Modeling Task Systems Using Parameterized Partial Orders

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***Abstract*—Inspired by work on model-based design of printers, the notion of a parametrized partial order (PPO) was introduced recently. PPOs are a simple extension of partial orders, expressive enough to compactly represent large task graphs with repetitive behavior. We present a translation of the PPO subclass to timed automata and prove that the transition system induced by the Uppaal models is isomorphic to the configuration structure of the original PPO. Moreover, we report on a series of experiments which demonstrates that the resulting Uppaal models are more tractable than handcrafted models of the same systems used in earlier case studies.**

I. Introduction

The complexity of today's embedded systems and their development trajectories is increasing rapidly. At the same time, development teams are expected to produce high-quality and cost-effective products, while meeting stringent time-to-market constraints. A common challenge during development is the need to explore extremely large design spaces, involving multiple metrics of interest (timing, resource usage, energy usage, or cost). The number of design parameters (number and type of processing cores, sizes and organization of memories, interconnect, scheduling and arbitration policies) is typically very large. Moreover, the relation between parameter settings and design choices on the one hand and metrics of interest on the other hand is often difficult to determine. Given these observations, embedded-system design trajectories require a systematic approach, that should be automated as far as possible. To achieve high-quality results, design process and tooling need to be model-driven.

Many methods and tools for design-space exploration (DSE) of embedded systems follow the Y-chart pattern [1], [2]. This pattern is based on the observation that embedded systems development typically involves the co-development of a set of applications, a platform, and the mapping of the applications onto the platform. In the Y-chart pattern, specification of applications, platforms and mappings are separated. This allows independent evaluation of various alternatives of one of these system aspects while fixing the others. For example, various platform and mapping options are often investigated for a fixed (set of) application(s). Diagnostic information is used to, automatically or manually, improve application, platform, and/or mapping.

Applications are typically described in terms of task graphs representing partially ordered sets of tasks. In practice, we frequently see that certain tasks need to be executed repetitively, for a finite number of times, and that there exists a hierarchical relationship between tasks. For instance, a manufacturing order of a beer brewery consists of several pallets, containing several crates, each containing several bottles of beer. Another example concerns a wafer scanner manufacturing system from the semiconductor industry. Wafers are produced in batches (lots). A wafer scanner projects a mask on a wafer, using light. Eventually, the projected masks result in Integrated Circuits (ICs). On one wafer, multiple ICs and types of ICs are manufactured. Multiple types of ICs involve multiple masks, and multiple masks are placed on a reticle. As a final example, we mention a copier machine, which has to process a certain number of copies of a file, which in turn consists of a certain number of pages. Due to the nested, repetitive behavior, task graphs tend to become very large and no longer practical for specification and analysis of application behavior. Following [3], [4], we argue that repetitive task structure of applications plays an important role in embedded systems design, and needs to be addressed in methods for specifying and reasoning about such systems. Repetitive execution of tasks leads to finite repetitive patterns in schedules. In practice, execution of the first few instances and last few instances of a task differ slightly from the rest. This is a large difference with unlimited repetitive ('periodic') behavior, which has received much attention in the scheduling literature.

Within concurrency theory, several semantic models have been proposed that are based on partial ordering of events such as Mazurkiewicz [5] traces, pomsets (partially-ordered multisets) [6], and event structures [7]), but these models do not incorporate an explicit notion of repetitive events. Partial orderings of events with repetition can be defined using Colored Petri Nets [8], [9], but this is an extremely rich and expressive formalism, which may be considered too complicated for the task at hand.

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The Octopus project has developed a Design-Space Exploration (DSE) toolset [10] that aims to leverage existing modeling, analysis, and DSE tools to support model-driven DSE for embedded systems [11]. Recently, inspired by work on model-based design of printers, the Octopus project has introduced the notion of parametrized partial orders [12]. PPOs are a simple extension of partial orders, but expressive enough to compactly represent large task graphs with repetitive behavior.

The Octopus DSE toolset is centered on an intermediate representation, DSEIR (Design-Space Exploration Intermediate Representation), to capture design alternatives. DSEIR models can be exported to various analysis tools. This facilitates reuse of models across tools and provides model consistency between analyses. The use of an intermediate representation also supports domain-specific abstractions and reuse of tools across application domains. The current version of the Octopus DSE toolset integrates CPN Tools [8], [9] for stochastic simulation of timed systems, SDF3 [13] for worst-case throughput calculation, and Uppaal [14] for model checking and schedule optimization. The toolset translates high-level PPO models of an application into the intermediate DSEIR representation, which in turn can be translated into the input formats of CPN Tools and Uppaal. A translation of PPOs to CPN Tools has recently been described in [12]. In this paper, we define a restricted version of PPOs that is more amenable to model checking. Moreover, we give a translation into timed automata, the semantic model underlying Uppaal.

Uppaal [14] is a model checker for networks of timed automata [15]. It has been successfully used in many domains, e.g. for finding optimal solutions for scheduling problems [16], performance analysis of real-time distributed systems [17], [18], protocol verification [19] and controller synthesis [20]. Within the Octopus project, we aim at harnessing the verification power of Uppaal for DSE of embedded systems. We have applied Uppaal for DSE of industrial printer designs, in particular for computing and optimizing schedules, latencies, and controller strategies [21], [22], [23]. Although these case studies demonstrate that Uppaal is able to handle industrial sized designs, the tool is really pushed to its limits. Therefore, it is crucial to have a translation from PPOs to Uppaal that is maximally efficient. By unfolding a PPO into a task graph and introducing a separate automaton for each task in the unfolding, we obtain a general translation of PPOs to Uppaal. However, especially when we have many repetitive events (e.g. a print job with 300 pages) the translation becomes intractable. Based on the observation that in practice the PPOs often contain tasks that are not auto-concurrent and precedence relations between task instances obey certain monotonicity conditions, we define a subclass of PPOs that allows a much more efficient translation. This brings us to the two main results of this paper: (a) a definition of a PPO subclass and its translation to Uppaal together with a correctness proof (the transition system induced by the Uppaal model is isomorphic to the configuration structure of the PPO), and (b) a series of experiments which demonstrates that Uppaal models obtained through this translation are in fact more tractable than handcrafted models of the same systems used in [21]. Our results boost the verification power of the Octopus toolset but, due to the omnipresence of finite repetitive tasks in embedded systems design, their applicability is much broader.

The structure of this paper is as follows. The next section recalls some preliminary definitions regarding labeled transition systems, the underlying notion used throughout the paper. Section III defines PPOs and their semantics, and the translation of a subset of PPOs into networks of timed automata together with a proof of its correctness. Section IV explains the timed automata models generated on this theory. Section V presents performance evaluation results of models generated by comparing them with handcrafted Uppaal models presented before in our papers[1]. Concluding remarks and future work follow in Section VI.

II. Preliminaries

We use $\mathbb{R}_{\geq 0}$ and $\mathbb{R}_{>0}$ to denote the sets of nonnegative and positive real numbers, respectively, and $\mathbb{N}$ to denote the set of natural numbers.

If X and Y are sets then we write $X \hookrightarrow Y$ for the set of partial functions from X to Y. Given a partial function $f \in X \hookrightarrow Y$, we write $f(x)\downarrow$ if $f(x)$ is defined, and $f(x)\uparrow$ if $f(x)$ is undefined, for $x \in X$.

A *labeled transition system (LTS)* is a tuple $\mathcal{L} = (S, s_0, \Sigma, \rightarrow)$, where:

- S is a set of states,
- $s_0 \in S$ is an initial state,
- Σ is a set of action labels, and
- $\rightarrow \subseteq S \times \Sigma \times S$ is a transition relation.

We write $s \xrightarrow{a} s'$ iff $(s, a, s') \in \rightarrow$ and $s \rightarrow s'$ if there exists an action $a \in \Sigma$ such that $s \xrightarrow{a} s'$. A *path* of $\mathcal{L}$ is a sequence of states $\pi = s_0 s_1 \cdots s_n$ such that, for all $0 \leq i < n$, $s_i \rightarrow s_{i+1}$. In this case we say π is a path from s_0 to s_n. A state $s \in S$ is *reachable* in $\mathcal{L}$ if there exists a path from s_0 to s.

Two labeled transition systems $\mathcal{L}_1 = (S_1, s_0^1, \Sigma_1, \rightarrow_1)$ and $\mathcal{L}_2 = (S_2, s_0^2, \Sigma_2, \rightarrow_2)$ are *isomorphic* if $\Sigma_1 = \Sigma_2$ and there exists a bijective function $f : S_1 \rightarrow S_2$ such that:

- $f(s_0^1) = s_0^2$ and
- $s \xrightarrow{a}_1 s' \Leftrightarrow f(s) \xrightarrow{a}_2 f(s')$, for all $s, s' \in S_1$, $a \in \Sigma_1$.

Given an LTS $\mathcal{L} = (S, s_0, \Sigma, \rightarrow)$, $\mathsf{reach}(\mathcal{L}) = (S', s_0, \Sigma, \rightarrow')$ is the LTS with S' equal to the set of reachable states of $\mathcal{L}$ and $\rightarrow' = \{(s, a, s') \mid s, s' \in S' \wedge s \xrightarrow{a} s'\}$.

III. Parameterized Partial Orders

A *parametrized partial order (PPO)* is a partial order that comes equipped with some extra structure to capture repetitive behavior. In [12], a PPO is defined at task level and assumes a precedence relation between tasks. In this paper, we view a PPO from a different angle where tasks are decomposed into events and a PPO imposes a partial order relation at event level. This perspective allows us to introduce a subclass

[1] The models generated for Section V and all the proofs are available at `http://www.mbsd.cs.ru.nl/publications/papers/fvaan/HIV11.`

of PPOs that can be efficiently translated into networks of automata, and later in this section we establish the correctness of this translation.

A. Definition of PPOs

Tasks in a PPO may be executed repeatedly: each task has a collection of parameters and each valuation of these parameters defines a task instance. The events in a PPO are structured and correspond to either the start or the end of a task instance.

Formally, we assume a universe $\mathcal{P}$ of typed variables called *parameters*. A *valuation* of a set $P \subseteq \mathcal{P}$ of parameters is a function that maps each parameter in P to an element of its domain. We assume that the domain of each parameter is a nonempty set. We write $V(P)$ for the set of valuations of variables in P.

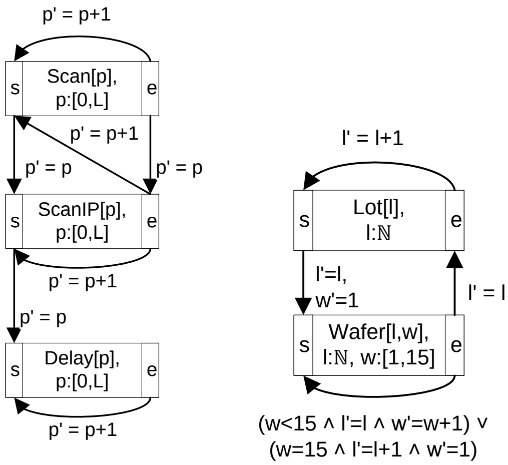
A *parameterized partial order (PPO)* is a tuple $\mathcal{A} = (\mathcal{T}, \mathcal{M}, E, U)$ where

- $\mathcal{T}$ is a finite set of *tasks*. Let $\mathcal{E} = \{s, e\} \times \mathcal{T}$. Projection functions $\mathsf{task} : \mathcal{E} \to \mathcal{T}$ and $\mathsf{type} : \mathcal{E} \to \{\mathsf{s}, \mathsf{e}\}$ are given by $\mathsf{task}((t, T)) = T$ and $\mathsf{type}((t, T)) = t$, and embeddings $\mathsf{start} : \mathcal{T} \to \mathcal{E}$ and $\mathsf{end} : \mathcal{T} \to \mathcal{E}$ are given by $\mathsf{start}(T) = (s, T)$ and $\mathsf{end}(T) = (e, T)$, with $t \in \{s, e\}$, and $T \in \mathcal{T}$.
- $\mathcal{M}$ is a function that assigns to each task T a finite set of parameters in $\mathcal{P}$; we write $V(T)$ as a shorthand for $V(\mathcal{M}(T))$.
- $E \subseteq \mathcal{E} \times \mathcal{E}$ is a set of *edges*. We require, for each $T \in \mathcal{T}$, $(\mathsf{start}(T), \mathsf{end}(T)) \in E$.
- For each edge $p = (A, B) \in E$, $U(p) : V(\mathsf{task}(A)) \hookrightarrow V(\mathsf{task}(B))$ is a *precedence function*. We write $A \overset{u}{\to} B$ if $(A, B) \in E$ and $U(A, B) = u$. We require that the start of a task instance precedes the end of that instance, that is, for each task $T \in \mathcal{T}$ and valuation $v \in V(T)$, $U((\mathsf{start}(T), \mathsf{end}(T)))(v) = v$.

Below, we present two examples that illustrate how PPOs can be used to model scheduling applications.

Example 1 (Printer): Figure 1a depicts a part of an application encountered in the printer domain (see [21]). There are three tasks: Scan, ScanIP and Delay, represented by rectangles. The corresponding start and end event types are indicated by subrectangles inscribed with s and e. Edges show the dependencies between event types (the edges from start to corresponding end are not shown). All three tasks have one parameter: p of type $[0, \ldots, L]$ representing the number of the current page processed. The constant $L \in \mathbb{N}$ is a bound for the parameter p. A precedence function $A \overset{u}{\to} B$ is represented by a predicate that may contain both the parameters of $\mathsf{task}(A)$ and primed versions of the parameters of $\mathsf{task}(B)$. For instance, the predicate $\mathsf{p}' = \mathsf{p} + 1$ on the edge from ScanIP to Scan represents the precedence function that maps a valuation v of the ScanIP parameters to the unique valuation v' of the Scan parameters that satisfies $v'(\mathsf{p}) = v(\mathsf{p}) + 1$.

An instance v of ScanIP may start as soon as its corresponding v instance of Scan has started. These task instances of Scan and ScanIP may then proceed in parallel. However, the next instance of Scan may only start after the current instances of both Scan and ScanIP have ended. Between the ScanIP and Delay tasks, there is a sequential dependency: after the occurrence of the start event in the ScanIP task, the start event of the corresponding Delay task may occur.



(a) Printer

(b) Wafer production

Fig. 1: PPO representation

Example 2 (Wafer production): The PPO displayed in Figure 1b describes the production of an infinite series of lots, where each lot is composed of 15 wafers. This example is inspired by [4]. After the start of each lot, 15 wafer tasks are executed in sequence, followed by the end of the lot.

B. From PPOs to Configuration Structures

The semantics of a PPO can be described in terms of a labeled transition system, referred to as the *configuration structure* of the PPO (see [7], [24]). The states of a configuration structure are *configurations*, finite sets of events that have already occurred. Each transition marks the occurrence of a single new event for which all the immediate predecessors have occurred.

Formally, an *event* is a pair (A, v) where A is an event type and $v \in V(\mathsf{task}(A))$ is a valuation of its task parameters. We write $\mathsf{ev\_type}((A, v)) = A$ and $\mathsf{task}((A, v)) = \mathsf{task}(A)$. Also, we write $\mathsf{ev}(\mathcal{A})$ for the set of events of a PPO $\mathcal{A}$. We call event (B, w) an *immediate predecessor* of event (A, v), notation $(B, w) \mapsto (A, v)$, if $(B, A) \in E \wedge U(B, A)(w) = v$.

Let $C \subset \mathsf{ev}(\mathcal{A})$ and $\alpha \in \mathsf{ev}(\mathcal{A})$ with $\alpha \notin C$. We say that C *enables* α, and write $C \vdash \alpha$, if all immediate predecessors of α are in C.

Let $\mathcal{A}$ be a PPO. The set $\mathsf{conf}(\mathcal{A})$ of *configurations* of $\mathcal{A}$ is the smallest subset of the power set $\wp(\mathsf{ev}(\mathcal{A}))$ of events of $\mathcal{A}$ such that:

1) $\emptyset \in \mathsf{conf}(\mathcal{A})$,
2) if $C \in \mathsf{conf}(\mathcal{A})$, and $C \vdash \alpha$ then $C \cup \{\alpha\} \in \mathsf{conf}(\mathcal{A})$.

The *configuration structure* of $\mathcal{A}$ is the LTS $\mathcal{C}(\mathcal{A}) = (\mathsf{conf}(\mathcal{A}), \emptyset, \mathcal{E}, \rightsquigarrow)$, where $(C, A, C \cup \{\alpha\}) \in \rightsquigarrow$ iff $C \in \mathsf{conf}(\mathcal{A})$, $\mathsf{ev\_type}(\alpha) = A$ and $C \vdash \alpha$. We write $C \overset{A}{\rightsquigarrow} C'$ if

$(C, A, C') \in \rightsquigarrow$. Also, we sometimes write $C \overset{\alpha}{\rightsquigarrow} C'$ to denote that $C \overset{\mathsf{ev\_type}(\alpha)}{\rightsquigarrow} C'$ and $C' = C \cup \{\alpha\}$.

The above definition implies that each configuration $C \in \mathsf{conf}(\mathcal{A})$ has a *securing*, that is, a sequence $\alpha_1, \ldots, \alpha_n$ of events such that $C = \{\alpha_1, \ldots, \alpha_n\}$ and, for each $1 \leq i \leq n$, $\{\alpha_j \mid j < i\} \in \mathsf{conf}(A)$ and $\{\alpha_j \mid j < i\} \vdash \alpha_i$.

In a PPO there are no conflicts between events: it is not possible that the occurrence of one event disables the occurrence of another event. In fact, it is easy to prove that the set of configurations of a PPO is closed under union: if $C \in \mathsf{conf}(\mathcal{A})$ and $C' \in \mathsf{conf}(\mathcal{A})$ then $C \cup C' \in \mathsf{conf}(\mathcal{A})$. We call an event *reachable* if it occurs in some configuration, and write $\mathsf{rev}(\mathcal{A})$ for the set of reachable events of $\mathcal{A}$. Note that, since in a PPO we allow cyclic predecessor relations, it may occur that some (or even all) events are not reachable. If α and β are in $\mathsf{rev}(\mathcal{A})$, we write $\alpha \leq_{\mathcal{A}} \beta$, if for each configuration $C \in \mathsf{conf}(\mathcal{A})$, $\beta \in C$ implies $\alpha \in C$. The technical lemma belowstates that the $\leq_{\mathcal{A}}$ contains the immediate predecessor relation:

Lemma 1: Let $\mathcal{A}$ be a PPO with events α and β such that $\alpha \mapsto \beta$. Then $\beta \in \mathsf{rev}(\mathcal{A})$ implies $\alpha \in \mathsf{rev}(\mathcal{A})$ and $\alpha \leq_{\mathcal{A}} \beta$.

Proof: If $\beta \in \mathsf{rev}(\mathcal{A})$, then there is a configuration $C \in \mathsf{conf}(\mathcal{A})$ that contains β. Furthermore, this configuration has a securing, that is, a sequence $\alpha_1, ..., \alpha_n$ such that $C = \{\alpha_1, ..., \alpha_n\}$ and there is a configuration C_β that we can construct with some of the events of C that enables β. Since $C_\beta \vdash \beta$, C_β contains all immediate predecessors of β. Let α be an immediate predecessor of β. Since $\alpha \in C_\beta$, then $\alpha \in \mathsf{rev}(\mathcal{A})$. Because $C_\beta \subset C$, then $\alpha \in C$, namely in any configuration that contains β, therefore $\alpha \leq_{\mathcal{A}} \beta$. ■

The following lemmastates that a parametrized partial order (PPO) induces a partial ordering relation on its (reachable) events.

Lemma 2: Let $\mathcal{A}$ be a PPO, then $\leq_{\mathcal{A}}$ is a partial order on $\mathsf{rev}(\mathcal{A})$.

Proof:

1) (Reflexivity). We need to prove that $\alpha \leq_{\mathcal{A}} \alpha$ is true, for any $\alpha \in \mathsf{rev}(\mathcal{A})$. This is obviously true since for each configuration C that contains α, the event α from the left side of the $\leq_{\mathcal{A}}$ relation is also in C.
2) (Antisymmetry). Let $\alpha, \beta \in \mathsf{rev}(\mathcal{A})$. We should prove that if $\alpha \leq_{\mathcal{A}} \beta$, and $\beta \leq_{\mathcal{A}} \alpha \implies \alpha = \beta$. Assume that $\alpha \neq \beta$. If $\alpha \leq_{\mathcal{A}} \beta$, there exists a securing that contains a configuration C_β, with $\alpha \in C_\beta$ that enables β. Further, if $\beta \leq_{\mathcal{A}} \alpha$, there exists a securing that contains a configuration C_α, with $\beta \in C_\alpha$ that enables α and $C_\beta \subset C_\alpha$. This implies that $\alpha \in C_\alpha$ and $C_\alpha \vdash \alpha$, which is impossible.
3) (Transitivity). Let $\alpha, \beta, \gamma \in \mathsf{rev}(\mathcal{A})$. We should prove that if $\alpha \leq_{\mathcal{A}} \beta$, and $\beta \leq_{\mathcal{A}} \gamma \implies \alpha \leq_{\mathcal{A}} \gamma$. Assuming that $\alpha \leq_{\mathcal{A}} \beta$, this implies that any configuration that contains β, also contains α. Further, if $\beta \leq_{\mathcal{A}} \gamma$, any configuration that contains γ also contains β. Since any configuration that contains β, also contains α, this allows us to conclude that any configuration that contains γ also contains α, and therefore $\alpha \leq_{\mathcal{A}} \gamma$.

■

C. Restricted PPOs

We explore the behavior of PPOs using the Uppaal model checker, and for this we need to translate PPOs to the input language of Uppaal. Here we describe a translation of a subclass of PPOs in which no two instances of a task can run concurrently. It is possible to translate arbitrary PPOs to Uppaal (provided the parameter domains are finite) but this translation leads to networks of automata that are much harder to analyze.

We call a PPO $\mathcal{A}$ *restricted* if it satisfies the following five conditions, for all tasks T and T', for all precedence functions $A \overset{u}{\rightarrow} B$ with $\mathsf{task}(A) = T$ and $\mathsf{task}(B) = T'$, and for all valuations $v, w \in V(T)$:

- **C0**: The only edges between events of the same task are the one from the start event to the end event, and the one from the end event to the start event:
$$\mathsf{task}(A) = \mathsf{task}(B) \quad \Rightarrow \quad ((A, B) \in \mathcal{E} \Leftrightarrow A \neq B)$$
We write $\mathsf{next}(T)$ for the function $U((\mathsf{end}(T), \mathsf{start}(T))$, and let $<_T$ be the least transitive relation on valuations in $V(T)$ satisfying $v <_T \mathsf{next}(T)(v)$. Write $v \leq_T w$ iff $v <_T w$ or $v = w$.
- **C1**: There is exactly one valuation of the parameters of T that does not appear in the range of $\mathsf{next}(T)$. This valuation is referred to as the *initial valuation* of T, and is written v_T^0.
- **C2**: $\mathsf{next}(T)$ is injective
- **C3**: u is only defined for reachable valuations:
$$u(v) \downarrow \quad \Rightarrow \quad v_T^0 \leq_T v$$
- **C4**: u is *monotonic*:
$$v \leq_T w \wedge u(w) \downarrow \quad \Rightarrow \quad u(v) \downarrow \wedge\, u(v) \leq_{T'} u(w)$$

Axioms **C0**, **C1** and **C2** impose precedence restrictions between event instances of the same task that exclude auto-concurrency. Axiom **C0** implies that we have an edge from the end event type of a task to the corresponding start event type. Axiom **C1** implies that, for each task, there is only one event that does not depend on some other event of the same task: necessarily this is going to be the first event of the task that will occur. Axiom **C2** implies that each event from a task, except the initial one, has a unique immediate predecessor event that belongs to the same task. Axioms **C0-C2** still allow cyclic precedence edges between events of the same task, but axiom **C3** implies that u is not defined for such "ghost events". Axiom **C4**, finally, states that a precedence function that links events of different tasks is monotonic w.r.t the event ordering within tasks. The reader may check that the PPOs of Examples 1 and 2 are restricted.

Lemma 3: Let $\mathcal{A}$ be a restricted PPO with task T and valuation v. Then

1) $(\mathsf{end}(T), v) \in \mathsf{rev}(\mathcal{A})$ implies $(\mathsf{start}(T), v) \in \mathsf{rev}(\mathcal{A})$ and $(\mathsf{start}(T), v) \leq_{\mathcal{A}} (\mathsf{end}(T), v)$.

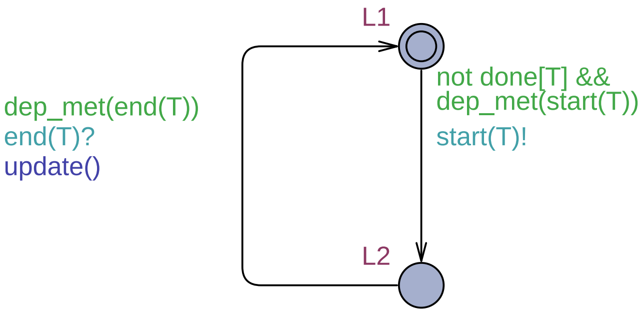


Fig. 2: Automaton for task T

2) $(\mathsf{start}(T), \mathsf{next}(T)(v)) \in \mathsf{rev}(\mathcal{A})$ implies $(\mathsf{end}(T), v) \in \mathsf{rev}(\mathcal{A})$ and $(\mathsf{end}(T), v) \leq_{\mathcal{A}} (\mathsf{start}(T), \mathsf{next}(T)(v))$.
3) $\leq_{\mathcal{A}}$ is a total ordering on the set $\{\alpha \in \mathsf{rev}(\mathcal{A}) \mid \mathsf{task}(\alpha) = T\}$ of reachable events of T.

Proof: (1) and (2) are immediate consequences of Lemma 1.
For (3), first observe that $\leq_{\mathcal{A}}$ is a partial order on $\mathsf{rev}(\mathcal{A})$ by Lemma 2. Hence it is also a partial order on the subset of reachable events of T. Let $\alpha, \beta \in \mathsf{rev}(\mathcal{A})$ with $\mathsf{task}(\alpha) = T$ and $\mathsf{task}(\beta) = T$. It suffices to prove that either $\alpha \leq_{\mathcal{A}} \beta$ or $\beta \leq_{\mathcal{A}} \alpha$. Assuming that $\alpha = (t_\alpha, v_\alpha), \beta = (t_\beta, v_\beta)$, with $t_\alpha, t_\beta \in \{\mathsf{end}(T), \mathsf{start}(T)\}$. If $v_\alpha = v_\beta$, then by applying the first case of this lemma, if follows that $\alpha \leq_{\mathcal{A}} \beta$ or $\beta \leq_{\mathcal{A}} \alpha$. If not, without loss of generality, we assume $v_\alpha <_T v_\beta$. Then using the first two cases of this lemma, if follows that $(t_\alpha, v_\alpha) \leq_{\mathcal{A}} (\mathsf{start}(T), \mathsf{next}(T)(v_\alpha)) \leq_{\mathcal{A}} \ldots \leq_{\mathcal{A}} (t_\beta, v_\beta)$, which by transitivity implies that $\alpha \leq_{\mathcal{A}} \beta$. ∎

D. From Restricted PPOs to Networks of Automata

We will show how each restricted PPO can be translated into a Uppaal-style parallel composition of a number of automata in such a way that (the reachable part of) the LTS induced by the composition of these automata is isomorphic to the configuration structure of the PPO. We refer the reader to [14] for an introduction to Uppaal.

Let $\mathcal{A}$ be a PPO as above. We define $\mathcal{N}(\mathcal{A})$ to be the LTS induced by the parallel composition of the Uppaal template displayed in Figure 2, for each task $T \in \mathcal{T}$. Below we explain the various predicates and functions occurring in Figure 2. The composed system $\mathcal{N}(\mathcal{A})$ has the following set of global shared variables:

$$\{T.\mathsf{p}, \mathsf{loc}[T], \mathsf{done}[T] \mid T \in \mathcal{T} \wedge \mathsf{p} \in \mathcal{M}(T)\}.$$

Variable $\mathsf{loc}[T]$ records the current location of the task automaton for T, which can be either L1 or L2. Boolean variable $\mathsf{done}[T]$ records whether the last event of T has been executed. Since different tasks may use the same parameter names, we make a copy $T.\mathsf{p}$ of each parameter $\mathsf{p} \in \mathcal{M}(T)$. As long as task T has not yet been completed, variable $T.\mathsf{p}$ gives the value of p in the next event of T that will occur. Variable $\mathsf{loc}[T]$ is initialized to L1, variable $\mathsf{done}[T]$ is initialized to false, and variable $T.\mathsf{p}$ is initialized to $v_T^0(\mathsf{p})$, for each parameter $\mathsf{p} \in \mathcal{M}(T)$.

For a given state of the automaton for task T, let function $\mathsf{val}(T)$ return the current valuation of the parameters of task T. For each event type A with $\mathsf{task}(A) = T$, function $\mathsf{done}(A)$ returns true iff the last event of A has occurred:

$$\begin{aligned} \mathsf{done}(A) \quad = \quad & \mathsf{done}[T] \vee (\mathsf{loc}[T] = \mathsf{L2} \wedge \\ & \mathsf{type}(A) = \mathsf{s} \wedge \mathsf{next}(T)(\mathsf{val}(T)) \uparrow) \end{aligned}$$

If the last event of A has not occurred, function $\mathsf{next}(A)$ gives the valuation of the parameters for the next event of A:

$$\mathsf{next}(A) = \begin{cases} \mathsf{next}(T)(\mathsf{val}(T)), & \text{if } \mathsf{loc}[T] = \mathsf{L2} \wedge \mathsf{type}(A) = \mathsf{s} \\ \mathsf{val}(T) & \text{, otherwise} \end{cases}$$

Suppose that the last event of type A has not occurred, then in order to decide whether the next event of A may occur, we check for each incoming precedence edge $B \overset{u}{\to} A$ whether the dependency induced by that edge has been met:

$$\begin{aligned} \mathsf{dep\_met}(A) \quad = \quad & \forall B, u : B \overset{u}{\to} A \wedge \mathsf{task}(B) \neq \mathsf{task}(A) \implies \\ & \mathsf{dep\_met}(B, u, A) \end{aligned}$$

Note that the task automaton already takes care of the dependencies induced by precedence functions between pairs of start and end events of T. In order to decide whether the dependencies induced by $B \overset{u}{\to} A$ are met, we first check if $\mathsf{done}(B)$ evaluates to true. If so then all events of B have occurred and hence all dependencies induced by $B \overset{u}{\to} A$ have been met. Next we check whether $u(\mathsf{next}(B))$ is defined. If not then, by monotonicity, all dependencies induced by $B \overset{u}{\to} A$ have been met. Finally, we check whether $\mathsf{next}(A)$ precedes $u(\mathsf{next}(B))$. If so, then for any immediate predecessor of $\mathsf{next}(A)$, that is, for any parameter valuation v of B with $u(v) = \mathsf{next}(A)$, monotonicity implies $v < \mathsf{next}(B)$. Formally,

$$\begin{aligned} \mathsf{dep\_met}(B, u, A) \quad = \quad & \mathsf{done}(B) \vee u(\mathsf{next}(B)) \uparrow \\ & \vee\ \mathsf{next}(A) <_T u(\mathsf{next}(B)) \end{aligned}$$

Finally, function $\mathsf{update}()$ sets $\mathsf{done}[T]$ to true if the last event for task T has occurred, and otherwise updates the parameters of T according to function $\mathsf{next}(T)$.

Lemma 4: For all reachable states s of $\mathcal{N}(\mathcal{A})$ and for all tasks $T \in \mathcal{T}$, the following invariant properties hold:

1) $v_T^0 \leq_T s.\mathsf{val}(T)$
2) $s.\mathsf{done}[T] \Rightarrow \mathsf{next}(s.\mathsf{val}(T)) \uparrow$
3) $s.\mathsf{done}[T] \Rightarrow s.\mathsf{loc}[T] = \mathsf{L1}$

Proof: Straightforward by induction on the length of the shortest path leading to s. ∎

Theorem 1: Let $\mathcal{A}$ be a PPO. Then $\mathcal{C}(\mathcal{A})$ and $\mathsf{reach}(\mathcal{N}(\mathcal{A}))$ are isomorphic.

Proof: Let $\mathcal{N}(\mathcal{A}) = (S, s_0, \mathcal{E}, \to)$. If $s \in S$ is a state and e is an expression containing variables of $\mathcal{N}(\mathcal{A})$, then we write $s.e$ for the result of evaluating expression e in state s. For each event type $A \in \mathcal{E}$, we define a function $\Re_A : S \to 2^{\mathsf{ev}(\mathcal{A})}$ that associates to each state of $\mathcal{N}(\mathcal{A})$ a set of events of type A. Intuitively, this is the set of events of type A that have occurred before reaching state s. Suppose $\mathsf{task}(\mathcal{A}) = T$. Then

$$\begin{array}{rcl} \Re_A(s) & = & \textbf{if } s.\mathsf{done}(A) \textbf{ then} \\ & & \qquad \{(A,v) \in \mathsf{ev}(\mathcal{A}) \mid v \leq_T s.\mathsf{val}(T)\} \\ & & \textbf{else} \\ & & \qquad \{(A,v) \in \mathsf{ev}(\mathcal{A}) \mid v <_T s.\mathsf{next}(A)\} \\ & & \textbf{fi} \end{array}$$

Let function $\Re : S \to 2^{\mathsf{ev}(\mathcal{A})}$ be defined by:

$$\Re(s) \quad = \quad \bigcup_{A \in \mathcal{E}} \Re_A(s)$$

We will prove that $\Re$ is an isomorphism from $\mathsf{reach}(\mathcal{N}(\mathcal{A}))$ to $\mathcal{C}(\mathcal{A})$.

Claim 1. $\Re(s_0) = \emptyset$.

Proof: Let A be an event type. Let $\mathsf{task}(A) = T$. By definition of s_0 we have $s_0.\mathsf{done}(A) = \mathsf{false}$ and $s_0.\mathsf{next}(A) = v_T^0$. Hence, by definition of $\Re_A$, $\Re_A(s_0) = \{(A,v) \mid v <_T v_T^0\}$. But since, by condition **C1**, v_T^0 does not appear in the range of $\mathsf{next}(T)$, there exists no v such that $v <_T v_T^0$. Hence $\Re_A(s_0) = \emptyset$. Since A was chosen arbitrarily, it follows that also $\Re(s_0) = \emptyset$. ■

Claim 2. If s is a reachable state and $s \xrightarrow{A} s'$ then $\Re(s) \vdash (A, s.\mathsf{val}(T))$.

Proof: Let $v = s.\mathsf{val}(T)$. Assume that $s \xrightarrow{A} s'$ and assume that (B,w) is an immediate predecessor of (A,v). It suffices to prove that $(B,w) \in \Re_B(s)$.

If $\mathsf{task}(B) = \mathsf{task}(A)$ and $A = \mathsf{start}(T)$ then, by **C0**, $B = \mathsf{end}(T)$ and $\mathsf{next}(T)(w) = v$. Since $s \xrightarrow{A} s'$, $s.\mathsf{done}[T] = \mathsf{false}$. This implies $s.\mathsf{done}(B) = \mathsf{false}$. Also $s.\mathsf{next}(B) = s.\mathsf{val}(T) = v$. We infer that

$$\Re_B(s) \quad = \quad \{(B,x) \in \mathsf{ev}(\mathcal{A}) \mid x <_T v\}$$

Since $w <_T v$ it follows that $(B,w) \in \Re_B(s)$, as required.

If $\mathsf{task}(B) = \mathsf{task}(A)$ and $A = \mathsf{end}(T)$ then $B = \mathsf{start}(T)$ and $w = v$. If $s.\mathsf{done}(B)$ holds then $(B,w) \in \Re_B(s)$ and we are done. If $s.\mathsf{done}(B)$ does not hold then $\mathsf{next}(T)(\mathsf{val}(T))) \downarrow$ and $\mathsf{next}(B) = \mathsf{next}(T)(\mathsf{val}(T)))$. It follows that $(B,w) \in \Re_B(s)$.

We may therefore assume that $\mathsf{task}(B) \neq \mathsf{task}(A)$. Let $U(B,A) = u$ and $\mathsf{task}(B) = T'$. Then $u(w) = v$. Since $s \xrightarrow{A} s'$, $s.\mathsf{dep\_met}(B,u,A)$ holds. This means that one of the following three cases applies:

- $s.\mathsf{done}(B)$.
  Using the first invariant of Lemma 4, we infer $v_{T'}^0 \leq_{T'} s.\mathsf{val}(T')$. Using the second invariant of Lemma 4, we infer that $\mathsf{next}(T')(s.\mathsf{val}(T')) \uparrow$. Condition **C3** implies that $v_{T'}^0 \leq_{T'} w$. It follows that $w \leq_{T'} s.\mathsf{val}(T')$. Hence $(B,w) \in \Re_B(s)$, as required.
- $s.\mathsf{done}(B) = \mathsf{false}$ and $u(s.\mathsf{next}(B)) \uparrow$.
  By monotonicity imposed by condition **C4**, we do not have $s.\mathsf{next}(B) <_{T'} w$. Condition **C3** implies $v_{T'}^0 \leq_{T'} w$, and Lemma 4 implies $v_{T'}^0 \leq_{T'} s.\mathsf{next}(B)$. Hence $w <_{T'} s.\mathsf{next}(B)$ and thus $(B,w) \in \Re_B(s)$.
- $s.\mathsf{next}(A) <_T u(s.\mathsf{next}(B))$.
  Since $s \xrightarrow{A} s'$, $s.\mathsf{next}(A) = s.\mathsf{val}(T) = v$. As in the previous case, we use conditions **C3**, **C4** and Lemma 4 to argue that $w <_{T'} s.\mathsf{next}(B)$, and thus $(B,w) \in \Re_B(s)$.

■

Claim 3. If $s \xrightarrow{A} s'$ then $\Re(s') = \Re(s) \cup \{(A, s.\mathsf{val}(T))\}$.

Proof: Assume $s \xrightarrow{A} s'$. It is easy to check that for all event types B with $\mathsf{task}(B) \neq \mathsf{task}(A)$, $\Re_B(s') = \Re_B(s)$. Let $\overline{\_} : \mathcal{E} \to \mathcal{E}$ be the function given by $\overline{\mathsf{start}(T)} = \mathsf{end}(T)$ and $\overline{\mathsf{end}(T)} = \mathsf{start}(T)$, for all T. We claim that $\Re_A(s') = \Re_A(s) \cup \{(A, s.\mathsf{val}(T))\}$ and $\Re_{\overline{A}}(s') = \Re_{\overline{A}}(s)$. We consider four cases:

- $A = \mathsf{start}(T)$ and $\mathsf{next}(T)(s.\mathsf{val}(T)) \uparrow$.
  Since $s \xrightarrow{A} s'$, $s.\mathsf{next}(A) = s.\mathsf{val}(T)$ and $s.\mathsf{done}(A) = \mathsf{false}$. Hence
  $$\Re_A(s) \quad = \quad \{(A,v) \in \mathsf{ev}(\mathcal{A}) \mid v <_T s.\mathsf{val}(T)\}$$
  Since $s \xrightarrow{A} s'$, $s'.\mathsf{loc}[T] = \mathsf{L2}$ and $s'.\mathsf{val}(T) = s.\mathsf{val}(T)$. Thus $\mathsf{next}(T)(s'.\mathsf{val}(T)) \uparrow$ and $s'.\mathsf{done}(A)$. Hence
  $$\Re_A(s') \quad = \quad \{(A,v) \in \mathsf{ev}(\mathcal{A}) \mid v \leq_T s.\mathsf{val}(T)\}$$
  Thus $\Re_A(s') = \Re_A(s) \cup \{(A, s.\mathsf{val}(T))\}$. Since $s \xrightarrow{A} s'$, $s.\mathsf{done}(\mathsf{end}(T)) = \mathsf{false}$ and $s'.\mathsf{done}(\mathsf{end}(T)) = \mathsf{false}$. Moreover $s'.\mathsf{next}(\mathsf{end}(T)) = s.\mathsf{next}(\mathsf{end}(T)) = s.\mathsf{val}(T)$. Hence
  $$\begin{array}{rcl} \Re_{\overline{A}}(s') & = & \Re_{\overline{A}}(s) \\ & = & \{(\overline{A},v) \in \mathsf{ev}(\mathcal{A}) \mid v <_T s.\mathsf{val}(T)\} \end{array}$$
- $A = \mathsf{start}(T)$ and $\mathsf{next}(T)(s.\mathsf{val}(T)) \downarrow$.
  Since $s \xrightarrow{A} s'$, $s.\mathsf{next}(A) = s.\mathsf{val}(T)$ and $s.\mathsf{done}(A) = \mathsf{false}$. Hence
  $$\Re_A(s) \quad = \quad \{(A,v) \in \mathsf{ev}(\mathcal{A}) \mid v <_T s.\mathsf{val}(T)\}$$
  Since $s \xrightarrow{A} s'$, $s'.\mathsf{loc}[T] = \mathsf{L2}$ and $s'.\mathsf{val}(T) = s.\mathsf{val}(T)$. Thus $\mathsf{next}(T)(s'.\mathsf{val}(T)) \downarrow$, $s'.\mathsf{done}(A) = \mathsf{false}$, and $s'.\mathsf{next}(A) = \mathsf{next}(T)(s'.\mathsf{val}(T))$. Hence
  $$\Re_A(s') \quad = \quad \{(A,v) \in \mathsf{ev}(\mathcal{A}) \mid v <_T \mathsf{next}(T)(s.\mathsf{val}(T))\}$$
  By **C2**, $\Re_A(s') = \Re_A(s) \cup \{(A, s.\mathsf{val}(T))\}$. Since $s \xrightarrow{A} s'$, $s.\mathsf{done}(\mathsf{end}(T)) = \mathsf{false}$ and $s'.\mathsf{done}(\mathsf{end}(T)) = \mathsf{false}$. Moreover $s'.\mathsf{next}(\mathsf{end}(T)) = s.\mathsf{next}(\mathsf{end}(T)) = s.\mathsf{val}(T)$. Hence
  $$\begin{array}{rcl} \Re_{\overline{A}}(s') & = & \Re_{\overline{A}}(s) \\ & = & \{(\overline{A},v) \in \mathsf{ev}(\mathcal{A}) \mid v <_T s.\mathsf{val}(T)\} \end{array}$$
- $A = \mathsf{end}(T)$ and $\mathsf{next}(T)(s.\mathsf{val}(T)) \uparrow$.
  Since $s \xrightarrow{A} s'$, $\mathsf{done}(A) = \mathsf{false}$ and $s.\mathsf{next}(A) = s.\mathsf{val}(T)$. Hence
  $$\Re_A(s) \quad = \quad \{(A,v) \in \mathsf{ev}(\mathcal{A}) \mid v <_T s.\mathsf{val}(T)\}$$
  Moreover, $s'.\mathsf{done}[T]$, $s'.\mathsf{done}(A)$ and $s'.\mathsf{val}(T) = s.\mathsf{val}(T)$. Hence
  $$\Re_A(s') \quad = \quad \{(A,v) \in \mathsf{ev}(\mathcal{A}) \mid v \leq_T s.\mathsf{val}(T)\}$$

Thus $\mathfrak{R}_A(s') = \mathfrak{R}_A(s) \cup \{(A, s.\mathsf{val}(T))\}$. By the assumptions, $s.\mathsf{done}(\overline{A})$. We can also infer $s'.\mathsf{done}(\overline{A})$. Hence

$$\begin{aligned}\mathfrak{R}_{\overline{A}}(s') &= \mathfrak{R}_{\overline{A}}(s)\\ &= \{(\overline{A}, v) \in \mathsf{ev}(\mathcal{A}) \mid v \leq_T s.\mathsf{val}(T)\}\end{aligned}$$

- $A = \mathsf{end}(T)$ and $\mathsf{next}(T)(s.\mathsf{val}(T)) \downarrow$.
  Since $s \xrightarrow{A} s'$, $\mathsf{done}(A) = \mathsf{false}$ and $s.\mathsf{next}(A) = s.\mathsf{val}(T)$. Hence

  $$\mathfrak{R}_A(s) \;=\; \{(A, v) \in \mathsf{ev}(\mathcal{A}) \mid v <_T s.\mathsf{val}(T)\}$$

  Moreover, $s'.\mathsf{done}(A) = \mathsf{false}$, $s'.\mathsf{next}(A) = s'.\mathsf{val}(T)$ and $s'.\mathsf{val}(T) = \mathsf{next}(T)(s.\mathsf{val}(T))$. Hence

  $$\mathfrak{R}_A(s') \;=\; \{(A, v) \in \mathsf{ev}(\mathcal{A}) \mid v <_T \mathsf{next}(T)(s.\mathsf{val}(T))\}$$

  By **C2**, $\mathfrak{R}_A(s') = \mathfrak{R}_A(s) \cup \{(A, s.\mathsf{val}(T))\}$. By the assumptions, $s.\mathsf{done}(\overline{A}) = \mathsf{false}$ and $s'.\mathsf{done}(\overline{A}) = \mathsf{false}$. Moreover

  $$s.\mathsf{next}(\overline{A}) = \mathsf{next}(T)(s.\mathsf{val}(T)) = s'.\mathsf{val}(T) = s'.\mathsf{next}(\overline{A})$$

  This implies

  $$\mathfrak{R}_{\overline{A}}(s') \;=\; \mathfrak{R}_{\overline{A}}(s)$$

It follows that $\mathfrak{R}(s') = \mathfrak{R}(s) \cup \{(A, s.\mathsf{val}(T))\}$. ■

Claim 4. If s is a reachable state of $\mathcal{N}(\mathcal{A})$ then $\mathfrak{R}(s) \in \mathsf{conf}(\mathcal{A})$.

Proof: Straightforward, by induction on the length of the shortest path to s, using Claims 1-3. ■

Claim 5. If s, s' are reachable states of $\mathcal{N}(\mathcal{A})$ and $s \xrightarrow{A} s'$ then $\mathfrak{R}(s) \overset{A}{\rightsquigarrow} \mathfrak{R}(s')$.

Proof: Straightforward, by combining Claims 2, 3 and 4. ■

In order to prove that $\mathfrak{R}$ is bijective, we define an inverse function $\mathfrak{S}$ that maps configurations of $\mathcal{A}$ to states of $\mathcal{N}(\mathcal{A})$. Let C be a configuration and let T be a task. Write C_T for the subset of C of events of type T. We consider four cases:

1) If $C_T = \emptyset$ then variable $\mathsf{loc}[T]$ is set to $\mathsf{L1}$, variable $\mathsf{done}[T]$ is set to false, and variable $T.\mathsf{p}$ is set to $v_T^0(\mathsf{p})$, for each parameter $\mathsf{p} \in \mathcal{M}(T)$.
2) If $C_T \neq \emptyset$ and the unique maximal event of C_T (cf Lemma 3) is of the form $(\mathsf{start}(T), v)$, then variable $\mathsf{loc}[T]$ is set to $\mathsf{L2}$, variable $\mathsf{done}[T]$ is set to false, and variable $T.\mathsf{p}$ is set to $v(\mathsf{p})$, for each parameter $\mathsf{p} \in \mathcal{M}(T)$.
3) If $C_T \neq \emptyset$, the unique maximal event of C_T is of the form $(\mathsf{end}(T), v)$ and $\mathsf{next}(T)(v) \downarrow$, then variable $\mathsf{loc}[T]$ is set to $\mathsf{L1}$, variable $\mathsf{done}[T]$ is set to false, and variable $T.\mathsf{p}$ is set to $\mathsf{next}(T)(v)(\mathsf{p})$, for each parameter $\mathsf{p} \in \mathcal{M}(T)$.
4) If $C_T \neq \emptyset$, the unique maximal event of C_T is of the form $(\mathsf{end}(T), v)$ and $\mathsf{next}(T)(v) \uparrow$, then variable $\mathsf{loc}[T]$ is set to $\mathsf{L1}$, variable $\mathsf{done}[T]$ is set to true, and variable $T.\mathsf{p}$ is set to $v(\mathsf{p})$, for each parameter $\mathsf{p} \in \mathcal{M}(T)$.

The following claim directly implies that $\mathfrak{R}$ is injective.

Claim 6. For each reachable state of $\mathcal{N}(\mathcal{A})$, $\mathfrak{S}(\mathfrak{R}(s)) = s$.

Proof: Routine checking. ■

Claim 7. If s is reachable, $\mathfrak{R}(s) = C$, $C \overset{A}{\rightsquigarrow} C'$ and $s' = \mathfrak{S}(C')$ then $s \xrightarrow{A} s'$.

Proof: By Claim 6, $\mathfrak{S}(C) = s$. Let $\mathsf{task}(A) = T$. By Lemma 3 and the definition of $\mathfrak{S}$, if $A = \mathsf{start}(T)$ then $s.\mathsf{loc}[T] = \mathsf{L1}$ and if $A = \mathsf{end}[T]$ then $s.\mathsf{loc}[T] = \mathsf{L2}$. Moreover, since $C \overset{A}{\rightsquigarrow} C'$, $s.\mathsf{done}[T] = \mathsf{false}$. Hence, in order to prove that s enables an A-transition, it suffices to establish that $\mathsf{dep\_met}(A)$ holds in s. For this, in turn, it suffices to prove, for any incoming precedence edge $B \xrightarrow{u} A$ with $\mathsf{task}(B) \neq \mathsf{task}(A)$, that $\mathsf{dep\_met}(B, u, A)$ holds in s. Let $C' = C \cup \{\alpha\}$ with $\alpha = (A, v)$. Since $C \vdash \alpha$, all immediate predecessors of α are in C. Let $\mathsf{task}(B) = T'$. The proof now proceeds with a routine case distinction in which 4x4 cases are considered, following the case distinction in the definition of $\mathfrak{S}$ for both T and T'. We conclude that s enables an A-transition. Suppose $s \xrightarrow{A} s''$. Then, by Claim 5, $\mathfrak{R}(s) \overset{A}{\rightsquigarrow} \mathfrak{R}(s'')$. Since C has only one outgoing A-transition, $\mathfrak{R}(s'') = C'$. Hence, by Claim 6, $s'' = s'$, as required. ■

Claim 8. $\mathfrak{R}$ is a bijection from the reachable states of $\mathcal{N}(\mathcal{A})$ to $\mathsf{conf}(\mathcal{A})$.

Proof: Straightforward using Claims 1, 4, 6 and 7. ■

The theorem now follows by combination of the claims. ■

IV. Generated Uppaal models

As mentioned in the introduction, we have developed a toolset for exploring embedded system designs, that is centered around an intermediate, Y-chart based representation. In this representation, a system is described as a combination of three modules: applications, platform and one of their possible mappings. Applications are described as PPOs and the platform as a collection of resources. Each resource is characterized by two parameters: total capacity and pace per time unit. The latter parameter might change at runtime e.g. the pace of a bus depends on the number of tasks that use the bus at some point in time. In this pattern, the mapping module contains details about the number of resources that some tasks claim and release at the beginning and at the end of their execution, respectively.

The generated Uppaal models have a simple structure: each task and resource that appears in the Y-chart representation instantiates a task template or a resource template, respectively, that we detail below.

The task template is shown in Figure 3. This is an extended version of the untimed task template of Figure 2. The task template of Figure 3 is enriched with timing and resource constraints, like a parameter for the amount of data (the cSize variable) that should be processed, a tPace() function which returns the pace at which the data is processed per time unit, that is dependent on the current paces of resources claimed. From these, a task duration can be computed which is the amount of time between the start event of a task and the corresponding end event.

Formally, the templates of Figures 2 and 3 can be related through the notion of a timed step simulation introduced in

[25]. Since timed step simulations are compositional [25], we can associate to any reachable state of our timed model a configuration of the PPO that represents the application part of it. Note however that, due to the imposed timing constraints, certain configurations of a PPO cannot be reached in the timed setting.

In Figure 3, the transition from the Idle to the Running location encodes a start event, and the reverse transition between these two locations encodes an end event. A start event can occur if the done function returns false, meaning that the last start event has not occurred yet, the dependencies induced by the event precedence functions are satisfied (the dep_met function) and all the resources claimed have enough capacity available to process the task (the canClaim condition). Tasks are scheduled using either a greedy or a lazy policy. If a resource changes its pace, then a throttle channel is urgently enabled in each task automaton that uses the resource at that moment. On the transition between the committed and Running locations, the remaining amount of data unprocessed is computed (the cSize variable) and also the task pace is updated. For these updates, a select statement is used in order to under-approximate the time elapsed from the latest pace modification recorded by the x clock (of real type) to the closest integer (the i variable) below this value. This approximation is necessary because clock variables cannot be utilized in expressions. Finally, the transition between the Running and Idle states, that encodes an end event, will fire when the dependencies of the end event are satisfied (dep_met(end(t))) and the task duration has elapsed. On this transition, the valuations of the parameters for the next task instance are computed.

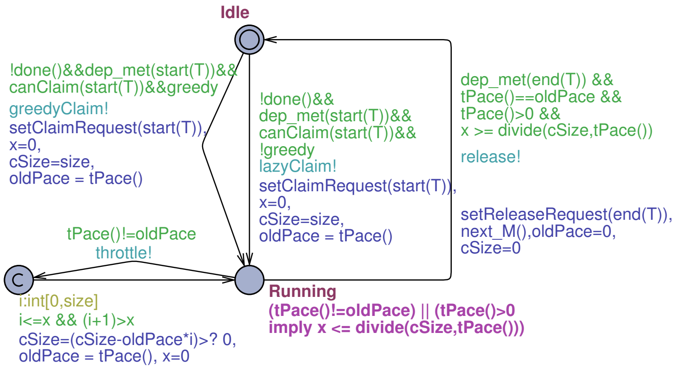


Fig. 3: Task Template

The resource template has a simpler structure as shown in Figure 4. The transitions with the greedyClaim and lazyClaim channels fire at the occurrence of a start event, whereas the transition with the release channel fires when an end event occurs. The channels in this template are broadcast which implies that all resources that instantiate this, interact with any task where an event occurs. The resMessages array is shared between tasks and resources, and it is updated by a task in the setClaimRequest and setReleaseRequest functions with the amount of resource capacities claimed or released, respectively. At the occurrence of an event in a task, the resources that are used by the task have a non-zero value placed at their position in the resMessages array. Further, they subtract or add this value from their available current capacity.

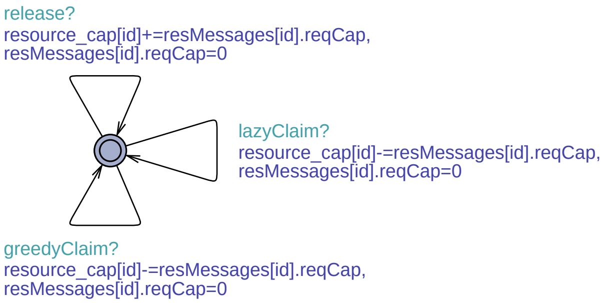

release?
resource_cap[id]+=resMessages[id].reqCap,
resMessages[id].reqCap=0
lazyClaim?
resource_cap[id]-=resMessages[id].reqCap,
resMessages[id].reqCap=0
greedyClaim?
resource_cap[id]-=resMessages[id].reqCap,
resMessages[id].reqCap=0

Fig. 4: Resource template

V. Experiments

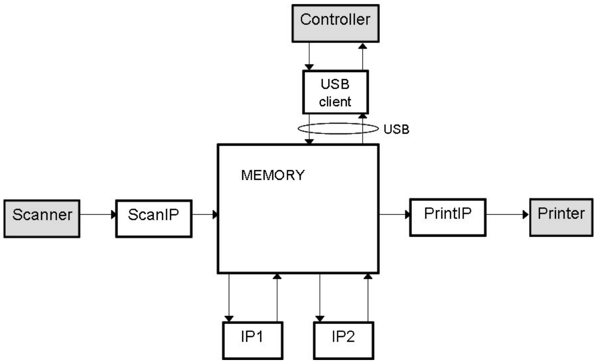


Fig. 5: Océ printer architecture

We now turn to an experimental evaluation of our translation from PPOs to timed automata. We compare generated Uppaal models with handcrafted models that were previously described in [21] for printing systems. The latter category of models contains a distinct template for each application (use case) type and each transition there encodes an event that can occur in that application. All the experiments are performed with Uppaal, version 4.1.2, on a Sun Fire X4440 server with 16 cores (AMD Opteron 8356, 2.3GHz) and 128 Gb of DDR2 RAM. The printer architecture is depicted in Figure 5. The USB has two modes: *static*, when the bus pace is not influenced by the number of simultaneous tasks transfered and *dynamic* when the pace is lower in case of bidirectional bus usage.

The cases explored here are depicted in Appendix A. We use the same notations as in Figure 1. In addition the rectangles contain between parentheses task durations. The arrow labels give the precedence function and resources handed over. The circles encode resources and the parentheses contain their maximum capacity available (one if not mentioned). The dashed lines represent resource claims. The difference between the amount claimed and the one that is handed over is released at the end of a task.

TABLE I: Direct Copy(DC) ‖ Simple Print(SP) Case - Comparison Improved Manual Models(grey) vs. Generated Models (O.M. - out of memory)

DC	SP	Peak Mem(KB)	Running Time(s)	Makespan (s)	States Explored
2	3	4500	0.50	23	1130
		5432	0.60	23	413
7	10	5480	1.60	71	10578
		5748	1.60	71	3050
35	50	12808	11.31	367	149926
		9572	17.00	367	48196
70	100	26568	27.92	737	433816
		18480	51.42	737	155491
334	500	598996	279.70	3585	6843592
		282732	961.98	3585	3038099
667	1000	2321768	1304.87	7166	25206064
		1076552	3964.64	7166	11704000
903	1355	4165896	1937.88	9705	45225661
		1962576	7805.70	9705	21272017
904	1356	O.M.	O.M.	O.M.	O.M.
		1965192	7655.01	9715	21302397
1460	1960	O.M.	O.M.	O.M.	O.M.
		4052524	18199.70	15117	44117751

TABLE II: Direct Copy(DC) ‖ Process from Store (PFS) Case - Comparison Improved Manual Models(grey) vs. Generated Models (O.M. - out of memory)

DC	PFS	Peak Mem(KB)	Running Time(s)	Makespan(s)	States Explored
1	2	4456	0.40	15	704
		5916	0.60	15	411
10	20	7540	4.10	114	47551
		7332	7.90	114	20118
25	50	21352	19.51	279	334606
		14420	48.03	279	135453
120	240	586172	384.66	1324	8255421
		244920	1122.34	1324	3269408
240	480	2555392	1857.78	2644	33327861
		1008512	4824.23	2644	13162088
303	606	4077452	2419.45	3337	53223828
		1573952	8196.21	3337	21007415
304	608	O.M.	O.M.	O.M.	O.M.
		1583572	8155.78	3348	21146664
480	960	O.M.	O.M.	O.M.	O.M.
		4057584	23394.71	5284	52819448

In each experiment we computed the fastest time in which all tasks are completed (also called makespan) by using greedy scheduling. Three performance metrics were used to evaluate each experiment: the peak memory usage (the 'Mem' column) and running time (the 'Time' column) of Uppaal, and the total number of states explored during analysis. For the comparison we considered the following two scenarios: Direct Copy in parallel with Simple Print (Table I), and then Direct Copy in parallel with Process from Store (Table II). The first two columns in each table indicate the maximum number of instances per use case. In fact these numbers also indicate how many task instances are processed in total per use case (e.g. 7 DC means that there are seven task instances that are processed in total for each task that composes the direct copy use case).

To combat state space explosion, we used the sweep line method of Uppaal [26]. For this, we specified progress measures in each model (in our case, the task instance) that Uppaal uses during the analysis to store only the states where the progress measures are weakly monotonically increasing, or occasionally decreasing.

The state space of the generated models is significantly smaller than the state space of the handcrafted models (reduced between 41% and 71%). There are two important reasons that cause this. Firstly, in the handcrafted models the resource template has one extra internal state in addition to the Idle and Running states, that is used to model a recovery phase that some resources require like the scanner. In the models generated, the recovery phase is modeled as an additional task. The second difference is caused by modeling tasks that need more than one resource for processing. In the generated models, one can naturally specify multi-resource claims per task, as described in the previous section. By contrast, in the handcrafted models, a multi-resource claim was modeled by a third party automaton placed between the application automaton and each resource required. This extra automaton would register the claim, wait until the resource became available, grab it for some duration, wait for the resource to signal the end event and send it back to its corresponding application automaton. Even more, the use of this extra automaton does not guarantee that all resources booked started to process a task at the same time. In the generated models, multi-resource claims or releases are easily specified by using broadcast channels, as detailed in the previous section.

The Tables I and II also show up to a 61% decrease in the peak memory used by Uppaal during the analysis. However, analysis of the generated models requires more time. Due to the parametric representation that characterizes the generated models, a lot of details were encoded into functions. Furthermore, some of these functions require a lot of time to be evaluated due to the conditions or function calls that they incorporate. However, the state space explosion problem emerges later in the analysis of these models, and we could analyze a higher number of tasks in comparison with the handcrafted models.

VI. Conclusions

PPOs are a simple extension of partial orders, but expressive enough to compactly represent large task graphs with repetitive behavior. We have presented a translation from a subclass of PPOs to Uppaal, together with a correctness proof that the transition system induced by a Uppaal model is isomorphic to the configuration structure of a PPO. We also presented experiments which demonstrate that the Uppaal models obtained through this translation are in fact more tractable than handcrafted models of the same systems used in earlier case studies.

As explained in this paper, when the applications (use cases) of an embedded system design are described using PPOs, then we have a well-defined partial order structure on the corresponding events. Due to competition for resources and timing constraints, only a fragment of all the interleavings of this partial order will be possible in the full system model. Nevertheless, it will be interesting to see if partial order reduction techniques [27], [28] will allow us to exploit the inherent structure of PPOs to alleviate the state space explosion problem when analyzing the full system model.

Another interesting topic for future research is to adapt the results of [4] to the PPO settings. This approach reduces the complexity of scheduling problems by exploiting the repetitive structure of tasks: it reduces a scheduling problem to a problem containing a minimal number of identical repetitions, and after solving this much smaller scheduling problem, the computed schedule is expanded to a schedule for the original, more complex scheduling problem.

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Appendix

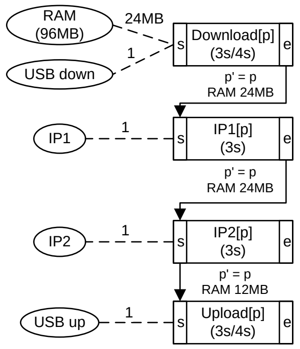

Fig. 6: Process from Store Case

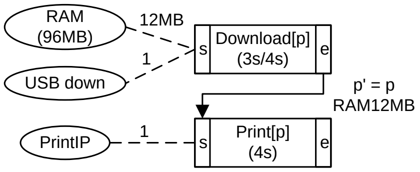

Fig. 7: Simple Print Case

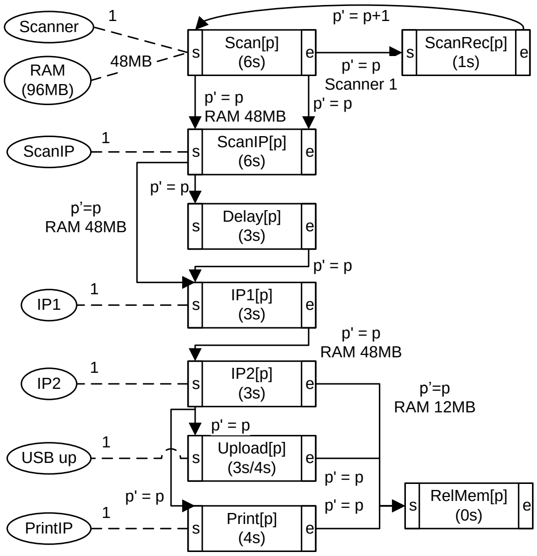

Fig. 8: Direct Copy Case Case