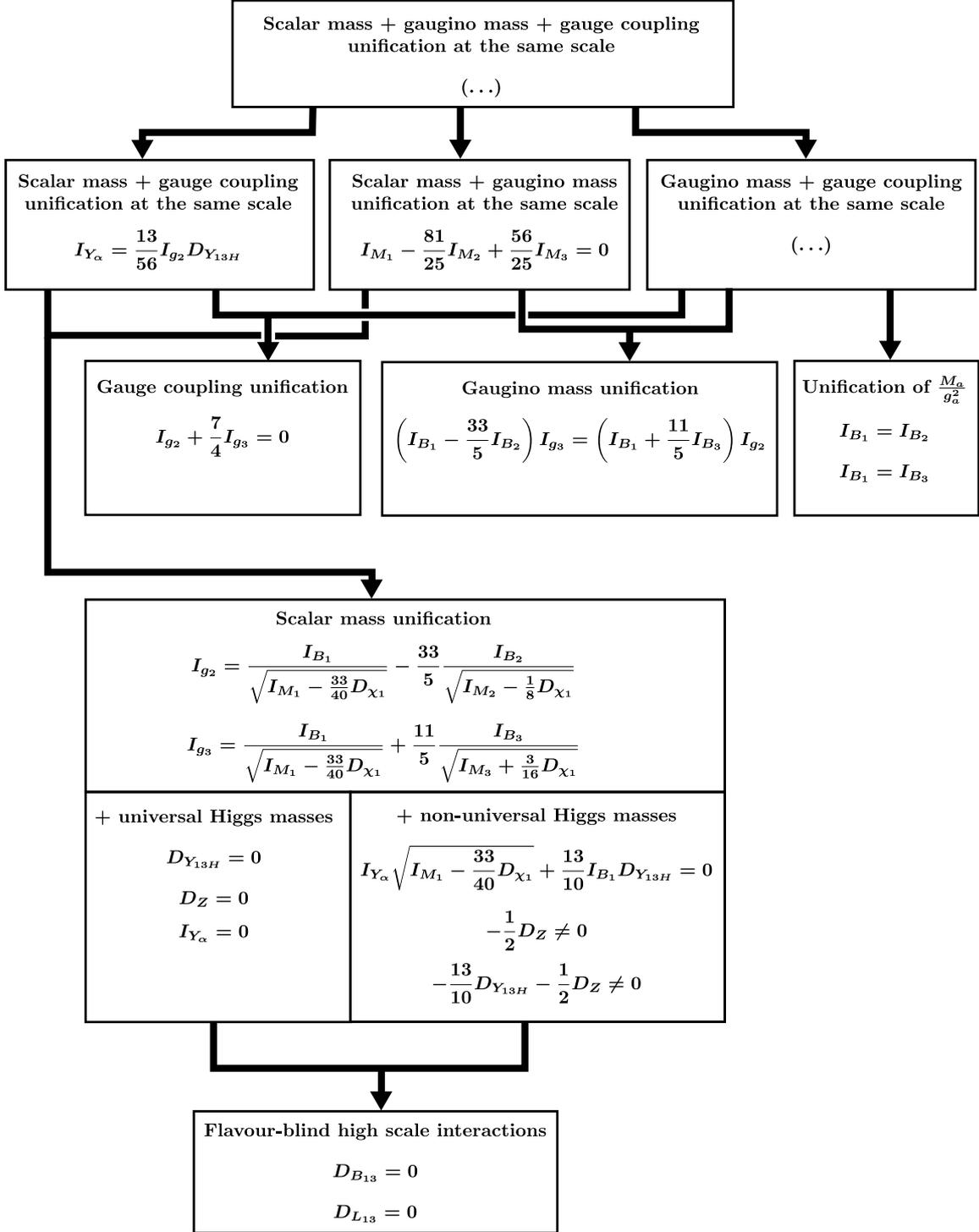


Figure 6. Scheme for testing hypotheses about the spectrum at the new-physics threshold. For a given scenario, the arrows point towards its underlying hypotheses. To test a specific hypothesis, check whether the corresponding sum rules are satisfied. Then follow the arrows downwards all the way to the bottom and for each sum rule along the way, check whether it is satisfied.

The remaining RGI have the following values:

$$D_{Y_{13H}} = -\frac{10}{13}(\delta_u - \delta_d), \quad (4.21a)$$

$$D_Z = -2\delta_d, \quad (4.21b)$$

$$I_{Y_\alpha} = g_1^{-2}(\delta_u - \delta_d), \quad (4.21c)$$

$$I_{B_a} = B_a \quad (a = 1, 2, 3), \quad (4.21d)$$

$$I_{M_1} = g_1^4 \left(B_1^2 + \frac{33}{10}A_1 \right), \quad (4.21e)$$

$$I_{M_2} = g_2^4 \left(B_2^2 + \frac{1}{2}A_2 \right), \quad (4.21f)$$

$$I_{M_3} = g_3^4 \left(B_3^2 - \frac{3}{2}A_3 \right), \quad (4.21g)$$

$$I_{g_2} = g_1^{-2} - \frac{33}{5}g_2^{-2}, \quad (4.21h)$$

$$I_{g_3} = g_1^{-2} + \frac{11}{5}g_3^{-2}, \quad (4.21i)$$

where the gauge couplings are understood to be evaluated at the messenger scale. This amounts to eleven equations with eleven unknowns, hence no additional sum rules can be constructed. Note that we can again verify non-universality in the Higgs sector using equations (4.11)–(4.12).

Gauge-coupling unification at the messenger scale. If both gauge-coupling unification and GGM are compatible with experimental data, we may ask ourselves if the messenger scale equals the scale of gauge-coupling unification. If we insert $g_1 = g_2 = g_3 = g$ into equations (4.21), we get eleven equations with nine unknowns. Hence we can make two more sum rules. This includes equation (4.2) for gauge-coupling unification. Hence, there is only *one additional model-specific sum rule*:

$$I_{Y_\alpha} = \frac{13}{56}I_{g_2}D_{Y_{13H}}. \quad (4.22)$$

Note that this sum rule happens to be identical to equation (4.18), which was constructed for testing a completely different concept: simultaneous scalar-mass and gauge-coupling unification (i.e. scenario 15). Therefore we have marked this sum rule as being model-specific. Also note that this sum rule becomes redundant in the case of universal Higgs masses, since $I_{Y_\alpha} = D_{Y_{13H}} = 0$ in that case.

Minimal gauge mediation. Recall that MGM is a GGM model restricted to a subset of the GGM parameter space defined by $A_a = A$, $B_a = B$ and $A = 2B^2$. Inserting this into the RGI values (4.21) of GGM, we find eleven non-vanishing RGIs that depend on six parameters. Hence we can construct five sum rules. These include (4.4)–(4.5) for M_a/g_a^2

unification. There are *three additional model-specific sum rules*:⁶

$$0 = I_{Y_\alpha} + \frac{13}{10} D_{Y_{13H}} I_{B_1} \sqrt{\frac{38}{5I_{M_1}}}, \quad (4.23)$$

$$0 = I_{B_1} \sqrt{\frac{38}{5I_{M_1}}} - \frac{33}{5} I_{B_1} \sqrt{\frac{2}{I_{M_2}}} - I_{g_2}, \quad (4.24)$$

$$0 = I_{B_1} \sqrt{\frac{38}{5I_{M_1}}} + \frac{11}{5} I_{B_1} \sqrt{\frac{-2}{I_{M_3}}} - I_{g_3}. \quad (4.25)$$

Note that (4.23) becomes redundant in the case of universal Higgs masses.

4.2.2 Anomaly mediation.

The RG boundary conditions for AMSB are given in section 3.6 in terms of 4 model parameters at the scale M_{AMSB} where supersymmetry breaking occurs: $m_{3/2}$ and $g_a(M_{\text{AMSB}})$ for $a = 1, 2, 3$. If we insert these RG boundary conditions into the RGIs, we immediately find *nine model-specific sum rules*:

$$D_{B_{13}} = D_{L_{13}} = D_{\chi_1} = D_{Y_{13H}} = D_Z = I_{Y_\alpha} = I_{M_1} = I_{M_2} = I_{M_3} = 0. \quad (4.26)$$

Note that $D_{B_{13}}$ and $D_{L_{13}}$ vanish although the sfermion masses are not flavour-universal! The non-vanishing RGIs have the values:

$$I_{B_1} = \frac{33}{5} \frac{m_{3/2}}{16\pi^2}, \quad (4.27a)$$

$$I_{B_2} = \frac{m_{3/2}}{16\pi^2}, \quad (4.27b)$$

$$I_{B_3} = -3 \frac{m_{3/2}}{16\pi^2}, \quad (4.27c)$$

$$I_{g_2} = g_1^{-2} - \frac{33}{5} g_2^{-2}, \quad (4.27d)$$

$$I_{g_3} = g_1^{-2} + \frac{11}{5} g_3^{-2}, \quad (4.27e)$$

where the gauge couplings should be evaluated at the scale of supersymmetry breaking. This amounts to five equations with four unknowns, but we can do better: equations (4.27a)–(4.27c) constitute three equations with one unknown. This yields *another two model-specific sum rules*:

$$0 = I_{B_1} - \frac{33}{5} I_{B_2}, \quad (4.28)$$

$$0 = I_{B_1} + \frac{11}{5} I_{B_3}. \quad (4.29)$$

⁶Note that in MGM, we can safely divide by I_{M_a} : if one of the I_{M_a} vanished, then $B = 0$ and the gaugino masses would vanish at the messenger scale. Their β -functions, being proportional to the gaugino masses, would vanish as well. Then at one-loop order gauginos would be massless at all scales (only through two-loop effects the masses will be non-vanishing). In that case we would have observed them already. Thus the I_{M_a} cannot vanish.

Gauge-coupling unification at the scale of supersymmetry breaking. If both AMSB and gauge-coupling unification turn out to be consistent with experimental data, we may ask ourselves whether supersymmetry breaking occurs at the scale of gauge-coupling unification. In that case we should insert $g_1 = g_2 = g_3 = g$ into (4.27). But this will only affect equations (4.27d) and (4.27e), which we have not used to make the above sum rules. This amounts to two equations with only one parameter, so we get one more sum rule. This must be the sum rule (4.2) for gauge-coupling unification, hence there are no sum rules that specifically test whether gauge-coupling unification occurs at the scale of supersymmetry breaking.

Minimal anomaly mediation. Recall that in minimal AMSB, a universal additional term m_0^2 is added to the soft scalar masses. If we insert this into the RGI expressions, we immediately find *five model-specific sum rules*:

$$D_{B_{13}} = D_{L_{13}} = D_{Y_{13H}} = D_Z = I_{Y_\alpha} = 0. \quad (4.30)$$

The non-vanishing RGIs have the values:

$$D_{\chi_1} = 5m_0^2, \quad (4.31a)$$

$$I_{B_1} = \frac{33}{5} \frac{m_{3/2}}{16\pi^2}, \quad (4.31b)$$

$$I_{B_2} = \frac{m_{3/2}}{16\pi^2}, \quad (4.31c)$$

$$I_{B_3} = -3 \frac{m_{3/2}}{16\pi^2}, \quad (4.31d)$$

$$I_{M_1} = \frac{33}{8} m_0^2, \quad (4.31e)$$

$$I_{M_2} = \frac{5}{8} m_0^2, \quad (4.31f)$$

$$I_{M_3} = -\frac{15}{16} m_0^2, \quad (4.31g)$$

$$I_{g_2} = g_1^{-2} - \frac{33}{5} g_2^{-2}, \quad (4.31h)$$

$$I_{g_3} = g_1^{-2} + \frac{11}{5} g_3^{-2}, \quad (4.31i)$$

where again the gauge couplings should be evaluated at the scale of supersymmetry breaking. This adds up to nine equations with five unknowns, so we expect to find four additional sum rules. However, if we leave out equations (4.31h)-(4.31i), we are left with seven equations with only two unknowns. This yields *another five model-specific sum rules*:

$$0 = I_{B_1} - \frac{33}{5} I_{B_2}, \quad (4.32)$$

$$0 = I_{B_1} + \frac{11}{5} I_{B_3}, \quad (4.33)$$

$$0 = D_{\chi_1} - \frac{40}{33} I_{M_1}, \quad (4.34)$$

$$0 = D_{\chi_1} - 8 I_{M_2}, \quad (4.35)$$

$$0 = D_{\chi_1} + \frac{16}{3} I_{M_3}. \quad (4.36)$$

Here equations (4.32)-(4.33) also hold for AMSB. Equations (4.34)-(4.36) are automatically satisfied in AMSB because D_{χ_1} and I_{M_a} vanish.

4.3 Discussion

In this section, we have found (a) sum rules that test general properties of the RG boundary conditions and (b) sum rules that test the consistency of specific model spectra. Comparing both sets of sum rules will help us to determine how good the sum rules are at distinguishing between several properties and model spectra.

Sum-rule ambiguities and how to eliminate them. In the sum rules we observe the following ambiguities:

- If the sum rules (4.2) for gauge-coupling unification and (4.4)–(4.5) for M_a/g_a^2 unification are both satisfied, then the sum rule (4.3) for gaugino-mass unification is automatically satisfied. But gaugino-mass unification is implied by gauge-coupling unification and M_a/g_a^2 unification only if both unifications occur at the same scale! Hence, if (4.2), (4.4) and (4.5) are satisfied by experimental data, then we cannot determine unambiguously whether the gaugino masses unify. At this point, we should use the bottom-up method to examine the running of the parameters. Then we could see whether the unification scales are the same.
- Equation (4.18) checks whether scalar masses and gauge couplings unify at the same scale. Equation (4.22) checks whether the gauge couplings unify at the messenger scale in GGM. Yet these sum rules happen to be the same. However, this does not mean we cannot distinguish between these two scenarios. The former scenario also requires that the sum rules (4.8)–(4.10) for scalar-mass unification are valid. In the latter scenario, these sum rules are not satisfied. Thus the double role of (4.18) poses no problem.
- In AMSB and mAMSB, the sum rules (4.6)–(4.7) for flavour-universality are satisfied, although the sfermion masses in these models are clearly non-universal. Fortunately, (m)AMSB has a lot more sum rules, which could help discern these models from flavour-universal ones. For example, the vanishing of $D_{Y_{13H}}$, D_Z and I_{Y_α} is typical for (m)AMSB. Equations (4.8)–(4.9) then help us discern (m)AMSB from scalar-mass unification with universal Higgs masses. Again, satisfying a single sum rule may be ambiguous, but other sum rules eliminate this ambiguity.
- Because $D_{Y_{13H}}$ and I_{Y_α} vanish in (m)AMSB, the sum rule (4.18) for simultaneous gauge-coupling and scalar-mass unification is automatically satisfied. However, the sum rules for gauge-coupling unification and scalar-mass unification again help us distinguish between both scenarios.
- The vanishing of I_{M_a} in AMSB and the sum rules (4.34)–(4.36) of mAMSB both imply that the sum rule (4.19) for simultaneous scalar-mass and gaugino-mass unification is satisfied. However, the sum rules for scalar-mass unification and gaugino-mass unification help us distinguish between both scenarios.

If we only consider the spectrum properties and breaking mechanisms that we discussed in this section, our sum rules work surprisingly well. Many of the sum rules are not unambiguous by themselves, but in most cases the other sum rules remove the ambiguity. Only when the data are consistent with both gauge-coupling unification and M_a/g_a^2 unification, we have to resort to other methods (such as the bottom-up method) to determine whether the gaugino masses also unify (or equivalently, whether both unifications occur at the same scale).

Of course, it is possible that a new supersymmetry-breaking model is concocted in the future, and that some of its corresponding sum rules introduce similar ambiguities. These may or may not be resolved by other sum rules. Therefore, we should keep in mind that if the sum rules of a model or hypothesis are satisfied, this is not a confirmation that this model or hypothesis is correct. *The true power of our sum rules is their falsifying power: the failure to satisfy just one sum rule implies that the corresponding hypothesis or model is incorrect.*

Now that we have an idea of the quality of the RGI sum rules, we can finally examine the advantages and limitations of the RGI method.

Advantages of RGIs

- The RGI method requires less input than the other methods we have discussed. We only need the values of all soft masses and gauge couplings at one scale. These are sufficient to reconstruct the values of the RGIs in table 3. In contrast to the bottom-up method, we do not need the values of the Yukawa couplings, soft trilinear couplings and μ , b because we could not use them anyway. Also, the value of the new-physics threshold does not have to be known.
- The RGI method is very simple: it is entirely algebraical and does not require the numerical integration of renormalisation group equations. Therefore it avoids the complicated propagation of errors between the collider scale and the new-physics threshold. Also, it is not as time-consuming as the top-down method.
- As long as just a few of the relevant soft masses have been measured experimentally, the sum rules can be exploited as a fast means of identifying theoretically interesting regions in the remaining parameter space (e.g. regions with specific unifications).

Limitations and challenges of the RGI method

- As we mentioned before, RG invariance only holds up to a certain loop level. The RGIs in table 3 have been determined using the one-loop RG equations. Higher-order loop effects will certainly spoil RG invariance. We could of course try to find RGIs for the MSSM at a higher loop order. But already at the two-loop level the RG equations for the MSSM (see e.g. [40]) are too complicated to retain the simplicity of this method, if it is possible to find RGIs at all.

However, the relevant question is to what extent we should worry about this approximate RG invariance. It has been demonstrated in [5] that two-loop contributions to

the RGIs are smaller than the expected experimental errors of the one-loop RGIs, even in the optimistic scenario of 1% experimental uncertainties in the determination of soft masses at the collider scale. Thus for all practical purposes we can safely treat the one-loop RGIs as true invariants.

- It may seem like the RGI method magically reduces the uncertainties of the running parameters, compared to RG-evolved parameters. However, we have paid a price for this reduction, namely *information*. We can directly see this from table 3: we started with 18 running parameters (12 scalar masses, 3 gaugino masses and 3 gauge couplings) and have reduced them to only 14 invariants.

We can easily understand why we have to give up information to gain smaller errors. Consider for example the RG equations for $m_{\tilde{Q}_1}^2$ and $m_{\tilde{Q}_3}^2$ (see appendix A for the definitions of D_Y, X_t, X_b):

$$16\pi^2 \frac{dm_{\tilde{Q}_1}^2}{dt} = -\frac{2}{15}g_1^2 M_1^2 - 6g_2^2 M_2^2 - \frac{32}{3}g_3^2 M_3^2 + \frac{1}{5}g_1^2 D_Y, \quad (4.37a)$$

$$16\pi^2 \frac{dm_{\tilde{Q}_3}^2}{dt} = X_t + X_b - \frac{2}{15}g_1^2 M_1^2 - 6g_2^2 M_2^2 - \frac{32}{3}g_3^2 M_3^2 + \frac{1}{5}g_1^2 D_Y. \quad (4.37b)$$

Note that in the RG equations of all soft masses, dependence on the gaugino mass M_2 occurs only as terms proportional to $g_2^2 M_2^2$. Hence we can eliminate the M_2 dependence by taking suitable linear combinations of MSSM parameters. For example, the RG equation for the quantity $m_{\tilde{Q}_1}^2 - m_{\tilde{Q}_3}^2$ (which occurs in $D_{B_{13}}$) does not depend on M_2 any more, so its experimental uncertainty will spread less under RG flow. However, in this process we have thrown away information about the value of $m_{\tilde{Q}_1}^2 + m_{\tilde{Q}_3}^2$. Thus we have to reduce the number of independent quantities to reduce the spread of uncertainties under RG flow.

This may become a limitation of the RGI method in the following sense. A minimal model such as mSUGRA, with only three parameters that govern the soft masses plus gauge couplings at the GUT scale ($m_0, M_{1/2}, g \equiv g_a(M_{\text{GUT}})$), allows us to construct sum rules because we have more RGIs than mSUGRA has parameters. However, if we have a not-so-minimal model with (say) 15 parameters that determine the high-scale spectrum, we do not have enough RGIs to make any sum rules.⁷ Hence, despite the simplicity of the method, we are still limited to models with few parameters.

- The applicability of the RGI method to the study of supersymmetry breaking depends crucially on the assumption that the MSSM renormalisation group equations are valid all the way up to the scale of supersymmetry breaking. But suppose that in Nature a new field Φ (or possibly more than one) enters the theory at a high scale μ_Φ that is not the scale of supersymmetry breaking; instead supersymmetry is broken at an even higher scale μ_{SUSY} . Then at μ_Φ the physical RG trajectories of the running parameters will be deflected from their MSSM trajectories. Thus we might mistakingly see gaugino-mass unification where it is absent, or vice versa. Hence, if

⁷That is, unless a subset of n RGIs accidentally depends on less than n model parameters.

we want to study supersymmetry breaking directly from RGIs, we have to assume that new physics, if present, does not alter the one-loop RG equations for the MSSM up to the scale of supersymmetry breaking.

- In order to make conclusive statements based on sum rules, the values of all RGIs should be reconstructed. To achieve that, all soft masses and gauge couplings need to be known at one energy scale. This may prove difficult in practice. First of all, due to mixing effects the gauge eigenstates do not always correspond to the mass eigenstates. Reconstructing the soft masses from measured pole masses will introduce additional uncertainties. Furthermore, determining all soft masses and gauge couplings is one thing, but determining all of them *at the same energy scale* may prove challenging. Note however that the bottom-up method also suffers from these complications.

5 Conclusions and Outlook

We advocate to employ sum rules in terms of renormalisation group invariants as a simple yet powerful method to probe high-scale physics in lower-energy experiments. This method has been worked out in detail for the study of supersymmetry-breaking mechanisms in the context of the Minimal Supersymmetric Standard Model. It has been argued that important clues about the supersymmetry-breaking mechanism are to be found in patterns between the high-scale soft-supersymmetry-breaking parameters. The renormalisation group is the prime tool to extract such information on the high-scale spectrum from lower-energy data. Several methods have been discussed to do this and a new strategy has been proposed to make effective use of renormalisation group invariants. Assuming that the Minimal Supersymmetric Standard Model is an appropriate effective field theory beyond the Standard Model, a model-independent set of renormalisation-group-invariant sum rules has been constructed that test properties that are common in supersymmetry-breaking models, such as unifications and flavour-universality. If a certain property is realised in Nature, all corresponding sum rules must be satisfied. Since none of these sum rules refer to any parameters that are specific to some supersymmetry-breaking mechanism, they are useful regardless of the way supersymmetry has been broken in Nature.

In addition, sum rules that are tailor-made for testing specific supersymmetry-breaking mechanisms have been considered. Their primary use was to determine the effectiveness and ambiguities associated with the model-independent sum rules. It was found that some sum rules do not provide unambiguous checks by themselves; however, in almost all cases other sum rules lift the ambiguity. Hence, for the currently known supersymmetry-breaking mechanisms the proposed model-independent sum rules are surprisingly effective. In the exceptional case when they are not, one may have to resort to other methods to resolve the ambiguity, such as a bottom-up analysis. It is possible that new breaking mechanisms will be proposed in the future, and that their corresponding sum rules introduce new ambiguities. Therefore, it should be kept in mind that the main strength of invariant sum rules is their falsifying power. If we are able to determine all soft masses and gauge couplings, the compatibility of the sum rules with experimental data will put severe constraints on any realistic model of supersymmetry breaking .

It is possible that the next effective field theory beyond the Standard Model is not the Minimal Supersymmetric Standard Model. It may as well be a non-minimal supersymmetric extension of the Standard Model, or even a non-supersymmetric theory. Nevertheless, the proposed scheme for probing high-scale properties of running parameters may be applied just the same. In order to perform an analogous study, one needs to determine the particle content, interactions and β -functions of the appropriate effective field theory. Then one should determine all independent renormalisation group invariants for this effective field theory and construct sum rules in a way similar to what has been worked out in this study. However, there is a large amount of structure in the β -functions of the Minimal Supersymmetric Standard Model in view of the limited number of combinations in which running parameters appear in them. So, an interesting topic is to determine the form of the (one-loop) β -functions for a theory that is more general than the Minimal Supersymmetric Standard Model, and to see what renormalisation group invariants can be found for such a general theory. We leave these issues to future work.

Apart from establishing the validity of the Minimal Supersymmetric Standard Model, the main obstacle to using our invariant sum rules is the necessity of knowing all soft masses and gauge couplings at one scale. Therefore, an important topic for future study will be to determine how well this can be done and how the sum rules can be exploited as a fast means of identifying theoretically interesting regions in as yet unconstrained parameter space. Another important issue is to find out how the soft mass parameters can be reconstructed from the mass eigenstates of the sparticles.

Acknowledgments

We thank Jari Laamanen and Irene Niessen for useful discussions. This work has been supported in part by the Foundation for Fundamental Research of Matter (FOM), program 104 “Theoretical Particle Physics in the Era of the LHC”.

A One-loop RG equations for the MSSM

In this appendix we give the renormalisation group equations of the MSSM that have been used in this study. They have been taken from [20] and are one-loop equations that have been simplified by the assumptions for the pMSSM (see section 3.3). For general two-loop RG equations, see e.g. [40]. It is convenient to use the β -functions, which differ from their corresponding RG equations by a constant:

$$\beta(p) \equiv 16\pi^2 \frac{dp}{dt}. \quad (\text{A.1})$$

Here p is a running parameter and $t \equiv \log(\mu/\mu_0)$, where μ is the renormalisation scale and μ_0 an (arbitrary) energy scale that makes the argument of the logarithm dimensionless. Under the approximations of the pMSSM we are left with the following running parameters:

g_a	$(a = 1, 2, 3)$	Gauge couplings
M_a	$(a = 1, 2, 3)$	Soft-supersymmetry-breaking gaugino masses
$m_{\tilde{Q}}^2, m_{\tilde{u}}^2, m_{\tilde{d}}^2, m_{\tilde{L}}^2, m_{\tilde{e}}^2$		Soft-supersymmetry-breaking sfermion masses
$m_{H_u}^2, m_{H_d}^2$		Soft-supersymmetry-breaking Higgs mass parameters
y_t, y_b, y_τ		Yukawa couplings for the third-generation (s)fermions
A_t, A_b, A_τ		Soft-supersymmetry-breaking trilinear couplings for the third-generation sfermions
μ		Supersymmetry-respecting Higgs mixing parameter
B		Soft-supersymmetry-breaking Higgs mixing parameter

Here we use the soft Higgs mixing parameter $B = b/\mu$ rather than b because its β -function is simpler. For the sfermion masses, we denote the first and third generation with a subscript 1 and 3 respectively.

The following notation is used for parameters that enter the RG equations through common combinations of Dynkin indices and quadratic Casimir invariants:

$$b_a = \left(\frac{33}{5}, 1, -3 \right) \quad \text{for } a = 1, 2, 3. \quad (\text{A.2})$$

It is also convenient to define the following combination of running parameters, which appears in the RG equations of the sfermion masses:

$$D_Y \equiv \text{Tr}(Ym^2) = \sum_{\text{gen}} \left(m_{\tilde{Q}}^2 - 2m_{\tilde{u}}^2 + m_{\tilde{d}}^2 - m_{\tilde{L}}^2 + m_{\tilde{e}}^2 \right) + m_{H_u}^2 - m_{H_d}^2. \quad (\text{A.3})$$

Here the trace runs over all chiral multiplets and the sum runs over the three sfermion generations. Note that D_Y is often called S in the literature. Furthermore, we define the useful combinations:

$$X_t = 2|y_t|^2 \left(m_{H_u}^2 + m_{\tilde{Q}_3}^2 + m_{\tilde{u}_3}^2 + |A_t|^2 \right), \quad (\text{A.4a})$$

$$X_b = 2|y_b|^2 \left(m_{H_d}^2 + m_{\tilde{Q}_3}^2 + m_{\tilde{d}_3}^2 + |A_b|^2 \right), \quad (\text{A.4b})$$

$$X_\tau = 2|y_\tau|^2 \left(m_{H_d}^2 + m_{\tilde{L}_3}^2 + m_{\tilde{e}_3}^2 + |A_\tau|^2 \right). \quad (\text{A.4c})$$

Then the resulting β -functions for the MSSM are:

$$\beta(g_a) = b_a g_a^3 \quad (a = 1, 2, 3), \quad (\text{A.5a})$$

$$\beta(M_a) = 2b_a g_a^2 M_a \quad (a = 1, 2, 3), \quad (\text{A.5b})$$

$$\beta(m_{\tilde{Q}_{1,2}}^2) = -\frac{2}{15}g_1^2 M_1^2 - 6g_2^2 M_2^2 - \frac{32}{3}g_3^2 M_3^2 + \frac{1}{5}g_1^2 D_Y, \quad (\text{A.5c})$$

$$\beta(m_{\tilde{u}_{1,2}}^2) = -\frac{32}{15}g_1^2 M_1^2 - \frac{32}{3}g_3^2 M_3^2 - \frac{4}{5}g_1^2 D_Y, \quad (\text{A.5d})$$

$$\beta(m_{\tilde{d}_{1,2}}^2) = -\frac{8}{15}g_1^2 M_1^2 - \frac{32}{3}g_3^2 M_3^2 + \frac{2}{5}g_1^2 D_Y, \quad (\text{A.5e})$$

$$\beta(m_{\tilde{L}_{1,2}}^2) = -\frac{6}{5}g_1^2 M_1^2 - 6g_2^2 M_2^2 - \frac{3}{5}g_1^2 D_Y, \quad (\text{A.5f})$$

$$\beta(m_{\tilde{e}_{1,2}}^2) = -\frac{24}{5}g_1^2 M_1^2 + \frac{6}{5}g_1^2 D_Y, \quad (\text{A.5g})$$

$$\beta(m_{\tilde{Q}_3}^2) = X_t + X_b - \frac{2}{15}g_1^2 M_1^2 - 6g_2^2 M_2^2 - \frac{32}{3}g_3^2 M_3^2 + \frac{1}{5}g_1^2 D_Y, \quad (\text{A.5h})$$

$$\beta(m_{\tilde{u}_3}^2) = 2X_t - \frac{32}{15}g_1^2 M_1^2 - \frac{32}{3}g_3^2 M_3^2 - \frac{4}{5}g_1^2 D_Y, \quad (\text{A.5i})$$

$$\beta(m_{\tilde{d}_3}^2) = 2X_b - \frac{8}{15}g_1^2 M_1^2 - \frac{32}{3}g_3^2 M_3^2 + \frac{2}{5}g_1^2 D_Y, \quad (\text{A.5j})$$

$$\beta(m_{\tilde{L}_3}^2) = X_\tau - \frac{6}{5}g_1^2 M_1^2 - 6g_2^2 M_2^2 - \frac{3}{5}g_1^2 D_Y, \quad (\text{A.5k})$$

$$\beta(m_{\tilde{e}_3}^2) = 2X_\tau - \frac{24}{5}g_1^2 M_1^2 + \frac{6}{5}g_1^2 D_Y, \quad (\text{A.5l})$$

$$\beta(m_{H_u}^2) = 3X_t - \frac{6}{5}g_1^2 M_1^2 - 6g_2^2 M_2^2 + \frac{3}{5}g_1^2 D_Y, \quad (\text{A.5m})$$

$$\beta(m_{H_d}^2) = 3X_b + X_\tau - \frac{6}{5}g_1^2 M_1^2 - 6g_2^2 M_2^2 - \frac{3}{5}g_1^2 D_Y, \quad (\text{A.5n})$$

$$\beta(y_t) = y_t \left[6|y_t|^2 + |y_b|^2 - \frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2 \right], \quad (\text{A.5o})$$

$$\beta(y_b) = y_b \left[6|y_b|^2 + |y_t|^2 + |y_\tau|^2 - \frac{7}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2 \right], \quad (\text{A.5p})$$

$$\beta(y_\tau) = y_\tau \left[4|y_\tau|^2 + 3|y_b|^2 - \frac{9}{5}g_1^2 - 3g_2^2 \right], \quad (\text{A.5q})$$

$$\beta(\mu) = \mu \left[3|y_t|^2 + 3|y_b|^2 + |y_\tau|^2 - \frac{3}{5}g_1^2 - 3g_2^2 \right], \quad (\text{A.5r})$$

$$\beta(A_t) = 12A_t|y_t|^2 + 2A_b|y_b|^2 + \frac{26}{15}g_1^2 M_1 + 6g_2^2 M_2 + \frac{32}{3}g_3^2 M_3, \quad (\text{A.5s})$$

$$\beta(A_b) = 12A_b|y_b|^2 + 2A_t|y_t|^2 + 2A_\tau|y_\tau|^2 + \frac{14}{15}g_1^2 M_1 + 6g_2^2 M_2 + \frac{32}{3}g_3^2 M_3, \quad (\text{A.5t})$$

$$\beta(A_\tau) = 8A_\tau|y_\tau|^2 + 6A_b|y_b|^2 + \frac{18}{5}g_1^2 M_1 + 6g_2^2 M_2, \quad (\text{A.5u})$$

$$\beta(B) = 6A_t|y_t|^2 + 6A_b|y_b|^2 + 2A_\tau|y_\tau|^2 + \frac{6}{5}g_1^2 M_1 + 6g_2^2 M_2. \quad (\text{A.5v})$$

B Deriving the one-loop RGIs for the MSSM

In this appendix we will derive a maximal set of independent RGIs for the MSSM. First we will determine invariants that contain the running parameters μ and $B = b/\mu$. We will see that there is only one independent RGI for each of them, making them useless for our study. Then we will argue that we are restricted to RGIs containing only soft masses and/or gauge couplings. We will derive all of them systematically; our approach will be globally the same as in [5], but using different arguments to show that we do indeed find all RGIs.

Let us consider the parameter μ . The only β -function containing μ is that of μ itself. Note that we can write $\beta(\mu)$ more conveniently as:

$$\beta(\log \mu) = 3|y_t|^2 + 3|y_b|^2 + |y_\tau|^2 - \frac{3}{5}g_1^2 - 3g_2^2. \quad (\text{B.1})$$

The only other β -functions containing terms linear in $|y_t|^2, |y_b|^2, |y_\tau|^2$ are those of the logarithms of the Yukawa couplings:

$$\beta(\log y_t) = 6|y_t|^2 + |y_b|^2 - \frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2, \quad (\text{B.2a})$$

$$\beta(\log y_b) = |y_t|^2 + 6|y_b|^2 + |y_\tau|^2 - \frac{7}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2, \quad (\text{B.2b})$$

$$\beta(\log y_\tau) = 3|y_b|^2 + 4|y_\tau|^2 - \frac{9}{5}g_1^2 - 3g_2^2. \quad (\text{B.2c})$$

The terms in the β -functions proportional to g_a^2 can be eliminated by taking linear combinations with logarithms of gauge couplings, of which we can rewrite the β -functions as:

$$\beta(\log g_a) = b_a g_a^2 \quad (a = 1, 2, 3). \quad (\text{B.3})$$

Hence, μ can only appear in an RGI through a linear combination of $\log \mu, \log y_t, \log y_b, \log y_\tau, \log g_1, \log g_2$ and $\log g_3$.⁸ We have seven β -functions with six different terms to eliminate (namely terms linear in $|y_t|^2, |y_b|^2, |y_\tau|^2, g_1^2, g_2^2$ or g_3^2), so we can make one RG invariant linear combination of them. Using elementary linear algebra we find that the linear combination

$$\begin{aligned} & -\frac{27}{61} \log y_t - \frac{21}{61} \log y_b - \frac{10}{61} \log y_\tau + \log \mu - \frac{1}{61} \cdot \frac{73}{33} \log g_1 \\ & \quad + \frac{9}{61} \log g_2 + \frac{1}{61} \cdot \frac{256}{3} \log g_3 \\ = & \log \left(\mu \left[\frac{g_2^9 g_3^{256/3}}{y_t^{27} y_b^{21} y_\tau^{10} g_1^{73/33}} \right]^{1/61} \right) \end{aligned} \quad (\text{B.4})$$

⁸We could also include logarithms of gaugino masses in these linear combinations, since their β -functions are also proportional to g_a^2 . However, in a moment we will construct RGIs from the gauge couplings and gaugino masses only. Any RGI that contains both μ and the gaugino masses will be a function of those RGIs and the one we are constructing now.

has a vanishing β -function. Thus we can choose the only independent RGI containing μ to be

$$I_2 \equiv \mu \left[\frac{g_2^9 g_3^{256/3}}{y_t^{27} y_b^{21} y_\tau^{10} g_1^{73/33}} \right]^{1/61}, \quad (\text{B.5})$$

using the notation of [4]. To summarise, we have found a set of independent RGIs containing μ (in this case only one) by considering what terms in the MSSM β -functions could cancel each other. This will be our general strategy for finding all RGIs of the MSSM, because the running parameters only enter the β -functions in a very limited number of combinations (e.g. the soft scalar masses only appear in the linear combinations D_Y , X_t , X_b and X_τ).

Now we turn to the parameter B . It does not appear in any of the MSSM β -functions itself. Its β -function contains only terms linear in $A_t|y_t|^2$, $A_b|y_b|^2$, $A_\tau|y_\tau|^2$, $g_1^2 M_1$, $g_2^2 M_2$ and $g_3^2 M_3$. The β -functions of A_t , A_b , A_τ , M_1 , M_2 and M_3 also contain only these terms, so B should always appear in RGIs in a linear combination of these parameters. This gives us seven β -functions with six different terms to eliminate, so again we can make one RG-invariant linear combination. Using elementary linear algebra this combination is found to be

$$I_4 \equiv B - \frac{27}{61}A_t - \frac{21}{61}A_b - \frac{10}{61}A_\tau - \frac{256}{183}M_3 - \frac{9}{61}M_2 + \frac{73}{2013}M_1. \quad (\text{B.6})$$

Indeed we have found only one independent RGI containing μ and one containing B . As was argued in section 3.10, RGIs are only useful as long as their constituent running parameters also appear in other RGIs. This is not the case for I_2 and I_4 , so we are restricted to RGIs that contain neither μ nor B . But in the above procedure, we needed their β -functions to eliminate the $|y_i|^2$ and $A_i|y_i|^2$ dependence respectively from the β -function of the RGI under construction. If we wish to construct RGIs containing the Yukawa couplings without using μ , we have to eliminate three different $|y_i|^2$ terms using three β -functions, so we cannot make any RG-invariant combinations. Similarly, we cannot make any RGIs containing the soft trilinear couplings without using B , because we have to eliminate three different $A_i|y_i|^2$ terms using three β -functions.

Thus, if we want to construct RGIs without using μ and B , we cannot use the Yukawa and soft trilinear couplings either: we do not have enough equations to eliminate all terms from the β -function of the RGI under construction. Therefore, from now on we will only consider RGIs that are functions of soft masses (fifteen parameters) and/or gauge couplings (three parameters).

Let us begin with RGIs constructed from the gauge couplings only. First we rewrite their β -functions into a more convenient form:

$$\beta(g_1^{-2}) = -\frac{66}{5}, \quad (\text{B.7a})$$

$$\beta(g_2^{-2}) = -2, \quad (\text{B.7b})$$

$$\beta(g_3^{-2}) = 6. \quad (\text{B.7c})$$

This gives us three equations to eliminate a single term (namely a constant), hence we can

make two independent RGIs out of them. In accordance with [5], we choose them to be

$$I_{g_2} \equiv g_1^{-2} - \frac{33}{5}g_2^{-2}, \quad (\text{B.8})$$

$$I_{g_3} \equiv g_1^{-2} + \frac{11}{5}g_3^{-2}. \quad (\text{B.9})$$

Now we turn to the gaugino masses. First we rewrite their β -functions as follows:

$$\beta(\log M_1) = \frac{66}{5}g_1^2, \quad (\text{B.10a})$$

$$\beta(\log M_2) = 2g_2^2, \quad (\text{B.10b})$$

$$\beta(\log M_3) = -6g_3^2. \quad (\text{B.10c})$$

Together with (B.3) this gives six equations with three different terms (namely those proportional to g_a^2) to eliminate. Hence, we get three new RGIs by taking linear combinations of $\log M_a$ and $\log g_a$:

$$0 = \beta(\log M_1 - 2 \log g_1) = \beta(\log \frac{M_1}{g_1^2}), \quad (\text{B.11a})$$

$$0 = \beta(\log M_2 - 2 \log g_2) = \beta(\log \frac{M_2}{g_2^2}), \quad (\text{B.11b})$$

$$0 = \beta(\log M_3 - 2 \log g_3) = \beta(\log \frac{M_3}{g_3^2}). \quad (\text{B.11c})$$

Thus we can choose the three independent RGIs to be:

$$I_{B_1} \equiv \frac{M_1}{g_1^2}, \quad (\text{B.12})$$

$$I_{B_2} \equiv \frac{M_2}{g_2^2}, \quad (\text{B.13})$$

$$I_{B_3} \equiv \frac{M_3}{g_3^2}. \quad (\text{B.14})$$

Now let us consider RGIs constructed solely from the twelve soft scalar masses. First we eliminate the Yukawa terms X_t, X_b, X_τ and the gaugino-mass terms $g_1^2 M_1^2, g_2^2 M_2^2, g_3^2 M_3^2$ from the β -function. Since we have to eliminate six terms using twelve equations, we can make six independent linear combinations of the soft scalar masses that have a β -function proportional to $g_1^2 D_Y$. Then we can make linear combinations of these quantities such that five of them have a vanishing β -function and the sixth quantity still runs with $g_1^2 D_Y$. In accordance with [5], we choose the five RGIs to be:⁹

$$D_{B_{13}} \equiv 2 \left(m_{\tilde{Q}_1}^2 - m_{\tilde{Q}_3}^2 \right) - m_{\tilde{u}_1}^2 + m_{\tilde{u}_3}^2 - m_{\tilde{d}_1}^2 + m_{\tilde{d}_3}^2, \quad (\text{B.15})$$

⁹The notation used for the RGIs may look odd here. In [5] they are related to symmetries of the MSSM Lagrangian. In this context, the D -term D_i of a charge Q_i is defined as $D_i \equiv \text{Tr}(Q_i m^2)$, with the trace running over all chiral multiplets. Then one should interpret $D_{B_{13}}$ as $D_{B_1} - D_{B_3}$, where the subscripts 1 and 3 mean that the trace is restricted to sfermions of the first and third generation respectively. See [5] for an explanation of the nomenclature for the remaining RGIs.

$$D_{L_{13}} \equiv 2 \left(m_{L_1}^2 - m_{L_3}^2 \right) - m_{\tilde{e}_1}^2 + m_{\tilde{e}_3}^2, \quad (\text{B.16})$$

$$D_{\chi_1} \equiv 3 \left(3m_{\tilde{d}_1}^2 - 2 \left(m_{\tilde{Q}_1}^2 - m_{L_1}^2 \right) - m_{\tilde{u}_1}^2 \right) - m_{\tilde{e}_1}^2, \quad (\text{B.17})$$

$$D_{Y_{13H}} \equiv m_{\tilde{Q}_1}^2 - 2m_{\tilde{u}_1}^2 + m_{\tilde{d}_1}^2 - m_{L_1}^2 + m_{\tilde{e}_1}^2 - \frac{10}{13} \left(m_{\tilde{Q}_3}^2 - 2m_{\tilde{u}_3}^2 + m_{\tilde{d}_3}^2 - m_{L_3}^2 + m_{\tilde{e}_3}^2 + m_{H_u}^2 - m_{H_d}^2 \right), \quad (\text{B.18})$$

$$D_Z \equiv 3 \left(m_{\tilde{d}_3}^2 - m_{\tilde{d}_1}^2 \right) + 2 \left(m_{L_3}^2 - m_{H_d}^2 \right). \quad (\text{B.19})$$

The sixth quantity, which runs with $g_1^2 D_Y$, can be chosen to be D_Y itself, because

$$\beta(D_Y) = \frac{66}{5} g_1^2 D_Y. \quad (\text{B.20})$$

Note that $\log D_Y$ runs with g_1^2 , so using (B.3) we find

$$\beta(\log D_Y - 2 \log g_1) = \beta(\log \frac{D_Y}{g_1^2}) = 0. \quad (\text{B.21})$$

This gives us another independent RGI:

$$I_{Y_\alpha} \equiv \frac{D_Y}{g_1^2} = \frac{1}{g_1^2} \left(m_{H_u}^2 - m_{H_d}^2 + \sum_{\text{gen}} \left(m_Q^2 - 2m_u^2 + m_d^2 - m_L^2 + m_{\tilde{e}}^2 \right) \right). \quad (\text{B.22})$$

Finally, we look for RGIs constructed from both scalar masses and gaugino masses. Note that the gaugino-mass β -functions can be rewritten as:

$$\beta(M_a^2) = 4b_a g_a^2 M_a^2 \quad (a = 1, 2, 3). \quad (\text{B.23})$$

Combining the gaugino masses and scalar masses, we have fifteen β -functions with seven terms to eliminate, so we can construct eight RGIs by taking linear combinations of the squared gaugino masses and the scalar masses. Five of them can be made from the scalar masses alone, so there must be three new RGIs. In accordance with [5], we take them to be

$$I_{M_1} \equiv M_1^2 - \frac{33}{8} \left(m_{\tilde{d}_1}^2 - m_{\tilde{u}_1}^2 - m_{\tilde{e}_1}^2 \right), \quad (\text{B.24})$$

$$I_{M_2} \equiv M_2^2 + \frac{1}{24} \left(9 \left(m_{\tilde{d}_1}^2 - m_{\tilde{u}_1}^2 \right) + 16m_{L_1}^2 - m_{\tilde{e}_1}^2 \right), \quad (\text{B.25})$$

$$I_{M_3} \equiv M_3^2 - \frac{3}{16} \left(5m_{\tilde{d}_1}^2 + m_{\tilde{u}_1}^2 - m_{\tilde{e}_1}^2 \right). \quad (\text{B.26})$$

These complete the list of independent one-loop RGIs for the MSSM.

References

- [1] [ATLAS Collaboration], *Combined search for the Standard Model Higgs boson using up to 4.9 fb-1 of pp collision data at sqrt(s) = 7 TeV with the ATLAS detector at the LHC*, [[arXiv:1202.1408](https://arxiv.org/abs/1202.1408) [[hep-ex](#)]].

- [2] S. Chatrchyan *et al.* [CMS Collaboration], *Combined results of searches for the standard model Higgs boson in pp collisions at $\sqrt{s} = 7$ TeV*, [[arXiv:1202.1488](#) [[hep-ex](#)]].
- [3] J. Ellis, J. R. Espinosa, G. F. Giudice, A. Hoecker and A. Riotto, *The Probable Fate of the Standard Model*, *Phys. Lett.* **B679** (2009) 369 [[arXiv:0906.0954](#) [[hep-ph](#)]].
- [4] D. A. Demir, *Renormalization group invariants in the MSSM and its extensions*, *JHEP* **0511** (2005) 003 [[hep-ph/0408043](#)].
- [5] M. Carena, P. Draper, N. R. Shah and C. E. M. Wagner, *Determining the Structure of Supersymmetry-Breaking with Renormalization Group Invariants*, *Phys. Rev.* **D82** (2010) 075005 [[arXiv:1006.4363](#) [[hep-ph](#)]].
- [6] M. Carena, P. Draper, N. R. Shah and C. E. M. Wagner, *SUSY-Breaking Parameters from RG Invariants at the LHC*, *Phys. Rev.* **D83** (2011) 035014 [[arXiv:1011.4958](#) [[hep-ph](#)]].
- [7] J. Jaeckel, V. V. Khoze and C. Wymant, *RG Invariants, Unification and the Role of the Messenger Scale in General Gauge Mediation*, *JHEP* **1105** (2011) 132 [[arXiv:1103.1843](#) [[hep-ph](#)]].
- [8] B. Delamotte, *A Hint of renormalization*, *Am. J. Phys.* **72** (2004) 170 [[hep-th/0212049](#)].
- [9] K. G. Wilson and J. Kogut, *The renormalization group and the ϵ -expansion*, *Phys. Rept.* **12** (1974) 75.
- [10] K. G. Wilson, *The renormalization group and critical phenomena*, *Rev. Mod. Phys.* **55** (1983) 583.
- [11] A. Pich, *Effective field theory: Course*, [[hep-ph/9806303](#)].
- [12] R. Lafaye, T. Plehn and D. Zerwas, *SFITTER: SUSY parameter analysis at LHC and LC*, [[hep-ph/0404282](#)].
- [13] B. K. Gjelsten, D. J. Miller and P. Osland, *Measurement of SUSY masses via cascade decays for SPS 1a*, *JHEP* **0412** (2004) 003 [[hep-ph/0410303](#)].
- [14] C. G. Lester, M. A. Parker and M. J. White, *Determining SUSY model parameters and masses at the LHC using cross-sections, kinematic edges and other observables*, *JHEP* **0601** (2006) 080 [[hep-ph/0508143](#)].
- [15] I. Niessen, *Supersymmetric Phenomenology in the mSUGRA Parameter Space*, [[arXiv:0809.1748](#) [[hep-ph](#)]].
- [16] G. A. Blair, W. Porod and P. M. Zerwas, *Reconstructing supersymmetric theories at high energy scales*, *Phys. Rev.* **D63** (2000) 017703.
- [17] N. Arkani-Hamed, G. L. Kane, J. Thaler and L. T. Wang, *Supersymmetry and the LHC inverse problem*, *JHEP* **0608** (2006) 070 [[hep-ph/0512190](#)].
- [18] J. L. Kneur and N. Sahoury, *Bottom-Up Reconstruction Scenarios for (un)constrained MSSM Parameters at the LHC*, *Phys. Rev.* **D79** (2009) 075010 [[arXiv:0808.0144](#) [[hep-ph](#)]].
- [19] K. Nakamura *et al.* (Particle Data Group Collaboration), *Review of particle physics*, *J. Phys.* **G37** (2010) 075021 and 2011 partial update for the 2012 edition.
- [20] S. P. Martin, *A Supersymmetry primer*, [[hep-ph/9709356](#)].
- [21] I. J. R. Aitchison, *Supersymmetry and the MSSM: An Elementary introduction*, [[hep-ph/0505105](#)].

- [22] S. Coleman and J. Mandula, *All Possible Symmetries of the S Matrix*, *Phys. Rev.* **159** (1967) 1251.
- [23] R. Haag, J. Lopuszanski and M. Sohnius, *All possible generators of supersymmetries of the S-matrix*, *Nucl. Phys.* **B88** (1975) 257.
- [24] S. Dimopoulos and D. W. Sutter, *The Supersymmetric flavor problem*, *Nucl. Phys.* **B452** (1995) 496 [[hep-ph/9504415](#)].
- [25] A. Djouadi *et al.* [MSSM Working Group Collaboration], *The Minimal supersymmetric standard model: Group summary report*, [[hep-ph/9901246](#)].
- [26] G. Colangelo, E. Nikolidakis and C. Smith, *Supersymmetric models with minimal flavour violation and their running*, *Eur. Phys. J.* **C59** (2009) 75 [[arXiv:0807.0801](#) [[hep-ph](#)]].
- [27] H. P. Nilles, *Supersymmetry, supergravity and particle physics*, *Phys. Rep.* **110** (1984) 1.
- [28] A. H. Chamseddine, R. Arnowitt and P. Nath, *Locally Supersymmetric Grand Unification*, *Phys. Rev. Lett.* **49** (1982) 970.
- [29] R. Barbieri, S. Ferrara and C. Savoy, *Gauge models with spontaneously broken local supersymmetry*, *Phys. Lett.* **B119** (1982) 343.
- [30] J. R. Ellis, K. A. Olive and P. Sandick, *Varying the Universality of Supersymmetry-Breaking Contributions to MSSM Higgs Boson Masses*, *Phys. Rev.* **D78** (2008) 075012 [[arXiv:0805.2343](#) [[hep-ph](#)]].
- [31] L. Randall and R. Sundrum, *Out of this world supersymmetry breaking*, *Nucl. Phys.* **B557** (1999) 79 [[hep-th/9810155](#)].
- [32] G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, *Gaugino mass without singlets*, *JHEP* **9812** (1998) 027 [[hep-ph/9810442](#)].
- [33] G. L. Cardoso and B. A. Ovrut, *A Green-Schwarz mechanism for $D = 4$, $N = 1$ supergravity anomalies*, *Nucl. Phys.* **B369** (1992) 351.
- [34] G. F. Giudice and R. Rattazzi, *Theories with gauge mediated supersymmetry breaking*, *Phys. Rept.* **322** (1999) 419 [[hep-ph/9801271](#)].
- [35] P. Meade, N. Seiberg and D. Shih, *General Gauge Mediation*, *Prog. Theor. Phys. Suppl.* **177** (2009) 143 [[arXiv:0801.3278](#) [[hep-ph](#)]].
- [36] M. Buican, P. Meade, N. Seiberg and D. Shih, *Exploring General Gauge Mediation*, *JHEP* **0903** (2009) 016 [[arXiv:0812.3668](#) [[hep-ph](#)]].
- [37] T. Kobayashi, Y. Nakai and M. Sakai, *(Extra)Ordinary Gauge/Anomaly Mediation*, *JHEP* **1106** (2011) 039 [[arXiv:1103.4912](#) [[hep-ph](#)]].
- [38] K. Choi, K. S. Jeong and K. I. Okumura, *Phenomenology of mixed modulus-anomaly mediation in fluxed string compactifications and brane models*, *JHEP* **0509** (2005) 039 [[hep-ph/0504037](#)].
- [39] O. Lebedev, H. P. Nilles and M. Ratz, *A Note on fine-tuning in mirage mediation*, [[hep-ph/0511320](#)].
- [40] S. P. Martin and M. T. Vaughn, *Two loop renormalization group equations for soft supersymmetry breaking couplings*, *Phys. Rev.* **D50** (1994) 2282 [Erratum *ibid.* **D78** (2008) 039903] [[hep-ph/9311340](#)].