Competition of two-ion and single-ion anisotropy in rare-earth systems: Large anisotropy
example of Tb₅Ge₃

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The standard model of rare-earth magnetism assumes that magnetic anisotropy is mainly caused by single-ion effects. The importance of two-ion anisotropy is still disputed. For a number of materials this two-ion anisotropy is in the same order of magnitude and strongly influences the magnetic properties. Single crystalline Tb₅Ge₃ has a high moment and low symmetry and, because of this, a large anisotropy is expected. In the present study the influence of the two-ion interaction was investigated. Magnetization and magnetostriction were measured in high magnetic fields up to 60 T. The magnetic phase diagram constructed for the main crystallographic directions shows antiferromagnetism in zero field and a number of magnetic phases at higher fields. The exchange interaction was evaluated based on a model calculation of these phase diagrams. We conclude that a huge anisotropy in the two-ion interaction is present, which overwhelms the single-ion effects.

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1. INTRODUCTION

Most members of the R₃Ge₃ (R=rare earth) series are unconventional antiferromagnets (AFs) with complex magnetic structure and a strong interplay of crystallographic and magnetic properties. The R³⁺ moments are located on two different crystallographic sites: one third are on the 4d site and two thirds on the 6g site of the hexagonal structure (space group P6₃/mcm). In the special case of Tb₅Ge₃, the lattice parameters are a=8.495 Å and c=6.351 Å.1 The compound orders antiferromagnetically at Tₐ=85 K and the lower susceptibility along the hexagonal axis compared to that in the plane indicates that there is a magnetic easy plane.2 The magnetic structure at zero field was analyzed by neutron powder diffraction.3 It suggests a sinusoidally modulated collinear magnetic ordering at Tₐ with the moments oriented in the a [(1010) in hexagonal notation] direction when they are projected into the basal plane and slightly canted toward the (0001) direction. The propagation vector is \( \tau=\begin{pmatrix} 0 & 0.5 \end{pmatrix} \) which shifts with decreasing temperature, reaching a value of \( \tau=\begin{pmatrix} 0 & 0.4616 \end{pmatrix} \) at about \( T_1=50 \) K, which then stays constant down to low temperatures. The low-temperature phase is characterized by a flat spiral with a helical angle of 166.14° between second-nearest neighbors. New neutron-diffraction experiments on a single crystal conclude that the commensurate magnetic structure is stable up to \( T_1 \) with the propagation vector of about \( \tau=\begin{pmatrix} 0 & 0.49 \end{pmatrix} \).4 These complex magnetic structures make Tb₅Ge₃ an excellent example for a study of magnetic exchange interactions.

Magnetization studies of Tb₅Ge₃ (and Pr₅Ge₃) have been reported for the (1010) and (0001) directions.5 The magnetic phase diagram for fields along (1010) shows three more antiferromagnetic phases up to the saturation field of \( H_s=9 \) T. It is interesting to note that the compound does not undergo a second magnetic transition in zero field at \( T_1 \) as proposed in Ref. 5 as well as in Ref. 3. However, a wide variety of magnetic modifications below 50 K exist in fields parallel to (1010). All data for fields along the hexagonal (0001) axis clearly show the reorientation of the magnetic moments near \( T_1 \).

All these experimental facts require further experimental and theoretical investigation in order to understand the complexity of magnetism in more detail. Measurements of magnetization in all three main crystallographic directions are necessary to construct the magnetic phase diagrams for all these directions. In this way, not only the anisotropy between hexagonal axis and basal plane can be analyzed but also...
anisotropy effects in the plane itself. Furthermore, the theoretical evaluation of the exchange interaction, which takes into account isotropic exchange, the dipolar interaction and anisotropic two-ion exchange effects, gives insight into the microscopic properties. In particular, the ratio of the two-ion and single-ion (or crystal-field) anisotropy energies in rare-earth systems is of fundamental interest.

II. SAMPLES AND EXPERIMENTAL

The single-crystalline samples of Tb$_5$Ge$_3$ were grown by Czochralski method from single-phase polycrystalline samples using a tri-arc furnace. After a special heat treatment the output crystal was checked and found to be single phase (with impurities less than 1%) by x-ray powder diffraction. Details of the sample preparation are given in Ref. 2. The crystal was cut into smaller pieces with characteristic dimensions of about 2 mm for the magnetization and magnetostriction experiments at moderate fields and of about 0.5 mm for the high-field investigations.

The magnetic properties of Tb$_5$Ge$_3$ were studied by magnetization measurements in the temperature range between 2 and 100 K and in magnetic fields up to 14 T using a Quantum Design physical properties measurement system equipped with a vibrating sample magnetometer. The high-field magnetization measurements were carried out up to 60 T in a pulsed magnet with an inductive system with a coil diameter of 1 mm in order to have a high filling factor. The rise time of the pulse magnet was about 10 ms.

Sensitive capacitive dilatometers were used to determine the magnetoelastic properties: the thermal expansion between 1.5 and 300 K and magnetostriction in steady fields (superconducting 15 T magnet at the TU Dresden, resistive Bitter 33 T magnet at the HFML Nijmegen) was measured by a tilted plate capacitive dilometer with a resolution of relative length changes of $10^{-7}$, whereas the parallel-plate dilometer was the appropriate device for the pulsed field experiments (resolution $10^{-5}$).

A. Magnetization

The magnetization of Tb$_5$Ge$_3$ at a constant field of 2 T and as a function of temperature is shown in Fig. 1. The magnetization curves in a magnetic field oriented parallel to the three main crystallographic directions at various temperatures between 2 and 100 K are presented in Fig. 2. The maximum of the temperature-dependent magnetization agrees with the antiferromagnetic ordering temperature of 85 K. It is interesting to note that in (0001) direction this peak is split, characterized by two kinks at about 81 and 93 K. This may be a consequence of separate ordering of the Tb$^{3+}$ moments on the 4$d$ and 6$g$ sites caused by strong anisotropic terms in the two-ion R-R exchange interaction. The (0001) curve shows another hump at about 50 K which coincides with the change in the magnetic structure discussed below. Small kinks in the (10$ar{1}$0) and (1120) curves also are connected to magnetic structure modifications but these modifications only occur when a field of about 2 T is applied and are not present in zero field (not shown here).

![Fig. 1. (Color online) Magnetization of a Tb$_5$Ge$_3$ single crystal parallel to the main crystallographic axes $a\ [=(10\bar{1}0)]$, $b\ [=(11\bar{2}0)]$, and $c\ [=(0001)]$ as a function of temperature in a magnetic field of 2 T.](image-url)
B. Magnetostriction

The fundamental trend of the thermal expansion and longitudinal magnetostriction curves, shown in Figs. 4 and 5, agrees well with that of the magnetization. The transition temperatures and fields were again determined from anomalies, here kinks or bends in the measured curves, which could be analyzed in detail by taking a derivative of the curves with respect to temperature or magnetic field, respectively. In the thermal-expansion (=zero field) curves only one critical temperature \( T_N \) is observed. Summarizing the data along all directions, the spontaneous volume striction below \( T_N \) is small with an absolute value of about \( 10^{-5} \). The field-dependent transitions are well pronounced in the longitudinal magnetostriction data: it is possible to identify anomalies at 3.0, 4.9, 7.5, and 8.8 T [(10\(\bar{1}0\)] axis), 3.1, 8.0, 10.0, and 11.3 T [(1120) axis], and 4.2 T [(0001) axis] for increasing field scans at 5 K. A very large effect of about \( 10^{-3} \) occurs along the (10\(\bar{1}0\)) and (1120) axes at the highest of these fields which suggests a drastic change in the magnetic structures at these fields whereas the other, smaller, transitions can be modeled as variations in the zero-field structure. The (10\(\bar{1}0\)) axis transition at about 9 T is very broad which may be understood if the (10\(\bar{1}0\)) axis oriented moments undergo a spin-flop transition here. Again, the low-temperature transitions are hysteretic. Measurements of the magnetostriction made in high steady and pulsed magnetic fields up to 60 T show no further transitions for fields oriented along any of the principal axes.

III. MAGNETIC PHASE DIAGRAM

The magnetic phase diagrams of \( \text{Tb}_5\text{Ge}_3 \) (see Figs. 6–8) were determined using the measurements of magnetization and magnetostriction. The transition lines created by magnetization resemble those of magnetostriction very well.

The magnetic phase diagrams of the (10\(\bar{1}0\)) and (1120) directions show three ferrimagnetic phases below 50 K in addition to the antiferromagnetic low-temperature phase AF. Taking the proposed spiral structure as the basis, all these phases may be explained by modifications of the angles between neighboring moments. The phase line between AF and the lowest lying ferrimagnetic phase does not seem to be closed and stops at about 50 K. Because the anomalies are small, in magnetization as well as in magnetostriction, this low-field phase transition may be attributed to a separate ordering of the two \( \text{Tb}^{3+} \) sublattices (as detected by a number of marginal effects before). In contrast, the phase lines between the other ferrimagnetic phases and all others are closed and go directly into the induced ferromagnetic state. The vanishing of the spiral structure modifications above 50 K confirms the tendency to form a more linear magnetic structure at higher temperatures. However, the application of a magnetic field along (10\(\bar{1}0\)) does not effect the same be-
behavior as a magnetic field along (11\overline{2}0). As discussed above, a field of approximately 8 T along (10\overline{1}0) induces a spin-flop phase. Therefore, saturation in the (10\overline{1}0) direction requires a higher field (continuous transition into the induced ferromagnetism). This observed anisotropy in the basal plane of the hexagonal structure can be understood theoretically in a model including anisotropic two-ion interactions (see below). The phase diagram along the (0001) direction is characterized by a steady rotation of the magnetic moments. There is a transition line around 5 T, deduced from magnetization and magnetostriction jumps, but saturation needs very high fields because of the strong anisotropy of Tb$^{3+}$.

IV. THEORETICAL MODEL

A. Basic considerations of anisotropy

A first theoretical step to calculate the saturation fields can be performed using the high-temperature approximation of the susceptibility measurements,\(^2\) which gives the paramagnetic Curie temperatures to be $\Theta_a=60$ K and $\Theta_c=-100$ K, approximately. Following the formalism\(^8\) (which is strictly valid only for a one-atom unit cell and assumes that the difference of the paramagnetic Curie temperatures is determined by only the crystal-field parameter $B_0$) the paramagnetic Curie and the magnetic ordering temperatures can be written as

$$\Theta_a = \frac{J(J+1)}{3k_B} J_{aa}(q=0) + \frac{2}{5k_B} \left( J - \frac{1}{2} \right) \left( J + \frac{3}{2} \right) B_0^2,$$

$$\Theta_c = \frac{J(J+1)}{3k_B} J_{cc}(q=0) - \frac{4}{5k_B} \left( J - \frac{1}{2} \right) \left( J + \frac{3}{2} \right) B_0^2,$$

$$T_N = \frac{J(J+1)}{3k_B} J_{aa}(q=\pi) \left[ 1 + \frac{2}{5k_B} \left( J - \frac{1}{2} \right) \left( J + \frac{3}{2} \right) B_0^2 / k_BT_N \right].$$

Using at first an isotropic two-ion interaction for reasons of simplicity (i.e., $J_{aa}=J_{cc}=J$) and inserting the experimental values $\Theta_a=60$ K and $\Theta_c=-100$ K and $J=6$ for Tb$^{3+}$ in Eqs. (1) and (2) this leads to a value $B_0^2=0.28$ meV. Inserting the experimental value of the Neel temperature $T_N$ into Eq. (3) allows us to evaluate the Fourier transform of the two-ion interaction at the wave vector $\mathbf{q}=\pi$. Together with $J(q=0)$ calculated from Eqs. (1) and (2) the saturation field can be computed,\(^9\)

$$H^S = \frac{J}{g_J\mu_B} [J(q=\pi) - J(q=0)].$$

Thus the saturation field in (10\overline{1}0) direction is given by

$$H_{a}^S = 3k_BT_N - \Theta_a.$$
Inserting experimental values for $T_N$ and $\Theta_g$ gives $H_s^a = 23$ T. This value is much too high in comparison to the experimental data and illustrates the discrepancy between the experimental results and the (still simple) model, in other words: in the framework of Eqs. (1)–(5) no congruence of the saturation field and transition temperatures could be reached.

Because of this, the calculation was repeated assuming an anisotropic two-ion interaction $J_{aa}$ and $J_{cc}$ and neglecting (to avoid confusion between the different anisotropy parts) the single-ion anisotropy (i.e., $B_2^s = 0$). Using again Eqs. (1)–(3) with $B_2^s = 0$ the saturation fields can be estimated from Eq. (5) and the equivalent expression

$$H_s^s = 3k_B \frac{T_N - \Theta_g}{g_J \mu_B (J+1)},$$

which gives $H_s^s = 10$ T and $H_s^c = 79$ T. These values are in very good agreement with the experimental data. This good match suggests a very small crystal-field influence on the anisotropy in Tb$_3$Ge$_3$. Note that in this estimation of the saturation field values we also obtain values for the two-ion interaction: we get $J_{aa}(q=0) = 0.37$ meV, $J_{cc}(q=0) = -0.62$ meV, and $J_{aa}(q=\tau) = 0.52$ meV.

However, these very basic estimations are not sufficient to explain the magnetism of Tb$_3$Ge$_3$ in detail. In order to describe the anisotropy (especially within the $ab$ plane) and the magnetic structure we have to consider nonzero anisotropy values $B_i^s$ and the complicated crystal structure with different Tb$^{3+}$ positions. A numerical mean-field calculation incorporating this complexity was realized.

### B. Model calculation of the magnetic structure

In order to model the magnetic phase diagram of Tb$_3$Ge$_3$ for several crystallographic directions the standard model of rare-earth magnetism has been evaluated. The numerical calculations have been performed using the program package MCPHASE. To account for the observed anisotropy effects, the Hamiltonian was expressed as

$$\mathcal{H} = \sum_{i,m} B_i^s(i) \Theta_i^s(J_i) - \frac{1}{2} \sum_{ij} \left( J_{aa}(ij) 0 0 \right) \left( 0 J_{bb}(ij) 0 \right) \left( 0 0 J_{cc}(ij) \right) - \sum_{i} g_J \mu_B \mathbf{H} \cdot \mathbf{S}_i.$$

In Eq. (7) the different parts denote the single-ion or crystal-field (CEF) effect, the anisotropic two-ion interaction and the Zeeman term. The $B_i^s(i)$ denote the crystal-field parameters of the ion $i$, the $\Theta_i^s(J_i)$ are Stevens operator equivalents, $g_J$ is the Landé coupling factor, and $\mathbf{H}$ the applied magnetic field.

First, the single-ion anisotropy was considered. Since no inelastic neutron-scattering data are available, the main crystal-field parameters $B_i^s$ were estimated from a point-charge model. It is necessary to consider the Tb site symmetry properly because of the different $B_2^0$ and $B_2^2$ parameters for the 6g and 4d positions.

Assuming an isotropic magnetic exchange interaction to begin with, the exchange parameters $J_{\gamma}$ ($\gamma = aa, bb, cc$) are determined by a parametrized Bethe-Slater curve. However, as expected from the initial considerations above, such a model failed to reproduce the saturation field in plane. Therefore, magnetoelastic interactions were considered but also this model could not explain the magnitude of the magnetic saturation fields.

Therefore in a next step a full theoretical calculation according to Eq. (7) was realized assuming a strong anisotropy of the two-ion interaction. As already seen by the analysis following Eqs. (1)–(4), a solution consistent with the experimental magnetization data only exists for very small single-ion anisotropy. Hence in our model we took point charges of $-0.11 e$ on neighboring atoms and obtained a very small $B_2^s = 0.0344$ meV. In this case, according to Eq. (5), the transition field $H_s^c = 14.5$ T is much lower and also the correct temperatures $T_N = 85$ K and $\Theta_g = 60$ K and $\Theta_c = -100$ K are confirmed. In consequence, this means a negligible CEF anisotropy between the hexagonal $ab$ plane and the $c$ axis compared to that of the magnetic two-ion exchange.

Note that a nonzero CEF anisotropy is necessary within the $ab$ plane to describe the anisotropy in the transition fields. There is, for example, still a small $B_2^s$ on the Tb 6g sites in order to get a different magnetization along (1010) and (1120). This results in a saturation field $H_s^c = 11.5$ T. The lower saturation field for the (1120) and the larger one for (1010) is self-explanatory assuming a spin-flop transition in (1010) at about 8 T and the saturation at a field of only $H_s \approx 15$ T.

Finally, the simulation is based on a parametrized Bethe-Slater curve for the two-ion interaction,

$$J_{\gamma}(R) = A_{\gamma} \left( \frac{R^2}{B_{\gamma}^2} + \frac{R^4}{D_{\gamma}^4} \right) \exp(-\alpha_{R^2}/D_{\gamma}^2)$$

with $\gamma = aa, bb, cc$. We used $A_{aa}=A_{bb}=1.303$ meV, $D_{aa}=D_{bb}=6.248/\AA$, $\alpha_{aa}=\alpha_{bb}=2.404$, $A_{cc}=-0.0383$ meV, $D_{cc}=5.254/\AA$, and $\alpha_{cc}=1.248$. 

![Magnetic phase diagram of Tb$_3$Ge$_3$](image-url)
FIG. 9. Isothermal magnetization of a Tb$_5$Ge$_3$ single crystal parallel to the main crystallographic axes $a$ [(1010), left], $b$ [(1120), center], and $c$ [(0001), right] at different temperatures from 2 to 98 K as calculated by the MCPHASE package (the temperature steps between the presented curves are 3 K).

With this parametrization of Hamiltonian (7) the MCPHASE simulation produces the magnetization curves presented in Fig. 9. The calculation of isothermal magnetization resembles the main features of the experimental curves well. The magnetization jumps along (1010) are consistent with the anomalies and transitions shown in the phase diagrams. It should be noted that these transitions can have a wide tolerance (especially seen in the phase diagram from the hysteresis at low temperatures) which is connected to the helical magnetic structure with an angle of nearly but not exactly 180°. This special configuration may result in a spin-slip behavior when applying an external magnetic field with no sharp transition fields. Also the calculated temperature dependence of magnetization corresponds well to the experimental data. The Néel temperature has the correct value $T_N = 83$ K and additionally the transition at about 60 K and 2 T which is visible in the phase diagram is pronounced in the calculation.

On the strength of this model calculation it is possible to propose an explanation of the low-temperature magnetic structure of Tb$_5$Ge$_3$. Figure 10 shows the orientation of the magnetic Tb$^{3+}$ moments [projected into the (hexagonal) $ab$ plane] at 5 K and its development with external magnetic field parallel to (1010) (the structures at 0, 7, and 11 T are shown here). The preferred orientation of the moments parallel to the (1010) and (1120) axes, respectively, is in accord with the results of the new neutron-diffraction experiments$^4$ which showed a nearly antiparallel orientation of the moments (helical structure with a characteristic angle close to 180°) and a temperature-independent propagation vector $\tau = (0 0 0.49)$. However, the calculated structure is more complex and contains parts, deviating from an easy collinear behavior which more or less reflects the hexagonal symmetry: The moments of the ions at the crystallographic 4$d$ positions are collinear, whereas those of the ions at the 6$g$ positions form a hexagonal structure. Both configurations are characterized by a propagation vector $\tau = (0 0 0.5)$. Figure 10 also illustrates the development of the magnetic array with field. The spin-flop transition starts at about 7 T and is not completely finished at 11 T (see right picture). This confirms the idea that magnetic saturation along the (1010) axis is not reached below a field of about 15 T (see phase diagram above). Additionally, it becomes clear from the calculation of the magnetic structure that the (1010) and (1120) directions are not equivalent due to a small CEF anisotropy. Nevertheless, the main points of the magnetic properties are caused by different Tb$^{3+}$ positions combined with an anisotropic magnetic exchange. This may lead to the requirement of experimental reconsideration of the magnetic structure.

The application of a magnetic field turns the moments toward this field direction. The beginning of the spin-flop process along (1010) is visible in the 7 T graph. The 11 T graph illustrates that a complete saturation is not yet reached at this field value as found by the basic measurements of magnetization and magnetostriction.

It is worth emphasizing again that the mean-field simulation following Eq. (7) that assumed an isotropic two-ion in-
The investigation of the magnetic properties of the hexagonal rare-earth compound Tb$_5$Ge$_3$, experimentally based on measurements of magnetization and magnetostriiction at a single crystal, results in a complex phase diagram. The moments are oriented in the $ab$ plane, so there is an easy plane of magnetization. The compound has an anisotropy not only between hexagonal plane and axis but also in the $ab$ plane itself. The experimental and theoretical investigation confirms a temperature-dependent propagation vector of the magnetic structure of approximately $\tau=(0\ 0\ 0.5)$, as found by elastic neutron scattering in a single crystal. Because of this, the magnetic structure is likely to be of spiral type with a characteristic angle close to 180°.

Usually the anisotropy of rare-earth systems is believed to be due to single-ion effects. However, it is known from Gd$^{3+}$ compounds, which have no single-ion anisotropy, that other types of anisotropy (for example, an anisotropic bilinear interaction as of Dzyaloshinskii-Moriya type) are necessary for a reasonable explanation of the magnetic phenomena. The magnetic properties exhibited by Tb$_5$Ge$_3$ can only be understood introducing a two-ion anisotropy into the mean-field simulation. This anisotropy was found to be stronger than the single-ion one characterized by a nearly vanishing (averaged) CEF parameter $B_2^1$. Because of this fact Tb$_5$Ge$_3$ is a system in which the single-ion anisotropy seems to be negligible in comparison to a huge anisotropy in the two-ion interaction. This demonstrates the importance of the two-ion anisotropy effects in the analysis of magnetic properties of rare-earth compounds.

Further modeling including the influence of magnetoelastic and biquadratic interactions could possibly improve the description of the experimental data. It will be interesting to see if the model assumptions made here can be confirmed by other experimental studies, such as inelastic or diffuse neutron scattering, electron-spin resonance, or x-ray absorption.

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