Network theory approach for data evaluation in the dynamic force spectroscopy of biomolecular interactions

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Abstract. - Investigations of molecular bonds between single molecules and molecular complexes by the dynamic force spectroscopy are subject to large fluctuations at nanoscale and possible other aspecific binding, which mask the experimental output. Big efforts are devoted to develop methods for effective selection of the relevant experimental data, before taking the quantitative analysis of bond parameters. Here we present a methodology which is based on the application of graph theory. The force–distance curves corresponding to repeated pulling events are mapped onto their correlation network (mathematical graph). On these graphs the groups of similar curves appear as topological modules, which are identified using the spectral analysis of graphs. We demonstrate the approach by analyzing a large ensemble of the force–distance curves measured on: ssDNA–ssDNA, peptide–RNA (system from HIV1), and peptide–Au surface. Within our data sets the methodology systematically separates subgroups of curves which are related to different intermolecular interactions and to spatial arrangements in which the molecules are brought together and/or pulling speeds. This demonstrates the sensitivity of the method to the spatial degrees of freedom, suggesting potential applications in the case of large molecular complexes and situations with multiple binding sites.

Introduction. – It has been recognized recently [1] that the signals generated at a single molecule (or another nano-size object) differ from signals obtained in large-scale systems consisting of ensemble of molecules. In particular, enhanced fluctuations, randomness and irreproducibility of the signals are observed in single-molecule measurements. A representative example is the mechanical signal generated in the dynamic force spectroscopy (DFS) of molecular bonds [2, 3]. The force spectroscopy of single molecules and molecular complexes has become a leading methodology for measuring biomolecular unbinding forces, which form the bases of biologically relevant molecular processes [4]. For instance, some recently studied examples include measurements of fundamental biomolecular forces in DNA unzipping [5], ALCAM-ALCAM [6], peptide- antibody [7], RNA- protein [8] interactions, etc.

In a typical pulling experiment in DFS based on the Atomic Force Microscopy, the ligand and receptor molecule are attached via polymer linkers on the AFM tip and the solid support (e.g glass, mica, gold- surface). The molecules are brought close to each other for certain contact time allowing them to form a bond and then pulled away until the bond breaks. The process is repeated many times. In each pulling event, changes in the deflection of the AFM cantilever as a function of distance are measured. Knowing the spring constant of the cantilever, this data can be calculated into distance dependent forces, resulting in so called force-distance curves. From these curves different parameters can be obtained, such as force needed to break a certain bond and the force loading rate. Further quantitative analysis of these data sets requires an elaborated theoretical framework [2] in order to extract the parameters of binding potential, the bond strength and the survival time. The applied force reduces the barrier between the bound and dissociated states of the binding molecules, allowing to estimate the force at which the barrier disappears and to measure the dissociation rate $\lambda(F)$. Then the natural dissociation rate at vanishing force is ex-
tracted using an appropriate theoretical framework. Since the pioneering work [2], assuming the exponential dependence of the dissociation rate on the applied force, the theory of the dynamic force spectroscopy evolved in two major directions: (i) incorporating a phenomenological distribution of the bond parameters [9], and (ii) assuming a specific type of the binding potential, from which the dissociation rate \( \lambda (F) \) is computed exactly [10, 11].

The pulling–disruption process of the molecular bonds, for instance in constant-velocity experiments used in this work, is both nonlinear and stochastic. Nonlinearity, which is manifested in force-dependent loading rate \( \dot{F}(F) \), can be related to the internal degrees of freedom of the (large) molecules and the spacers. Fluctuations in the disruption force and loading rate originate from the diffusion of binding molecules in solvent, whose effects depend on the binding potential and pulling speed. Thus the bond parameters can be determined in terms of probability, using the appropriate theoretical concept. In addition to the molecular bond of interest (specific binding), other non-specific processes may occur at the same contact time and range of forces: mis-binding or binding at different site, or unspecified interaction of the molecule with linkers and surface. It is, therefore, of great importance to be able to select the force-distance curves which are related to true binding prior to the quantitative analysis of the bond parameters. It is a general belief (see also discussion in [12]) that, apart from the fluctuations, the force-distance curves originating from the same binding process share strong similarity. Given a large amount of data in a particular experiment, this is a technically demanding task, which requires automated computational approach. Recently two methods were proposed based on pattern-recognition [13] and master-curve fitting [12].

In this work we present a new approach for systematic selection of groups of mutually similar force–distance curves and demonstrate it on sets of data from peptide–RNA complex from HIV1. Our methodology is based on the theory of complex networks and their spectral analysis [14–16]. Mapping multichannel datasets onto a network representation has proved as a useful tool [17] in the analysis of many complex dynamical systems, for example stock-market time series [18], gene-expression signals [19, 20], neural activity signals [21–23], climate phase fluctuations [24], and information traffic time series [25].

Compared to these examples, where the time-series are fluctuation signals [24], and information traffic time series [25]. Compared to these examples, where the time-series are fluctuation signals [24], and information traffic time series [25]. Compared to these examples, where the time-series are fluctuation signals [24], and information traffic time series [25]. Compared to these examples, where the time-series are fluctuation signals [24], and information traffic time series [25]. Compared to these examples, where the time-series are fluctuation signals [24], and information traffic time series [25]. Compared to these examples, where the time-series are fluctuation signals [24], and information traffic time series [25]. Compared to these examples, where the time-series are fluctuation signals [24], and information traffic time series [25]. Compared to these examples, where the time-series are fluctuation signals [24], and information traffic time series [25]. Compared to these examples, where the time-series are fluctuation signals [24], and information traffic time series [25]. Compared to these examples, where the time-series are fluctuation signals [24], and information traffic time series [25]. Compared to these examples, where the time-series are fluctuation signals [24], and information traffic time series [25]. Compared to these examples, where the time-series are fluctuation signals [24], and information traffic time series [25].

Figure 1: Correlation matrix of different types of force curves, described in the text. Color intensity from blue (low) to red (high) indicates the strength of correlation \( C_{ij} \) between pairs of curves. Shown are correlations after matrix filtering and above a threshold \( C_{ij} > 0.5 \).

**Description of the Experimental Data.** We created the correlation matrix from all pairs of \( N = 1188 \) force curves, that were pre-processed in the following way: contact part was removed, curves were corrected to the baseline, and only the part where interactions are expected was kept (first 200nm). Curves for the correlation matrix were selected from four different experiments with two different experimental setups. The experimental setups differ in the number of flexible linkers used to couple molecules to the surface and the cantilever: either both molecules were coupled to the cantilever/surface via PEG spacers or one was coupled to the gold surface directly and the other to the cantilever via PEG spacer. In the correlation matrix (see Fig. 1 and text below), from left-to-right, the first block (I) consists of 200 curves taken from measurements on RNA–Rev peptide interaction (two linkers, velocity 581nm/s); second block (II) consists of 188 curves taken from RNA–Rev peptide interaction in the presence of neomycin (two linkers, velocity 581nm/s); third block (III) contains 200 curves measured on RNA–Rev peptide interaction (one linker, velocity: 2540nm/s); fourth block (IV) contains 200 curves measured on ssDNA–ssDNA interaction (two linkers, velocity 218nm/s); fifth block (V) has 200 curves of RNA–Rev peptide interaction (one linker, velocity 1160nm/s), and the last block (VI) consists of 200 curves of Rev peptide–Au interaction (one linker, velocity 1160nm/s).
Correlation Matrix and its Spectral Analysis.

In our dataset consisting of \( N = 1188 \) force-distance curves, \( \{ f_i(x) \}, i = 1, 2, 3, \ldots, N \), each curve is identified by a uniquely defined index \( i \), thus representing a separate pulling event. The elements of the correlation matrix \( C_{ij} \) are calculated as the Pearson’s correlation coefficient between each pair \((i, j)\) of curves as follows:

\[
C_{ij} = \frac{\sum x_i [f_i(x) - \langle f_i \rangle][f_j(x) - \langle f_j \rangle]}{\sigma_i \sigma_j}.
\]

where the distance \( x \) is given by a discrete set of measured values, and \( \sigma_i, \sigma_j \) stand for the standard deviation of the force signal \( f_i(x) \) and \( f_j(x) \). In Fig. 1, we show a 3D color plot of the correlation matrix of all force curves, after filtering out spurious correlations. For the filtering, we used the affinity transformation method [22, 25], where the element is enhanced as \( C_{ij} \rightarrow M_{ij}C_{ij} \) if the rows \( i \) and \( j \) correlate with the rest of the matrix in a similar way, yielding \( M_{ij} > 1 \), and diminished otherwise. The meta-correlation factor \( M_{ij} \) is computed as a Pearson’s coefficient of the rearranged elements \( \{ C_{ij}, C_{i1}, \ldots, C_{i-j}, C_{i+j}, \ldots, C_{iN} \} \) and \( \{ C_{ji}, C_{j1}, \ldots, C_{j-i}, C_{j+i}, \ldots, C_{jN} \} \) without diagonal.

For further discussion we notice that the matrix can be represented by a network (mathematical graph), where each matrix index \( i = 1, 2, \ldots, N \) defines network node and the matrix element \( C_{ij} \) — a link between nodes \( i \) and \( j \). In our case the links are symmetrical \( C_{ij} = C_{ji} \) by definition. Notice that the matrix and network representations are formally equivalent. The network picture is suitable for visualization and topological interpretation. In particular, the matrix in Fig. 1 makes a sparse network containing topological modules, i.e., groups of nodes with strong connections inside the group and sparse connections between them. As the Fig. 1 shows, correlations between different sets of data (blocks I through VI) shown as off-diagonal block-matrices can be as strong as correlations inside the same data set (diagonal blocks). This suggests that the network modules may contain curves from different data blocks! In the following we apply the eigenvalue spectral methods to identify these topological modules.

Here we perform spectral analysis of the normalized Laplacian operator \( L \) related to the filtered correlation \( C \) in Fig. 1, or more precisely, its binary form with the elements: \( A_{ij} = 1 \) whenever \( C_{ij} > C_0 \), or \( A_{ij} = 0 \) otherwise. The matrix elements of the Laplacian are given by [16, 27]

\[
L_{ij} = \delta_{ij} - \frac{A_{ij}}{\sqrt{q_i q_j}},
\]

where \( q_i, q_j \) are the number of links at nodes \( i \) and \( j \), respectively. Although the same conclusions can be reached using the adjacency matrix \( A_{ij} \) directly, the analysis of the Laplacian \( L \) in this paper is more convenient since its eigenvalue spectrum is limited in the range \( \lambda_i \in [0, 2] \). Moreover, its eigenvectors belonging to few lowest non-zero eigenvalues tend to localize along the network modules [15, 16]. This is a direct consequence of the orthogonality to the eigenvector belonging to zero eigenvalue of the Laplacian (or the largest eigenvalue of the adjacency matrix) which has all positive components (Perron-Frobenius theorem [28]). Precisely, the localization means that, among \( N \) components of the eigenvector, the nonzero (positive/negative) components have indexes which coincide with nodes in a network module (detailed analysis of spectra in modular networks is given in [16]). We use this property of the eigenvectors to identify nodes in different modules.

Identification of modules: Force-curves grouping.

When a modular structure occurs, the localization of eigenvectors belonging to smallest non-zero eigenvalues is manifested in a branched pattern of the scatterplot in the space of these eigenvectors [16]. In Fig. 2, we show the scatterplot of the eigenvectors \( \{ V_1, V_2, V_3 \} \) belonging for three smallest non-zero eigenvalues of the Laplacian \( L \), related to the correlation matrix in Fig. 1. In the scatterplot, each point carries one index, thus
Table 1: (upper part) Representative curves appearing at tips of four branches $G_1, G_2, G_3, G_4$ in the scatterplot in Fig. 2 and their distribution over original blocks of data I through VI. (Middle and bottom) Further splitting of groups $G_2$ and $G_3$.

<table>
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<tr>
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<th>II</th>
<th>III</th>
<th>IV</th>
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Fig. 4: Example of the correlation network constructed from the force curves in group $G_2$ from Fig. 2, exhibiting modular structure. Shown are only links above the threshold $C_0 = 0.91$.

indicating one network node, that is a force curve index in our original dataset. The plot in Fig. 2 shows four branches, marked by $G_1, G_2, G_3, G_4$, thus four groups of curves can be identified. The most representative curves in each group are those at the tips of branches with most distinct curves sitting at the opposite ends of the branches $G_2$ and $G_3$, whereas, the differences gradually diminishes closer to the center of the plot. By matching the indexes with curves in the original dataset, we identify representative curves at the tips of four branches (Table 1). Overlay of all curves in the $G_1 \cdots G_4$ groups is shown in Fig. 3.

Each of the groups contains number of curves from one or several blocks of the original datasets. Recalling the nature of the data in different blocks, we see that the module $G_2$ contains curves obtained predominantly on experimental setup with one PEG spacer (blocks III, V, and VI) and a fraction of data with two spacers (from blocks I and II), while excluding altogether the DNA-DNA interactions (block IV). On the other hand, the module $G_3$ contains data from DNA-DNA interactions and a number of curves from Rev-RNA and blocked Rev-RNA with neomycin, all data obtained using two spacers. $G_1$ has similar composition although it appears as a separate module, similar as the small group $G_4$ (see visual differences in Fig. 3).

We further analyze the group of curves in $G_2$ applying the same approach on the now reduced correlation matrix. The complete group consists of $N_2 = 597$ curves. The correlation network of these curves, shown in Fig. 4, exhibits modular structure, suggesting that smaller subgroups of the group $G_2$ can be identified. The eigenvalues of the Laplacian related to this correlation matrix are shown in the ranking order in Fig. 5, top panel. Compatible with the network modularity in Fig. 4 is the appearance of three eigenvalues in the gap between $\lambda = 0$ and the main part of the spectrum. The corresponding scatterplot in the space of three eigenvectors belonging to these eigenvalues is also shown in Fig. 5, bottom panel. In this case similarity between points, forming a half-helmet in $(V_1, V_2, V_3)$ space, is stronger compared to Fig. 2 however, groups can be identified by the end points of the half-ring in the horizontal plane and points with largest vertical distance, marked as $g_1, g_2, g_3$ and different symbols (colors) in Fig. 5.

The identity of these curves with respect to the original datasets is also indicated in Table 1, middle part. As mentioned above, curves in large group $G_2$ are on Rev peptide binding, whereas, their subgroups appear to be from different pulling velocities: $G_2 - g_1$ consists mostly
of curves in blocks V and VI, measurements at velocity of 1160 nm/sec, while the groups of curves in blocks V and VI, measurements at velocity associated to ends of the scatterplot ring in Fig. 5.

Fig. 6: Overlay of the selected force curves of low- and high-loading rate $\dot{\epsilon}$ to zero force values. Pulling velocity affects both force and velocities and correct assignments of the curves are important for the extrapolation of bonding parameters to zero force values. Pulling velocity affects both force loading rate $F(\dot{F})$ as well as the average disruption force $F_g$. Within our methods selected groups of high- and low-velocity curves, $G_2 - g2$ and $G_2 - g3$, respectively, are shown in Fig. 6. Similar analysis of the data contained in the group $G_3$ (not shown) leads to the curves identified in the lower part of the Table 1. Here data contain three different interactions (DNA-DNA, Rev-RNA, and Rev-RNA-with-neomycin), all measured in the setup with two spacers and low velocity. We find much larger mixing between the groups, indicating either increased number of aspecific binding or a dominant role of spacers.

Finally, in Fig. 7 we show histograms of the rupture forces made upon curves in groups $G_2$ and $G_3$ selected by our methodology. Detailed analysis of Rev–RNA bonding parameters is given in [26].

Conclusions. – We have shown that stochastic signals of different molecular systems (peptide–RNA, DNA–DNA) measured in the dynamic force spectroscopy experiments can be effectively selected according to their similarities. Our methodology uses the signal’s relevant correlation matrix mapped onto a mathematical graph. Then using the spectral analysis of these graphs the groups of similar curves are detected, which appear as topological modules on them. Although the method is essentially statistical, we have shown that strong regularities in groupings of the force curves occur (and can be effectively used for the signal evaluation), based on the pulling speeds and experimental setup and even type of the interaction measured. Note that for the demonstration purposes in this work we used raw data with minimal pre-processing. Further pre-processing, e.g., with curve-fitting [12], could be incorporated and increase selectivity in our method. Improved efficiency in the evaluation of force curves by this approach is also expected in the case of larger molecules or molecular complexes and the situations where distinction between many binding sites is applicable.

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