SU(5) orientifolds, Yukawa couplings, Stringy Instantons and Proton Decay

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Abstract: We construct a large class of SU(5) orientifold vacua with tadpole cancellation both for the standard and the flipped case. We give a general analysis of superpotential couplings up to quartic order in orientifold vacua and identify the properties of needed Yukawa couplings as well as the baryon number violating couplings. We point out that successful generation of the perturbatively forbidden Yukawa couplings entails a generically disastrous rate for proton decay from an associated quartic term in the superpotential, generated from the same instanton effects. We search for the appropriate instanton effects that generate the missing Yukawa couplings in the SU(5) vacua we constructed and find them in a small subset of them.

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1. Introduction

String vacua involving open strings [1] have been seriously considered for the SM search after non-perturbative string dualities indicated that the heterotic string did not have the monopoly of interesting and complex vacuum structure, [2]. Interest in such vacua was enhanced by the observation that the string scale was less tightly constrained than in heterotic ones, [3, 4, 5].

Orientifold vacua obtained a novel and important impetus after the realization [6, 7, 8] that they allowed a modular (bottom-up) approach in assembling the ingredients of the SM. This promoted local constructions of D-brane stacks that could carry the SM spectrum and could be them embedded in full- fledged string compactifications. There are many distinct ways of embedding the Standard Model group into that...
of quiver gauge theories, which appear in the context of orientifolds and these are
reviewed in [9]-[13].

The prototype of the modular construction approach was implemented via RCFT
techniques. The orientifolds were constructed from Gepner models (studied earlier
in [14]-[19]), using the algorithmic techniques of RCFT developed in [20]. In the first
search in [18], vacua realizing the Madrid incarnation [21] of the Standard Model
were analyzed. They provided the largest collection of vacua (tadpole solutions) to
date, chirally realizing the (supersymmetric) SM.

In the same context, a more general search was done where all possible embed­
ddings of the SM in four stack configurations was analyzed [22]. A total of 19345
chirally distinct top-down spectra were found, that comprise so far the most exten­sive
such list known in string theory [22]. For 1900 of these 19345 spectra at least one
tadpole solution was also found (no further attempts at solving tadpoles were made
once a solution was found for a given chirally distinct type). The wealth of tadpole
solutions can only be compared to a recent extensive list from the $Z_6$ orientifold,
[23], although even that set appears to cover far fewer distinct possibilities. Not all
regions of moduli space are rich in SM-like vacua though. The $Z_2 \times Z_2$ orientifolds
[24] and the free-fermionic orientifolds [25] although they contain a large number
of vacua, seem to be SM-free. Recently tachyon-free tadpole-free non-supersymmetric
vacua have been searched for [26] in Gepner models. No solutions were found, al­
though there do exist many tachyon-free non-supersymmetric “local” configurations
with uncancelled tadpoles.

On a different note, heterotic and orientifold vacua look generically different at
least in one direction. Although in heterotic vacua, there is a generic underlying
GUT structure, this is generically not the case in orientifold vacua. In a generic
orientifold construction the SU(3) and SU(2) groups originate in generically distinct
D-brane stacks, without an a priori relation of their respective gauge couplings at the
string scale. Moreover, as was first analyzed in detail in [6], the hypercharge gauge
group is always a linear combination of U(1)’s from different brane stacks, although
this linear combination may vary from orientifold to orientifold. The general such
hypercharge embedding was classified in [22] in terms of a real number $x$ that is
typically discrete.

This characteristic structure is responsible for some unique generic properties
of SM-like orientifold vacua in particular the presence of at least one anomalous
U(1) gauge boson in the standard model stack\(^1\), which mixes because of electroweak
symmetry breaking with the photons and $Z_0$. This mixing can be substantial and
observable if the associated anomalous gauge boson is light, [6, 27, 28, 29, 30]. This
can happen both when the string scale is at the TeV scale, as well as when it is higher,
if there are large cycles in the compactification manifold, [31]. Another property

\(^1\)The generic number is three, [6, 7, 22].
relates to the novelty and richness of patterns and mechanisms for the generation of the hierarchy of masses [32, 33, 34, 35, 36], a fact that is welcome in the search for realistic vacua in string theory.

It is admittedly true that there are several indications favoring embedding the SM in a unified gauge group. They include the apparent unification of coupling constants at a “GUT” scale $M_{\text{GUT}} \sim 10^{16}$ GeV as well as the appearance of the same scale in seemingly unrelated sectors (neutrino masses, dark matter etc). A direct attempt to generate the popular GUT groups² (SU(5) and its relative “flipped SU(5)”, SO(10), and E₆) in orientifolds indicates that only SU(5) is possible. Both SO(10) and E₆ contain spinor-like representations and these cannot be constructed perturbatively in the context of orientifolds. They can be constructed non-perturbatively however, as the duality with the heterotic string [2] and direct constructions have indicated [40]. Although this can be done, the models lose their weak-coupling appeal as the non-perturbative states are very heavy in perturbation theory.

On the other hand, SU(5) and related constructions can be achieved by embedding the SU(5) in a U(5) stack of branes. There are even other unified constructions that have not been discussed in the GUT literature but appear naturally in orientifolds. An interesting example is a U(6) super-unified tadpole solution found in [22] that through three distinct symmetry breakings can produce an SU(5), flipped SU(5) or Pati-Salam intermediate group and spectrum. An early supersymmetric SU(5) example was described in [41] and several others were constructed in [42]. They all contained chiral exotics. Other non-supersymmetric SU(5) examples were found but as usual in these cases tadpole conditions were not satisfied [43], [44]. The first supersymmetric examples without chiral exotics and satisfying all tadpole conditions were constructed in [22].

Although SU(5) orientifold vacua have a simple structure and produce easily the appropriate spectrum including right-handed neutrinos they suffer from an important ailment. It was pointed out in [43] (see also [45] and [22]), that the top Yukawa coupling is absent in perturbation theory as it carries a non-zero charge under $U(1)_5$ (the U(1) factor of the U(5) group).

There are two possible ways to generate such a Yukawa term. Both of them break in a way the $U(1)_5$ global symmetry. A first possibility is turning-on fluxes that break $U(1)_5$. Such a mechanism has not so far been explored in detail due to the difficulty of constructing realistic vacua with non-trivial fluxes. The second possibility relies on the fact that generically the $U(1)_5$ gauge symmetry is of the anomalous type and the associated global symmetry is expected to be broken by non-perturbative (instanton) effects. This is the route we will pursue here.

²Pati-Salam groups are natural in orientifolds and many such vacua have been found. The earliest examples are in [37, 38, 39] while the largest list of tadpole solutions without chiral exotics is in [22].
Non-perturbative instanton effects in string theory have been discussed early on, while the non-perturbative dualities were explored, [46]. In particular, D-instanton effects in open string theory could be mapped to perturbative string effects on a dual heterotic side, [47], and this gave the first glimpse into the structure of the D-instanton corrections (for an early review see [48]). Several years later, the structure of gauge instantons were elucidated using D-brane techniques, [49]. Lately, D-instantons have been argued to provide non-trivial contributions to couplings protected otherwise by anomalous U(1) symmetries, like neutrino masses, [50] motivating a resurgence of interest whose output has been reviewed in [51]. The first global example analyzed, that provided non-zero instanton contributions, was based on the $Z_3$ orientifold, [52]. It provided the generation of an ADS superpotential, mass terms for chiral multiplets, that together lead to supersymmetry breaking contributions if the closed string moduli are stabilized, [53]. Although at a general point in moduli space the gauge group is SU(4), there are enhanced regions where the group is SU(5) with a spectrum of three antisymmetric chiral multiplets, 3 5s as well as 3 pairs of $(5 + 5)$ Higgses, [54]. In this phase, the same instanton generates the top-like Yukawa couplings. Upon further Higgsing to SU(4) these match the instanton generated mass terms computed in [52]. Several further works analyzed the structure of instanton corrections further [55]-[58] and in particular the generation of the top Yukawa couplings in SU(5) orientifolds both at the local and global level [59, 60].

In the context of RCFT orientifold constructions of Madrid-like SM embeddings, [21], a search for instanton effects was done, in order to track neutrino mass generation. The experience from such a search is that RCFT vacua, having typically enhanced symmetries, possess instantons with typically large number of zero modes. Therefore instanton contributions to the superpotential are atypical, and indeed no single instanton contribution was found in [61].

The results of the paper can be summarized as follows:

- We analyze orientifold vacua with SU(5) gauge group, realizing SU(5) or flipped SU(5) grand unification. We construct many tadpole solutions from Gepner model building blocks using the algorithm developed in [22]. We found all such top-down constructions as well as tadpole-free vacua, with one extra observable brane of the U(1) or O(1) type. This is one small subset (but the simplest) of the SU(5) configurations found in [22].

- We give a general analysis of possible terms in the superpotential of such vacua, up to quartic order, and classify them according to their fatality ( baryon and lepton violating interactions which are relevant or marginal), and usefulness (Yukawa coupling). We have classified which terms can or must be generated by instanton effects. As is well known, the top Yukawa’s in SU(5) and the bottom in flipped SU(5) must be generated from instantons (in the absence of fluxes).
• We find that in flipped SU(5) vacua, B-L cannot be anomalous as it participates in the hypercharge. This forbids all dangerous terms, but it is necessarily broken when the SU(5) gets broken at the GUT scale. The proton decay generated is estimated to be typically small.

• In U(5) x U(1) vacua, instanton effects must generate the top Yukawa couplings, and at the same time they break the B-L symmetry. Successful vacua, have either a $Z_2$ remnant of the B-L symmetry acting as as R-parity and forbidding the dangerous terms, or such terms may have exponentially suppressed instanton contributions. In the second case they are viable if the exponential factors are sufficiently suppressed. We provide several tadpole solutions of the first case where instantons generate the top Yukawa’s, but preserve a $Z_2$ R-symmetry.

• U(5) x O(1) vacua are problematic on several grounds and need extra symmetries beyond those that are automatic, in order to have a chance of not being outright excluded. This is related to the absence of natural R-symmetries or gauge symmetries that will forbid the dangerous low-dimension baryon-violating interactions.

• A generic feature of all SU(5) vacua is that the same instanton the generates the non-perturbative quark Yukawa coupling also generates the 10 10 10 5 in the superpotential. This is a second source of proton decay, beyond the classic one emanating from the Higgs triplet times the appropriate Yukawa coupling. Generically, the size of this contribution to proton decay is $10^5 \frac{M_T}{M_H}$ larger than the conventional source in flipped SU(5) model, $(M_T$ is the triplet Higgs mass). This signals severe phenomenological trouble and calls for important fine tuning. In the SU(5) case the size is 30 times smaller, but that does not evade the need for fine tuning.

• We have searched for appropriate instantons that would generate the perturbatively forbidden quark Yukawa couplings in the SU(5) vacua we have constructed. We found the appropriate instantons with the correct number of zero modes in 6 relatives of the spectrum Nr. 2753. We have also searched for all other instantons that could generate the bad terms in the superpotential and found none. This translates into the existence of a $Z_2$ R-symmetry that protects from low-dimension baryon and lepton-violating couplings.

The structure of this paper is as follows:

In section 2 we describe the search for SU(5) vacua with tadpole cancellation with at most one extra observable stack using the RCFT of Gepner models.
In section 3 we give a general analysis of superpotential terms up to quartic order in such vacua, their relevance for baryon and lepton number violation as well as the possibility of their generation via non-perturbative effects.

In section 4 we give an analysis of the relevant instanton zero modes and their impact in the generation of terms in the superpotential.

In section 5 we provide an analysis of the relative effects in proton decay of two superpotential operators whose generation by instantons is correlated.

In section 6, we search the RCFT vacua found for the instantons appropriate to generate the missing quark Yukawa couplings.

Section 7 contains our conclusions.

In Appendix A we provide the complete spectrum of the class of tadpole solutions with the requisite instanton effects.

2. Explicit Constructions

In Ref. [22], a methodology for identifying self-consistent semi-realistic string models was developed. This methodology was employed on a set of string vacua constructed using RCFT techniques on Gepner models and a host of examples were presented. In this paper, we are primarily interested in orientifold vacua with an SU(5) GUT group (both standard and flipped). We are also interested on the possibility of generating the appropriate Yukawa couplings (forbidden perturbatively) by string instantons. Moreover, we will also analyze some issues related to this mechanism and in particular the issue of proton decay. For this we will revisit some of the models originally presented in [22].

The methodology developed in [22], starts with a variation the bottom-up approach developed in [6, 8]. Instead of geometric brane configurations, RCFT boundary state combinations are searched for that give rise to a spectrum of interest (usually the MSSM or a unified extension of the latter). Then an attempt is made to find additional boundary states that provide a “hidden sector” that can cancel the tadpoles. This method was pioneered in [18] and is based on the boundary state formalism presented in [67], which in its turn is based on earlier work, such as [68, 69] and [70]. This method provides bona-fide string vacua that have low-energy limits consistent with the MSSM. We shall briefly summarize the relevant points for the subset of these string models considered in the present work:

- The visible sector is required to consist of three or fewer stacks of branes where the $SU(5)$ arises from exactly one stack.

- The chiral spectrum for the visible sector should reduce to three generations of the MSSM, once the gauge group is reduced to $SU(3) \times SU(2) \times U(1)$. 
• No chiral exotics are present in the spectrum. Chiral exotics are defined as any states that are chiral with respect to the standard model gauge group and that do not fit in the usual three families.

Here “visible sector” is defined as the set of branes that contribute to the standard model gauge group and/or the charged quarks and leptons (some right-handed neutrinos may also originate from the visible sector, but are not required).

Note that the set of chiral states in the visible sector may be larger than just the three standard model families. We are only requiring that the superfluous ones become non-chiral under a group-theoretical reduction to $SU(3) \times SU(2) \times U(1)$. For an example, see the spectrum presented in table 5 below. However, the other examples considered in the present paper, and the vast majority of Madrid type models considered in [18] have no superfluous chiral states whatsoever.

In both [18] and [22] chiral states that are charged under both the visible and the hidden sector were forbidden, even if they reduce to non-chiral standard model states. This is a bit more restrictive than the conditions imposed on the visible sector. For example, the combination $(5, r_1) + (5, r_2)$, where $r_1$ and $r_2$ are distinct hidden sector representations of equal dimension, would not be allowed in both papers, even though it reduces to a vector-like $SU(5)$ representation. Obviously a mere group-theoretic reduction is not sufficient to give a mass to these vector-like state. One would have to get into the details of hidden sector dynamics in each individual case to see if such a model is viable, and for this reason lifting this requirement is unattractive. However, in order to be as complete as possible we have lifted this requirement in the present paper.

Explicit examples of models are presented below.

2.1 The $SU(5)$ orientifolds

In a previous study focusing on Gepner models [22], $SU(5)$ models satisfying the criteria listed above were presented. In the full set of 1900 chirally distinct tadpole solutions there are 494 cases where the $SU(3)$ and $SU(2)_W$ branes are identical. This implies an extension of the standard model group to at least $SU(5)$ (in some cases $SU(5)$ is a subgroup of a larger unitary group). Two spectra are called “chirally distinct” if the visible sector gauge groups are different, if the matter that is chiral with respect to the gauge group is different, or if different $U(1)$ bosons acquire a mass through axion mixing. Consequently, an $SU(5)$ model is regarded as distinct from the model obtained by splitting the $U(5)$ stack into a $U(3)$ and a $U(2)$ stack. In some cases, both possibilities exist.

The simplest orientifold realizations of $SU(5)$ models consist of one $U(5)$ brane stack, plus one additional brane, with Chan-Paton multiplicity 1, intersecting that stack. Matter in the $(10)$ of $SU(5)$ arises from chiral anti-symmetric tensors, where matter in the $(\bar{5})$ comes from intersections of the $U(5)$ brane and the additional
brane. The search criteria of [22] allow for even more general SU(5) models. Instead of getting three copies of the (5) from a triple intersection, one may obtain each family from a separate brane intersecting the U(5) stack, and/or get some of the family multiplicity by allowing higher CP-multiplicities. There is indeed a large variety of such more complicated spectra in the database of [22], but we consider here only the ones with a single additional brane.

The additional brane can be either O(1) or U(1). In the following table we list the distinct models and how often they occurred. Note that this refers to brane configurations prior to attempting to cancel tadpoles (named top-down models in [22]). We note also that counting the number of distinct models has some subtleties. It is likely that some of them are merely distinct points in the same moduli space. Moving around in the moduli space would then provide the differences in vector-like matter.

<table>
<thead>
<tr>
<th>Nr.</th>
<th>Frequency</th>
<th>CP group</th>
<th>Gauge group</th>
</tr>
</thead>
<tbody>
<tr>
<td>617</td>
<td>16845</td>
<td>U(5) × O(1)</td>
<td>SU(5)</td>
</tr>
<tr>
<td>2753</td>
<td>1136</td>
<td>U(5) × U(1)</td>
<td>SU(5)</td>
</tr>
<tr>
<td>2880</td>
<td>1049</td>
<td>U(5) × U(1)</td>
<td>SU(5) × U(1)</td>
</tr>
<tr>
<td>6580</td>
<td>146</td>
<td>U(5) × U(1)</td>
<td>SU(5)</td>
</tr>
<tr>
<td>14861</td>
<td>12</td>
<td>U(5) × U(1)</td>
<td>SU(5)</td>
</tr>
</tbody>
</table>

Table 1: List of the SU(5) models with a single additional brane. The second column show the number of such spectra (modulo non-chiral matter) that was found. The first column is the number used to refer to these spectra, and is equal to the position of the spectrum on the full list of [22], sorted by frequency.

The unitary phase of the U(5) stack is anomalous, and hence the corresponding gauge boson always acquires a mass. However, in some of the U(5) × U(1) models a linear combination of the U(5) and U(1) phases is anomaly-free, and may or may not acquire a mass. It remains massless in model type 2880, whereas it is anomaly-free, but not massless in model type 2753. The existence of this anomaly free combination corresponds to the possibility to interpret the model as a flipped SU(5) model. Hence model type 2880 has two distinct interpretations, either as a normal or as a flipped SU(5) model. In both interpretations there is necessarily an additional massless U(1) gauge boson in the exact string spectrum, corresponding to B − L. In the other four model types the gauge group is exactly SU(5), plus a hidden sector that may be required for tadpole cancellation.

Tadpole canceling hidden sectors were found for model types 617, 2753 and 2880. In [22] these solutions were not optimized for simplicity; for each model type just one tadpole solution was collected.

We have done a systematic analysis of all 16845 U(5) × O(1) models, and the results are as follows. First we tried to solve the tadpole conditions allowing chiral
matter between the observable and hidden sector. For 15499 of these 16845 we were able to show that no such solution exists, for 641 we did find a solution, and 705 cases were inconclusive: the tadpole cancellation equations were too complicated to decide if there is a solution. The algorithm used for solving the tadpole conditions consists of two parts: an exhaustive search for all solutions with up to four hidden branes, plus a different algorithm allowing in principle an arbitrary number of hidden branes. The latter search was limited in time. If it is terminated prematurely such a case is labeled “inconclusive”.

Next we tried to solve the tadpole conditions under the more restrictive condition that only non-chiral observable-hidden matter is allowed (the same condition as used in [22] and [18]). Of the 641 cases that had solutions in the previous search, 521 had no non-chiral solutions, 109 did have non-chiral solutions, and 11 were inconclusive. Of the 705 previously inconclusive cases, 508 had no non-chiral solutions, 64 had non-chiral solutions, and 133 were still inconclusive. These numbers give some idea about the success rates of attempts to solve the tadpole conditions.

The simplest solution found for model type 617 is shown in table 2.

<table>
<thead>
<tr>
<th>Multiplicity</th>
<th>Visible Sector</th>
<th>Hidden Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$U(5)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>3(3+0)</td>
<td>A</td>
<td>0</td>
</tr>
<tr>
<td>5(4+1)</td>
<td>V*</td>
<td>V*</td>
</tr>
<tr>
<td>8(4+4)</td>
<td>V</td>
<td>0</td>
</tr>
<tr>
<td>2(1+1)</td>
<td>V</td>
<td>0</td>
</tr>
<tr>
<td>8(4+4)</td>
<td>S</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>Adj</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2: The particle spectrum of the simplest $SU(5)$ model found. The visible sector consists of $U(5)$ and $O(1)$ with a hidden sector consisting only of $O(1)$. The transformation properties under the various groups are listed on the table where a “V” refers to a vector, “S” is a symmetric tensor, “A” is an antisymmetric tensor, “Adj” is an adjoint, and 0 is a singlet. In the first column “M+N” means M copies of the representation, plus N copies of its complex conjugate.

For model type 2880 the results are as follows. The 1049 configurations split into two classes: one where the dilaton tadpole condition is already saturated, so that there is no room for a hidden sector, and one where a hidden sector is required. In the former case one can only hope that the remaining tadpole conditions are solved...
as well. This class contains 437 configurations, and all of them turn out to satisfy all tadpole conditions! All of them are closely related, and occur for the same MIPF of tensor product (1,4,4,4,4). Only a few of these 437 spectra are distinct, and the simplest one is shown in table 3. Note that this is a different spectrum than the one presented in [22], because in that paper no attempt was made to look for the simplest version of a spectrum. The column “Variations” list the other values for the total multiplicity that were found (though not uncorrelated).

The other 612 configurations require a hidden sector, and in only 10 of them it can indeed be found. All of these have chiral hidden-observable matter, and therefore they do not satisfy the original requirements of [22]. Indeed, in that paper no such solution was found.

<table>
<thead>
<tr>
<th>Multiplicity</th>
<th>Variations</th>
<th>Visible Sector</th>
<th>$U(5)$</th>
<th>$U(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3(3+0)</td>
<td>3,7</td>
<td>$A$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>3(3+0)</td>
<td>3,5</td>
<td>$V^*$</td>
<td>$V^*$</td>
<td></td>
</tr>
<tr>
<td>3(3+0)</td>
<td>3,7,11</td>
<td>0</td>
<td>$S$</td>
<td></td>
</tr>
<tr>
<td>4(2+2)</td>
<td>4,8,12</td>
<td>$V$</td>
<td>$V^*$</td>
<td></td>
</tr>
<tr>
<td>8(4+4)</td>
<td>4,8</td>
<td>$S$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3,5,7,9</td>
<td>Adj</td>
<td>0</td>
<td>Adj</td>
</tr>
<tr>
<td>3</td>
<td>3,5</td>
<td>0</td>
<td>Adj</td>
<td></td>
</tr>
</tbody>
</table>

Table 3: The particle spectrum of the $SU(5) \times U(1)$ model nr. 2880. This satisfies all tadpole conditions without a hidden sector.

For model nr. 2753 only a rather complicated solution with seven hidden sector factors was found in [22]. This solution was just a sample, the first one that was encountered. We have now scanned all 1136 models of this type, and nothing simpler was found. Of the 1136 models, six turned out to admit a solution, and the other 1130 did not. All these solutions are similar to the first sample found. They all have a hidden sector $U(5) \times Sp(4) \times U(2) \times O(2)^2 \times U(1)^2$, a large number of non-chiral exotics, and some chiral fermions entirely within the hidden sector. Further details will not be presented here. Instead, we will present a spectrum for this model with instanton branes in section (6).

For the remaining two model types no tadpole solution was found. Below we display the “local” spectra of these models, i.e. the standard model configuration without tadpole cancellation. As always, this is just the first sample found, without any attempt to optimize or simplify the non-chiral spectrum. The spectrum for model nr. 6580 is remarkably simple and shown in table 4.

Note the complete absence of any non-chiral matter, an extremely rare feature. However, this also implies the absence of any Higgs candidates for breaking $SU(5)$ or for breaking $SU(2) \times U(1)$. The last model type, Nr. 14861, of which only 12
Table 4: The local spectrum of the $SU(5) \times U(1)$ model nr. 6580. No global version of this model was found.

Table 5: The local spectrum of the $SU(5) \times U(1)$ model nr. 14861. No global version of this model was found.

3. Yukawa terms and other relevant couplings

Table 6: The manner of embedding one generation of the SM into $SU(5)$ multiplets with their respective $U(1)_{1,5}$ charges for the typical string realization.

There are several couplings in the superpotential whose size is crucial for acceptable low-energy physics.\textsuperscript{3} We will list them below up to quartic order.

\textsuperscript{3}Several related issues about some of these terms have been discussed in [60].
• The $55_H$ term gives rise to relevant lepton number violating interactions that can kill models instantly. In model building it is typically forbidden by an R-symmetry.

• The $1 \ 55_H$ term is a Yukawa coupling. The $1$ is the singlet that plays the role of the right-handed neutrino in SU(5) models and the lepton singlet in flipped SU(5) models. In both cases this term should be present to generate the appropriate mass.

• The $1055$ term, where the $5$'s are matter, generates dimension four operators that break baryon and lepton number and are instantly fatal unless the coupling is exponentially suppressed. These terms are usually forbidden by advocating an R-symmetry.

• The $1055_H$ is a standard Yukawa coupling that must appear with appropriate coefficients, as it generates the masses of half of the quarks and leptons (which ones depends whether we are in SU(5) or flipped SU(5).)

• The $10\delta_5\delta_H$ is generating couplings between the light Higgs and the singlet. For a single Higgs this is zero by symmetry. It contributes in the presence of more than one pairs of Higgses.

• The $10105_H$ term is a standard Yukawa coupling that must appear with appropriate coefficients, as it generates the masses of the other half of the quarks and leptons (which ones again depends whether we are in SU(5) or flipped SU(5).)

• $1010105_H$ is a term that can be important for proton decay.

• There is also an associated term $1010105_H$. This term seems relatively innocuous as we will describe below.

We will now discuss the status of all of these terms in the various realizations of SU(5) orientifold models.

3.1 Flipped SU(5) from $U(5) \times U(1)$ orientifolds

The standard generic spectrum in an $U(5) \times U(1)$ orientifold is given in table 6. We may write the important U(1) symmetries in this context using the standard diagonal SU(5) generator

$$ W = \begin{pmatrix}
-\frac{1}{3} & 0 & 0 & 0 & 0 \\
0 & -\frac{1}{3} & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{3} & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2}
\end{pmatrix} \quad (3.1) $$
as

\[ Y = \frac{Q_x - W}{5}, \quad Q_{B-L} = \frac{Q_x + 4W}{5}, \quad Q_B = \frac{Q_5 - 2W}{5}, \quad Q_X = \frac{5Q_1 + Q_5}{2} \]  

Above, \( Q_5 \) is the \( U(1) \) charge of the \( U(5) \) stack, and \( Q_1 \) is the charge of the \( U(1) \) stack. Both are normalized to integers.

To assess further the possibility of using instantons to generate the missing Yukawa’s we will keep track of the \( U(1)_X \) charges. The reason is that the potential instanton effects violating \( U(1)_X \) are severely constrained by the fact that \( U(1)_X \) participates in the Standard Model hypercharge \( Y \).

The other generator participating in \( Y \) is the \( SU(5) \) generator \( W \) that is traceless, and cannot therefore become massive nor can be violated by instanton effects. As the same should be true for \( Y \), (otherwise this is not acceptable in the SM) we conclude that for a string vacuum to be a viable SM candidate, the \( Q_X U(1) \) symmetry must be massless and therefore it should not be violated by instanton effects\(^4\)

- The \( 55_H \) term has charge \( (-2, 0, -5) \) under \( (U(1)_1, U(1)_5, U(1)_X) \) and is therefore forbidden. As it is charged under \( U(1)_X \). It might be generated only if \( U(1)_X \) breaks spontaneously.

- The \( 1 \ 55_H \) term is a standard Yukawa coupling that gives mass to the lepton singlet and it is perturbatively allowed as it is uncharged under \( (U(1)_1, U(1)_5) \).

- The \( 1055 \) term has charge \( (-2, 0, -5) \) under \( (U(1)_1, U(1)_5, U(1)_X) \) and is therefore forbidden.

- The \( 1055_H \) term is a standard Yukawa coupling that gives mass to the top quark and is perturbatively allowed as it is uncharged under \( (U(1)_1, U(1)_5) \).

- The \( 1055_H \) term has charge \( (2, 0, 5) \) under \( (U(1)_1, U(1)_5, U(1)_X) \) and is therefore forbidden.

- The \( 10105 \) term gives masses to the bottom quark and the right-handed neutrino. It has charges \( (-1, +5, 0) \) under \( (U(1)_1, U(1)_5, U(1)_X) \). It is perturbatively forbidden and can only be generated by instantons.

- The \( 1010105 \) term has charges \( (-1, +5, 0) \) under \( (U(1)_1, U(1)_5, U(1)_X) \), and is therefore perturbatively forbidden. The charge structure is however the same as the previous Yukawa and later on we will argue that such a term is generated by the same instanton effects.

\(^4\)Masses for \( U(1) \) symmetries in string theory appear via the mixing with various closed string forms. The appearance of a mass breaks the \( U(1) \) gauge symmetry but not the global \( U(1) \) symmetry. However we do not expect exact global (internal) symmetries in string theory. Indeed, it is the defects charged under the same forms that appear as instanton effects violating the global \( U(1) \) symmetry, breaking it to a discrete subgroup.
• The $\mathbf{1010105}_H$ term has charges $(+1, +5, 5)$ under $(U(1)_1, U(1)_5, U(1)_X)$ and is forbidden by the $U(1)_X$ symmetry.

Of all the couplings that are perturbatively forbidden, $55_H$, $1055$, $105_H5_H$, $1010105_H$, $10105_H$, $1010105$, the first four have $Q_X = \pm 5$ while the two last ones have $Q_X = 0$. This is not a surprise as $Q_X$ is intimately related to B-L and therefore forbids the first four terms.

The upshot of the previous discussion is, that in flipped SU(5) orientifold vacua, if the hypercharge is massless, there is necessarily a massless $U(1)_{B-L}$ gauge boson associated to the gauged B-L symmetry. This can also be rephrased in the opposite way: in order for a $U(5) \times U(1)$ configuration to reproduce the SM, the $U(1)_{B-L}$ must be unbroken at high energy. At some energy, $U(1)_X$ and B-L must be broken by a vev. This is indeed what happens at the GUT scale as in flipped SU(5) models the appropriate GUT symmetry breaking happens when a Higgs multiplet that transforms as the 10 obtains a vev to break $SU(5) \times U(1)$ to $SU(3) \times SU(2) \times U(1)$. At the same time it breaks $U(1)_X$. We will denote by $10_H$ and $10_H$ such Higgs fields.

Let us consider the potential generation of the most relevant “unwanted term” in the superpotential namely $55_H$ with charges $(-2, 0, -5)$ under $(U(1)_1, U(1)_5, U(1)_X)$. As $U(1)_X$ is not broken by instantons the leading term in the superpotential that can generate $55_H$ after symmetry breaking is $55_H(10_H)^5$ with charges $(-2,10,0)$. This has now $Q_X$ charge zero and is therefore allowed, but is perturbatively forbidden by the anomalous $U(1)$ symmetry that forbids also the bottom Yukawa coupling $10105_H$ with charges $(-1,5,0)$. Therefore if there is an instanton that generates the relevant Yukawa coupling $h_{10105_H} \sim e^{-S_{\text{inst}}}$, it is also plausible (although it is not guaranteed$^5$) that there is also an instanton with charge violation $(-2,10,0)$ that will generate the $55_H (10_H)^5$ term.

Assuming this worst scenario case, we can therefore estimate its strength as $h_{10105_H}^2 \frac{105}{M^8_{\mu}}$, assuming that the instanton contribution is the square of an $(-1,5,0)$ instanton. This gives an effective scale

$$\mu_{55_H} \sim h_{10105_H}^2 \left( \frac{M_{\text{GUT}}}{M_\mu} \right)^4 M_{\text{GUT}} \sim 10^{-4} \text{ GeV}$$

(3.3)

To obtain the estimated value we have taken the Yukawa coupling of the strange quark as the central value for $h_{10105_H} \sim h_s \sim 4 \times 10^{-4}$, $\frac{M_{\text{GUT}}}{M_\mu} \sim 10^{-3}$ and $M_{\text{GUT}} \sim 10^{15}$ GeV. This term in the IR generates an $LH$ term that can be rotated into the lepton number violating marginal interactions by rotating in the space of lepton doublets and Higgs. If we assume a $\mu$ term of order the EW scale then the angle of rotation

$^5$In the brane picture of instantons this would amount to a $O(2)$ or Sp(2) bound state. Although this will have too many zero modes to contribute to the superpotential there could be points in its moduli space where the symmetry is broken and the associated zero modes are lifted along the lines of [71].
is $\theta \sim \mu_{55H} \sim 10^{-6}$ and therefore well below the limits on such couplings from lepton violation [72, 73].

Similar arguments apply to the baryon violating $10^{55}$ term that descends from the $10^{55}(10^H)^5$ term. The induced dimensionless effective coupling of the $10^{55}$ term in the superpotential is then

$$h_{1055}^{eff} \sim h_{10105H}^2 \left( \frac{M_{GUT}}{M_s} \right)^5 \sim 10^{-22} \tag{3.4}$$

This is much smaller that the proton decay bound that roughly requires $h_{1055}^{eff} \lesssim 10^{-14}$ assuming a relevant spartner mass of about 100 GeV.

The other two terms, $10^5H5H$, $101010^5H$, may be generated by instantons that do not violate the $U(1)_X$ symmetry and the discussion here is similar to the other cases and will be presented later.

There are further superpotential terms that may be needed for the phenomenological viability of flipped SU(5) GUT vacua. One of terms, namely $10^H10^H5H$ is necessary to give a large mass to the Higgs triplet. Indeed such a term gives $<10> \sim 3_H3_h$ where $3_H$ is the triplet contained in $10^H$ and $3_h$ is the standard EW triplet. Such a term has charges (-1,5,0) and is therefore allowed by $U(1)_X$ but not by the anomalous $U(1)$ symmetry. Therefore the same instanton that generates the bottom Yukawa couplings will also generate this term and we can estimate its size as $h_{10^H10^H5H} \sim h_{10105H}^2$ and the mass mixing term $<10> \sim 3_H3_h \sim h_{10105H}M_{GUT}3_H3_h$. This is the only contribution to the triplets’ mass, from which we conclude that $M_T \sim h_{10105H}M_{GUT}$. This implies that generically the triplets will be 1-3 orders of magnitude lighter than the GUT scale, a fact that can spell problems with proton decay.

On the other hand a see-saw mechanism can work since its needs couplings $10^H10^H$ with $\phi$ a singlet if $10^H$ and $10$ have the same quantum numbers.

Note that such extra $10^H+10^H$ multiplets may not exist in a given model. In the 437 flipped SU(5) vacua we have presented in the previous section there are either precisely 3 chiral (10)'s and hence no such Higgses, or there are two such Higgs pairs.

### 3.2 SU(5) from $U(5) \times U(1)$ orientifolds

In this case the relevant $U(1)$ generators are

$$Y = W \quad , \quad Q_{B-L} = \frac{Q_X + 4W}{5} \quad , \quad Q_B = \frac{Q_5 - 2W}{5} \quad , \quad Q_X = \frac{5Q_1 + Q_5}{2} \tag{3.5}$$

Compared to flipped SU(5) the spectrum is the same, it is just the hypercharge and the identification of particles that changed. Here $Q_X$ does not participate in $Y$. In the relevant vacua we found (and mentioned in section 2.1) $U(1)_{B-L}$ is massive. This implies that the associated gauge symmetry is violated. We then expect a violation of the global $U(1)_{B-L}$ symmetry from instanton effects. Typically this will break
B-L to a discrete subgroup and this remnant subgroup, if non-trivial, could play the role of a low energy R-symmetry.

In particular concerning the terms $55_H$, $1055$, $105H5H$ that are unwanted, there are two possibilities:

(a) They are not generated by the instantons of the relevant vacuum, although the $Q_X$ symmetry is violated. This is equivalent to the statement that the leftover discrete symmetry forbids such terms.

(b) They are generated by instantons. Then only if their coefficients are very suppressed can the vacuum pass the baryon and lepton number violation constraints. This is, in principle at least, possible.

Similar statements apply to the two terms $10105_H$, $1010105$. The first term we want to be non-zero as it gives mass to top quarks. It should be generated by instantons and we will find that it does so in some of our vacua. Then, we will also argue that the second term is also generated by the same instantons and contributes non-trivially to proton decay.

We will finally analyze the potential impact of the $1010105_H$ term. If this term is generated at all, it is generated by an instanton and therefore its size is dependent on the instanton factor. It contains the MSSM superpotential terms, $QQQH$ and $QUEcH$ as well as couplings to the heavy triplet Higgs $T$, $TUEc$ and $TUQQ$. The triplet related term will generate after integrating out $T$ terms that are too suppressed to worry us about proton decay. The MSSM terms $QQQH$ and $QUEcH$ give rise to dimension 4 terms proportional to $\frac{\langle H \rangle}{M_{OUT}}$ and are therefore innocuous. The dimension 5 terms involve the Higgs and therefore give a suppressed contribution to proton decay as they must go through a Higgs one-loop to generate the appropriate operators.

3.3 SU(5) from $U(5) \times O(1)$ orientifolds

In such vacua, as we described in section 2.1, the $U(1)$ brane is replaced with an $O(1)$ brane, that we will label as $O(1)_1$. Typically, candidate Higgs pairs end in the hidden sector in such models. This is the case for the solutions presented in section 2.1 where the hidden sector group is another $O(1)$ that we label $O(1)_2$. Although the hidden sector groups associated to the Higgs branes can be different, our arguments below apply with trivial modifications. For this reason we assume an $O(1)$ hidden sector brane in the sequel. Note that in table 2, there is another candidate Higgs pair from strings ending on the $O(1)_1$, which may also be interpreted as a mirror pair of $(5)$’s. We will assume that this pair will eventually become supermassive as it would be problematic otherwise, and since there are no quantum numbers available that would distinguish one the Higgses from a $(5)$.

The role of “$O(1)$” groups in selection rules requires some discussion. First of all, there is no disk diagram that contains an odd number of open string fields ending on any given brane. This implies that there is a perturbative $Z_2$ symmetry associated
with any $O(1)$ matter brane. This symmetry can be viewed as the special case $N = 1$ for $O(N)$ matter branes, and hence it is natural to call it $O(1)$, as if it were a gauge symmetry.

Non-perturbatively these $Z_2$ symmetries may be broken. Instantons may generate zero-mode interactions $\epsilon_{i_1 \ldots i_N} \psi^{i_1} \ldots \psi^{i_N}$, which for odd $N$ (the case of interest here; for even $N$ the $Z_2$ symmetry acts on an odd number of fermions as an $O(N)$ reflection, and is also broken by instantons) clearly violate this symmetry. Such an $\epsilon$ tensor cannot be generated perturbatively. Therefore in general one expects this symmetry to be broken from $O(N)$ to $SO(N)$. This is completely analogous to the breaking of the $O(32)$ symmetry of the ten-dimensional type-I string [40].

The extrapolation of the $\epsilon$ tensor to $N = 1$ may seem tricky, but is best understood by observing that for any $N$, the full contraction of the $\epsilon$ tensor is equal to $N!$, and $1! = 1$. Hence the analog of the $\epsilon$ tensor for $O(1)$ is 1.

Alternatively, one may simply observe in examples that instantons exist which intersect certain $O(1)$ matter branes an odd number of times, and hence, unlike disk diagrams, there is no obvious obstruction for generating the required $O(1)$-violating couplings.

The foregoing discussion might be confusing because it mentions zero-mode interactions involving an odd number of fermions. However, it is only the number of zero-modes involving a given $O(1)$ matter brane that is odd. The total number of zero-modes for a given instanton must in fact be even. For $O1$ instanton branes (the only ones without superfluous universal zero-modes) this is in fact guaranteed by the K-theory constraints. These instanton branes are Euclideanized symplectic matter branes. The K-theory constraints imply as a necessary condition that all symplectic matter branes must have an even number of intersections with any consistent brane configuration. This must be true even if the symplectic brane does not itself participate in that configuration, i.e even if it has vanishing Chan-Paton multiplicity. This is known as the “probe-brane” constraint [74]. Since this condition was checked for all configurations in the database of [22], we cannot encounter any $O1$ instanton branes with an odd number of fermions. Indeed, the additional instanton required (in comparison to $U(5) \times U(1)$ models) to generate the down quark Yukawa couplings must violate both $O(1)_1$ and $O(1)_2$. Note that the K-theory constraints do not impose restrictions on $O$-type matter branes, so the required instantons are in principle allowed to exist.

We describe the couplings of interest below

- The $\overline{5}5_H$ term has charge $(0, 1, 1)$ under $(U(1)_5, O(1)_1, O(1)_2)$ and is therefore still perturbatively forbidden. There can be in principle instantons that violate the $O(1)'s$ and generate this term though.

---

6To avoid confusion of instanton branes and matter branes, we use the notation “O1” for the instanton brane, and “O(1)” for a matter brane
The fate of the $55^H_{15}$ term depends on what plays the role of the right-handed neutrino singlet. There are three possibilities for the singlet seen in table 2, namely $(S,0)$, $(V,V)$ and $(0,S)$ under $(O(1)_1, O(1)_2)$. It is only the choice $(V,V)$ that makes this coupling, and therefore the right-handed neutrino mass perturbatively allowed.

The $1055$ term has charge $(0,0)$ under $(O(1)_1, O(1)_2)$ and is therefore perturbatively allowed. This could spell a disaster as such a term strongly violates lepton and baryon number. A vacuum in this class is viable if some other reason forbids this term. One possibility is to have three distinct observable $O(1)$ branes and each 5 family ends on a different $O(1)$. Because of antisymmetry the two terms $\bar{U} \bar{D} \bar{D}$ and $E_c LL$ must involve different families and therefore are forbidden perturbatively by $O(1)$ charge conservation. However, this does not seem to be the case for the $QDL$ term. We conclude that in the absence of some additional discrete symmetry that forbids these terms, such vacua are ruled out by proton decay.

The $1055^H_{15}$ term is a standard Yukawa coupling that gives mass to the bottom quark and is now perturbatively forbidden because it is charged as $(0,1,1)$ under $(U(1)_5, O(1)_1, O(1)_2)$. The only viable possibility is that it is generated by instantons.

The $105^H_{105}$ term is uncharged under $(U(1)_5, O(1)_1, O(1)_2)$ and is therefore perturbatively allowed. If we have a single higgs pair then such a term does not exist because of antisymmetry. With more pairs then this term is non-trivial but provides mild constraints. However with more than one Higgs pairs FCNC are a generic problem to be addressed.

The $10105^H_{10}$ term gives masses to the top quark. It has charges $(+5,0,1)$ under $(U(1)_5, O(1)_1, O(1)_2)$ It is perturbatively forbidden and can only be generated by instantons.

The $1010105^H_{10}$ term has charges $(+5,1,0)$ under $(U(1)_5, O(1)_1, O(1)_2)$ and is perturbatively forbidden.

Finally the $1010105$ term has charges $(+5,0,1)$ under $(U(1)_5, O(1)_1, O(1)_2)$ the same as the previous Yukawa and later we will argue that they are generated by the same instanton effects.

4. Instanton zero modes

As discussed previously, there are necessary Yukawa couplings that are perturbatively forbidden in orientifold $SU(5)$ models. In Ref. [60], the necessary conditions for
generating these couplings were derived. In this section, we shall briefly review these conditions and then consider other possible operators that would be induced by the same instanton.

In our consideration of the necessary conditions for instantonically inducing the forbidden Yukawa couplings, we shall concentrate on the fermionic zero-modes. As mentioned earlier, we are only considering O1 branes as these contain no extra universal zero-modes that would need to be lifted. As such, we should concentrate on the necessary charged-zero mode content to induce the operator that we are interested in. To zeroth order, the easiest way in order to determine the necessary charged zero-mode content is merely to examine the net charge of the operator desired and then add the minimum number of charged-zero modes to exactly compensate for the charge. As the operator in question, $10_{ij} \bar{5}_H$, has net $U(1)$ charges of $(-1,+5)$ in terms of $(U(1)_1, U(1)_5)$, we would expect that we need to integrate over a set of charged zero-modes with net charges $(+1,-5)$ where the factor of five in $U(1)_5$ comes from the brane multiplicity associated with $U(5)$.

However, we do need to keep in mind that ultimately we are evaluating disc amplitudes. As such, we shall examine which disc amplitudes are of interest. As we shall be considering operators that are perturbatively forbidden and because they should be induced directly into the superpotential, we are interested in perturbatively allowed disc amplitudes containing exactly two charged-zero modes. A quick examination of the $U(1)$ charges in table 6 reveals that one can write down a disc amplitude containing the following states: $10_{ij} \bar{\eta}^i \bar{\eta}^j$, where the $\bar{\eta}$ are zero-modes transforming in a 5 of $SU(5)$ and the $SU(5)$ indices have been left explicit. In addition to this trilinear disc amplitude, another disc amplitude is required in order to generate the operator containing the Yukawa coupling. This other disc amplitude involves: $5_{Hm} \bar{\eta}^m \nu$ where $\nu$ is a charged zero-mode stretched between the instanton brane and the $U(1)_1$ brane. Only considering these two classes of diagrams, we find that

$$ S_{disc} = a \ 10_{ij} \bar{\eta}^i \bar{\eta}^j + b \ 5_{Hm} \bar{\eta}^m \nu + ... $$

(4.1)

where the coefficients $a, b$ would be determined by the explicit evaluation of the aforementioned disc amplitudes and are moduli dependent and the ... refers to higher order terms. We have suppressed the family indices for the moment and we will return to this at the end.

Considering the two classes of disc amplitudes only in Eq. 4.1, we would find upon integration over the set of zero-modes,

$$ \int \prod_{i=1}^{5} d\bar{\eta}^i d\nu e^{-S_{disc}} \sim a^2 b \ \epsilon_{ijklm} 10_{ij} 10_{kl} 5_m, $$

(4.2)

and so we would conclude that, indeed, the expectation of five charged zero-modes for the $U(5)$ stack and one charged zero-mode for the $U(1)$ stack is correct. As mentioned earlier, this analysis was originally performed in Ref. [60].
It is interesting to note that the disc amplitudes that we considered previously are merely the lowest order disc amplitudes possible. In fact, there are higher order amplitudes that can be considered which involve the exact same zero-mode content. These higher order amplitudes would induce other higher order terms in $S_{disc}$ which, in turn, would correspond to higher order terms induced in the superpotential. We shall now consider the next lowest order term induced in $S_{disc}$.

When considering the next lowest order disc amplitude the question of which fields to consider is relatively important. We shall restrict our attention only to the fields contained in table 6. The next lowest order disc amplitude involving $\eta \nu$ is $10_{mn} 5^m \eta^m \nu$. This stems from the fact that $10^5$ has identical $U(1)$ charges to $5_H$ which is generic for these models given two assumptions. The first assumption is that the Yukawa coupling contained in $10^5 5_H$ is perturbatively allowed. This sets the $U(1)$ charges of $5$ relative to $5_H$. The second assumption is that the $U(1)$ charges for $5_H$ and $\tilde{5}_H$ are opposite. This is a fairly generic phenomenon in the models that we examined. Combining these two assumptions we find that the $U(1)$ charges of $5$ should be the same as $5_H$ and the $U(1)_5$ opposite. Therefore, one can always trade a $10 \cdot 5$ for a $5_H$ at the level of $U(1)$ charges. For the other class of disc amplitude, the one involving the charged zero modes $\eta^i \eta^j$, the next lowest order invariant involves three additional fields $(5_H 5_H 1)$ and, as such, we shall ignore it.

Including this new class of disc amplitude we find,

$$S_{disc} \sim a 10_{ij} \eta^i \eta^j + b 5_H \eta^m \nu + c 10_{mn} 5^m \eta^m \nu + ... \quad (4.3)$$

and we find that after integrating out the charged zero-modes,

$$\int \prod_{i=1}^{5} d\eta^i d\nu e^{-S_{disc}} \sim \epsilon_{ijklm} (C_3 10_{ij} 10_{kl} 5_m + C_4 10_{ij} 10_{kl} 10_{mn} 5^m), \quad (4.4)$$

where $C_3 = a^2 b$ and $C_4 = a^2 c$. There is no a priori reason why the coefficient will be zero, and we will proceed assuming that the coefficient $C_4$ in Eq. 4.4 is non-zero and examine the potential phenomenological consequences of inducing this higher order term.

Another important ingredient is the family structure of these terms. Assuming a single Higgs doublet, we can obtain the family structure by the substitutions

$$a_{10_{ij}} \rightarrow \sum_{I=1}^{3} a_s^J 10_{ij}^I, \quad c_{10_{ij} 5^i} \rightarrow \sum_{I,J=1}^{3} a_s^J 10_{ij}^I 5_J^i \quad (4.5)$$

in (4.3) where $I, J$ are family indices, and $s$ is an index that labels different generating instanton configurations. Such configurations are generated by the instanton brane wrapping different possible rigid cycles. This is important for the structure of the effective couplings as it was first pointed out in [52, 53] and subsequently discussed in [60].
Taking all of this into account we can write the final results for the instanton generated couplings as
\[
\delta W = \sum_s e^{-S_s} \left[ C_s^{IJ} 10^I 10^J 5_H + D_s^{JJKL} 10^J 10^K 5^L \right]
\] (4.6)
with
\[
C_s^{IJ} = \sum_{IJ=1}^3 a_s^I a_s^J b_s, \quad D_s^{JJKL} = \sum_{J,K,L=1}^3 a_s^I a_s^J c_s^{KL}
\] (4.7)

5. Effective Field Theory Analysis of proton decay operators

As we have shown in Sect. 4, the generation of perturbatively forbidden Yukawa couplings via stringy instantons will typically generate other operators as well. In this section, our goal is to analyze the potential phenomenological implications of one of these additionally generated operators. We shall consider the phenomenological implications of the operator 1010105. In particular, our goal is to compare the size of this incidentally instantonically induced operator to the size of other operators which contribute to identical low-energy operators in an effective field theory valid below the GUT scale. In an effective field theory valid below the GUT scale, the operator 1010105 contains two separate contributions to proton decay, namely QQQL and UUDE. These are both dimension five operators that have been extensively considered in the literature\[75\]. We shall concentrate on QQQL as the analysis for UUDE is very similar.

Before we proceed with the size comparison, we first should note that QQQL has some symmetry considerations to take into account. This operator is in the superpotential and, as such, should be symmetric under the exchange of all indices (i.e. all SU(3), SU(2), and flavor indices). If we explicitly write the flavor indices as \( Q_i Q_j Q_k L \) and as this term should be invariant under \( SU(3) \) and \( SU(2) \) gauge transformations, the (suppressed) gauge indices corresponding to these groups should be anti-symmetric under exchanges. We therefore conclude that if \( i = j = k \) then this term is vanishing by gauge invariance. This implies that \( D_{iiii} \) from Eq. 4.6 is actually zero and that \( D_{1122} \) would be the leading contribution to proton decay. We shall proceed assuming that this coefficient is nonzero. We also note that the symmetry considerations for UUDE are different but similar in nature.

We shall now consider other sources of QQQL for effective field theories of \( SU(5) \) GUT models. In the absence of the instantonically generated 1010105, the primary source of QQQL is the exchange of the triplet associated with the Higgs. If the Yukawa’s, \( 1055^H \) and \( 10105^H \), are non-zero then upon integrating out triplet QQQL is generated. Thus, in our effective theory we have,
\[
(G_{\text{eff}} + G_{\text{np}})QQQL,
\] (5.1)
where $G_{np}$ is the instantonically generated term and $G_{eff}$ is the term arising from integrating out the Higgs triplet. Our goal is to compare the relative sizes of $G_{eff}$ and $G_{np}$.

From Ref. [75], we have an estimate for the size of $G_{eff}$. The size of $G_{np}$ can be estimated using standard methods as well. We find,

$$G_{eff} = \frac{h_u h_s}{M_T}$$
$$G_{np} = \frac{\tilde{\det} e^{-S}}{M_s}$$

(5.2)

where, $h_u, h_s$ are the Yukawa couplings for the up and strange quark respectively, $M_T$ is the mass of the triplet, and $\tilde{\det} e^{-S}$ is an estimate of the size of the instanton contribution that is generating the 1010105, where $e^{-S}$ is the classical instanton factor and $\det$ stands for the determinant of fluctuations around the instanton.

As was shown earlier, the instantonic zero-modes that generate the Yukawa coupling are the same as those that generate 1010105. We therefore expect that the details of the instantonic contribution should mostly drop out. In the case of flipped $SU(5)$ we find,

$$G_{np} = \frac{\tilde{\det} M_T}{\det M_s h_u} \sim 10^5 \frac{M_T}{M_s}$$

(5.3)

where we have assumed that the ratio of determinants for the instantonic contributions will amount to only $O(1)$ effects.

If the triplet obtains its mass from the standard mechanism described in section 3.1, then its related to $M_{GUT}$ by the square of a Yukawa coupling. This implies that it is several orders of magnitude below the GUT scale and therefore the primary source of proton decay, is deadly. If on the other hand there is another source of mass for the triplet so that it is $\gtrsim M_{GUT}$, then (5.3) implies that the contribution of the 1010105 operator to proton decay is deadly. There seems to be no way out except some form of fine tuning.

There could be several ways that such a fine tuning could arise:

- The associated determinants relevant for the two coupling are hierarchically different in size, by carefully tuning relevant moduli.
- The generalized volumes of the relevant instanton cycles are not very large so multi-instanton corrections are comparable. This may have as an effect that the effective instanton effect is much smaller than what indicated by the one-instanton result.

The corresponding ratio (5.3) for standard Georgi-Glashow $SU(5)$ is smaller by a factor of $\frac{h_u}{h_s}$ and is thus, better by a factor of $\sim 30$. In this case of course fine-tuning is needed to make $M_T \gtrsim M_{GUT}$. In view of this the 1010105 operator is still
highly problematic and additional fine-tuning is needed of the type described above for flipped SU(5).

Thus, if the GUT scale and the string scale are not separated by five orders of magnitude in energy, we conclude that these non-perturbative effects could be quite important, and can easily rule out SU(5) vacua.

6. Search for Instanton branes in string vacua

<table>
<thead>
<tr>
<th>Model Type</th>
<th>All instanton branes</th>
<th>Yukawa generators</th>
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</thead>
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<td>Total</td>
<td>U</td>
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<tr>
<td>2753</td>
<td>1136</td>
<td>4.9 x 10^5</td>
</tr>
<tr>
<td>2880</td>
<td>1049</td>
<td>2.1 x 10^5</td>
</tr>
<tr>
<td>6580</td>
<td>146</td>
<td>7.0 x 10^4</td>
</tr>
<tr>
<td>14861</td>
<td>12</td>
<td>1190</td>
</tr>
<tr>
<td>617</td>
<td>16845</td>
<td>3.5 x 10^6</td>
</tr>
</tbody>
</table>

Table 7: Summary of instanton branes

In table (7) we list the number of candidate instanton branes, divided into unitary, symplectic and orthogonal, for all models combined. Only instantons of type O have a chance of having exactly the right number of zero-modes, but to get an idea of how common these are we have listed the other types as well. The fifth column indicates how many of all these candidates have exactly the correct number of zero-modes.

The first four rows in the table refer to the various kinds of U(5) x U(1) models discussed earlier. Here zero-modes from intersections of the instanton brane with U(5) x U(1) as well as self-intersections were taken into account. The final step is to find a hidden sector that cancel all tadpoles, and does not intersect the candidate instanton brane, so that no additional zero-modes are introduced.

This turned out to be possible in precisely six cases, although only at a price: we had to allow chiral hidden-observable matter. In [22] such matter was always required to be non-chiral, but it turns out that none of the tadpole solutions described admit an additional instanton brane. This is not surprising as intuition from the constructions and earlier searches [61] that such instanton branes are very rare in RCFT models with a high degree of symmetry as here. By allowing chiral hidden-observable matter we enlarge the set of available models, and hence the chance of success.

The six cases are all very similar, but not all identical, and occur for the same MIPF as the six solutions (without instantons) for model 2753 described above. They
have an hidden sector group with even more factors, $O(4) \times O(3) \times O(2)^3 \times O(1)^2 \times U(1)^2$, and rather amazingly none of these intersects the instanton brane.

<table>
<thead>
<tr>
<th>Multiplicity</th>
<th>Visible Sector</th>
<th>$U(5)$</th>
<th>$U(1)$</th>
<th>O1 Instanton</th>
<th>Hidden</th>
</tr>
</thead>
<tbody>
<tr>
<td>5(4+1)</td>
<td>A</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3(3+0)</td>
<td>V*</td>
<td>V*</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3(3+0)</td>
<td>0</td>
<td>S</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2(1+1)</td>
<td>V</td>
<td>V*</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1(1+0)</td>
<td>0</td>
<td>V</td>
<td>V</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1(1+0)</td>
<td>V*</td>
<td>0</td>
<td>V</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>V</td>
<td>0</td>
<td>0</td>
<td>V</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>V*</td>
<td>0</td>
<td>0</td>
<td>V</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>V</td>
<td>0</td>
<td>V</td>
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<tr>
<td>5</td>
<td>0</td>
<td>V*</td>
<td>0</td>
<td>V</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Adj</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>Adj</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

**Table 8:** The spectrum of the model with exactly the correct instanton brane.

The spectrum of this model is shown in (8), without details of the hidden sector, and without purely hidden matter (matter with trivial $U(5) \times U(1)$ quantum numbers). The detailed hidden sector and the observable-hidden matter is presented in appendix A. The bi-fundamentals in lines 7...10 are the chiral observable-hidden matter multiplets. Although their net chirality in $U(5)$ and $U(1)$ is – necessarily – zero, they are chiral because they end on distinct hidden sector branes. Only after a breakdown of most of the hidden sector gauge group can these particles acquire a mass.

We have searched the same models for instantons that may generate the unwanted couplings *(i.e. those that violate R-parity)* mentioned in section 3, and we found none. This is not terribly surprising: “good” instantons with precisely the correct zero modes are very rare, and hence one may expect exact “bad” instantons to be rare as well. In this particular case the large number of hidden sectors is very likely to yield superfluous zero-modes, but it was not even necessary to check that, because already the number of zero-modes from intersections with the $U(5)$ and $U(1)$ was too large. These statements are true in the exact RCFT point in moduli space, where we do our computations. Outside that point some of the zero-modes may be lifted, but it is possible that a kind of R-parity survives in the form of a restriction on instanton zero modes.

The exponential suppression of the instanton contribution is determined by the size of $\frac{1}{g^2}$, where $g$ is the gauge coupling. This quantity in its turn is determined by the coupling of the dilaton to the instanton brane. Since the instanton brane is not
a matter brane, there is at least a chance that the gauge coupling is large, and hence
the instanton contribution is not too suppressed. In this particular example the
ratio of the $U(5)$ and instanton brane dilaton coupling is 4.38. This means that the
instanton contribution is indeed considerably larger than those of standard model
instantons (at the GUT scale), but still far too small to give the right top quark
Yukawa coupling (which should be of order 1). But as above, this statement is valid
in the exact RCFT point. In this context, such considerations are qualitative only.
In order to get quantitative agreement, one would have to move far away from the
RCFT point into a region that is non-perturbative in the instanton brane coupling.

The last line in table (7) describes the results for $U(5) \times O(1)$ models, with the
$O(1)$ factor treated analogously as the $U(1)$ factor in the other models. In other
words, the column “Correct zero modes” list instantons that would generate top
quark Yukawa couplings if the Higgs comes from $U(5)$ and $O(1)$ intersections, just
as the (5). As explained earlier, this is an undesirable option, but the only one we
can investigate without knowing the hidden sector.

A better option would be to have an additional “Higgs brane” like the $O(1)_2$
factor mentioned in section (3.3). Note that this $O(1)_2$ is not part of the Standard
Model brane configuration according to the criteria used in [22]. These criteria only
take into account chiral standard model matter (quarks and leptons), and not the
vector-like (M)SSM Higgs pair. The configurations considered in [22] have either
two, three or four brane stacks, and include only those branes contributing to chiral
matter. Indeed, the group $O(1)_2$ described above came out coincidentally as a hidden
sector.

The reason for organizing the search in that manner was that in general it is
undesirable to have a separate Higgs brane (even though in this particular case it may
still be the best option). A separate Higgs brane would imply that all couplings with
a single Higgs fields (and hence all Yukawa couplings) are perturbatively forbidden,
and can at best be generated non-perturbatively. This is precisely the problem we
are facing here.

Since the $O(1)_2$ brane is, by the definition of [22], not part of the standard model
brane configuration, we do not have a systematic database at our disposal for such
model. However, as explained in section (2.1), we did perform a complete hidden
sector search for all 16845 models in this class. There are a few more case with just
a single Higgs fields (and hence all Yukawa couplings) are perturbatively forbidden,
and can at best be generated non-perturbatively. This is precisely the problem we
are facing here.

Although the scan of the 16845 $U(5) \times O(1)$ models was for just one sample of
the hidden sector per configuration, we are certain that all single-brane hidden sector
were found, since they appear first. In other cases one might consider to use one of the various hidden sector branes as the Higgs brane. However, this would require a systematic enumeration of all possible hidden sectors for each of the 16845 standard model configurations. In addition, the chances for finding perfect solutions seem small: Not only would one have to find two instantons, both for up and for down type couplings, but also their intersections with all the other hidden sector branes would have to vanish. With a large enough sample, solutions will probably exist, but given the success rate in other cases it is unlikely that the set of 16845 models from [22] is large enough. For these reasons we did not pursue these models further.

7. Conclusions

We have analyzed orientifold vacua with SU(5) gauge group, realizing SU(5) or flipped SU(5) grand unification. Many tadpole solution have been constructed from Gepner model building blocks using the algorithm developed in [22]. We found all such top-down constructions as well as tadpole-free vacua, with one extra observable brane of the U(1) or O(1) type. This is one small subset (but the simplest) of the SU(5) configurations found in [22].

We gave a general analysis of possible terms in the superpotential of such vacua, up to quartic order, and classified them according to their fatality (baryon and lepton violating interactions which are relevant or marginal), and usefulness (Yukawa coupling). We have classified which terms can or must be generated by instanton effects. As is well known the top Yukawa’s in SU(5) and the bottom in flipped SU(5) must be generated from instantons (in the absence of fluxes).

In flipped SU(5) vacua, B-L cannot be anomalous as it participates in the hypercharge. It forbids all dangerous terms, but it is necessarily broken when the SU(5) gets broken at the GUT scale. We have estimated that the proton decay generated is typically small.

In U(5)×U(1) vacua, instanton effects must generate the top Yukawa couplings, and at the same time they break the B-L symmetry. Successful vacua, have either a $Z_2$ remnant of the B-L symmetry acting as as R-parity and forbidding the dangerous terms, or such term may have exponentially suppressed instanton contributions. In the second case they are viable if the exponential factors are sufficiently suppressed. We provide several tadpole solutions of the first case where instantons generate the top Yukawa’s, but preserve a $Z_2$ R-symmetry.

Finally U(5)×O(1) vacua are problematic on several grounds and need extra symmetries beyond those that are automatic, in order to have a chance of not being outright excluded. This is related to the absence of natural R-symmetries or gauge symmetries that will forbid the dangerous low-dimension baryon-violating interactions.
A generic feature of all SU(5) vacua is that the same instanton that generates the non-perturbative quark Yukawa coupling also generates the $10 \times 10 \times 10 \times 5$ in the superpotential. This is a second source of proton decay, beyond the classic one emanating from the Higgs triplet times the appropriate Yukawa coupling. Generically, the size of this contribution to proton decay is $10^5 \frac{M_T}{M}$ larger than the conventional source in the flipped SU(5) model, ($M_T$ is the triplet Higgs mass). This signals severe phenomenological trouble and calls for important fine-tuning. In the SU(5) case, the size is 30 times smaller, but that does not evade the need for fine-tuning.

We have searched for appropriate instantons that would generate the perturbatively forbidden quark Yukawa couplings in the SU(5) vacua we have constructed. We found the appropriate instantons with the correct number of zero modes in 6 relatives of the spectrum Nr. 2753. We have also searched for all other instantons that could generate the bad terms in the superpotential and found none. This translates into the existence of a $Z_2$ R-symmetry that protects from low-dimension baryon and lepton-violating couplings.

A related formalism that provides orientifold vacua with a non-perturbative description for some of their features is F-theory, [62]. This is a new area for model building and recently bottom-up constructions of SM stacks of D7 branes were explored, [63, 64]. Global constructions are in their infancy, [65, 66] but despite this, phenomenologically interesting global GUT vacua were recently described in [66].

Like orientifolds as long as the appropriate U(1) symmetry that forbids the top Yukawa coupling is present, then the coupling can be generated only by instantons. In such a case, our discussion, estimates and conclusion remain unaltered. However, in F-theory there is the option of breaking the offending U(1) symmetry non-perturbatively by considering enhanced symmetry singularities. This now allows the top Yukawa coupling at triple intersections of appropriate singularities. The offending $1010105$ term may be now generated via three possible sources: (a) Mediation by higher triplets (for example KK triplets). (b) Potential $D_3$-instantons effects. (c) String instantons stretched between four appropriate divisors.

The (a) contribution is phenomenologically dangerous and an idea to avoid it has been advanced in [63] but putting the up and down Higgses on different divisors. Contribution (b) is no-longer guaranteed to exist but if it does, it is no longer related to the top Yukawas. It is generically exponentially suppressed. Our arguments in section 5 imply that if the coupling generated here is much smaller than about $10^{-12}$ in string units then we do not need to worry about it. Otherwise a detailed analysis is necessary.

Contribution (c) is also generically exponentially suppressed. The reason is that four-point intersections of divisors are non-generic. However, the quantitative statements and constraints in (b) are also valid in this case.

Finally we should mention that in special cases extra PQ-like (anomalous) symmetries may forbid the $1010105$ term, while allowing the top Yukawa. An example
based on such a symmetry emanating from $E_6$ was described in [63].

Acknowledgements


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A. APPENDIX

In this appendix we present the exact spectrum of one of the models that have an exact instanton brane (the other models are nearly identical). The first column gives an ad-hoc number we use for referring to the various massless states. The second column gives the total multiplicity (for representation plus its conjugate), the last column the chiral multiplicity (\textit{i.e} the multiplicity for the representation minus its conjugate).

The spectrum is divided in the table into the following segments: Quarks and leptons (1-3), the third row contributes 5 symmetric tensors of $U(1)$, with a net chirality 3; these can play the rôle of right-handed singlet neutrinos), the Higgs pair (4), the instanton zero modes (5-6), chiral observable-hidden matter (7-12), non-chiral observable-hidden matter (13-16), non-chiral observable rank two tensors (17-20), chiral matter within the hidden sector (21-29), and non-chiral matter within the hidden sector (30-49).

The chiral exotics may acquire masses via symmetry breaking in the hidden sector. In view of the size of the hidden sector such a analysis lies beyond the scope of the present paper.
Table 9: The complete spectrum of the model with exactly the correct instanton brane.

References


[ArXiv:hep-th/0502005].


