Double parton interactions in $\gamma + 3$ jet events in $pp$ collisions at $\sqrt{s} = 1.96$ TeV

(The D0 Collaboration)
We have used a sample of $\gamma + 3$ jets events collected by the D0 experiment with an integrated luminosity of about $1 \text{ fb}^{-1}$ to determine the fraction of events with double parton scattering ($\delta p$) in a single $pp$ collision at $\sqrt{s} = 1.96 \text{ TeV}$. The DP fraction and effective cross section ($\sigma_{\text{eff}}$), a process-independent scale parameter related to the parton density inside the nucleon, are measured in three intervals of the second (ordered in $p_T$) jet transverse momentum $p_T^{j2}$ within the range $15 < p_T^{j2} < 30 \text{ GeV}$. In this range, $f_{\text{DP}}$ varies between $0.23 < f_{\text{DP}} < 0.47$, while $\sigma_{\text{eff}}$ has the average value $\sigma_{\text{eff}} = 16.4 \pm 0.3(\text{stat}) \pm 2.3(\text{syst}) \text{ mb}$.

PACS numbers: 14.20.Dh, 13.85.Qk, 12.38.Qk

I. INTRODUCTION

Many features of high energy inelastic hadron collisions depend directly on the parton structure of hadrons. The inelastic scattering of nucleons need not to occur only through a single parton-parton interaction and the contribution from double parton (DP) collisions can be significant. A schematic view of a double parton scattering event in a $pp$ interaction is shown in Fig. 1. The rate of events with multiple parton scatterings depends on how the partons are spatially distributed within the nucleon. Theoretical discussions and estimations [1–5] stimulated measurements [6–9] of DP event fractions and DP cross sections. The latter can be expressed as

$$\sigma_{\text{DP}} \equiv \frac{m \sigma_A \sigma_B}{2 \sigma_{\text{eff}}},$$

where $\sigma_A$ and $\sigma_B$ are the cross sections of two independent partonic scatterings $A$ and $B$. The factor $m$ is equal to unity when processes $A$ and $B$ are indistinguishable while $m = 2$ otherwise [5, 10, 11].

14.20.Dh, 13.85.Qk, 12.38.Qk
section. Its relation to the spatial distribution of partons within the proton has been discussed in [1, 3–5, 10, 11].

The ratio $\sigma_B/\sigma_{\text{eff}}$ can be interpreted as the probability for partonic process $B$ to occur provided that process $A$ has already occurred. If the partons are uniformly distributed inside the nucleon (large $\sigma_{\text{eff}}$), $\sigma_{\text{DP}}$ will be rather small and, conversely, it will be large for a highly concentrated parton spatial density (small $\sigma_{\text{eff}}$). The implication and possible correlations of parton momenta distribution functions in (1) are discussed in [12–14].

In addition to constraining predictions from various models of nucleon structure and providing a better understanding of non-perturbative QCD dynamics, measurements of $f_{\text{DP}}$ and $\sigma_{\text{eff}}$ are also needed for the accurate estimation of backgrounds for many rare new physics processes as well as for Higgs boson searches at the Tevatron and LHC [15, 16].

To date, there have been only four dedicated measurements studying double parton scattering: by the AFS Collaboration in $pp$ collisions at $\sqrt{s} = 63$ GeV [6], by the UA2 Collaboration in $pp$ collisions at $\sqrt{s} = 630$ GeV [7], and twice by the CDF Collaboration in $pp$ collisions at $\sqrt{s} = 1.8$ TeV [8, 9]. The four-jet final state was used in the measurements to extract values of $\sigma_{\text{DP}}$ and then $\sigma_{\text{eff}}$, and the $\gamma + 3$ jets final state was used in [9] to extract $f_{\text{DP}}$ and then $\sigma_{\text{eff}}$. The obtained values of $\sigma_{\text{eff}}$ by those experiments are $\sigma_{\text{eff}} \approx 5 \text{ mb (AFS)}$, $\sigma_{\text{eff}} > 8.3 \text{ mb at the 95\% C.L. (UA2)}$, $\sigma_{\text{eff}} = 12.1^{+10.4}_{-5.1} \text{ mb (CDF, four-jet)}$ and $\sigma_{\text{eff}} = 14.5 \pm 1.7^{+1.7}_{-2.3} \text{ mb (CDF, } \gamma + 3 \text{ jets)}$. Table I summarizes all previous measurements of $\sigma_{\text{eff}}, \sigma_{\text{DP}}$, and $f_{\text{DP}}$.

This paper presents an analysis of hard inelastic events with a photon candidate (denoted below as $\gamma$) and at least 3 jets (referred to below as $\gamma + 3$ jets events) collected with the D0 detector [17] at the Fermilab Tevatron Collider with $\sqrt{s} = 1.96$ TeV and an integrated luminosity of $1.02 \pm 0.06 \text{ fb}^{-1}$. In this final state, DP events are caused by two partonic scatterings, with $\gamma + j$ events production in the first scattering and dijet production in the second. Thus, the rate of $\gamma + 3$ jets events and their kinematics should be sensitive to a contribution from additional parton interactions. Differences in the types of the two final states ($\gamma + j$ and dijets) and better energy measurement of photons as compared with jets facilitate differentiation between the two DP scatterings as compared with the 4-jet measurements. Also, it was shown in [18] that a larger fraction of DP events is expected in the $\gamma + 3$ jets final state as compared with the 4-jet events. The large integrated luminosity allows us to select $\gamma + 3$ jets events at high photon transverse momentum, $60 < p_T^\gamma < 80 \text{ GeV (vs. } p_T^\gamma > 16 \text{ GeV in CDF [9]), with a larger photon purity [19]. The choice of a high threshold on the photon momentum provides (a) a clean separation between the jet produced in the same parton scattering from which the photon originates and the jets originating from additional parton scatterings and (b) a better determination of the energy scale of the $\gamma + j$ process. Also, in contrast to [9], the jet transverse momenta are corrected to the particle level. Other differences in the technique used for extracting $\sigma_{\text{eff}}$ are described below.

This paper is organized as follows. Section II briefly describes the technique used to extract the $\sigma_{\text{eff}}$ parameter. Section III provides the description of the data samples and selection criteria. Section IV describes the models used for signal and background events. In Section V we introduce the variables which allow us to distinguish DP events from other $\gamma + 3$ jets events and determine their fraction. The procedure for finding the fractions of DP events is described in Section VI. Section VII describes the determination of other parameters needed to calculate $\sigma_{\text{eff}}$. Results of the measurement are given in Section VIII with their application to selected models of parton density.

II. TECHNIQUE FOR EXTRACTING $\sigma_{\text{eff}}$ FROM DATA

In the 4-jet analyses [6–8], $\sigma_{\text{eff}}$ was extracted from measured DP cross sections using Monte Carlo (MC) modeling for signal and background events and QCD predictions for the dijet cross sections. Both MC modeling and the QCD predictions suffer from substantial uncertainties leading to analogous uncertainties in $\sigma_{\text{eff}}$. Another technique for extracting $\sigma_{\text{eff}}$ was proposed in [9]. It uses only quantities determined from data and thus minimizes the impact of theoretical assumptions. Here we follow this method and extract $\sigma_{\text{eff}}$ without theoretical predictions of the $\gamma + j$ and dijets cross sections by comparing the number of $\gamma + 3$ jets events produced in DP interactions in single $pp$ collisions to the number of $\gamma + 3$ jets events produced in two distinct hard interactions occurring in two separate $pp$ collisions in the same beam crossing. The latter class of events is referred to as double interaction (DI) events. Assuming uncorrelated parton scatterings in the DP process [1–5, 11], DP and DI events should be kinematically identical. This assumption is discussed in Appendix A.

Measurements of dijet production with jet $p_T \gtrsim 12 - 15 \text{ GeV [20]}$ in both central and forward rapidity [21] regions indicate that the contribution from single and double diffraction events represents $\lesssim 1\%$ of the total dijet cross section. Therefore $\gamma + j$ and dijet events with jet...
TABLE I: Summary of the results, experimental parameters, and event selections for the double parton analyses performed by the AFS, UA2 and CDF Collaborations.

| Experiment | $\sqrt{s}$ (GeV) | Final state $p_T^{\text{jet}}$ (GeV) | $|y|_{\text{jet}}$ range | $\sigma_{\text{eff}}$ | $\sigma_{\text{DP}}$, $f_{\text{DP}}$ |
|------------|------------------|---------------------------------|----------------------|------------------|---------------------|
| AFS (pp), 1986 [6] | 63 | 4 jets $p_T^{\text{jet}} > 4$ | $|y|_{\text{jet}} < 1$ | $\sim 5$ mb | $\sigma_{\text{DP}}/\sigma_{\text{dijet}} = (6.1 \pm 1.5)$% |
| UA2 (pp), 1991 [7] | 630 | 4 jets $p_T^{\text{jet}} > 15$ | $|y|_{\text{jet}} < 2$ | $> 8.3$ mb (95% C.L.) | $\sigma_{\text{DP}} = 0.49 \pm 0.20$ nb |
| CDF (pp), 1993 [8] | 1800 | 4 jets $p_T^{\text{jet}} > 25$ | $|y|_{\text{jet}} < 3.5$ | $12.1^{+10.7}_{-6.4}$ mb | $\sigma_{\text{DP}} = (63.3^{+39.2}_{-28})$ nb, $f_{\text{DP}} = (5.4^{+1.6}_{-2.0})$% |
| CDF (pp), 1997 [9] | 1800 | 4 jets $p_T^{\text{jet}} > 6$ | $|y|_{\text{jet}} < 3.5$ | $14.5 \pm 1.7$ mb | $f_{\text{DP}} = (52.6 \pm 2.5)$% |

$pt > 15$ GeV are produced predominantly as a result of inelastic non-diffractive (hard) $p\bar{p}$ interactions. In a $p\bar{p}$ beam crossing with two hard collisions the probability for a DI event in that crossing can be expressed as

$$P_{\text{DI}} = 2 \frac{\sigma_{\gamma \gamma} \sigma_{jj}}{\sigma_{\text{hard}} \sigma_{\text{hard}}}.$$  \hspace{1cm} (2)

Here $\sigma_{\gamma \gamma}$ and $\sigma_{jj}$ are the cross sections to produce the inclusive $\gamma +$ jets and dijet events, which combined give the $\gamma +$ 3 jets final state, and $\sigma_{\text{hard}}$ is the total hard $p\bar{p}$ interaction cross section. The factor 2 takes into account that the two hard scatterings, producing a $\gamma +$ jets or dijet event, can be ordered in two ways with respect to the two collision vertices in the DI events. The number of DI events, $N_{\text{DI}}$, can be obtained from $P_{\text{DI}}$, after correction for the efficiencies to pass geometric and kinematic selection criteria $\epsilon_{\text{DI}}$, the two-vertex event selection efficiency, $\epsilon_{\text{2vtx}}$, and the number of beam crossings with two hard collisions, $N_{\text{2coll}}$:

$$N_{\text{DI}} = 2 \frac{\sigma_{\gamma \gamma} \sigma_{jj}}{\sigma_{\text{hard}} \sigma_{\text{hard}}} N_{\text{2coll}} \epsilon_{\text{DI}} \epsilon_{\text{2vtx}}.$$  \hspace{1cm} (3)

Analogously to $P_{\text{DI}}$, the probability for DP events, $P_{\text{DP}}$, in a beam crossing with one hard collision, is

$$P_{\text{DP}} = \frac{\sigma_{\text{DP}}}{\sigma_{\text{hard}}} = \frac{\sigma_{\gamma \gamma} \sigma_{jj}}{\sigma_{\text{eff}} \sigma_{\text{hard}}},$$  \hspace{1cm} (4)

where we used Eq. (1). Then the number of DP events, $N_{\text{DP}}$, can be expressed from $P_{\text{DP}}$ with a correction for the geometric and kinematic selection efficiency $\epsilon_{\text{DP}}$, the single-vertex event selection efficiency $\epsilon_{\text{1vtx}}$, and the number of beam crossings with one hard collision, $N_{\text{1coll}}$:

$$N_{\text{DP}} = \frac{\sigma_{\gamma \gamma} \sigma_{jj}}{\sigma_{\text{eff}} \sigma_{\text{hard}}} N_{\text{1coll}} \epsilon_{\text{DP}} \epsilon_{\text{1vtx}}.$$  \hspace{1cm} (5)

The ratio of $N_{\text{DP}}$ to $N_{\text{DI}}$ allows us to obtain the expression for $\sigma_{\text{eff}}$ in the following form:

$$\sigma_{\text{eff}} = \frac{N_{\text{DI}} \epsilon_{\text{DP}} \epsilon_{\text{1vtx}}}{N_{\text{DP}} \epsilon_{\text{DI}}} R_c \sigma_{\text{hard}},$$  \hspace{1cm} (6)

where $R_c \equiv (1/2)(N_{\text{1coll}}/N_{\text{2coll}})(\epsilon_{\text{1vtx}}/\epsilon_{\text{2vtx}})$. The $\sigma_{\gamma \gamma}$ and $\sigma_{jj}$ cross sections do not appear in this ratio and all the remaining efficiencies for DP and DI events enter only as ratios, resulting in a reduction of the impact of many correlated systematic uncertainties.

Figure 2 shows the possible configurations of signal $\gamma +$ 3 jets DP events produced in a single $pp$ interaction and having one parton scattering in the final state with a $\gamma$ and at least one jet, superimposed with another parton scattering into a final state with at least one jet. We define different event topologies as follows. Events in which both jets from the second parton scattering are reconstructed, pass the selection cuts and are selected as the second and third jets, in order of decreasing jet $pt$, are defined as Type I. In Type II events, the second jet in the dijet process is either lost due to the finite jet reconstruction efficiency of detector acceptance or takes the fourth position after the jet $pt$ ordering. We also distinguish Type III events, in which a jet from the second parton interaction becomes the leading jet of the final 3-jets system, although they are quite rare given the $pt$ range selected for the photon.

The main background for the DP events are single parton (SP) scatterings with hard gluon bremsstrahlung in the initial or final state $qq \rightarrow q'g'g$, $qq \rightarrow q'gg$ that give the same $\gamma +$ 3 jets signature. They are also shown in Fig. 2. The fraction of DP events is determined in this analysis using a set of variables sensitive to the kinematic configurations of the two independent scatterings of parton pairs (see Secs. V and VI).

The DI events differ from the DP events by the fact that the second parton scattering happens at a separate $pp$ collision vertex. The DI events, with the photon and at least one jet from one $pp$ collision, and at least one jet from another $pp$ collision are shown in Fig. 3 with a similar (to DP) set of DI event types. The background to DI events is due to two-vertex SP events with hard $\gamma + 3$ jets events from one $pp$ interaction with an additional soft interaction, i.e. having no reconstructed jets. The diagrams for these non-DI events are also shown in Fig. 3.

III. D0 DETECTOR AND DATA SAMPLES

The D0 detector is described in detail in [17]. Photon candidates are identified as isolated clusters of energy depositions in the uranium and liquid-argon sampling.
calorimeter. The central calorimeter covers the pseudorapidity [22] range \( |\eta| < 1.1 \) and two end calorimeters cover \( 1.5 < |\eta| < 4.2 \). The electromagnetic (EM) section of the calorimeter is segmented longitudinally into four layers and transversely into cells in pseudorapidity and azimuthal angle \( \Delta \eta \times \Delta \phi = 0.1 \times 0.1 \) (0.05 \( \times \) 0.05 in the third layer of the EM calorimeter). The hadronic portion of the calorimeter is located behind the EM section. The calorimeter surrounds a tracking system consisting of silicon microstrip and scintillating fiber trackers, both located within a 2 T solenoidal magnetic field.

The events used in this analysis should first pass triggers based on the identification of high \( p_T \) clusters in the EM calorimeter with loose shower shape requirements for photons. These triggers are 100% efficient for \( p_T > 35 \text{ GeV} \). To select photon candidates in our data samples, we use the following criteria [19]. EM objects are reconstructed using a simple cone algorithm with a cone size of 0.7. Jets must satisfy quality criteria which suppress background from leptons, photons, and detector noise effects. To reject background from cosmic rays and \( W \to \ell \nu \) decay, the missing transverse momentum in the event is required to be less than 0.7\( p_T \). All pairs of objects in the event, (photon, jet) or (jet, jet), also are required to be separated in \( \eta - \phi \) space by \( \Delta R > 0.7 \).

Each event must contain at least one \( \gamma \) in the rapidity region \(|y| < 1.0 \) or \( 1.5 < |y| < 2.5 \) and at least three jets with \( |y| < 3.0 \). Events are selected with \( \gamma \) transverse momentum \( 60 < p_T < 80 \text{ GeV} \), leading (in \( p_T \) jet) \( p_T > 25 \text{ GeV} \), while the next-to-leading (second) and third jets must have \( p_T > 15 \text{ GeV} \). The jet transverse momenta are corrected to the particle level. The high \( p_T \) scale (i.e. the scale of the first parton interaction) allows a better separation of the first and second parton interactions in momentum space.

Data events with a single \( pp \) collision vertex, which compose the sample of DP candidates (“1Vtx” sample), are selected separately from events with two vertices which compose the sample of DI candidates (“2Vtx” sample). The collision vertices in both samples are required to have at least three associated tracks and to be within 60 cm of the center of the detector along the beam (z) axis.

The \( p_T \) spectrum for jets from dijet events falls faster than that for jets resulting from initial or final state radiation in the \( \gamma + \) jets events, and thus DP fractions should depend on the \( p_T [1, 3, 4, 10] \). The DP fractions and \( \sigma_{\text{eff}} \) are determined in three \( p_T^{\text{jet}} \) bins: 15–20, 20–25, and 25–30 GeV. The total numbers of 1Vtx and 2Vtx \( \gamma + 3 \) jets events remaining in each of the three \( p_T^{\text{jet}} \) bins after all selection criteria are given in Table II.

### Table II: The numbers of selected 1Vtx and 2Vtx \( \gamma + 3 \) jets events in bins of \( p_T^{\text{jet}} \).

<table>
<thead>
<tr>
<th>Sample</th>
<th>( p_T^{\text{jet}} ) (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>15–20</td>
</tr>
<tr>
<td>1Vtx</td>
<td>2182</td>
</tr>
<tr>
<td>2Vtx</td>
<td>2026</td>
</tr>
</tbody>
</table>
IV. DP AND DI MODELS

To study properties of DP and DI events and calculate their fractions in the 1Vtx and 2Vtx samples, respectively, we construct DP and DI models by pairing data events. The DP model is constructed by overlaying in a single event one event of an inclusive sample of $\gamma + \geq 1$ jet events and one event of a sample of inelastic non-diffractive events selected with the minimum bias trigger and a requirement of at least one jet ("MB" sample) [24]. Both samples contain only single-vertex events. The jet $p_T$ from the MB sample is recalculated relative to the vertex of the $\gamma + j$ event. The resulting mixed events, with jets re-ordered in $p_T$, are required to pass the $\gamma + 3$ jets event selections described above. This model of DP events, called MixDP, assumes independent parton scatterings, with $\gamma + j$ and dijet final states, by construction. The mixing procedure is shown schematically in Fig. 4.

FIG. 4: Description of the mixing procedure used to prepare the MixDP signal sample. Two combinations of mixing $\gamma + 1$ jet and two jets from dijet events (a) and $\gamma + 2$ jets and one jet from dijet events (b) are considered. The dotted line represents a jet failing the selection requirements.

In the DI model, called MixDI, each event is constructed by mixing one event of the $\gamma + \geq 1$ jet sample and one event of the $\gamma + 1$ jet MB sample. Both events are exclusively selected from the two-vertices events sample. In the case of $\geq 2$ jets in any component of the MixDI mixture, the first two jets, leading in $p_T$, are required to originate from the same vertex using the position along the beam axis of the point of closest approach to a vertex for the tracks associated to each jet and a cut on the minimal jet charged particle fraction, as discussed in Appendix B. We consider the two-vertex $\gamma + j$ and dijet events, components of the MixDI model, to better take into account the underlying energy, coming from the soft interactions of the spectator partons. The amount of this energy is different for single- and two-vertex events and causes a difference in the photon and jet identification efficiencies in the DP and DI events (see Section VII). As a background to the DI events, we consider the two-vertex $\gamma + 3$ jets sample without a hard interaction at the second vertex (Bkg2Vtx sample), obtained by imposing the direct requirement that all three jets originate from the same vertex using the jet track information.

The fractions of Type I (II) events in the MixDP and MixDI samples are the same within 1.5% for each $p_T^{j\ell\ell}$ bin and vary for both samples from 26% (73%) at $15 < p_T^{j\ell\ell} < 20$ GeV to (14–15)% [(84–86)%] at $25 < p_T^{j\ell\ell} < 30$ GeV. Type III events are quite rare and their fraction does not exceed 1%. The MixDP and MixDI samples have similar kinematic ($p_T$ and $\eta$) distributions for the photon and all the jets. They differ only by the amount of energy coming from soft parton interactions in either one or two $pp$ collisions, which may affect the photon and the jet selection efficiencies.

V. DISCRIMINATING VARIABLES

A distinctive feature of the DP events is the presence of two independent parton-parton scatterings within the same $pp$ collision. We define variables sensitive to the kinematics of DP events, specifically to the difference between the $p_T$ imbalance of the two object pairs in DP and SP $\gamma + 3$ jets events as [4]:

$$\Delta S \equiv \Delta \phi (\vec{p}_T (\gamma, i), \vec{p}_T (j, k)),$$ (7)

where the indices $i, j, k (= 1, 2, 3)$ run over the jets in the event. Here $\vec{p}_T (\gamma, i) = \vec{p}_T^\gamma + \vec{p}_T^{j\ell\ell}$ and $\vec{p}_T (j, k) = \vec{p}_T^{j\ell\ell} + \vec{p}_T^{j\ell\ell}$, where the two object pairs, ($\gamma, j, i$) and (jet $j$, jet $k$), are selected to give the minimal $p_T$ imbalance. These pairs are found by minimizing $S_{p_T}$, $S_{p_T^\gamma}$, or $S_{\phi}$ defined as

$$S_{p_T} = \frac{1}{\sqrt{2}} \sqrt{(\frac{p_T (\gamma, i)}{\delta p_T (\gamma, i)})^2 + (\frac{p_T (j, k)}{\delta p_T (j, k)})^2}, \quad (8)$$

$$S_{p_T^\gamma} = \frac{1}{\sqrt{2}} \sqrt{(\frac{p_T^\gamma (\gamma, i)}{|p_T (\gamma, i)|})^2 + (\frac{p_T (j, k)}{|p_T^\gamma (j, k)|})^2}, \quad (9)$$

$$S_{\phi} = \frac{1}{\sqrt{2}} \sqrt{(\frac{\Delta \phi (\gamma, i)}{\delta \phi (\gamma, i)})^2 + (\frac{\Delta \phi (j, k)}{\delta \phi (j, k)})^2}. \quad (10)$$

In Eq. (10) $\Delta \phi (\gamma, i) = |\pi - \phi (\gamma, i)|$ is the supplement to $\pi$ of the minimal azimuthal angle between the vectors $\vec{p}_T^\gamma$ and $\vec{p}_T^{j\ell\ell}$, $\phi (\gamma, i)$.

The uncertainties $\delta p_T (\gamma, i)$ in Eq. (8) and $\delta \phi (\gamma, i)$ in Eq. (10) are calculated as root-mean-square values of the $|\vec{p}_T (\gamma, i)|$ and $\Delta \phi (\gamma, i)$ distributions using the signal MixDP sample for each of the three possible pairings. Azimuthal angles and uncertainties for jets $j$ and $k$ are defined analogously to those for the photon and jet $i$. Any of the S-variables in Eqs. (8)–(10) represents a significance of the pairwise $p_T$-imbalance. On average, it should be higher for the SP events than for the DP events. Also, each S-variable effectively splits the $\gamma + 3$ jets system into $\gamma + j$ and dijet pairs, based on the best pairwise balance.

The two best $p_T$-balancing pairs, which give the minimal $S$ for each of three variables in Eqs. (8)–(10),
are used to calculate the corresponding $\Delta S$ variables, $\Delta S_{pt}$, $\Delta S_{pT}$, and $\Delta S_\phi$, according to Eq. (7). The $\Delta S_{pt}$, $\Delta S_{pT}$ variables are also used in [7, 9], while the $\Delta S_\phi$ is first introduced in this measurement.

Figure 5 illustrates a possible orientation of the transverse momenta vectors of the photon and jets as well as their $p_T$ imbalances vectors, $\vec{P}_1$ and $\vec{P}_2$, in $\gamma + 3$ jets events. In SP events, the topologies with the two radiation jets emitted close to the leading jet (recoiling against the photon direction in $\phi$) are preferred. The resulting peak at $\Delta S = \pi$ is smeared by the effects of additional gluon radiation and detector resolution. For a simple model of DP events, we have exact pairwise balance in $p_T$ and thus $\Delta S$ will be undefined. The exact $p_T$ balance in the pairs can be violated due to either detector resolution or additional gluon radiation. Both effects introduce an additional random contribution to the azimuthal angle between the $\gamma$+jet and the dijet $p_T$ imbalance vectors, broadening the $\Delta S$ distribution (see also Fig. 9 below).

![FIG. 5: A possible orientation of photon and jets transverse momenta vectors in $\gamma + 3$ jets events. Vectors $\vec{P}_1$ and $\vec{P}_2$ are the $p_T$ imbalance vectors of $\gamma$+jet and jet-jet pairs. The figure illustrates a general case for the production of $\gamma + 3$ jets +X events.](image)

### VI. FRACTIONS OF DP AND DI EVENTS

#### A. Fractions of DP events

To extract the fractions of DP events, we exploit the difference in the $p_T$ spectrum of DP and radiation jets, mentioned in Sec. III, and consider data in two adjacent $p_T^{jet2}$ intervals: DP-enriched at smaller $p_T^{jet2}$ and DP-depleted at larger $p_T^{jet2}$ [1, 3, 4]. The distribution for each $\Delta S$ variable in data (D) can be expressed as a sum of signal (DP) and background (SP) distributions:

$$D_1 = f_1 M_1 + (1 - f_1) B_1$$  \hfill (11)

$$D_2 = f_2 M_2 + (1 - f_2) B_2,$$  \hfill (12)

where $M_i$ and $B_i$ stand for the signal MixDP and background distributions, $f_i$ is the DP fraction, $(1 - f_i)$ is the SP fraction, and indices 1, 2 correspond to the DP-enriched and DP-depleted data sets. Multiplying (12) by $\lambda K$ and subtracting from (11) we obtain:

$$D_1 - \lambda K D_2 = f_1 M_1 - \lambda K C f_1 M_2,$$  \hfill (13)

where $\lambda = B_1 / B_2$ is the ratio of the background distributions, and $K = (1 - f_1) / (1 - f_2)$ and $C = f_2 / f_1$ are the ratios of the SP and DP fractions between the DP-enriched and DP-depleted samples, respectively. In contrast to [9], we introduce a factor $\lambda$ that corrects for the relative difference of $\Delta S$ shapes for the SP distributions in adjacent $p_T^{jet2}$ intervals. It is obtained using Monte Carlo (MC) $\gamma + 3$ jets events generated with PYTHIA [25] without multiple parton interactions and with a full simulation of the detector response and is found to be in the range 0.95 – 1.3 for different bins of $\Delta S$. The factor $C$ is extracted using ratios of the numbers of events in data and MixDP samples in the adjacent bins by

$$C = \left( \frac{N_{MixDP}^2}{N_{data}^2} \right) / \left( \frac{N_{MixDP}^1}{N_{data}^1} \right),$$  \hfill (14)

i.e. without actual knowledge of DP fractions in those bins. Thus, the only unknown parameter in Eq. (13) is the DP fraction $f_1$. It is obtained from a $\chi^2$ minimization of Eq. (13) using MINUIT [26]. The fit was performed for each pair of $p_T^{jet2}$ bins (15 – 20/20 – 25 GeV and 20 – 25/25 – 30 GeV) and for each of $\Delta S$ variables (8)-(10). The DP fractions in the last bin, 25 < $p_T^{jet2}$ < 30 GeV, are calculated from $f_2 = C f_1$. The extracted DP fractions are shown in Fig. 6. The DP fractions, averaged over the three $\Delta S$ variables (with uncertainties), are summarized in Table III. The location of the points in Fig. 6 corresponds to the mean $p_T^{jet2}$ for the DP model in a given bin. They are also shown in Table III as $\langle p_T^{jet2} \rangle$. The uncertainties are mainly caused by the statistics of the data and MixDP samples (used in the fitting) and partially by the determination of $\lambda$ (2–5)%.

<table>
<thead>
<tr>
<th>$p_T^{jet2}$ (GeV)</th>
<th>$(p_T^{jet2})$ (GeV)</th>
<th>$f_{DP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 – 20</td>
<td>17.6</td>
<td>0.466 ± 0.041</td>
</tr>
<tr>
<td>20 – 25</td>
<td>22.3</td>
<td>0.334 ± 0.023</td>
</tr>
<tr>
<td>25 – 30</td>
<td>27.3</td>
<td>0.235 ± 0.027</td>
</tr>
</tbody>
</table>

Since each component of a MixDP signal event may contain two jets, where one jet may be caused by an
additional parton interaction, the MixDP sample should simulate the properties of the double plus triple parton (TP) interactions (DP+TP), and thus the fractions in Table III take into account a contribution from triple interactions as well. In this sense, the DP cross section calculated using Eqs. (1) and (6) is inclusive [27, 28].

Figure 7 shows tests of the fit results for $f_1$ using the $\Delta S_\phi$ variable for the combination of two $p_T^{\text{jet2}}$ bins, $15 - 20$ GeV and $20 - 25$ GeV. Figure 7(a) shows the $\Delta S_\phi$ distributions for the SP model (shaded area) and the DP model (points). Figure 7(b) shows the $\Delta S_\phi$ distributions for the DP-enriched data set in data (points), corrected to remove SP contribution, and the DP model (shaded area) as a difference between the corresponding distributions of (a) and (b); (c) shows the prediction for DP from data (points), corrected to remove SP contribution; (d) shows the extracted SP distributions in the two bins.

The error bars in (a) and (b) are only statistical, while in (c) and (d) they represent total (statistic and systematic) uncertainty. It can be concluded that the regions of small $\Delta S_\phi$ (< 1.5) is mostly populated by signal events with two independent hard interactions. Figure 7(c) shows the difference between the data distributions of Figs. 7(a) and 7(b), corrected to remove the SP contribution by the factor $\lambda K$ (the factor $\lambda$ corrects for the relative difference of the $\Delta S_\phi$ shapes and $K$ corrects for the difference in the SP fractions in the two adjacent $p_T^{\text{jet2}}$ bins) [left side of Eq. (13)] and compared to the MixDP prediction [right side of Eq. (13)]. As expected, the difference is always positive since the fractions of DP events drop with $p_T^{\text{jet2}}$. The DP model provides an adequate description of the data. In Fig. 7(d) we extract the SP distributions by subtracting the estimated DP contributions from the data: $(D_1 - f_1 M_1)/(1 - f_1)$ for the DP-enriched data set and $(D_2 - f_2 M_2)/(1 - f_2)$ for the DP-depleted data sets. Figure 8 shows the analogous test of the fit results for the other pair of $p_T^{\text{jet2}}$ bins, $20 - 25$ GeV and $25 - 30$ GeV.

Predictions for SP events are obtained using PYTHIA. The $\Delta S_{p_T^{\text{jet2}}}$ distribution for $\gamma + 3$ jets events simulated with initial and final state radiation (ISR and FSR) and without multiple parton interactions (MPI) is shown in Fig. 9 for the interval $15 < p_T^{\text{jet2}} < 20$ GeV. Since the $p_T^{\text{jet2}}$ imbalance of the two additional jets should compensate the $p_T^{\text{jet2}}$ imbalance of the “$\gamma$ + leading jet” system, the $\Delta S_{p_T^{\text{jet2}}}$ distribution is shifted towards $\pi$. This distribution shows good agreement with the results for the SP sample shown in Fig. 7(d). The DP $\gamma + 3$ jets events are

FIG. 8: Same as in Fig. 7 but for the other combination of $p_T^{\text{jet2}}$ bins, $20 - 25$ GeV and $25 - 30$ GeV.

FIG. 9: $\Delta S_{p_T^{\text{jet2}}}$ distributions for $\gamma + 3$ jets events simulated using PYTHIA with ISR/FSR but with MPI switched off (shaded region), as well as for $\gamma + 3$ jets events without ISR/FSR but MPI switched on using Tune A-CR (triangle markers). The bin $15 < p_T^{\text{jet2}} < 20$ GeV is considered.
also simulated without ISR and FSR and using the MPI model corresponding to the PYTHIA parameters Tune A-CR [25]. In this case, the two subleading jets may originate only from the second parton interaction (as in DP events of Type I, see Fig. 2). As expected, the \( \Delta S_{\nu_T} \) distribution for these events is uniform, since the two \( p_T \) balance vectors for the two systems, \( \gamma \) + jets and dijets, are independent from each other.

Another source of background to the single-vertex \( \gamma + 3 \) jets DP events is caused by double \( pp \) collisions close to each other along the beam direction, for which a single vertex is reconstructed. This was estimated separately and found to be negligible with a probability < 10^{-3}.

### B. Fractions of DI events

The DI fractions, \( f_{\text{DI}} \), are extracted by fitting the shapes of the \( \Delta S \) distributions of the MixDI signal and Bkg2Vtx background samples to that for the 2Vtx data using the technique described in [29]. Uncertainties are mainly caused by the fitting procedure and by building Bkg2Vtx and MixDI (in case of Type I events) models. To estimate the uncertainty due to the Bkg2Vtx or MixDI backgrounds, we vary a cut on the minimal jet charged particle fraction (see Appendix B) from 0.5 to 0.75. The fitted \( f_{\text{DI}} \) in this case varies in different \( p_T^{\text{jet2}} \) bins within \((3 - 10)/\%\), which is taken as the uncertainty. The final \( f_{\text{DI}} \) values with total uncertainties are 0.189 ± 0.029 for \( 15 < p_T^{\text{jet2}} < 20 \) GeV, 0.137 ± 0.027 for \( 20 < p_T^{\text{jet2}} < 25 \) GeV, and 0.094 ± 0.025 for \( 25 < p_T^{\text{jet2}} < 30 \) GeV. The relative \( f_{\text{DI}} \) uncertainties grow with increasing \( p_T^{\text{jet2}} \). This is caused by a decreasing probability for a jet to originate from a second \( pp \) collision vertex. As a consequence, the sensitivity to DI events in the 2Vtx data sample becomes smaller.

Figure 10 shows the \( \Delta S_\phi \) distributions for the two-vertex \( \gamma + 3 \) jets events selected in three \( p_T^{\text{jet2}} \) intervals, \( 15 - 20 \) GeV, \( 20 - 25 \) GeV and \( 25 - 30 \) GeV, for the DI model (MixDI) and the total sum of MixDI and Bkg2Vtx distributions, weighted with their fractions found from the fit, compared to 2Vtx data (black points). The shown uncertainties are only statistical.

![Figure 10: \( \Delta S_\phi \) distributions for two-vertex \( \gamma + 3 \) jets events in the three \( p_T^{\text{jet2}} \) intervals](image)

**DIFFERENT AMOUNTS OF SOFT UNCLUSTERED ENERGY IN THE SINGLE AND DOUBLE \( pp \) COLISION EVENTS.** This could lead to a difference in the jet reconstruction efficiencies, due to the different probabilities of passing the jet selection requirement \( p_T > 6 \) GeV (applied during jet reconstruction) and different photon selection efficiencies, due to different amount of energy in the track and calorimeter isolation cones around the photon.

To estimate these efficiencies, we use \( \gamma + \) jets and dijet MC events and also MixDI and MixDP data samples. The MC events are generated with PYTHIA [25] and processed through a GEANT-based [30] simulation of the DO detector response. In order to accurately model the effects of multiple proton-antiproton interactions and detector noise, data events from random \( pp \) crossings are overlaid on the MC events using data from the same time period as considered in the analysis. These MC events are then processed using the same reconstruction code as for the data. We also apply additional smearing to the reconstructed photon and jet \( p_T \) so that the measurement resolutions in MC match those in data. The MC events are preselected with the vertex cuts and split into the single- and two-vertex samples.

The efficiencies for the photon selection criteria are estimated using \( \gamma + \) jets MC events. We found that the ratio of photon efficiencies in single-vertex (\( \varepsilon_{\gamma,v}^{1v} \)) to that in two-vertex samples (\( \varepsilon_{\gamma,v}^{2v} \)) does not have a noticeable dependence on \( p_T^{\text{jet2}} \) and can be taken as \( \varepsilon_{\gamma,v}^{1v}/\varepsilon_{\gamma,v}^{2v} = 0.96 \pm 0.03 \). The purity of \( \gamma \) + jets events in the interval of \( 60 < p_T^\gamma < 80 \) GeV in data is expected to be about 75% [19], and the remaining events are mostly dijet events with...
one jet misidentified as photon. An analogous analysis of
the MC dijet events gives the ratio of the efficiencies for
jets to be misidentified as photons equal to 0.99 ± 0.06,
which does not change the $\varepsilon_{\gamma}^1/\varepsilon_{\gamma}^2$ value found with the
signal $\gamma +$ jets sample.

The ratio of jet efficiencies is calculated in two steps.
First, the efficiencies are estimated with respect to a re­
quirement to have at least three jets with $p_T^{\text{jett}>25}$ GeV,
$p_T^{\text{jett}>15}$ GeV, and $p_T^{\text{jett}>15}$ GeV. These efficiencies
are calculated using MC $\gamma +$ jets and dijet events mixed
according to the fractions of the three main MixDP and
MixDI event types, described in Sec. IV. The ratio of
efficiencies for other jet selections (e.g., to get into the
$p_T^{\text{jett}}$ interval and satisfy $\Delta R$ and jet rapidity selections) has been calculated using MixDP and MixDI signal data samples. The total ratio of DP/DI jet efficiencies is
found to be stable for all $p_T^{\text{jett}}$ bins and equal to 0.93
with ~5% uncertainty. Thus, the overall ratio of photon
and jet DP/DI selection efficiencies $\varepsilon_{\text{DP}}/\varepsilon_{\text{DI}}$ is about 0.90
with uncertainties in the three $p_T^{\text{jett}}$ bins varying within
(5.6 — 6.5)%.

B. Vertex efficiencies

The vertex efficiency $\varepsilon_{\text{Vtx}} (\varepsilon_{2\text{vtx}})$ corrects for the single
(double) collision events that are lost in the DP (DI)
candidate sample due to the single (double) vertex cuts
($|z_{\text{Vtx}}| < 60$ cm and $\geq 3$ tracks). The ratio $\varepsilon_{\text{Vtx}}/\varepsilon_{2\text{vtx}}$ is
calculated from the data and found to be 1.08 ± 0.01 for
all $p_T^{\text{jett}}$ bins. The probability to miss a hard interaction
event with at least one jet with $p_T > 15$ GeV due to
a non-reconstructed vertex is calculated in $\gamma +$ jets and
minimum bias data and found to be (0.2 — 0.4)%.

C. Calculating $\sigma_{\text{hard}}, N_{\text{coll}}$ and $N_{\text{2coll}}$

The numbers of expected events with one ($N_{\text{coll}}$) and
two ($N_{\text{2coll}}$) $pp$ collisions resulting in hard interactions
are calculated from the known instantaneous luminosity
spectrum of the collected data ($L_{\text{inst}}$), the frequency of
beam crossings ($f_{\text{cross}}$) for the Tevatron [17], and the hard
$pp$ interaction cross section ($\sigma_{\text{hard}}$).

The value of $\sigma_{\text{hard}}$ at $\sqrt{s} = 1.96$ TeV is ob­
tained in the following way. We use the inelastic
cross section calculated at $\sqrt{s} = 1.96$ TeV,
$\sigma_{\text{inst}}(1.96$ TeV) = 60.7 ± 2.4 mb [31], found from
averaging the inelastic cross sections measured by the
CDF [32] and ES811 [33] Collaboration at $\sqrt{s} = 1.8$ TeV
and extrapolated to 1.96 TeV. To calculate single diffrac­tive
(SD) and double diffractive (DD) cross sections at
$\sqrt{s} = 1.96$ TeV, $\sigma_{\text{SD}}(1.96$ TeV) and $\sigma_{\text{DD}}(1.96$ TeV), we
use SD and DD cross sections measured at $\sqrt{s} = 1.8$ TeV
(8$\sigma_{\text{SD}}(1.8$ TeV) = 9.46 ± 0.44 mb [32] and
$\sigma_{\text{DD}}(1.8$ TeV) = 6.32 ± 0.03(stat) ± 1.7(syst) mb) [34]
and extrapolate them to $\sqrt{s} = 1.96$ TeV using the slow
asymptotic behaviour predicted in [35]. We find
$\sigma_{\text{hard}}(1.96$ TeV) = $\sigma_{\text{inst}}(1.96$ TeV) — $\sigma_{\text{SD}}(1.96$ TeV)
− $\sigma_{\text{DD}}(1.96$ TeV) = 44.76 ± 2.89 mb (15).

We also do analogous estimates by calculating first $\sigma_{\text{hard}}$
at $\sqrt{s} = 1.8$ TeV and then extrapolating it to $\sqrt{s} = 1.96$
TeV using [35]. This method results in $\sigma_{\text{hard}}(1.96$ TeV) =
43.85 ± 0.83 mb which agrees well with Eq. (15).

In each bin of the $L_{\text{inst}}$ spectrum, we calculate
the average number of hard $pp$ interactions $\langle n \rangle =
(L_{\text{inst}}/f_{\text{cross}})\sigma_{\text{hard}}$ and then $N_{\text{coll}}$ and $N_{\text{2coll}}$ are
determined from $\langle n \rangle$ using Poisson statistics. Summing
over all $L_{\text{inst}}$ bins, weighted with their fractions, we get
$N_{\text{coll}}/(2N_{\text{2coll}}) = 1.169$ and thus $R_c\sigma_{\text{hard}} = 56.45 ± 0.88$
mb. Here we take into account that $R_c$ and $\sigma_{\text{hard}}$
enter Eq. (6) for $\sigma_{\text{eff}}$ as a product. Any increase of $\sigma_{\text{hard}}$
leads to an increase of $\langle n \rangle$ and, as a consequence, to a
decrease in $R_c$, and vice versa. Specifically, while the
found value of $\sigma_{\text{hard}}$ has a 6.5% relative uncertainty, the
product $R_c\sigma_{\text{hard}}$ has approximately 2% uncertainty.

VIII. RESULTS

A. Effective cross section

The calculation of $\sigma_{\text{eff}}$ is based on Eq. (6) of Sec. I.
The numbers $N_{\text{DP}}$ and $N_{\text{DI}}$ in each $p_T^{\text{jett}}$ bin are ob­
tained from the numbers of the 1Vtx and 2Vtx $\gamma + 3$ jets
events in Table II, multiplying them by $f_{\text{DP}}$ and $f_{\text{DI}}$.
The determination of all other components of Eq. (6) are described in Sec. VII. The resulting values of $\sigma_{\text{eff}}$
with total uncertainties (statistical and systematic are
summed in quadrature) are shown in Fig. 11 and given
in Table IV for the three $p_T^{\text{jett}}$ bins. The location of the
points in Fig. 11 corresponds to the mean $p_T^{\text{jett}}$ for the
DP model in a given bin (the mean $p_T^{\text{jett}}$ values for DI
model are the same within 0.15 GeV). These values are also
shown in Table IV. Table V summarizes the main
sources of uncertainties for each $p_T^{\text{jett}}$ bin. The main sys­
tematic uncertainties are related to the determinations
of the DI fractions (dominant uncertainty), DP fractions,
the $\varepsilon_{\text{DP}}/\varepsilon_{\text{DI}}$ ratio, jet energy scale (JES), and $R_c\sigma_{\text{hard}}$, giving a total systematic uncertainty of (20.5 — 32.2)%.

TABLE IV: Effective cross section $\sigma_{\text{eff}}$ in the three $p_T^{\text{jett}}$ bins.

<table>
<thead>
<tr>
<th>$p_T^{\text{jett}}$ (GeV)</th>
<th>$\langle p_T^{\text{jett}} \rangle$ (GeV)</th>
<th>$\sigma_{\text{eff}}$ (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 — 20</td>
<td>17.6</td>
<td>18.2 ± 3.8</td>
</tr>
<tr>
<td>20 — 25</td>
<td>22.3</td>
<td>16.3 ± 3.7</td>
</tr>
<tr>
<td>25 — 30</td>
<td>27.3</td>
<td>13.9 ± 4.5</td>
</tr>
</tbody>
</table>
The measured $\sigma_{\text{eff}}$ values in the different $p_T^{\text{jet12}}$ bins agree with each other within their uncertainties, however a slow decrease with $p_T^{\text{jet12}}$ can not be excluded. The $\sigma_{\text{eff}}$ value averaged over the three $p_T^{\text{jet12}}$ bins is

$$\sigma_{\text{eff}}^{\text{ave}} = 16.4 \pm 0.3(\text{stat}) \pm 2.3(\text{syst}) \, \text{mb.} \quad (16)$$

![Graph](image)

**FIG. 11:** Effective cross section $\sigma_{\text{eff}}$ (mb) measured in the three $p_T^{\text{jet12}}$ intervals.

### B. Models of parton spatial density

In this section we study the limits that can be obtained on the parameters of three phenomenological models of parton spatial density using the measured effective cross section (16). In the discussion below we follow a simple classical approach. For a given parton spatial density inside the proton or antiproton $\rho(r)$, one can define a (time-integrated) overlap $O(\beta)$ between the parton distributions of the colliding nucleons as a function of the impact parameter $\beta$ [10]. The larger the overlap (i.e. smaller $\beta$), the more probable it is to have at least one parton interaction in the colliding nucleons. The single hard scattering cross sections (for example, $\gamma + \text{jets}$ or di-jet production) should be proportional to $O(\beta)$ and the cross section for the double parton scattering is proportional to the squared overlap, both integrated over all impact parameters $\beta$ [28, 36]:

$$\sigma_{\text{eff}} = \frac{\int_0^\infty O(\beta)^2 2\pi\beta \, d\beta \, d\beta}{\int_0^\infty O(\beta)^2 2\pi\beta \, d\beta}.$$  \hspace{1cm} (17)

First, we consider the “solid sphere” model with a constant density inside the proton radius $r_p$. In this model, the total hard scattering cross section can be written as $\sigma_{\text{hard}} = 4\pi r_p^2$ and $\sigma_{\text{eff}} = \sigma_{\text{hard}}/f$. Here $f$ is the geometrical enhancement factor of the DP cross section. It is obtained by solving Eq. (17) for two overlapping spheres with a boundary conditions that the parton density $\rho(r) = \text{constant}$ for $r \leq r_p$ and $\rho(r) = 0$ for $r > r_p$ and found to be $f = 2.19$. The role of the enhancement factor can be seen better if we rewrite Eq. (1) as $\sigma_{\text{DP}} = f \sigma_A \sigma_B / \sigma_{\text{hard}}$. The harder the single-parton interaction is the more it is biased towards the central hadron-hadron collision with a small impact parameter, where we have a larger overlap of parton densities and, consequently, higher probability for a second parton interaction [5]. Using the measured $\sigma_{\text{eff}}$, for the solid sphere model we extract the proton radius $r_p = 0.53 \pm 0.06 \, \text{fm}$ and proton rms-radius $R_{\text{rms}} = 0.41 \pm 0.05 \, \text{fm}$. The latter is obtained from averaging $r^2$ as $R_{\text{rms}}^2 \equiv \int_0^\infty r^2 4\pi r^2 \rho(r) \, dr = 4\pi \int_0^\infty \rho(r) r^4 \, dr$ [37]. The results are summarized in the line “Solid Sphere” of Table VI. The Gaussian model with $\rho(r) \propto e^{-r^2/2\sigma^2}$ and exponential model with $\rho(r) \propto e^{-r/\lambda}$ have been also tested. The relationships between the scale parameter $(r_p, \sigma$ or $\lambda)$ and rms-radius for all the models are given in Table VI. The relationships between the effective cross section $\sigma_{\text{eff}}$ and parameters of the Gaussian and exponential models are taken from [38], neglecting the terms that represent correlations in the transverse space. The scale parameters and rms-radii for both models are also given in Table VI. In spite of differences in the models, the proton rms-radii are in good agreement with each other, with average values varied as 0.41 – 0.47 and with about 12% uncertainty. On the other hand, having obtained rms-radius from other sources (for example, [39]) and using the measured $\sigma_{\text{eff}}$, the size of the transverse correlations [38] can be estimated.

### IX. SUMMARY

We have analyzed a sample of $\gamma + 3 \, \text{jets}$ events collected by the DO experiment with an integrated luminosity of about 1 fb$^{-1}$ and determined the fraction of events with hard double parton scattering occurring in a single $pp$ collision at $\sqrt{s} = 1.96 \, \text{TeV}$. These fractions are measured in three intervals of the second (ordered in $p_T$) jet transverse momentum $p_T^{\text{jet2}}$ and vary from 0.466 ± 0.041 at $15 \leq p_T^{\text{jet2}} \leq 20 \, \text{GeV}$ to 0.235 ± 0.027 at $25 \leq p_T^{\text{jet2}} \leq 30 \, \text{GeV}$.

In the same three $p_T^{\text{jet2}}$ intervals, we calculate an ef-
fective cross section $\sigma_{\text{eff}}$, a process-independent scale parameter which provides information about the parton spatial density inside the proton and define the rate of double parton events. The measured $\sigma_{\text{eff}}$ values agree for the three $p^\text{jet}_{T}$ intervals with an average $\sigma_{\text{eff}} = 16.4 \pm 0.3^{\text{stat}} \pm 2.3^{\text{syst}} \text{mb}$. We note that this average value is in the range of those found in previous measurements [7–9] performed at different parton interaction energy scales, and may indicate stable behavior of $\sigma_{\text{eff}}$ with respect to the considered energy scales.

Using the measured $\sigma_{\text{eff}}$ we have calculated scale parameters and rms-radii of the proton for three models of the parton matter distribution.

Acknowledgements

We would like to thank T. Sjöstrand and P. Skands for very useful discussions. We also thank the staffs at Fermilab and collaborating institutions, and acknowledge support from the DOE and NSF (USA); CEA and CNRS/IN2P3 (France); FASI, Rosatom and RFBR (Russia); CNPq, FAPERJ, FAPESP and FUNDUNESP (Brazil); DAE and DST (India); Colciencias (Colombia); CONACyT (Mexico); KRF and KOSEF (Korea); CONICET and UBACyT (Argentina); FOM (The Netherlands); STFC and the Royal Society (United Kingdom); MSMT and GACR (Czech Republic); CRC Program, CFI, NSERC and WestGrid Project (Canada); BMBF and DFG (Germany); SFI (Ireland); The Swedish Research Council (Sweden); and CAS and CNSF (China).

X. APPENDIX A

In this measurement we assume that the two parton interactions in the DP $\gamma + 3$ jets events can be considered to be independent from each other. Possible correlation may appear both in momentum space, since the two interactions have to share the same proton momentum, and at the fragmentation stage.

In the hypothesis of two independent scatterings, the kinematic properties of SP dijet events should be very similar to those produced in the second parton interaction in the DP $\gamma + 3$ jets events. We compare the $p_T$ and $\eta$ distributions for the two cases using the PYTHIA event generator, which includes momentum and flavor correlations among the partons participating in MPI. It also provides the possibility of choosing different MPI models. In our comparison we use the PYTHIA parameters Tune A-CR, which is usually considered as an example of a model with a strong color reconnection with an extreme prediction for track multiplicities and/or average hadron $p_T$ [40]. As a model for the DP events, we simulate $\gamma + 3$ jets events using Tune A-CR but with ISR and FSR effects turned off and applied all selection criteria as described in Sec. III. This configuration of the event generator guarantees that the two jets produced in addition to the leading jet (and $\gamma$) in the $\gamma + 3$ jets final state arise only from additional parton interactions. The $\Delta S$ distribution for these events is shown in Fig. 9 (by triangles). The SP dijets events are also generated without ISR and FSR. Figure 12(a) compares the $p_T$ spectra of the first (in $p_T$) jet from the second partonic collision in DP events (second jet in $\gamma + 3$ jets events) and the first jet in the SP dijet events, while Fig. 12(b) make analogous comparisons of the next (in $p_T$) jet in both event types. Figures 12(c) and 12(d) compare the $\eta$ distributions of
we consider all tracks inside a jet radius (R = 0.7 in our case) and calculate the p_T-weighted position in z of all the tracks ("jet_z"). Here the track z position is calculated at the point of closest approach of this track to the beam (z) axis. For each jet in the 2Vtx data sample (Sec. III) we can estimate the distance between the jet_z and the pp vertex closest in z, Δz(Vtx, jet_i). These distributions are shown in Fig. 13 for each jet in the γ+3 jets 2Vtx sample. About (95-96)% ([97-99%]) of events have Δz(Vtx, jet_i) < 1.5 (2.0) cm.

We also use an algorithm that is based on a jet charged particle fraction (CPF) and define a discriminant which measures the probability that a given jet originates from a particular vertex (a jet, having originated from a vertex, may still have tracks coming from another vertex). The CPF discriminant is based on the fraction of charged transverse energy in each jet i (in the form of tracks) originating from each identified vertex j in the event:

$$\text{CPF}(\text{jet}_i, Vtx_j) = \frac{\sum_k p_T(\text{trk}_k^{\text{jet}_i}, Vtx_j)}{\sum_n \sum_k p_T(\text{trk}_k^{\text{jet}_i}, Vtx_n)}.$$  (18)

To confirm that a given jet originate from a vertex, we require Δz < 2.0 and CPF > 0.5. These requirements being applied to two (or three) jets in the 2Vtx events allow to build the signal and background DI models described in Section IV.

XI. APPENDIX B

In building signal and background DI models in Sec. IV, we take into account information about tracks associated with jets. We use two algorithms. In the first, we consider all tracks inside a jet radius (R = 0.7 in our case) and calculate the p_T-weighted position in z of all the tracks ("jet_z"). Here the track z position is calculated at the point of closest approach of this track to the beam (z) axis. For each jet in the 2Vtx data sample (Sec. III) we can estimate the distance between the jet_z and the pp vertex closest in z, Δz(Vtx, jet_i). These distributions are shown in Fig. 13 for each jet in the γ+3 jets 2Vtx sample. About (95-96)% ([97-99%]) of events have Δz(Vtx, jet_i) < 1.5 (2.0) cm.

We also use an algorithm that is based on a jet charged particle fraction (CPF) and define a discriminant which measures the probability that a given jet originates from a particular vertex (a jet, having originated from a vertex, may still have tracks coming from another vertex). The CPF discriminant is based on the fraction of charged transverse energy in each jet i (in the form of tracks) originating from each identified vertex j in the event:

$$\text{CPF}(\text{jet}_i, Vtx_j) = \frac{\sum_k p_T(\text{trk}_k^{\text{jet}_i}, Vtx_j)}{\sum_n \sum_k p_T(\text{trk}_k^{\text{jet}_i}, Vtx_n)}.$$  (18)

To confirm that a given jet originate from a vertex, we require Δz < 2.0 and CPF > 0.5. These requirements being applied to two (or three) jets in the 2Vtx events allow to build the signal and background DI models described in Section IV.

[a] Visitor from Augustana College, Sioux Falls, SD, USA.
[b] Visitor from The University of Liverpool, Liverpool, UK.
[c] Visitor from SLAC, Menlo Park, CA, USA.
[d] Visitor from ICREA/IFAE, Barcelona, Spain.
[e] Visitor from Centro de Investigación en Computacion - IPN, Mexico City, Mexico.
[f] Visitor from ECFM, Universidad Autonoma de Sinaloa, Culiacán, Mexico.
[g] Visitor from Universität Bern, Bern, Switzerland.
FIG. 13: Normalized distribution of the number of events as a function of the distance along the z-axis between jet_j (see text) and the pp vertex closest in z-position for the (a) first, (b) second and (c) third jets in the 2Vtx data sample.

[27] It differs from [9], where the measured inclusive double parton fractions are corrected for the fraction of triple parton interactions, what makes $\sigma_{\text{eff}}$ and then double parton cross section exclusive [28].


[40] D. Wicke and P.Z. Skands, arXiv:0807.3248 [hep-ph] (Fig.3).