Rovibrational states of the H$_2$O–H$_2$ complex: An ab initio calculation

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All bound rovibrational levels of the H$_2$O–H$_2$ dimer are calculated for total angular momentum $J = 0$–5 on two recent intermolecular potential surfaces reported by Valiron et al. [J. Chem. Phys. 129, 134306 (2008)] and Hodges et al. [J. Chem. Phys. 120, 710 (2004)] obtained through ab initio calculations. The method used handles correctly the large amplitude internal motions in this complex; it involves a discrete variable representation of the intermolecular distance coordinate $R$ and a basis of coupled free rotor wave functions for the hindered internal rotations and the overall rotation of the dimer. The basis is adapted to the permutation symmetry associated with the para/ortho (p/o) nature of both H$_2$O and H$_2$ as well as to inversion symmetry. Dimers containing oH$_2$ are more strongly bound than dimers with pH$_2$, as expected, with dissociation energies $D_0$ of 33.57, 36.63, 53.60, and 59.04 cm$^{-1}$ for pH$_2$–pH$_2$, oH$_2$O–pH$_2$, pH$_2$O–oH$_2$, and oH$_2$O–oH$_2$, respectively, on the potential of Valiron et al. that corresponds to a binding energy $D_e$ of 235.14 cm$^{-1}$. Rovibrational wave functions are computed as well and the nature of the bound states in the four different dimer species is discussed. Converged rovibrational levels on both potentials agree well with the high-resolution spectrum reported by Weida and Nesbitt [J. Chem. Phys. 110, 156 (1999)]; the hindered internal rotor model that was used to interpret this spectrum is qualitatively correct. © 2011 American Institute of Physics.

I. INTRODUCTION

Collisions between water, H$_2$O, which is a predominant molecular species in interstellar clouds, and H$_2$, the most abundant molecule in the universe, play a critical role in important astrophysical phenomena. One such phenomenon, for example, involves the formation of the so-called H$_2$O masers, whereby the combination of long path lengths and inverted H$_2$O internal state distributions in regions of star formation yields intense, amplified spontaneous emission on rotational lines of H$_2$O. How such highly nonequilibrium rotational distributions of H$_2$O are formed is still not clear, but one putative mechanism is that this internal excitation is formed by collisions of hot H$_2$ gas outflows in star formation regions with H$_2$O in the interstellar medium. Essential to any first principles understanding of such maser formation processes will clearly be a detailed knowledge of the underlying bimolecular collision dynamics, which in turn requires a rigorously accurate understanding of the H$_2$O–H$_2$ intermolecular potential.

A second example from astrophysics is the recombination of H atoms to form H$_2$ on icy interstellar dust grains. This represents a critical step in balancing photofragmentation with molecule reformation in the interstellar medium, which requires an understanding of the large amplitude quantum mechanics of how the newly formed H$_2$ interacts with and ultimately dissociates from the H$_2$O ice surface itself. Indeed, the dynamics of such H$_2$–H$_2$O ice grain species represents internally excited, quasibound levels of the weakly bound H$_2$O–H$_2$ complex. This highlights another key area in which a quantitatively benchmarked intermolecular potential for H$_2$O–H$_2$ plays a critically important role.

As yet another example of terrestrial as well as interstellar relevance, there have been some recent measurements by Dick et al. on the temperature collisional broadening of the rotational lines of H$_2$O by H$_2$. These studies exploit high resolution millimeter wave absorption spectroscopy of H$_2$O in a cold H$_2$ gas chamber, where the accessible temperatures range from 20 to 200 K. By detailed least squares fits of these THz transitions to a pressure broadened line shape, they infer the quantum state resolved pressure broadening coefficient for collisions of H$_2$O with He and H$_2$ as a function of temperature. The broadening coefficients with He exhibit only a simple and modest dependence on temperature. However, the trend for collisions of H$_2$O with H$_2$ as the colliding gas is qualitatively different: it exhibits an inverse power law rise for $T_{coll}$ from 200 K down to 50 K, followed by an entirely unexpected, precipitous drop in the broadening rate below 50 K, with this behavior reiterated in all six rotational transitions studied. These results are currently somewhat controversial, but if they are borne out by further experimental confirmation, this would be strong indication of novel collisional dynamics between H$_2$O and H$_2$ at low temperatures. This speaks to the importance of having high quality ab initio potential surfaces for H$_2$O–H$_2$ collisional interactions, which in turn must be rigorously tested by benchmarking against accurate experimental data.

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comparison of fully converged energy levels from quantum dynamical calculations on high quality \textit{ab initio} potential surfaces against the known energy level structure obtained from high resolution spectroscopy of the weakly bound H$_2$O–H$_2$ complex.

Though there is much in the interstellar medium to motivate exploration into such fascinating collisional energy transfer dynamical phenomena, the H$_2$O–H$_2$ complex is both interesting and challenging in its own right in our world of chemical physics. For example, the quantum dynamics of large amplitude rearrangement of nuclei within the three atom (e.g., O + H$_2$) and four atom (e.g., H + H$_2$O) systems have only recently become possible in full dimensionality. However, this first required multiple years of efforts to (i) calculate high quality \textit{ab initio} potential surfaces, (ii) find suitably efficient analytical representations of these potentials as well as (iii) develop sufficiently powerful quantum mechanical tools for accurately predicting energy level structures and large amplitude motion in such quantum states. Critical to this process has been the availability of high quality experimental data for rigorously testing the accuracy of such predicted surfaces, for example, from cluster, crossed molecular beam, photofragmentation, or photodetachment studies.

One important step up in complexity arises from the intermolecular dynamics of 5 atom systems (e.g., four H atoms + one heavy O atom), which currently represent a state-of-the-art frontier for both theory and experiment. As a case of particular relevance to interstellar chemistry, we tackle the weakly bound 5 atom H$_2$O–H$_2$ complex. Even under reduced dimensionality constraints associated with the covalently intact HOH and HH bonds, this is a challenging problem, requiring both (i) a high quality \textit{ab initio} potential energy surface as well as (ii) the tools for extraction of quantum state energy levels on such a surface. Crucial to this work, therefore, has been the \textit{ab initio} computation of interaction energies by a number of groups, which provide a first glimpse of the weakly bonded interaction. Specifically, there are multiple global and local minima, one corresponding to a hydrogen-bonded structure with the H$_2$ being called the “proton donor” (in which the H$_2$ axis points along the C$_2$ axis of H$_2$O) as well as two “proton acceptor” sites (at which a OH points into the H$_2$ bond). Of special relevance to the calculations in this paper has been the recent discovery of a high level \textit{ab initio} surface by Valiron \textit{et al.}, which maps out the potential in full dimensionality (9D) as a function of all internal degrees of freedom. Furthermore, this 9D potential has been suitably averaged over quantum motion associated with the high frequency stretching and bending degrees of freedom, yielding a high quality 5D potential energy surface explicitly tailored to a given intramolecular H$_2$O(v$_1$, v$_2$, v$_3$) and H$_2$(v) vibrational state. This potential has already been used in classical and quantum scattering calculations of rotationally and vibrationally inelastic H$_2$O–H$_2$ collision sections and collisional line broadening. In conjunction with sophisticated large basis set variational methods designed for obtaining quantum energy levels of weakly bound complexes, it permits us to obtain detailed predictions of intermolecular rovibrational energy levels for H$_2$O–H$_2$. Finally, these results in turn can be compared with previous experimental results from Weida and Nesbitt on rovibrationally resolved diode laser spectroscopy of this weakly bound complex in the HOH bending region.

The first complete 5D potential for H$_2$O–H$_2$ computed in 1992 by Zhang \textit{et al.} turned out to be not so accurate by comparison with potential reported by Valiron \textit{et al.} The potential computed in 2004 by Hodges \textit{et al.} with rigid monomers in their vibrationally averaged geometries rather closely resembles the 5D ground state vibrationally averaged potential reported by Valiron \textit{et al.} We found it worthwhile to compute the rovibrational energy levels of the complex also on the potential reported by Hodges \textit{et al.}

The organization of this paper is as follows. Section II outlines the computational method, the two potential surfaces used, and the symmetry properties applied to simplify the calculations and in the comparison of the results with the experimental data. In Sec. III, we present and discuss our results and compare them with the high-resolution spectroscopic data by Weida and Nesbitt. Section IV summarizes our conclusions.

II. COMPUTATIONAL METHOD, POTENTIAL SURFACE, AND SYMMETRY

A. Method

The method used in the calculation of the rovibrational levels of H$_2$O–H$_2$ is similar to the method developed for the water dimer; it is described in detail in Ref. 25. It starts from the Hamiltonian of a rotating dimer consisting of two rigid monomers, expressed in body-fixed (BF) dimer coordinates:

\[ H = T_A + T_B + \frac{1}{2\mu_{AB} R^2} \left[ -\hbar^2 \frac{\partial^2}{\partial R^2} + J^2 + j_{AB}^2 - 2j_{AB} \cdot J \right] + V(R, \beta_A, \gamma_A, \beta_B, \alpha). \]  (1)

The H$_2$O and H$_2$ monomers are labeled \( A \) and \( B \), respectively. The BF frame has its \( z \)-axis along the vector \( R \) that points from the center of mass of \( A \) to that of \( B \). The coordinate \( R \) is the length of this vector and \((\Theta, \Phi)\) are the polar angles of \( R \) with respect to a space-fixed (SF) frame. The Euler angles \( \omega_A \equiv (\alpha_A, \beta_A, \gamma_A) \) define the orientation of H$_2$O with respect to the BF frame, \( \omega_B \equiv (\beta_B, \alpha_B) \) are the polar angles of the H$_2$ axis relative to this frame. The planar reference geometry with angles \( \omega_A = (0, 0, 0) \) and \( \omega_B = (0, 0) \) is shown in Fig. 1(a); it lies in the \( xy \)-plane. This structure corresponds to the global minimum in the potential surface used, see Sec. II B. The potential \( V(R, \beta_A, \gamma_A, \beta_B, \alpha) \) depends on the dihedral angle \( \alpha = \alpha_B - \alpha_A \) while \((\Phi, \Theta, \alpha_A)\) are the overall rotation angles of the complex.

The operator \( J \) represents the total angular momentum, \( j_{AB} = j_A + j_B \) is the sum of the monomer angular momenta, and \( \mu_{AB} = \mu_{H_2O} \mu_{H_2}/(\mu_{H_2O} + \mu_{H_2}) \) is the dimer reduced mass. The components of the angular momentum operators in Eq. (1) are defined relative to the BF frame. The monomer kinetic energy operators for H$_2$O and H$_2$ are given by

\[ T_A = A_\perp (j_{A\perp}^M)^2 + A_\parallel (j_{A\parallel}^M)^2 + A_\cap (j_{A\cap}^M)^2 \]  (2)
by adding a linear potential with slope $\zeta$. The choice of the parameter $\zeta$ was guided by minimizing the energy of the lower rovibrational states in full five-dimensional calculations with the lowest ten eigenfunctions $\chi_n(R)$ of the radial Hamiltonian with the potential $V_{\text{eff}}(R)$. The selected value is $\zeta = 10^{-4}$ atomic units = $21.947$ cm$^{-1}/\hbar_0$. In the final calculations we used 20 basis functions $\chi_n(R)$ obtained from the radial eigenvalue equation with this optimized parameter.

The angular basis functions are coupled products of symmetric rotor functions—Wigner functions $D_{m_1m_2}(\alpha, \beta, \gamma)^*$ (Ref. 31) and (Racah normalized) spherical harmonics $C_{jm}(\beta, \alpha)$. It is the same as the basis used in earlier work on the NH$_3$ dimer$^{32–34}$ the water dimer$^{25, 35, 38}$ and benzene dimer$^{39}$ except for the replacement of the symmetric rotor functions by spherical harmonics for the H$_2$ monomer $B$. The expression $\langle j_A m_A; j_B m_B | j_{AB} K \rangle$ is a Clebsch–Gordan coupling coefficient$^{31}$ The quantum numbers $J$ and $M$ are exact quantum numbers and are fixed.

The kinetic energy operator is diagonal in this basis, except for the monomer asymmetric rotor terms for H$_2$O, cf. Eq. (2), and the small off-diagonal Coriolis coupling terms in the operator $2j_{AB} \cdot J$, see Ref. 32. In contrast with some of the earlier work on the water dimer$^{25, 35, 38}$ these off-diagonal Coriolis coupling terms were explicitly included. This implies that the angular momentum projection $K$ on the dimer axis $R$ is not an exact quantum number.

The potential is expanded in the same type of angular functions as used in the basis in Eq. (4). Since the potential is invariant under overall rotation only functions with $J = 0$ are needed to expand it. The potential is written as

$$V(R, \omega_A, \omega_B) = \sum_{L_AK_A,L_B} v_{L_AK_A,L_B}(R) A_{L_AK_A,L_B}(\omega_A, \omega_B)$$

with expansion functions

$$A_{L_AK_A,L_B}(\omega_A, \omega_B) = (-1)^{L_A+L_B+L} \sum_{M_A M_B} \left( \begin{array}{ccc} L & L_B & L \\ M_A & M_B & 0 \end{array} \right) \times D^{L_A}_{M_A K_A}(\omega_A)^* C_{L_A M_B}(\omega_B)$$

and coefficients

$$v_{L_AK_A,L_B}(R) = \frac{(2L_A + 1)(2L_B + 1)(2L + 1)}{16\pi^2} \times \langle A_{L_AK_A,L_B}(\omega_A, \omega_B) | V(R, \omega_A, \omega_B) \rangle.$$  

With this expansion all angular matrix elements of the potential over the basis in Eq. (4) can be expressed in terms of $3-j$ and $9-j$ symbols.$^{25}$

Tests of the convergence of the bound energy levels in full 5D calculations showed that the increase of $j_{\text{max}}$ from 8 to 10 and of $j_B_{\text{max}}$ from 6 to 8 lowers the bound energy levels by about $10^{-7}$ to $10^{-5}$ cm$^{-1}$. The levels of dimers with OH$_2$ and OH$_3$ are the most sensitive, which is not surprising since they require basis functions with odd $k_A$ and $j_B$ that are effectively truncated at maximum values of $7$ and $5$ when $j_{\text{max}} = 8$ and $j_{\text{max}} = 6$, for instance. In the final calculations we chose $j_{\text{max}} = 10$ for H$_2$O and $j_{B_{\text{max}}} = 8$ for H$_2$. The error introduced by using a contracted radial basis of 15 functions $\chi_n(R)$

\begin{align*}
T_B &= B j_B^2. \\
\text{The parameters } A_A, A_Y, A_Z &\text{ are the rotational constants of H}_2O \text{ and } B \text{ is the rotational constant of } H_2. \text{ The superscript MF (molecule-fixed) implies that } x, y, \text{ and } z \text{ refer to the components of } j_A \text{ along the principal axes of the H}_2O \text{ monomer. We use the ground state experimental values for the rotational constants, i.e., } A_x = A_0 = 27.8806 \text{ cm}^{-1}, A_Y = C_0 = 9.2778 \text{ cm}^{-1}, \text{ and } A_Z = B_0 = 14.5216 \text{ cm}^{-1} \text{ for H}_2O \text{ (Ref. 27) and } B = B_0 = 59.3398 \text{ cm}^{-1} \text{ for } H_2. \text{ The atomic masses are 1.007 825 u for H and 15.994 915 u for O.}

The basis in which the eigenstates of the Hamiltonian are computed is

$$|n, j_A, k_A, j_B, j_{AB}, K; J, M\rangle = \chi_n(R) \left( \frac{(2j_A + 1)(2j_B + 1)(2J + 1)}{128\pi^4} \right)^{1/2} \times \sum_{m_A m_B} D_{m_A k_A}(\alpha) C_{j A m_B}(\omega_B) \times \langle j_A m_A; j_B m_B | j_{AB} K \rangle D^{(J)}_{M K}(\Phi, \Theta, 0)^*.$$  

The radial basis functions $\chi_n(R)$ are computed by a sinc function discrete variable representation (DVR) (Refs. 29 and 30) on an equidistant grid of 96 points in the range from $R = 4$ to $26 a_0$. At first, a one-dimensional radial Schrödinger equation was solved with a potential $V_0(R)$ that is a cut of the full five-dimensional potential surface through the minimum at angles $(\beta_A, \gamma_A, \beta_B, \alpha) = (0, 0, 0, 0)$. This potential has only four bound states, however, and we need a complete set of radial basis functions, so we constructed an alternative radial potential

$$V_{\text{eff}}(R) = V_0(R) + \zeta R.$$
instead of the full DVR grid was found to be about $10^{-4}$ cm$^{-1}$; in the final calculations we included 20 contracted radial basis functions. The radial grid had to be extended to the rather large $R$ value of 26 $a_0$ in order to also converge bound states close to the dissociation limit. The dimension of the Hamiltonian matrices with the symmetry-adapted basis for $0 \leq J \leq 5$ ranges from about 20 000 to more than 160 000. The lowest 20 eigenvalues and eigenvectors of these matrices were obtained by means of the iterative algorithm of Davidson.40

B. Potential surface

A potential surface that includes all nine internal degrees of freedom of H$_2$O–H$_2$ was calculated ab initio by Valiron et al.16 with the use of the CCSD(T)-R12 method (coupled-cluster with singles, doubles, and perturbative triples, explicitly correlated). In the same paper they also presented a 5D potential surface that includes only the five intermolecular degrees of freedom. The 5D potential was obtained by averaging the 9D potential over the ground state vibrational wave functions of H$_2$O and H$_2$. The global minimum in this potential corresponds to the planar geometry depicted in Fig. 1(a). According to Ref. 16 a local minimum occurs for the non-planar geometry shown in Fig. 1(b). Both of these geometries may be considered as hydrogen bonded: in Fig. 1(a) the H$_2$ monomer is the donor and H$_2$O the acceptor and in Fig. 1(b) H$_2$O is the donor and H$_2$ the acceptor.

Figure 2 shows a two-dimensional cut of the 5D potential for planar geometries as a function of the orientations of the H$_2$O symmetry axis and the H$_2$ bond axis. At each angular point the energy was minimized by optimizing $R$. This cut passes through the global minimum. Starting from the global minimum in Fig. 1(a) there is a narrow valley that slowly rises toward a planar geometry which resembles the local minimum in Fig. 1(b), except that the dihedral angle $\alpha = 0$, whereas it is 90° at the local minimum. As one will see below, the bound state wave functions tend to follow this valley, even though the intermolecular zero point vibrational energy is so high that they become strongly delocalized.

Valiron et al.16 expressed their 5D potential in the form of an expansion,

$$V(R, \theta, \phi, \theta', \phi') = \sum_{l_1 m_1 l_2} v_{l_1 m_1 l_2}(R) \tilde{T}_{l_1 m_1 l_2}(\theta, \phi, \theta', \phi').$$

(9)

The expansion functions are

$$\tilde{T}_{l_1 m_1 l_2}(\theta, \phi, \theta', \phi') = \alpha_{l_1 m_1 l_2}(1 + \delta_{m_1 0})(l_1)_{r_1} (l_2)_{r_2} Y_{l_1 r_1}(\theta', \phi') Y_{l_2 r_2}(\theta, \phi)$$

$$\times \left[ \delta_{m_1 r_1} + (-1)^{l_1 + l_2 + \frac{1}{2}} \delta_{-m_1 r_1} \right]$$

(10)

with the normalization factor

$$\alpha_{l_1 m_1 l_2} = \left[ 2(1 + \delta_{m_1 0})^{-1}(2l_1 + 1)^{-1} \right]^{-1/2}$$

(11)

and $m_1 \geq 0$. Their coordinates are defined relative to a frame fixed to the H$_2$O monomer with the origin at the H$_2$O center of mass, the $z$-axis along the twofold symmetry axis with O at positive $z$, and the molecule lying in the $xz$-plane. The coordinates $(\theta, \phi)$ are the polar angles of the intermolecular vector $\mathbf{R}$ and $(\theta', \phi')$ are the polar angles of the H$_2$ axis. The coefficients in the expansion of Eq. (9) were obtained by least squares fitting to a number of ab initio calculated interaction energies. They were cubic spline interpolated as functions of $R$ from $R = 3$ to $15 a_0$ and extrapolated to a larger $R$ with the correct asymptotic $R^{-6}$ behavior. Only 149 expansion coefficients $v_{l_1 m_1 l_2}(R)$ were needed to produce an accurate representation of the potential.

The angular coordinates of the potential in Eqs. (9) and (10) are different from the angular coordinates that describe the orientations of the monomers in our BF frame. In our BF
frame the orientation of the H₂O monomer is given by the Euler angles \((\alpha_\text{H}, \beta_\text{A}, \gamma_\text{A})\) and the inverse Euler rotation over these angles rotates it back to the orientation that it has in the H₂O fixed frame of Valiron et al. It follows easily that the polar angles \((\theta, \phi)\) of \(\mathbf{R}\) in the latter frame are equal to the angles \((-\beta_\text{A}, -\gamma_\text{A})\) or, equivalently, \((\beta_\text{A}, \pi - \gamma_\text{A})\) in our BF frame. With the explicit use of the inverse Euler rotation matrix also the angles \((\theta', \phi')\) can be expressed in terms of our Euler angles \((\alpha_\text{B}, \beta_\text{A}, \gamma_\text{A})\) and \((\alpha_\text{B}, \beta_\text{B})\). Then, the potential can be calculated in terms of our angular coordinates.

It is also possible, however, to derive directly the relation between the expansion functions in Eq. (10) and those in Eq. (7). We show this by starting with the expansion functions in general SF coordinates, see Ref. 32,

\[
A_{L_A K_A L_B L} (\omega_A^\text{SF}, \omega_B^\text{SF}, \Theta, \Phi) = (-1)^{L_A + L_B + L} \sum_{M_A M_B M} \left( L_A \atop M_A \right) \left( L_B \atop M_B \right) L \left( \Theta, \Phi \right) \times D^{L_{A+1}}_{M_A K_A} (\omega_A^\text{SF}) C_{L_B M_B} (\omega_B^\text{SF}) C_{LM} (\Theta, \Phi).
\]

(12)

The angles \(\omega_A^\text{SF}\) and \(\omega_B^\text{SF}\) define the orientations of the monomers with respect to the SF frame. The potential is a scalar function and, hence, the expansion functions are coupled to a scalar. In other words, as written above, we use coupled angular basis functions with \(J = 0\) in the expansion of the potential. Therefore, any frame rotation leaves the form of the expansion functions unaltered and the functions in different frames are simply obtained by substitution of the angular coordinates for each frame. In our BF frame the monomer angles are \((\omega_A, \omega_B) \equiv (\omega_A^\text{BF}, \omega_B^\text{BF})\) and the polar angles \((\Theta, \Phi)\) of the vector \(\mathbf{R}\) are \((0, 0)\). Use of the relation \(C_{LM}(0, 0) = \delta_{M0}\) produces the expansion functions of Eq. (7). In the H₂O fixed frame the angles \(\alpha_\text{A}\) are \((0, 0, 0)\), the angles \(\alpha_\text{B}\) are \((\theta', \phi')\), and the polar angles \((\Theta, \Phi)\) of \(\mathbf{R}\) were called \((\theta, \phi)\). Then one can use that \(D^{L_{A+1}}_{M_A K_A}(0, 0, 0) = \delta_{M_A K_A}\), which yields the expansion functions in the H₂O fixed frame

\[
(-1)^{L_A + L_B + L} \sum_{M_A M_B M} \left( L_A \atop K_A \right) \left( L_B \atop M_B \right) L \left( \Theta, \Phi' \right) C_{LM} (\Theta, \Phi).
\]

(13)

Comparing this with Eq. (10) and taking into account the different normalizations of the spherical harmonics \(C_{lm}\) and \(Y_{lm}\), one finds the relation between our expansion coefficients in Eq. (6) and those in Eq. (9) that define the potential of Valiron et al.,

\[
v_{L_A K_A L_B L}(\mathbf{R}) = \begin{cases} (-1)^{l_1 + l_2 + 1} f_{1, m_1, l_2} v_{1, m_1, l_2}(\mathbf{R}), & \text{for } K_A \geq 0 \\ (-1)^{m_1} f_{1, m_1, l_2} v_{1, -m_1, l_2}(\mathbf{R}), & \text{for } K_A < 0 \end{cases}
\]

(14)

with factors

\[
f_{1, m_1, l_2} = \frac{((2l_1 + 1)(2l_2 + 1)(2l_1 + 1)(1 + \delta_{m_1 0})/2)^{1/2}}{4\pi}
\]

and \(L_A = l_1, K_A = \pm m_1, L_B = l_2, L = l\).

Another potential for H₂O–H₂ that includes all five intermolecular degrees of freedom was calculated by Hodges et al.\(^\text{24}\) with the use of scaled perturbation theory. This potential is not very different from the 5D ground state averaged potential of Valiron et al. It also has a global and a local minimum for dimer structures similar to those in Figs. 1(a) and 1(b). The binding energy \(D_s\) is slightly larger, 241.18 \(\text{cm}^{-1}\) at \(R_s = 5.77 a_0\) against 235.14 \(\text{cm}^{-1}\) at \(R_s = 5.82 a_0\) for the potential of Valiron et al., while the depth of the local minimum is 214.48 \(\text{cm}^{-1}\) at \(R = 5.96 a_0\) and \(\beta_A = 117^\circ\) instead of 199.40 \(\text{cm}^{-1}\) at \(R = 6.07 a_0\) and \(\beta_A = 119^\circ\).

Before we can use this potential to calculate the rovibrational states of H₂O–H₂, it must be transformed to center-of-mass coordinates—Hodges et al. placed the H₂O origin on the O atom—and expanded according to Eq. (6). The angular integrals in the expansion coefficients given by Eq. (8) were evaluated numerically for each value of \(R\) by Gauss–Legendre quadrature for the angles \(\beta_A, \beta_B\) and Gauss–Chebyshev quadrature for the angles \(\gamma_A, \alpha\). All expansion coefficients \(v_{L_A K_A L_B L}(R)\) up to maximum \(L_A, L_B\) values of 12, 6 and \(L = 18\) were included. The quadrature grid contained 13 points for \(\beta_A\), 7 points for \(\beta_B\), and 26 points for \(\gamma_A\) and for \(\alpha\). This expansion of the potential of Hodges et al. represents it around the global and local minima with an accuracy better than 0.01 \(\text{cm}^{-1}\), in the long range the relative accuracy is even better, and in the repulsive region at \(R = 4.5 a_0\) the expansion differs from the original potential by about 1%. Most of the results in Sec. III were obtained with the potential reported by Valiron et al. because we believe its quality is somewhat higher; some results obtained with the potential reported by Hodges et al. are given for comparison.

C. Symmetry

The permutation–inversion (PI) or molecular symmetry group\(^\text{41}\) of the H₂O–H₂ dimer with rigid monomers is isomorphic with the point group \(D_{2h}\) and may be denoted as \(G_8 \equiv D_{2h}(M)\). It is generated by the permutation \(P_1\) that interchanges the H nuclei in H₂O, the permutation \(P_3\) that interchanges the H nuclei in H₂, and inversion \(E^*\). Inversion has the following effect on the basis defined in Eq. (4),

\[
E^*[n, j_A, k_A, j_B, j_{AB}; K, J, M] = (-1)^{j_A+j_B-j_{AB}+k_A} [n, j_A, -k_A, j_B, j_{AB}, -K; J, M].
\]

(16)

Basis functions with even/odd values of \(k_A\) are even/odd under the permutation \(P_{12}\), basis functions with even/odd values of \(j_B\) are even/odd under \(P_{34}\). Irreducible representations (irreps) of \(G_8\) that are even and odd under \(P_{34}\) are labeled \(A\) and \(B\), respectively, and irreps that are even or odd under \(P_{12}\) get the subscripts 1 or 2. The superscript \(+/-\) denotes the even/odd parity under \(E^*\). We adapted the basis to the irreps of \(G_8\) and performed calculations for each irrep separately, which yields a substantial reduction of the computer time needed. Table I shows the relation between the \(G_8\) irreps and the quantum numbers \(k_A\), which determines the para/ortho (p/o) H₂O nature of the states, and \(j_B\), which determines whether the states belong to para or ortho H₂. The corresponding nuclear spin statistical weights are also listed. In our analysis of the rovibrational states we found it
TABLE I. Irreducible representations of $G_A$, quantum numbers $k_A$ and $j_B$ relevant for symmetry, para/ortho (p/o) nature of the monomers in H$_2$O–H$_2$, and nuclear spin statistical weights.

<table>
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<th>irrep</th>
<th>$k_A$</th>
<th>H$_2$O</th>
<th>$j_B$</th>
<th>H$_2$</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
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<td>$A^+_1$</td>
<td>even</td>
<td>$p$</td>
<td>even</td>
<td>$p$</td>
<td>1</td>
</tr>
<tr>
<td>$A^+_2$</td>
<td>even</td>
<td>$p$</td>
<td>even</td>
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</tr>
<tr>
<td>$A^+_3$</td>
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<td>$o$</td>
<td>even</td>
<td>$p$</td>
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<tr>
<td>$A^-_1$</td>
<td>odd</td>
<td>$o$</td>
<td>even</td>
<td>$p$</td>
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</tr>
<tr>
<td>$B^+_1$</td>
<td>even</td>
<td>$p$</td>
<td>odd</td>
<td>$o$</td>
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</tr>
<tr>
<td>$B^+_2$</td>
<td>even</td>
<td>$p$</td>
<td>odd</td>
<td>$o$</td>
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</tr>
<tr>
<td>$B^-_1$</td>
<td>odd</td>
<td>$o$</td>
<td>odd</td>
<td>$o$</td>
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</tr>
<tr>
<td>$B^-_2$</td>
<td>odd</td>
<td>$o$</td>
<td>odd</td>
<td>$o$</td>
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III. RESULTS, COMPARISON WITH EXPERIMENT

A. Energy levels

Tables II–V show the energy levels calculated for $J = 0$–$5$. They contain all bound levels for these values of $J$; blank entries in the tables imply that the corresponding states are not bound. Although $K$ is not an exact quantum number one can see in these tables that many states can be labeled with specific (approximate) $K$ values. States with $K = 0$, $K = 1$, and $K = 2$ are called $\Sigma$, $\Pi$, and $\Delta$, respectively. Since we did not find any bound levels with $K$ values higher than 2, we omitted basis functions with $K$ higher than 3 from the calculations for $J = 4$ and 5. Tests for $J = 3$ in which we omitted the basis functions with $K = 3$ show that such a restriction of the basis causes negligibly small errors if the lowest $K$ omitted from the basis exceeds the highest $K$ value of the bound states by at least 2. The percentage of $\Sigma$, $\Pi$, or $\Delta$ character of a state was obtained by summation of the squared eigenvector components with a given $K$ over all other basis set labels. In addition we extracted effective $m_A$ and $m_B$ values for each $K = m_A + m_B$ from the eigenvectors by transforming the coefficients of the coupled basis functions in Eq. (4) to an uncoupled product basis labeled by $m_A$ and $m_B$.

In Table II one observes that only $\Sigma$ states of spectroscopic parity $e$ are bound for pH$_2$O–pH$_2$. The dissociation energy $D_0 = 33.57 \text{ cm}^{-1}$ is much smaller than the binding energy $D_e = 235.14 \text{ cm}^{-1}$. Thus, the zero-point energy due to the intermolecular vibrations is a large fraction of the binding energy. As one will see below, the corresponding ground state wave function is strongly delocalized. This is, of course, a consequence of the small mass and large rotational constant of H$_2$, but also the rotational constants of H$_2$O are relatively large compared to those of most molecules. Just one excited intermolecular vibrational state is bound for pH$_2$O–pH$_2$, although barely and only for $J \leq 2$. When looking at the wave functions, below, it becomes clear that it is the intermolecular stretch mode that is excited. The end-over-end rotational constant $B$ of the complex that can be extracted from the ground state levels for $J = 0$–3 is 0.6923 cm$^{-1}$, see Table VI. The value of $B$ for the excited state is much smaller. Obviously the excited state is much more diffuse, which is not surprising since it is stretch excited and close to dissociation.

For H$_2$O–pH$_2$ more states are bound, see Table III, of both parities $e$ and $f$. The ground state of this species has $\Sigma$ character. Since H$_2$O and pH$_2$ have different nuclear spin states and the monomer nuclear spins are conserved at the time scale of the experiment the ground state of H$_2$O–pH$_2$ dimer dissociates into H$_2$O with $j_A = 1$, $k_A = \pm 1$ and energy 23.7994 cm$^{-1}$ and ground state pH$_2$. The corresponding dissociation energy $D_0$ is 36.63 cm$^{-1}$, slightly higher than for pH$_2$O–pH$_2$. The projection $m_A$ of the monomer angular momentum $j_A$ on the intermolecular axis $R$ is (approximately) 0 and also the total angular momentum projection $K = m_A + m_B$ equals 0. About 6 cm$^{-1}$ above the $\Sigma$ ground state there is a $\Pi$ state with $K = 1$. From the contributions $m_A$ and $m_B$ of the individual monomers to the $K = 1$ value we conclude that $m_A$ is approximately 1 in this $\Pi$ state, while $m_B$ is nearly zero. This is naturally related to the $j_A = 1$ and $j_B = 0$ nature of the H$_2$O and pH$_2$ monomers from which this dimer is formed. The $\Pi$ levels occur in pairs, of $e$ and $f$ parities. In our calculations the parity splittings originate from off-diagonal Coriolis terms in the Hamiltonian that couple the $K = 1$ levels with $K = 0$ levels. If one considers the dimer as a (prolate) near-symmetric rigid rotor this splitting is the asymmetry doubling. One can see in Table III that $K$ becomes more strongly mixed for some of the excited states.

TABLE II. Rovibrational levels of pH$_2$O–pH$_2$ (in cm$^{-1}$), dissociation limit 0, $D_0 = 33.57 \text{ cm}^{-1}$. All states have more than 99% $\Sigma$ character. The parity $e/f$ is the spectroscopic parity.

<table>
<thead>
<tr>
<th>$\Sigma(K = 0)$</th>
<th>Parity</th>
<th>Energy</th>
</tr>
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<tr>
<td>$J = 0$</td>
<td>$</td>
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</tr>
<tr>
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<td>$</td>
<td>e</td>
</tr>
<tr>
<td>$J = 2$</td>
<td>$</td>
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<tr>
<td>$J = 5$</td>
<td>$</td>
<td>e</td>
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<td>f</td>
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<td>$J = 1$</td>
<td>$</td>
<td>f</td>
</tr>
<tr>
<td>$J = 2$</td>
<td>$</td>
<td>f</td>
</tr>
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</table>
The amount of mixing depends on the occurrence of $K = 0$ and $K = 1$ levels near to each other.

The number of bound levels for pH$_2$O–oH$_2$ is about the same as for oH$_2$O–pH$_2$, cf. Tables IV and III. Again, the ground state has $\Sigma$ character. The ground state pH$_2$O–oH$_2$ dimer dissociates into ground state ($j_A = 0$) pH$_2$O and oH$_2$ with $j_B = 1$, energy 118.6796 cm$^{-1}$, and its dissociation energy $D_0$ with respect to this asymptotic limit is 53.60 cm$^{-1}$. This value of $D_0$ is substantially higher than for oH$_2$O–pH$_2$ and pH$_2$O–pH$_2$. The lowest $\Pi$ state is now about 15 cm$^{-1}$ higher than the $\Sigma$ ground state. All of this indicates that the H$_2$ monomer likes to align itself parallel to the intermolecular axis $\mathbf{R}$, rather than perpendicular to it (as it is in the $\Pi$ state) or in a near-spherically averaged orientation (as it is in the dimers with pH$_2$). This provides the most attractive configuration of the H$_2$O dipole and the (positive) H$_2$ quadrupole and yields the strongest hydrogen bond with H$_2$ as the donor and H$_2$O as the acceptor. The $e$–$f$ parity splitting (asymmetry doubling) of all $\Pi$ states in pH$_2$O–oH$_2$ is considerably smaller than in oH$_2$O–pH$_2$. Table V shows that oH$_2$O–oH$_2$ has by far the largest number of bound states of all species. Also the dissociation energy $D_0$ of 59.04 cm$^{-1}$ with respect to the asymptotic energy of 23.7994 + 118.6796 = 142.4790 cm$^{-1}$ of $(j_A = 1, k_A = \pm 1)$ oH$_2$O and $(j_B = 1)$ oH$_2$ is the highest. This is, of course, related with the fact that both monomers in their ortho states with $j_A = 1$ and $j_B = 1$ can be aligned and adopt the most favorable structure. Also the occurrence of $\Delta$ states with $K = m_A + m_B = 2$ can thus be understood; we checked that these correspond to projection angular momenta $m_A = 1$ and $m_B = 1$ of both monomers. Another feature by which this species is different from all others is that the ground state has $\Pi(K = 1)$ rather than $\Sigma(K = 0)$ character. In this $\Pi$ state it is the oH$_2$O monomer that has $m_A = 1$, while the oH$_2$ monomer has $m_B = 0$. Again, this can be understood by looking at the average orientations of oH$_2$O in its rotational state with $j_A = 1, |k_A| = 1, |m_A| = 1$ and of oH$_2$ in its rotational state with $j_B = 1, m_B = 0$. We already mentioned that the oH$_2$ monomer is mostly aligned along the bond axis $\mathbf{R}$ when $m_B = 0$. The orientation of the oH$_2$O monomer
A. van der Avoird and D. J. Nesbitt


TABLE V. Rovibrational levels of oH2O–oH2 (in cm⁻¹), dissociation limit 142.4790 cm⁻¹, and D₀ = 59.04 cm⁻¹. In parentheses the Σ, Π, or Δ character; if not indicated it is higher than 99%.

<table>
<thead>
<tr>
<th>J = 0</th>
<th>J = 1</th>
<th>J = 2</th>
<th>J = 3</th>
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<td>138.7024</td>
<td>140.5341</td>
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<td>141.6250</td>
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Δ(Κ = 2)

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Δ(Κ = 3)

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</table>

For J ≥ 3 this state becomes dominant Π character: 50%, 51%, 52% for J = 3, 4, 5, respectively.

For J = 5 this state becomes dominant Π character: 51%.

For J ≥ 3 this state becomes dominant Σ character: 48%, 46%, 43% for J = 3, 4, 5, respectively.

For J = 5 this state becomes dominant Σ character: 57%.

is more difficult to visualize, but it is clear that the rotational angular momentum vector j_A of oH2O with j_A = 1, |k_A| = 1 is nearly parallel to its C₂ symmetry axis. The vector j_A, and therefore also the C₂ axis of H₂O, is best aligned with the BF z-axis R when |m_A| = 1. Hence, the rotational state of oH₂ with j_A = 1, |k_A| = 1, |m_A| = 1 provides the most favorable orientation for the oH₂O monomer to act as the acceptor in the hydrogen bond with (aligned) H₂. The lowest Σ state is higher than the ground Π state by about 10 cm⁻¹ and the Δ state is higher by about 27 cm⁻¹. It is interesting that our conclusion that in oH₂O–oH₂ the Π state is the ground state and the lowest Σ state is higher in energy was already predicted from a simple model by Weida and Nesbitt.²³

Table VI lists the rotational constants of H₂O–H₂ that can be extracted from the levels calculated for J = 0–3 by fitting them in the same way as the experimentally determined levels in Ref. 23. As a brief reminder of this analysis procedure, a single Σ ← Σ band was observed and assigned to pH₂O–pH₂, whereas two bands out of the same lower state (Π ← Π and Σ ← Π) were observed for the oH₂O–oH₂ species. The lack of experimental evidence for either oH₂O–pH₂ or pH₂O–pH₂ complexes with pH₂ was attributed to differential stabilization of the oH₂ versus pH₂ dimers, which from detailed balance considerations leads to exponentially different equilibrium constants at the low temperatures of a supersonic expansion. The 11 cm⁻¹ proximity between the two upper states for the oH₂O–oH₂ species suggested the importance of significant Coriolis interactions. This was taken into account via a simple 2 × 2 Hamiltonian in the unperturbed Σ, Π state basis,

$$H = \begin{pmatrix} E_{\Sigma}^{(J)} & \beta \sqrt{J(J+1)} \\ \beta \sqrt{J(J+1)} & E_{\Pi}^{(J)} \end{pmatrix},$$  (17)

<table>
<thead>
<tr>
<th></th>
<th>Valiron et al. potential</th>
<th>Hodges et al. potential</th>
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</thead>
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<td></td>
<td>$B$</td>
<td>$D$</td>
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<tr>
<td>oH$_2$O–pH$_2$</td>
<td>$\Sigma$</td>
<td>0.6923</td>
</tr>
<tr>
<td></td>
<td>$\Pi$</td>
<td>0.6719</td>
</tr>
<tr>
<td>oH$_2$O–oH$_2$</td>
<td>$\Sigma$</td>
<td>0.7085</td>
</tr>
<tr>
<td></td>
<td>$\Pi$</td>
<td>0.7024</td>
</tr>
</tbody>
</table>

$\Pi(K = 1)$ energy gap is comparable for oH$_2$O–pH$_2$ and oH$_2$O–oH$_2$. We apply an identical Coriolis analysis to theoretical predictions for both of these complexes, as summarized in Table VI. These fits to the theoretical data offer several interesting insights into the internal rotor dynamics. For example, the Coriolis coupling constants $\beta$ obtained can be compared with predictions in the simple free rotor limit. In this limit the monomer angular momenta $j_A$ and $j_B$, their projections $m_A$ and $m_B$ on the intermolecular axis $R$, and the projection $k_A$ of $j_A$ on the H$_2$O monomer $z$-axis are good quantum numbers. Let us remind the reader that $K = m_A + m_B$. The Coriolis coupling term $-2(j_A + j_B) \cdot J/2\mu_{AB}R^2$ in the Hamiltonian of Eq. (1) couples the $\Sigma(K = 0)$ and $\Pi(K = 1)$ levels through the shift operators,

$$-j_A \pm j_B \neq j_B \pm j_A \neq 2\mu_{AB}R^2. \tag{18}$$

The first term shifts $m_A$ and $K$ by $\pm 1$ and yields coupling matrix elements

$$-\left\langle \chi(R) \left| \frac{1}{2\mu_{AB}R^2} \chi(R) \right. \rightangle \times \sqrt{j_A(j_A + 1) - m_A(m_A \pm 1)} \sqrt{(J + 1) - K \pm 1} \tag{19}$$

between primitive uncoupled dimer basis functions with $j'_A = j_A, k'_A = k_A, j'_B = j_B, m'_A = m_A \pm 1, m'_B = m_B$, and $K = K \pm 1$. Similarly, the second term shifts $m_B$ and $K$. In the free internal rotor limit the radial and angular motions are decoupled, the $R$-dependent factor in the Coriolis coupling operator can be averaged over the radial wave function, and the radial matrix elements in Eq. (19) can be replaced by the end-over-end rotational constant of the dimer $B = (1/(2\mu_{AB}R^2))$.

As discussed above, the ground $\Sigma(K = 0)$ state of oH$_2$O–pH$_2$ is dominated by internal rotor basis functions with $j_A = 1, k_A = \pm 1, m_A = 0$, and $j_B = m_B = 0$. The lowest $\Pi(K = 1)$ state has similar approximate quantum numbers, except for $m_A$ and $K$ which are $\pm 1$. Substitution of these values into Eq. (19) shows that, in the free internal rotor limit, the coupling matrix element between normalized parity-adapted wave functions of the $\Sigma$ and $\Pi$ states becomes $-2B\sqrt{J(J + 1)}$, i.e., the Coriolis coupling constant equals $2B$. The values of $B$ for oH$_2$O–pH$_2$ are about 0.68 cm$^{-1}$, which yields $2B = 1.36$ cm$^{-1}$. The Coriolis coupling constant $\beta$ of 1.35 cm$^{-1}$ from the combined fit of the $\Sigma$ and $\Pi$ levels calculated for $J = 0$ is very close to this value. So we may conclude that oH$_2$O–pH$_2$ dimers behave close to the free internal rotor limit. It should be mentioned, however, that the error of the fit is relatively large in this case.

For oH$_2$O–oH$_2$ the approximate internal rotor quantum numbers of the ground $\Pi(K = 1)$ state are $j_A = 1, k_A = \pm 1, m_A = \pm 1, j_B = 1, m_B = 0$. The approximate quantum numbers of the lowest $\Sigma$ state are similar, except that $m_A = K = 0$. The free internal rotor limit with these values of the quantum numbers again yields a Coriolis coupling constant of $2B$. The value of $B$ is about 0.70 cm$^{-1}$ in this case, which gives $2B = 1.40$ cm$^{-1}$, while the fit of the calculated dimer levels yields a Coriolis coupling constant $\beta$ of 1.16 cm$^{-1}$. So, the actual value is nearly 20% smaller than the free internal rotor value, which indicates that the internal rotations in oH$_2$O–oH$_2$ are more strongly hindered than in oH$_2$O–pH$_2$.

The energy levels calculated with the potential of Hodges et al. are similar to those in Tables II–V. The wells in this potential are slightly deeper than in the potential of Valiron et al. and the rovibrational states are slightly lower in energy. This is reflected by the dissociation energies $D_0$, which are 36.11 cm$^{-1}$ for pH$_2$O–pH$_2$, 40.43 cm$^{-1}$ for oH$_2$O–pH$_2$, 57.07 cm$^{-1}$ for pH$_2$O–oH$_2$, and 62.71 cm$^{-1}$ for oH$_2$O–oH$_2$, i.e., about 7% higher than with the potential of Valiron et al. For some of the higher states with energies not far below the dissociation limit this implies that they remain bound up to a higher $J$ value than the levels in Tables II–V. Furthermore, it implies that there exists a (barely) bound state also for pH$_2$O–oH$_2$. The end-over-end rotational constants $B$ are slightly larger than with the potential of Valiron et al., see Table VI, in accordance with the somewhat smaller equilibrium distance $R_e$ in the potential of Hodges et al.

B. Wave functions

The character of the various bound states of the different nuclear spin species can be determined by looking at their wave functions. Wave function plots also give insight in the nature of the excited intermolecular vibrations. The ground state $J = 0$ wave function of pH$_2$O–pH$_2$ plotted as a function of $R$ at the equilibrium angles $\omega_A = (0, 0, 0)$ and $\omega_B = (0, 0)$ has a maximum near $R = 6.3 \alpha_0$. Angular plots
of the \( J = 0 \) wave functions at this \( R \) value for planar geometries are shown in Fig. 3. In these angular plots one can nicely see the different characteristics of the wave functions for para/ortho \( \text{H}_2\text{O} \) and para/ortho \( \text{H}_2 \) monomers. All these wave functions are for \( J = 0 \), so they all belong to \( \Sigma \) states with \( K = 0 \). The ground state wave function of \( \text{pH}_2\text{O}–\text{pH}_2 \) does not have any nodes. It is strongly delocalized, but still one can see clearly that it has its largest amplitude in the valley of the potential surface, cf. Fig. 2, that gradually rises from the global minimum in the direction of the local minimum. For \( \text{oH}_2\text{O}–\text{pH}_2 \) the wave function contains only functions with odd \( k_A \) and must have nodes at \( \beta_A = 0^\circ \) and \( 180^\circ \).

This restriction on the wave function pushes its maximum away from \( \beta_A = 0^\circ \) to a point further up into the potential valley. The wave function of \( \text{pH}_2\text{O}–\text{oH}_2 \) contains only functions with odd \( j_B \) and must change sign between \( \beta_B = 0^\circ \) and \( 180^\circ \). For \( \beta_A = 0^\circ \) and \( 180^\circ \) this implies that this wave function must have a node at \( \beta_B = 90^\circ \), as one can see. In Fig. 3 one can also see that this symmetry restriction tends to push the wave function of \( \text{pH}_2\text{O}–\text{oH}_2 \) away from the higher part of the potential valley toward the global minimum. The wave function of \( \text{oH}_2\text{O}–\text{oH}_2 \) must both have a node at \( \beta_A = 0^\circ \) and \( 180^\circ \) and change sign between \( \beta_B = 0^\circ \) and \( 180^\circ \). In the corresponding panel of Fig. 3 this is clearly visible.
A radial cut of the wave function of the second (barely) bound state of pH₂O–pH₂ for \( J = 0 \) shows that it has as a node in \( R \), so this state is radially excited. This implies that the intermolecular stretch has a fundamental frequency of 31.5 cm\(^{-1}\). The same holds for the lowest excited \( J = 0 \) parity \( e \) bound states of oH₂O–pH₂ and pH₂O–oH₂; the corresponding stretch frequencies are 34.8 and 45.0 cm\(^{-1}\), respectively. The ratio of these three stretch frequencies is well in line with the dissociation energies \( D_0 \) of the three dimer species, which are 33.57, 36.63, and 53.60 cm\(^{-1}\), in the same order. For oH₂O–oH₂ the situation is more complicated; all three excited states of parity \( e \) for \( J = 0 \) have a radial node. The second excited state seems to be mostly stretch excited, since it has considerable amplitude both inside and outside of the node. The first excited state has only little amplitude outside the radial node, and the third excited state has only small amplitude inside.

If we consider the second excited state as the stretch fundamental the corresponding stretch frequency is 43.8 cm\(^{-1}\), but it is clear that in oH₂O–oH₂ the radial excitations are more strongly mixed with excitations in the angular coordinates.

We saw in Tables II–V that H₂O–H₂ dimers except for pH₂O–pH₂ also have bound states of \( \Pi \) character, i.e., with approximately \( K = 1 \). For oH₂O–oH₂ the ground state is of \( \Pi \) type. Figure 4 shows the \( J = 1 \) wave functions of the
lowest $\Pi$ states for the three nuclear spin species that possess bound $\Pi$ states and the second $J = 0$ $\Sigma$ state for $\text{H}_2\text{O}−\text{pH}_2$. As discussed above, the latter is stretch excited and it is illustrated in the upper left panel of Fig. 4 that it does not have a node in the angular coordinates. There is some arbitrariness in the plots of $J = 1$ wave functions since they depend on the overall rotation angles ($\Phi, \Theta, \alpha_{\text{A}}$). We chose $M = 0$ and $\Theta = 90^\circ$, $\alpha_{\text{A}} = 0^\circ$, since the real part of the overall rotation wave function with $(J, M, K) = (1, 0, 1)$ has its maximum amplitude at these angles and does not depend on $\Phi$. The contour plots of the $\Pi$ states in Fig. 4 show typical differences with the corresponding plots in Fig. 3 for the $\Sigma$ states. One may notice, in particular, that the angles $\beta_{\text{A}}$ or $\beta_{\text{B}}$ where the nodal planes in the wave functions meet the edges of the contour plots are typically shifted by $90^\circ$ for the $\Pi$ states relative to the $\Sigma$ states. Again, one may observe that the amount of localization of the wave functions and the geometries where they have their maximum amplitudes are determined to some extent by the nodal plane structure imposed by the symmetry. This illustrates that the change of the overall rotation quantum number $K$ from 0 to 1 significantly affects the (hindered) internal rotations of the monomers.

In order to get an idea of the amplitude of the bound state wave functions in the region of the nonplanar metastable structure displayed in Fig. 1(b), we show plots (in Fig. 5) of the same wave functions as in Fig. 3 for geometries that start from a planar structure with the same coordinates $\beta_{\text{A}} = 119^\circ$, $\gamma_{\text{A}} = 0^\circ$, $\beta_{\text{B}} = 90^\circ$ as the metastable geometry. The system is planar when $\alpha = 0^\circ$ and arrives at the local minimum in the potential when $\alpha = 90^\circ$. One observes that the wave functions of $\text{pH}_2$ complexes have considerable amplitude at the local minimum, slightly higher than at the corresponding planar structure and not much lower than the amplitude at their maximum, cf. Fig. 3. The amplitudes of the wave functions of $\text{oH}_2$ complexes are substantially smaller in this region due to the neighborhood of a nodal plane. One can see in Fig. 5 that this nodal plane actually passes through the local minimum at $\alpha = 90^\circ$, as it must because of symmetry requirements. This does not imply, however, that the presence of a relatively deep local minimum is less important for the properties of the $\text{oH}_2$ complexes.

Wave functions of spectroscopic parity $f$ have a nodal plane for planar geometries, including the equilibrium geometry. For this reason the $f$ states with $K = 0$ may be considered as excited out-of-plane intermolecular vibrations, but it should be realized that they have rather low energies and look more like hindered rotations. As already mentioned, the $e/f$ parity doublets for $K > 0$ are split by (off-diagonal) Coriolis coupling between the internal and overall rotations of the dimer but may also be considered as asymmetry doublets if we regard the dimer as a prolate near-symmetric rotor.

**C. Comparison with experiment**

A high resolution spectrum of the $\text{H}_2\text{O}−\text{H}_2$ complex was measured in the region of the $\nu_1$ bend origin of $\text{H}_2\text{O}$ at 1594.7 cm$^{-1}$.23 In scans from 1582 to 1622 cm$^{-1}$ three bands were observed which were all assigned to transitions in the two dimer species containing $\text{oH}_2$. The band between 1589 and 1603 cm$^{-1}$ corresponds to $\Sigma \leftrightarrow \Sigma$ transitions in $\text{pH}_2\text{O}−\text{oH}_2$ and the band between 1590 and 1594 cm$^{-1}$ corresponds to $\Pi \leftrightarrow \Pi$ transitions in $\text{oH}_2\text{O}−\text{pH}_2$. The end-over-end rotational and distortion constants extracted from the rotationally resolved band for $\text{pH}_2\text{O}−\text{oH}_2$ are $B = 0.7182$ cm$^{-1}$, $D = 3.57 \times 10^{-4}$ cm$^{-1}$, and $H = 8.9 \times 10^{-7}$ cm$^{-1}$. From a fit of the levels calculated for $J = 0$ to 5 we obtain $B = 0.7080$ cm$^{-1}$, $D = 3.75 \times 10^{-4}$ cm$^{-1}$, and $H = 10.4 \times 10^{-7}$ cm$^{-1}$, in very good agreement with the experimental data. The fact that even the distortion constants agree so well implies in particular that the long range part of the potential surface of Valiron et al. is accurate.

The band between 1590 and 1594 cm$^{-1}$ assigned to the $\Pi \leftrightarrow \Pi$ transitions in $\text{oH}_2\text{O}−\text{oH}_2$ shows $e/f$ parity splittings that are inconsistent with a semirigid rotor behavior of the complex. Hence, the transitions in this band were analyzed23 with the aid of a model that includes the hindered internal rotations. In such a model the parity splittings between the $\Pi$ levels of parity $e/f$ are due to Coriolis coupling of the $\Pi$ states to nearby $\Sigma$ states, as in our calculations. Also $\Sigma \leftrightarrow \Pi$ transitions in combination with the $\nu_3$ monomer transitions were predicted on the basis of this model and were actually found at frequencies between 1603 and 1622 cm$^{-1}$. An analysis of all the experimental data gave rotational constants $B$ of 0.7055 and 0.7113 cm$^{-1}$ for the $\Sigma$ and $\Pi$ states, respectively. From the same type of analysis of our levels calculated with the potential of Valiron et al. we obtain 0.7013 and 0.7024 cm$^{-1}$. The observed energy difference between the lowest $\Sigma$ and $\Pi$ energy levels in $\text{oH}_2\text{O}−\text{oH}_2$ is 10.55 cm$^{-1}$, while our calculations yield 10.93 cm$^{-1}$. Thus, our calculations not only reproduce the correct ordering of the $\Pi$ and $\Sigma$ states in $\text{oH}_2\text{O}−\text{oH}_2$ but also the energy gap between them is very realistic, which in turn depends sensitively on the anisotropy of the potential surface. Also the Coriolis coupling
constant of the $\Pi$ and $\Sigma$ levels could be extracted: the mea-
sured data yield 1.166 cm$^{-1}$ and we calculated 1.156 cm$^{-1}$. A
fit of the levels calculated with the potential of Hodges et al.
gave rotational constants $B$ of 0.7171 and 0.7156 cm$^{-1}$ for the
$\Sigma$ and $\Pi$ states, respectively, an energy gap of 10.06 cm$^{-1}$
between those states, and a Coriolis coupling constant of
1.158 cm$^{-1}$. Obviously, the agreement between our calcula-
tions with both potentials and experiment is very good, which
implies that both the potential surfaces of Valiron et al. and
of Hodges et al. are accurate.

IV. CONCLUSIONS

This paper describes the calculation of the rovibrational
states of the H$_2$O–H$_2$ dimer on two recently developed 5D
intermolecular potential surfaces obtained through high level
*ab initio* calculations. The variational method employed in-
volves a discrete variable representation of the intermolecular
distance $R$ and a basis of coupled free rotor wave functions
for the hindered internal rotations and the overall rotation of
the dimer. The basis is adapted to the permutation symme-
try associated with the para/ortho (p/o) nature of both H$_2$O
and H$_2$ as well as to inversion symmetry. All bound rovibra-
tional levels of the four different nuclear spin species: pH$_2$O–
pH$_2$, oH$_2$O–pH$_2$, pH$_2$O–oH$_2$, and oH$_2$O–oH$_2$, are computed
for both parities and $J = 0$–5.

We found that a large fraction of the binding energy $D_e$, which is 235.14 cm$^{-1}$ in the potential of Valiron et al. goes
into the intermolecular vibrational zero-point energy: the dis-
soociation energies $D_0$ on this potential are only 33.57, 36.63,
53.60, and 59.04 cm$^{-1}$ for pH$_2$O–pH$_2$, oH$_2$O–pH$_2$, pH$_2$O–
oH$_2$, and oH$_2$O–oH$_2$, respectively. Wave functions, which are
also calculated, show that the complex is very floppy and that
the intermolecular vibrations are more like hindered internal
rotations. Still, they show a marked preference of the system
to explore the low energy region of the potential that ex-
tends from the hydrogen-bonded structure at the global mini-
imum geometry, with the H$_2$ monomer as the donor and H$_2$O
as the acceptor, to the only slightly less stable local minimum
geometries, which are hydrogen-bonded with H$_2$O as the
donor and with H$_2$ as the acceptor. The observation that ortho
monomers lead to stronger binding than para monomers is
not surprising, but in excellent confirmation of experiment. Specifically, pH$_2$O and pH$_2$ can only orient themselves un-
der the influence of the anisotropic interaction potential when
their $j = 0$ ground states mix with excited rotational states.
By way of contrast, however, both oH$_2$O and oH$_2$ have de-
genarate $j = 1$ ground states in which the monomers can be
aligned even without any mixing. Especially for pH$_2$ with its
large rotational constant of nearly 60 cm$^{-1}$ and lowest excited
state with $j = 2$, this leads to a much smaller value of $D_0$ than
for oH$_2$. The ground state wave functions indeed show that
the monomers want to orient themselves as much as they can
for the given nuclear spin species, such that the dimer resem-
bles the most stable hydrogen-bonded structure with C$_2v$
symmetry. The wave functions have considerable amplitude also
at the higher lying local minima, however.

All four species have a bound excited intermolecular
stretch state, with fundamental frequencies that are not much
lower than the dissociation energies. The species that include
at least one ortho monomer also have excited hindered inter-
nal rotor states of different $\Sigma$, $\Pi$, and $\Delta$ character, i.e., with
$K = 0$, 1, and 2. The ground state is $\Sigma$, except for oH$_2$O–oH$_2$
where it is of $\Pi$ type. Also the greater stability of the $\Pi$ state
in oH$_2$O–oH$_2$ can be explained by considering the optimum
orientations of the monomers, as discussed above.

The converged rovibrational levels can be compared with
the high-resolution spectrum of the H$_2$O–H$_2$ dimer measured
by Weida and Nesbitt. Three bands were observed in this
spectrum, in the range of the H$_2$O monomer bond mode; all
transitions were assigned to pH$_2$O–oH$_2$ and oH$_2$O–oH$_2$. The
data that could be extracted from the measured spectrum and
compared with our calculated energy levels are the rotational and
distortion constants of both species and, for oH$_2$O–oH$_2$, the
energy gap between the $\Sigma$ and $\Pi$ states that are mixed by
Coriolis coupling, as well as the coupling constant. We find
that an agreement with experiment for all of these data for the
levels calculated on the potential of Valiron et al., which
confirms that this potential is accurate. Also for the levels cal-
culated on the somewhat older potential of Hodges et al. we
get good agreement, which is perhaps not so surprising since
the two potentials are quite similar. The advantage of the
potential of Valiron et al. is that it is 9D and includes also
the coupling between the inter- and intramolecular degrees
of freedom. The 5D potential that we used was obtained by
averaging the 9D potential over the ground state vibrational
wave functions of the H$_2$O and H$_2$ monomers. It can also be
averaged over excited vibrational states of the monomers and
can be used in calculations on dimers that are excited both in
the intramolecular and intermolecular modes. Finally, we may
conclude from our calculations that the hindered internal rotor
model used by Weida and Nesbitt to interpret their experi-
mental spectrum, although simplified, is qualitatively correct.

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