THE THERMAL STATE OF THE ACCRETING WHITE DWARF IN AM CANUM VENATICORUM BINARIES

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ABSTRACT

We calculate the heating and cooling of the accreting white dwarf (WD) in the ultracompact AM Canum Venaticorum (AM CVn) binaries and show that the WD can contribute significantly to their optical and ultraviolet emission. We estimate the WD’s effective temperature, $T_{\text{eff}}$, using the optical continuum for a number of observed binaries, and show that it agrees well with our theoretical calculations. Driven by gravitational radiation losses, the time averaged accretion rate, $\langle M \rangle$, decreases monotonically with increasing $P_{\text{orb}}$, covering six orders of magnitude. If the short period ($P_{\text{orb}} < 10$ min) systems accrete at a rate consistent with gravitational radiation via direct impact, we predict their unpulsed optical/UV light to be that of the $T_{\text{eff}} > 50,000$ K accreting WD. At longer $P_{\text{orb}}$ we calculate the $T_{\text{eff}}$ and absolute visual magnitude, $M_V$, that the accreting WD will have during low accretion states, and find that the WD naturally crosses the pulsational instability strip. Discovery and study of pulsations could allow for the measurement of the accumulated helium mass on the accreting WD, as well as its rotation rate. Accretion heats the WD core, but for $P_{\text{orb}} > 40$ minutes, the WD’s $T_{\text{eff}}$ is set by its cooling as $\langle M \rangle$ plummets. For the two long period AM CVn binaries with measured parallaxes, GP Com and CE 315, we show that the optical broadband colors and intensity are that expected from a pure helium atmosphere WD. This confirms that the WD brightness sets the minimum light in wide AM CVn binaries, allowing for meaningful constraints on their population density from deep optical searches, both in the field and in Globular Clusters.

Subject headings: binaries: close—gravitational waves—novae, cataclysmic — variables— white dwarfs

1. INTRODUCTION

The AM Canum Venaticorum (AM CVn) class of ultracompact binaries provides an opportunity for probing the accretion-induced heating of a white dwarf (WD) over six orders of magnitude in the time-averaged accretion rate, $\langle M \rangle$. In these very tight ($P_{\text{orb}} < 80$ min) binaries (see Warner 1995 for an overview), the donor star is a very low mass ($M < 0.1 M_\odot$) helium WD and the accretor is either a He or a C/O WD. Though there are a few distinct AM CVn formation scenarios (e.g., NElemanS et al. 2001 and references therein), they all evolve as mass-transferring binaries driven by the loss of angular momentum from gravitational radiation (e.g., Deloye, Bildsten & NElemanS 2005).

This paper is an outgrowth of Townsley and Bildsten’s (2004, hereafter TB) work on the reheating of WDs in H-accreting cataclysmic variables. However, the case of WD reheating in AM CVn binaries is different because the accreted material is pure helium, making flashes an isolated rather than constant occurrence, and the binary evolution proceeds only under angular momentum loss by gravitational radiation, leading to a plummeting $\langle M \rangle$ with increasing $P_{\text{orb}}$. We have found that the WD is heated in the earliest phases of binary evolution and cools at later times. There is a critical decoupling accretion rate of $M_{\text{dec}} \approx (1 - 3) \times 10^{-14} M_\odot \text{yr}^{-1}$ below which the reheated WD will cool even while accreting. We find the amount of time that has passed since the binary had this $\langle M \rangle$ by using the evolutionary calculations of Deloye et al. (2005). Simple cooling after this point predicts the WD luminosity, as well as the expected WD $T_{\text{eff}}$ and absolute visual magnitude, $M_V$ (assuming, of course, a WD mass, $M$, and radius, $R$). We show that the accreting WD can become the dominant broadband optical light source in those binaries with orbital periods in excess of $\approx 40$ minutes.

In §2, we discuss the physics of the WD heating due to accretion and discuss the uncertainty in the total helium layer mass expected on the WD. Since the WD luminosity, $L$, is nearly always less than the accretion luminosity, $L_{\text{acc}} = GM\langle M \rangle / R$, we explain in §3 how the binary’s behavior allows the WD to be directly detected at different orbital periods. At early times, during direct impact accretion (e.g., Marsh and Steeghs 2002), the unpulsed UV/optical light is from the hot WD, whereas at longer orbital periods, the disk is often quiescent, allowing for the underlying WD to dominate the optical continuum of a number of these binaries. We also speculate on what could be learned if one of these WDs...
was caught in the DB pulsational instability strip. We make direct comparisons to the observations in §4, and conclude in §5.

2. THE PHYSICS OF WHITE DWARF HEATING IN AM CVN BINARIES

The ability for accretion to heat the deep interior of a WD has been discussed extensively (Sion 1991; TB). Indeed, accretion of pure helium at rapid rates $M > 10^{-7} M_\odot$ yr$^{-1}$ is known to flash unstably and was studied as a Type Ia progenitor scenario (e.g. Nomoto & Sugimoto 1977; Nomoto 1982; Woosley & Weaver 1994). Though eventually ruled out as a major mechanism for Type Ia supernovae (Solheim & Yungelson 2005), AM CVn binaries accrete pure helium for a long enough time to reheat the accreting WD. An additional feature of these binaries is the large dynamic range in the time averaged accretion rate ($\langle M \rangle$), onto the WD, allowing us to test the theories of re-heated WDs, as discussed in §§3 & 4.

Here we calculate the accretor’s thermal evolution along with that of the binary for several AM CVn systems that represent the expected ranges for this population. Motivated by the expected mass distribution of He WD donors that evolve into AM CVn systems (Nelemans et al. 2001), we chose an initial donor mass of 0.24$M_\odot$. The thermal state of the donor is very important for setting the relationship between ($\langle M \rangle$) and $P_{\text{orb}}$ (Bildsten 2002; Deloye & Bildsten 2003; Deloye et al. 2005). For this study we chose two thermal states found by setting the initial entropy of the donor so that the system’s $P_{\text{orb}}$ at contact is greater than the $P_{\text{orb}}$ at contact of 30% and 90% of the systems in the Deloye et al. (2005) AM CVn population evolution calculation. This provides $\langle M \rangle$ as a function of time, and we then use the quasi-static envelope methods of TB to heat the accreting WD. For simplicity, this evolution is performed at constant $M$ and with a constant helium layer. We chose two masses $M = 0.65 M_\odot$ and $1.05 M_\odot$, each case with a 0.05$M_\odot$ accreted helium shell (justified below). Initial $M$ values for the binary evolution were chosen such that $M(t)$ matched that of the thermal calculations at late times (long $P_{\text{orb}}$) to ensure the correct $P_{\text{orb}}$-time relation.

Using these $\langle M \rangle$ trajectories (see Figure 1), we identified four relevant regimes for the state of the accreting WD. Chronologically since initiation of mass transfer (and in order of decreasing $\langle M \rangle$), these are: (1) rapid accretion and thermally unstable He ignition ($< \text{few} \times 10^5 \text{yr}$), (2) core heating during a phase in which $L$ is determined from “compressional” heating in the WD envelope ($\sim 3 \times 10^5$ to $3 \times 10^6$ yr), (3) a transition period where the core temperature is near an equilibrium value set by $\langle M \rangle$ ($3 \times 10^6$ to $10^7$ yr), and (4) the WD cooling regime ($> 10^8$ yr) where $L$ is determined by the time, $\Delta t$, since the heating stopped at a previous epoch where $\langle M \rangle \sim M_{\text{dec}}$, the decoupling mass-transfer rate. We now describe each regime.

At the very earliest times, when $\langle M \rangle \gtrsim 10^{-7} M_\odot$ yr$^{-1}$, the accreted helium is compressed rapidly and ignites in a thermally unstable manner, leading to recurrent weak explosions (Fujimoto & Sugimoto 1982). It is uncertain how much of the accreted material is ejected (see Kato & Hachisu 1999). As $\langle M \rangle$ drops, the compression happens more slowly so that the envelope temperature is lower, and the helium shell can become quite large. Thermal instabilities at this stage likely lead to detonations (Fujimoto & Sugimoto 1982), such that the last thermonuclear instability might be the most violent. However, once $\langle M \rangle < 2 \times 10^{-8} M_\odot$ yr$^{-1}$, the required unstable He ignition mass for 0.6–1.1$M_\odot$ WDs (Nomoto 1982; Limongi & Tornambe 1991; Goriely et al. 2002) exceeds the donor mass. Thus once a system reaches an orbital period of 10 minutes, it is unlikely to undergo further unstable helium ignition events. That is, $\langle M \rangle$ drops quickly enough that once $\langle M \rangle$ is low enough for a strong flash, the accreted layer never becomes thick enough for it to occur. Over the remaining course of the binary’s evolution, about 0.1$M_\odot$ is accreted by the WD, motivating us to use a fixed accreted layer mass of 0.05$M_\odot$ to calculate the thermal evolution of the accretor. Because $L$ depends only logarithmically on the accreted layer mass (TB), using this fixed value has only a small impact, especially when compared to the large variations in $L$ from the orders of magnitude variation in $\langle M \rangle$.

The period of time during which $\langle M \rangle > 10^{-7} M_\odot$ yr$^{-1}$ is so short ($\sim 10^5$ yr) that the accretor’s core is thermally unperurbed from its initial state. However, once the helium layer begins to accumulate, thermal diffusion both out of the surface (setting the exiting luminosity, $L$) and into the core (acting to increase the temperature there, $T_c$) occurs. We thus begin evolving the WD’s thermal state $10^7$ yr after contact.

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1 Extremely high accretion rates for which a red giant envelope can be supported, $\langle M \rangle \gtrsim 10^{-7} M_\odot$ yr$^{-1}$, are not consistent with the energy released from helium fusion ($\sim 0.6$ MeV/amu) only appearing at the brief ($< 10^3$ yr) and poorly understood onset of accretion.
Although the compressional energy release depends weakly on the accumulated He layer mass (i.e. logarithmically, TB), the \( I-T_c \) relation which determines the cooling is essentially independent of it. The evolution is shown in Figure 1 and exhibits all of the epochs (2)-(4) including reheating, decoupling and cooling. The heat capacity and crystallized fraction of the core was calculated using the equation of state of Potekhin & Chabrier (2000), and the late-time \( I-T_c \) relation (in the limit of minimal accretion) was adapted from Hansen (1999). We have included a latent heat of crystallization of 0.77\( kT \) per particle, but no fractionation.

At early times in the evolution, the WD core temperature, \( T_c \), is much lower than the maximum temperature reached at the base of the outer radiative envelope, \( T_{en} \). In this limit, TB showed that \( T_c \) becomes independent of \( T_{en} \) and is only a function of \( \langle M \rangle \) and \( M \). They found that the envelope state is determined self consistently by balancing the heat released in the outer (non-degenerate) envelope due to fluid motion down the entropy gradient, \( I_{comp} \approx 2\langle \dot{M} \rangle k T_{en}/\mu m_p \) with that carried through the envelope by radiative transfer \( I \approx L_0 (T_{en}/10^8 \text{K})^{-2} \), where \( \mu = 4/3 \) is the mean molecular weight in the He envelope, \( k \) is Boltzmann’s constant, and \( m_p \) is the baryon mass. This same state applies here when \( T_c < T_{en} \) and we have calculated this balance numerically using the quasi-static envelope methods outlined in TB for He accretion and found,

\[
I_c \approx 2.8 \times 10^{-2} L_0 \left( \frac{\langle \dot{M} \rangle}{10^{-9} M_0 \text{yr}^{-1}} \right)^{1.4} \left( \frac{0.6 M_0}{M} \right)^{0.3} \text{K} \left( \frac{T_{en}}{10^9 \text{M}_0 \text{yr}^{-1}} \right)^{-2}.
\]

This relation holds for \( T_c < T_{en} \approx 2 \times 10^7 \text{K} \langle \dot{M} \rangle (10^{-9} M_0 \text{yr}^{-1})^{1.5} \) and \( 10^{-10} M_0 \text{yr}^{-1} < \langle \dot{M} \rangle < 10^{-7} M_0 \text{yr}^{-1} \), although for our time histories \( T_c \approx T_{en} \) by the time \( \langle \dot{M} \rangle \approx 10^{-7} M_0 \text{yr}^{-1} \) so that this form only applies to Figure 1 for \( \langle \dot{M} \rangle > 10^{-7} M_0 \text{yr}^{-1} \). During this early phase, the WD core is being heated as \( \langle \dot{M} \rangle \) is dropping.\(^2\)

As \( \langle \dot{M} \rangle \) drops and \( T_c \) increases, the accreting WD approaches the equilibrium state where the outgoing luminosity is matched by the internal energy release (Nomoto 1982; TB). There is a brief transition from core heating to cooling which occurs when \( T_c = 1-2 \times 10^7 \text{K} \langle \dot{M} \rangle T_c/\mu m_p \) and is found,\(^3\) the WD cooling and the time since decoupling becomes \( \tau \) so that at longer orbital periods we can safely predict \( I \). Low-entropy donors reach the decoupling \( \langle \dot{M} \rangle \) at shorter orbital periods and evolve more slowly in \( P_{orb} \), so that the WDs at a given \( P_{orb} \) always look "older" (i.e. colder) if their donor was initially of low entropy. The track for the cool donor (30th

\(^2\) The only other calculation we could find that reported exiting luminosities are in Table 2 of Limongi & Tornambe (1991). While our values compares well to their calculations with \( M = 0.6 M_0 \), the starting WD mass, \( M = 0.8 M_0 \) and the accreted layer is thin, their values are as much as a factor of 50 lower for \( \langle \dot{M} \rangle \approx 10^{-8} M_0 \text{yr}^{-1} \). We believe that \( I_c \) is largely due to compressional heat release in the outer layers and was only of peripheral interest in their study, is not well determined by their modeling for small accreted layer masses at \( M < 0.85 M_0 \).

\(^3\) Using the values tabulated by Limongi & Tornambe (1991, hereafter LT91) for accretion of Helium onto a \( 0.8 M_0 \) WD at a fixed rate, we find that our determination of the heating time from the quasi-static model agrees well with their full numerical model. Table 2 in LT91 gives a detail of \( L_{out} \) (the maximum with respect to radius) and \( T_{center} \) for a mass history starting from \( 0.8 M_0 \) up to about \( 0.9 M_0 \). We compare our \( L_{cool} = L(t) \) for a \( 0.9 M_0 \) WD with a \( 0.05 M_0 \) Helium layer accreting at a constant rate to \( T_{cool} = (T_{center} + T_{max})/2 \) (representative of the heat content) from Table 2 of LT91, and find that in both cases, for this range of masses, \( dT_{cool}/dT \) is approximately constant. We find \( dT_{cool}/dT \approx 1.6 \times 10^7 \text{K} M_0^{-2} \) and \( 2.7 \times 10^8 K M_0^{-1} \) for \( \langle \dot{M} \rangle = 10^{-8} M_0 \text{yr}^{-1} \) and \( 5 \times 10^{-8} M_0 \text{yr}^{-1} \) respectively, compared to the values of \( 1.3 \times 10^8 K M_0^{-1} \) and \( 2.7 \times 10^8 K M_0^{-1} \) from LT91. Also, the \( T_{max} \) reached in the plateau (~ 4 \( \times 10^7 \text{K} \)) reflects the equilibrium \( T_c \) for \( \langle \dot{M} \rangle \) at this epoch ~ 5 \( \times 10^{-9} M_0 \text{yr}^{-1} \). This is consistent with the typical values of interior temperatures, \( \log T \approx 7.5-7.7 \), found by LT91 for a \( 0.8 M_0 \) WD accreting Helium steadily at \( \langle \dot{M} \rangle \approx 5 \times 10^{-10} M_0 \text{yr}^{-1} \).
percentile) gives a good maximum $\Delta t$, which for $P_{\text{orb}} > 40$ min can be fit by $\Delta t \approx 5 \times 10^3 P_{\text{orb}}^{0.4} \text{yr}$, and thus yields the relation for the coldest possible $0.65 M_\odot$ WD at these orbital periods, $L_{\text{min}} \approx 1.5 \times 10^{-3} L_\odot$ $(40 \text{ min/} P_{\text{orb}})^{4.8}$.

3. OBSERVABILITY OF THE ACCRETING WHITE DWARF

In order to assess the likelihood of observing the accreting WD, we must compare its luminosity to that from active accretion. When the WD is near equilibrium or simply emitting the compressional heating luminosity, $L \approx kT \langle M \rangle / m_p$ will be much less than the accretion luminosity, $L_{\text{acc}} \approx GM(M_1 + M_2) / R$. Seeing the hot WD then depends on either geometry (for example, an edge-on system like the newly discovered eclipsing binary SDSS J0926+3624; Anderson et al. 2005) or probes of different spectral regions. Four classes among observed systems are delineated in Table 1 by horizontal lines. The shortest period systems are in a phase where the accretion stream directly hits the WD (e.g. Marsh & Steeghs 2002; Israel et al. 2002), providing the first opportunity. In this case there is no disc and there are times when the direct impact location is out of sight, providing an excellent chance to see the (very hot) accreting WD. At slightly longer $P_{\text{orb}}$, around 20 minutes, all systems are expected to have a consistently bright accretion disc in the optical/UV that would energetically dominate the WD (e.g. Nelemans et al. 2004) as noted above. Indeed, Nelemans et al. (2004) showed that a simple accretion disc model at 15 minutes would give $M_2 \approx 7$ and temperatures of $\approx 30,000 \text{ K}$, swamping the hot WD. This model is shown by the thick green line on the bottom panel of Figure 2.

The third group of AM CVn systems, with again slightly longer orbital periods, show large brightness variations due to a thermal instability in the disc (Tsubara & Osaki 1997). Although at this $\langle M \rangle$ the disc absolute magnitude is expected to be brighter than the WD, the low state is a time when the accreting WD can be detected, just as in the hydrogen accreting CVs (Sion 1991; Townsley & Bildsten 2003).

The last group, with the longest orbital periods, again show hardly any brightness variations. In the context of the thermal instability interpretation, these systems could be on the lower, stable, branch of the instability curve and thus would be cool discs, consisting mainly of neutral helium. The steadily accreting luminosity is only $\approx 10$ times larger than the cooling WD (see §2), enhancing the prospects of seeing the accreting WD relative to shorter orbital periods, where the contrast is greater. In addition, the disc temperature will generally be cooler than the WD as shown in the bottom panel of Figure 2.

### 3.1. Non-Radial Oscillations of the Accreting WD

In addition to directly seeing the re-heated WD in the AM CVn binaries, there are also opportunities for seismological tests as the WD passes through the non-radial pulsation instability strip ($L_{\text{eff}} = 22,400 - 27,800 \text{ K}$ for isolated DBs; Beauchamp et al. 1999), which we plot in Figure 2. The pulsation periods are 200-1000 seconds (see Handler 2003 for a recent summary) and are typically $l = 1, 2$ g-modes of high radial order (e.g. Kotak et al. 2003).

We do not know, a priori, that the instability strip will be the same for a He accreting WD. The clear structural difference in the accreting WD is the mass of the helium envelope, which will likely exceed 0.05 $M_\odot$, and be seismically measurable. It seems unlikely that the thicker He envelope would alter the instability strip, since the relevant "pump" for the mode excitation occurs at low pressures (see Gautschy & Althaus 2002 for a recent study). However, active accretion onto the outer atmosphere might make a difference, especially if the convection zone (responsible for the mode excitation; e.g. Brickhill 1991) is altered. It also seems possible that the mode pump could become active during the cooling of the outer WD layers (Piro, Arras & Bildsten 2005) following an accretion disk outburst. Indeed, Wood et al. (1987) reported excess power at periods of 100-200 seconds when CR Boo had an optical magnitude of $V = 15.4$, which might well be the passing of the accreting WD through the instability strip following an accretion outburst.

The most dramatic alteration of the g-mode frequencies would be from WD rotation at periods comparable to the g-mode periods, as the WD has accumulated a significant amount of mass and angular momentum since the last mass ejecting He nova event. This angular momentum either resides solely in the He layer or has been shared with the underlying C/O WD in the $\sim 10^7$ years spent at the $P_{\text{orb}} \approx 20 - 40$ minutes where $L_{\text{eff}}$ is in the isolated DB stability strip (see Figure 2). Motivated by the likelihood that very weak internal magnetic fields can easily mediate angular momentum transport on this timescale (e.g. Spruit 2002), we presume that all accreted angular momentum after the last He nova event is
shared with the whole WD, yielding a lower limit to the rotation frequency of the He layer, where much of the g-mode energy resides.

We assume that helium accretes onto the WD with the specific angular momentum of the inner edge of the accretion disk, \((GM/R)^{1/2}\), and that the amount of accreted matter, \(\Delta M \ll M\). The final WD spin frequency is then \(2\pi/P_{\text{spin}} = (GM/R)^{1/2}\Delta M/I\), where \(I \approx 0.2MR^2\) is the WD moment of inertia, yielding \(2\pi/P_{\text{spin}} = 5\Delta M(G/MR^3)^{1/2}\).

\[
P_{\text{spin}} \approx 50 \text{ s} \left( \frac{0.05M_\odot}{\Delta M} \right)^{1/2} \left( \frac{M}{0.6M_\odot} \right) \left( \frac{R}{9 \times 10^8 \text{cm}} \right)^{3/2},
\]

which reaches about 100 seconds when \(M = 1.2M_\odot\). Hence, it appears very likely that the WD surface layer (and maybe even core) is rotating rapidly enough (i.e. \(P_{\text{spin}} < P_{\text{mode}}\)) that the seismology will be dramatically altered from the isolated DB case. The discovery and study of a DB pulsator in an AM CVn system would provide an unprecedented opportunity to measure the accumulated He layer mass on the C/O WD and the angular momentum transport mechanisms within it. The most probable orbital periods for this range from 20 to 35 minutes.

The rapid surface rotation will broaden any spectral features originating there, as the resulting minimum equatorial rotational velocity is \(\approx 1250 \text{ km s}^{-1} (\Delta M/0.05M_\odot)\), nearly independent of \(M\). The narrow, central line emission (i.e. spikes) seen in He lines of GP Com, CE 315 and SDSSJ1240 (Marsh 1999, Ruiz et al., 2001, Roelofs et al., 2005a) are generally believed to originate on the accreting WDs. However, their widths are much less than this estimate, suggesting either a different origin or an effective loss of angular momentum from the rapidly spinning accretor.

3.2. Atmospheric Properties

Another difference with single DB WDs is that \(^{14}\text{N}\) is present in the accreting material at a mass fraction of 1-2\% (due to CNO burning in the progenitor star of the He donor). This \(^{14}\text{N}\) would rapidly sink out of the WD photosphere were it not for the constant mixing provided by the underlying convection zone of mass \(M_{\text{conv}}\). It is thus the sedimentation time, \(t_{\text{sed}}\), at the base of the convection zone that sets the relevant timescale. Paquette et al. (1986) calculated \(t_{\text{sed}}\) for \(^{14}\text{N}\) in pure He atmospheres of isolated WDs, and defined \(M_{\text{sed}} = M_{\text{conv}}/t_{\text{sed}}\), below which the \(^{14}\text{N}\) in the convective envelope will be less abundant than in the accreting material. For \(M = 0.6M_\odot\) WDs, \(M_{\text{sed}} \approx 10^{-12}M_\odot\text{yr}^{-1}\). Though \(M_{\text{conv}}\) decreases rapidly with increasing \(M\) (see Fontaine & Van Horn 1976), so does the sedimentation time, making the actual value of \(M_{\text{sed}}\) not too dependent on \(M\) for the range of \(P_{\text{orb}}\) relevant here (see Figure 14 in Paquette et al. 1986). More recent calculations with updated opacities (MacDonald, Herrnanz, & Jose 1998) found larger \(M_{\text{conv}}\) values by factors of 4-6 from Paquette et al. (1986). Hence, for \(t_{\text{orb}} < 40\) minutes, the atmospheric N abundance will be comparable to that in the accreted material. At longer orbital periods, depending on the state of the donor, it seems plausible that \(M_{\text{conv}}\) will be less than \(M_{\text{sed}}\), and the \(^{14}\text{N}\) will tend to sediment. Detection and study of atmospheric N (i.e. not from the accretion disk) in GP Com, CE-315 and SDSSJ1240 could thus constrain the sedimentation physics.

The abundant presence of \(^{14}\text{N}\) may simplify the spectral modeling of these WDs relative to what has been done for pure He atmospheres (see Bergeron, Saumon & Wesemael 1995a). This is because the low first ionization potential of \(^{14}\text{N}\) will provide free electrons more readily than the He. This may well allow the He-free-free opacity to more readily dominate for the relevant \(T_{\text{eff}}\)’s, but only calculations will tell. This could also alter the location of the pulsational instability strip.

4. COMPARISON TO OBSERVATIONS

We tabulate relevant observed and inferred properties of the AM CVn binaries in Table 1. The minimum donor mass, \(M_{\text{c, min}}\), for each system is the mass of the Deloye et al. (2005) fully degenerate donor model that fills the Roche lobe at the system’s \(P_{\text{orb}}\). The minimum \(\langle M_{\text{c, min}} \rangle\) at fixed \(P_{\text{orb}}\) occurs when \(M_{\text{c}} = M_{\text{c, min}}\) (Deloye et al. 2005). We thus take \(M_{\text{c, min}}\) and calculate the tabulated values of \(\langle M_{\text{c, min}} \rangle\) as fol-
lows. For systems in which the mass ratio has been inferred (all the AM CVn binaries except RX J0806, V407 Vul, ES Cet, and SDSS J0926+3624), we use it and $M_{\text{min}}$ to find $M$ and thus $\langle M \rangle_{\text{min}}$. When there is no mass ratio constraint, we assume that $\langle M \rangle_{\text{min}}$ is given by the minimum possible value, which occurs at a mass ratio of $\approx 0.3$ (this determined the entries for RX J0806, V407 Vul, and ES Cet), except for SDSS J0926+3624, for which we use the mass ratio of CP Eri.

Our discussion in the previous sections outlined the opportunities for detecting the hot WD and we now discuss in more detail the implications of our calculations in regards to directly observing the WD in these binaries.

4.1. Short period systems

The evolutionary state of the two shortest period systems, RX J0806.3+1527 ($P_{\text{orb}} = 5.36$ min) and RX J11914.4+2456 ($= V407$ Vul, $P_{\text{orb}} = 9.48$ min) is uncertain and different models have been proposed (see e.g. Wu et al. 2002; Ramsay et al. 2002; Marsh & Steeghs, 2002; Marsh & Nelemans, 2005; Dall’Osso et al. 2005; Willems & Kalogera 2005). Marsh & Steeghs (2002) suggested that V407 Vul is a direct-impact accretor where the accretion stream directly strikes the accreting WD. Noting the earlier work of Townsley & Bildsten (2002), they suggested that much of the optical light could originate from the hot, accreting WD.

Our high $\langle M \rangle$ calculations allow us to address the direct impact scenario. Even though the accreting material arrives over a limited part of the WD, we expect complete spreading by the time it has reached depths relevant for convective heating (Piro & Bildsten, 2004), thus allowing our calculation to apply. At the minimum ($M \approx 10^{-3} M_\odot$ yr$^{-1}$) expected for V407 Vul (see Table 1), we predict that $L \approx L_\odot$. The predicted range of $L_{\text{eff}} \approx 50,000$–$100,000$ K and absolute magnitude, $M_V \approx 8$. The analysis of V407 Vul is complicated by the fact that the optical light is dominated by a G9V star (Steeghs et al., in prep). However, the visible part of the light clearly shows a blue color, with implied effective temperature $> 20,000$ K (see Table 1), consistent with our calculations. The estimated reddened visual magnitude is 16.9 (Ramsay et al., 2002) as being generated by a blue color, with implied effective temperature $> 20,000$ K. The apparent magnitude of 21.1 would put the system at 20 kpc, 8 kpc above the Galactic plane. A similar distance is needed to interpret the rather low X-ray flux (Israel et al., 2002) as being generated by the WD contributions in the low state are about 1 magnitude above the predicted range, and the radius either suggests a massive accretor or due to most of the accretion luminosity (including a substantial G-star contribution. Combined with the estimated $M_V \approx 8$, this suggests a distance greater than 500 pc if V407 Vul is a direct impact accretor.

For RX J0806.3+1527 ($P_{\text{orb}} = 5.36$ min), the situation is quite different. The shorter orbital period implies a luminosity of $\approx 44 L_\odot$, $T_{\text{eff}} \approx 140,000$ K and $M_V \approx 4.7$ for the accretor. The estimated $T_{\text{eff}}$ agrees with the observed very blue spectrum, for which Israel et al. (2002) find a lower limit of 40,000 K. The apparent magnitude of 21.1 would put the system at 20 kpc, 8 kpc above the Galactic plane. A similar distance is needed to interpret the rather low X-ray flux (Israel et al., 2002) as being generated by the WD.

It was recently suggested that the low X-ray flux could be reconciled with much smaller distances due to a temporarily lower $M \ll \langle M \rangle$ or due to most of the accretion luminosity being emitted in the UV (Marsh & Nelemans 2005; Willems & Kalogera, 2005). However, our calculations show that not only the X-ray luminosity, but also the optical flux tracks $\langle M \rangle$, so that the latter suggestion would still produce a much too bright WD. The compositional heating luminosity would track a drop in $M$ on the thermal time of the accreted layer, $\tau \sim c_T M (0.05 M_\odot) / L \sim 10^5 \sqrt{\langle M \rangle / 10^{-3} M_\odot} \text{yr}^{-1}$, where $T_m$ is the maximum temperature, and $L$ is given by the equation (1). This timescale is much longer than the timescale for which a temporarily lower $M$ can be sustained ($\approx 100$ yrs, Marsh & Nelemans 2005), leading to a bright WD even at low $M$. Therefore, for RX J0806.3+1527 to be a direct impact accretor, the only solution seems to be a very large distance.

The third system in the table, ES Cet, is clearly a helium accreting WD. Esplialla et al. (2005) suggest it is a direct impact accretor and find $T_{\text{eff}} \approx 130,000$ K and radius $R \approx 4.5 \times 10^8$ cm for a distance of 300 pc. They interpret this as the size of the impact spot, but our calculations suggest that the whole WD should be very hot. This observed temperature is in the expected range, and the radius either suggests a massive (1.1$M_\odot$) WD or a less massive WD at a larger distance than they assumed.

4.2. Outbursting sources

For the outbursting systems the spectra are dominated by the accretion disc when they are in their high state. The absolute magnitudes of the disc are at least 6–8, much brighter than the expected absolute magnitudes of the accreting WDs of about 11 (Figure 2). However, in the low state, the WD can contribute significantly to the spectra, potentially producing the observed continuum. Between 20 and 30 min, the expected disc absolute magnitude drops, while the WD $M_V$ stays roughly constant. Interestingly, the magnitude range observed in these outbursting systems decreases with increasing orbital period. While CR Boo has a range of 5 magnitudes, V803 Cen is less, at 4.2, and CP Eri only changes by 3.2 magnitudes.

To check if the low-state spectra could be dominated by the WD, we estimate the temperature from the shape of the 4000–6000 Å continuum from the few spectra that are available. We assume a black-body spectrum, which reasonably produces the broad band colors of theoretical DB white dwarf spectra (Bergeron, Wesemael & Beauchamp 1995b). Our temperature determinations in Table 1 are accurate at about the 10% level. For CR Boo and V803 Cen, the absolute magnitudes are also known. For CP Eri and V803 Cen, the temperatures are below the expected WD temperatures and indeed the absolute magnitudes in the low state are about 1 magnitude above the predicted WD brightness (see Figure 2), suggesting a significant contribution from the disc (which is cooler than the WD, but of larger area), even in the low-state. Using the slope of spectrum in the low state, we found a temperature of 30,000 K for CP Eri, whereas a recent UV spectrum suggests 17,000 K (Sion, priv. comm.). We have to be careful with these low-state estimates, though, as a second low-state spectrum of CR Boo (Patterson et al. 1997) shows a very different continuum shape, suggesting a cool or hybrid cool plus hot spectrum.

The recently discovered eclipsing AM CVn binary, SDSS J0926+3624, at $P_{\text{orb}} = 28.3$ min (Anderson et al. 2005) provides an opportunity for probing the distinct contributions of the hot WD and the disc. Anderson et al. (2005) report that the 1 minute duration of the eclipse is consistent with the 0.02$M_\odot$ secondary transiting a hot spot or the inner disk. It is also consistent with an eclipse of the hot WD. Fast photometry and phase resolved spectroscopy could confirm that the WD contributes and would tie the eclipse minimum to the
WD rather than the disk, yielding a more robust measurement of the orbital period change expected from gravitational wave driven mass transfer (Anderson et al. 2005).

4.3. Long period systems

For GP Com and CE 315 we derived the temperatures in the same way as described above and interpret the temperature as that of the accreting WD. For GP Com, using the known distance \((d = 67 \text{ pc}; \text{ Thorstensen 2003})\), we can derive the emitting area. Thorstensen (2003) gives a typical observed \(V\) magnitude of 16.1, but Warner (1995) lists the range 15.7–16.0. We therefore use \(V = 15.7–16.1\) as the observed range. Together with \(m - M = 4.2 \pm 0.2\) (Thorstensen 2003) this gives \(M_V = 11.7 \pm 0.4\). From Bergeron et al. (1995b) we use the bolometric correction for an 11,000 K DB model, being \(-0.57\). This gives a bolometric magnitude \(M_{bol} = 11.13 \pm 0.4\), i.e. a luminosity of \(2.78 \times 10^{-3}L_{\odot}\), with an error of 44\%. Together with the effective temperature, this gives the size of the emitting region of \(10^9\) cm, i.e. a typical WD radius!

Using the same argument for CE 315, with a distance of 77 pc (Thorstensen, priv. comm.), using a effective temperature of 9500 K and a bolometric correction of \(-0.34\), we find an emitting region of \(6 \times 10^8\) cm, implying a more massive WD of \(\approx 0.9 M_{\odot}\). Indeed, its absolute magnitude falls close to the track for a 1.0 \(M_{\odot}\) accretor (Fig. 2). Interestingly, two systems have been recently discovered that, in their low state, clearly show the broad absorption lines of a DB white dwarf, with estimated temperature of 17,000 K (SDSS J1240-01 and SN2003aw, Roelofs et al. 2005a; Warner & Woudt, 2003, Roelofs et al. 2005b). Based on an absolute magnitude of 11 (Fig. 2), these systems should have distances of 735 pc (SN2003aw with \(V = 20.3\)) and 525 pc (SDSS J1240-01, with \(V = 19.6\)).

5. SUMMARY AND FUTURE WORK

We have shown (both from theory and observation) that thermal emission from the accreting WD within an AM CVn binary is an important contributor to the observed optical emission. For the direct impact systems with the shortest orbital periods, the hot WD is visible when the hot spot is out of the line of sight. At longer orbital periods, the WD can either be seen when the disk is in a low state, or at even longer orbital periods (\(P_{orb} > 40\) min), the WD always dominates the accretion luminosity in the optical bandpass.

We compared our theoretical results to the temperature and brightness of observed systems. The accreting WD is the dominant contributor to the optical light for the direct impact accretors and constrains their distances. In particular, the expected brightness of the WD is so large for RX J0806 that it can only be in the direct impact phase if it is located at a distance of 20 kpc, far outside the Galactic disc. For \(P_{orb} = 20–40\) min, our predicted absolute magnitude of the WD is \(M_V \approx 11\). This allows for estimates of the distance to the two newly discovered AM CVn systems (2003aw and SDSSJ1240-01) in which the DB white dwarf is visible. The two outbursting systems (CR Boo and V803 Cen) with parallaxes have \(M_V \approx 10\) in quiescence, 1 magnitude brighter than our estimate for the WD. This suggests that even in the low state there is a disc contribution. For GP Com and CE-315, a combination of estimated effective temperature and known distance suggests that the optical continuum emission is dominated by emission from the WD, in agreement with our calculations. For these two systems we find consistency with a hot (high-entropy) donor. Whether this donor was born hot and has evolved at fairly constant entropy or is heated by ongoing processes is unknown but could be addressed by future theoretical work.

Our successful comparisons to observations gives us confidence to apply our work to predict just how faint these binaries can become. For example, either mass WD reaches \(M_V = 15\) at 5 Gyrs, and at 10 Gyrs, the absolute magnitudes are \(M_V > 20\). This will allow for meaningful constraints on the population density of such binaries from deep optical searches, both in the field (e.g. the 3 additional candidates found by SDSS, Anderson et. al. 2005) and in Globular Clusters.

Much work remains to be done on the theoretical side, the most important of which is to resolve better the nature and occurrence rate of He flashes at early stages in the evolution of these binaries. In addition to better constraining the remaining He mass on the accreting WD at late times, it would also allow us to predict the Ne novae rate and potentially constrain the population from the measurements of such events. Perhaps the most dramatic development would be the secure detection of non-radial oscillations in these WDs, allowing for measurements of both the WD rotation rate and accumulated He layer mass.

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