Modeling a leaky glottis

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The fact that the oral flow of both males and females normally contains an appreciable dc component during the "closed glottis" interval of vowel sounds produced at normal loudness levels indicates that glottal leakage is a very common phenomenon. In this paper the acoustic consequences of glottal leakage are studied by means of a computer simulation. The effects of two different types of leaks were studied, i.e., of (a) a linked leak: an opening (at least partly) situated in the membranous glottis and caused by abduction, and (b) a parallel chink: a leak that can be viewed as an opening which is essentially separated from (parallel to) the time-varying part of the glottis. The results of our simulations show that a moderate leak may give rise to appreciable source-tract interaction which becomes most apparent for a parallel chink. In the time domain it manifests itself as a ripple in the glottal flow waveform just after closure. In the frequency domain, the spectrum of the flow through a glottis with a leak (both linked and parallel) is characterized by zeros at the formant frequencies. The major difference in spectral effects of a linked leak and a parallel chink is the spectral slope. For a parallel chink it is of the same order of magnitude as in the no leakage case or even slightly flatter. In the case of a linked leak, the spectral slope falls off much more rapidly.

These findings suggest that the amount of dc flow alone is not a very good measure to use for voice efficiency measures.

1. Introduction

During the production of voiced speech sounds where the vocal folds are assumed to be adducted, most speakers do not close their glottis completely. As shown by Holmberg, Hillman & Perkell (1988), the oral flow of most speakers, both males and females, contains an appreciable dc component during the "closed" glottis

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interval (CGI) of vowel sounds produced at normal loudness levels. Yet, this leak flow does not necessarily give rise to a breathy voice quality.

Earlier attempts to explain dc flow during the CGI solely on the basis of incomplete adduction were not successful. In modeling experiments it appeared impossible to simulate relatively sudden changes in contact area [or equivalently, in electroglottographic waveforms (EGG)] simultaneously with realistic glottal flow waveforms (Cranen, 1991). Since an increased abduction does not only give rise to a larger leak, but also to an increased duration of the opening and closing epochs of the vocal folds (cf. Titze, 1984), one must assume almost complete adduction in order to obtain realistic EGG waveforms with a duty cycle of approximately 50% and a relatively steep slope at closure. With dc-offset values of approximately 100 cm$^3$/s as observed in real flow recordings (Holmberg et al., 1988) this leads to a contradiction that cannot be resolved under the assumption that all leakage is due to abduction.

Also, in more recent simulation studies we found indications that not all leak flow can be due to abduction: we were not able to simulate both realistic transglottal pressure signals and flow waveforms with the proper dc-offset by simply abducting the vocal folds. Either the variations in the transglottal pressure were too small, or the flow had too low a dc-offset. Again, the simulation problems appeared to be due to the fact that abduction goes hand in hand with a relatively gradual change in flow at the moments of opening and closure. Consequently, the acoustic excitation of the subglottal and supraglottal cavities will be weak (and consequently, the transglottal pressure variations small) when the folds are abducted.

In this paper, we describe a simulation experiment in which we studied an alternative explanation of the phenomena observed in real inverse-filtered mouth flow data by looking at the effects of two types of glottal leakage. One type of leakage comprises the effects of abduction; the other type takes into account that a separate orifice in the cartilaginous portion of the glottis may exist. Both types of leakage are known to occur quite frequently in practice (cf. Södersten & Lindestad, 1990), but we feel that the relation between glottal configuration and voice quality is not very well understood (also cf. Holmberg, 1993). We will show that in our computer model the two types of glottal leaks have quite different acoustic effects. Part of this work was inspired by, and is an extension to, earlier unpublished work by Ishizaka (1989).

2. Two types of glottal leakage

The first type of leak we will indicate as linked leak. It is defined as an opening that comprises at least part of the membranous glottis and equals the glottal area that remains when the folds have reached maximum closure. Consequently, a linked leak is mainly controlled by abduction [Fig. 1(a)].

The second type of leak, denoted by parallel chink, is defined as a leak that can be viewed as a duct or orifice which is essentially separated from (parallel to) the time-varying part of the glottis. The latter could thus be considered as an idealization of a posterior chink when the vocal processes completely touch [Fig. 1(b)]. Note that, except maybe for a small transition region, a linked leak and a parallel chink are mutually exclusive: as soon as the folds start to abduct the parallel chink will become a linked leak.
Abduction tends to make the actual opening and closing gesture more zipper-like (Titze, 1984). As a consequence, the corners of the glottal area waveforms at opening and closure will become more rounded (and the corresponding change in flow more gradual). The most important implication of this observation is that dc-offset flow through a glottis with a linked leak must co-occur with flow pulses that have relatively smooth corners, while in a glottal configuration with a parallel chink the membranous flow component may show abrupt changes regardless of the presence of a dc-offset component.

One might be tempted to think that the presence of a parallel chink simply adds a dc-offset to the membranous flow component and that a glottis with parallel chink is acoustically more or less equivalent to a glottis without a chink. However, one should realize that the formant ripple in the transglottal pressure is greatest during the interval that the membranous glottis is closed and that this pressure may cause an appreciable modulation of the flow through the parallel chink. In fact it is this source-tract interaction that appears to have some interesting consequences in our modeling results.

3. The acoustic impedance of a leaky glottis

In this paper, we study the acoustic effects of glottal leakage under the assumption that it is sufficient to model glottal geometry by solely specifying the glottal inlet and outlet area waveforms and that, for the purpose of speech synthesis, the distribution in space of resistance and inertance can be represented as discrete circuit elements. Thus, we assume that is is not necessary to model any two- or three-dimensional air flow behaviour (cf. Liljencrants, 1991; Iijima, Miki, & Nagai, 1992; Guo & Scherer, 1993), but that a similar approach as taken in the original work on the two-mass model (Ishizaka & Flanagan, 1972) can also be applied to a glottal configuration containing two openings.

In our simulation experiment, we have opted not to use a self-oscillating vocal fold model, but to use prototype waveforms for the glottal inlet and outlet area. The generation of the glottal inlet and outlet area waveforms has been based on a physiologically realistic description of the geometry of the vibrating vocal folds [i.e., they were generated on the basis of control parameters that bear a close resemblance to those proposed by Titze (1984)]. For reasons of compactness the
details will not be described here but in a separate paper (Cranen & Schroeter, 1995).

Although no glottal leakage was incorporated in the original two-mass model one could simply substitute the inlet and outlet areas of the glottis ($A_{gl1}$ and $A_{gl2}$, respectively) by the sum of a constant leak ($A_{l1}$ and $A_{l2}$, respectively) and a time-varying part ($A_{g1}$ and $A_{g2}$, respectively). This would in our view be a reasonable first order approximation to model abduction. The most important complication that arises in the description of the aerodynamics of a linked leak is due to the fact that it is unclear what area value should be used for the upper part of the glottal section ($A_{gl2}$) during the time interval that the lower part of the folds are already maximally closed. If a jet is formed in the posterior part of the glottis, it is unlikely that this jet will use for expansion the entire space that is left between the upper margins of the folds (Cranen & Boves, 1987). However, since there seems to be no straightforward method to account for this effect we assumed that $A_{gl2} = A_{g2} + A_{l2}$ throughout the entire glottal cycle.

Obviously, a glottal constriction consisting of two separate openings will cause the total flow to split into two components. As a consequence, the description of the aerodynamics of a glottis with a parallel chink (with an inlet opening $A_{cl}$ and an outlet opening $A_{c2}$) is slightly more complicated. However, it can also be handled using the same basic approach as applied to a glottis consisting of one tube (cf. Ishizaka & Flanagan, 1972) provided reasonable assumptions can be made about where the flow splitting takes place. It is not very realistic to assume that the stream lines will exactly follow the physical boundaries of the walls. At the inlet, the flow will probably split into two components some distance upstream of the actual openings. Moreover, it must be expected that the stream lines close to the entrance are no longer parallel: if the membranous portion of the glottis has almost closed,

![Figure 2. In a glottis with a parallel chink the flow will split up into two components at a certain distance upstream of the inlet. At the outlets, the two components will mix with the air in the pharynx and join to form a single flow component.](image-url)
that portion of the glottis becomes an "unattractive" flow path and air particles may be transported in a direction perpendicular to the axis of the trachea (Fig. 2). At the outlet there are two jets which will mix with the stagnant air in the pharynx.

Of course, one cannot hope to model any of these phenomena into any detail, but with a few simplifying assumptions we might be able to get a feeling for the main effects. In our simulation we have assumed that the particle velocity in the inlet region remains uniform as well as the pressure until the total flow ($U_{tot}$) has contracted to an area which we denote by $A_t$. Beyond this point we no longer require the pressure to be uniform (to account for possible flow in lateral direction) and we assume that there are two separate flow components: one component entering the membranous glottis ($U_g$) and one entering the parallel chink ($U_c$). The mixing is assumed to start directly at the glottal outlets (Fig. 2).

4. Synthesis

Due to space limitations we cannot include a comprehensive description of our entire synthesizer implementation. This will be done in a separate paper (Cranen & Schroeter, 1995). Instead, may it suffice to derive the three equations that form the heart of our synthesizer.

If the airstream splits at a certain distance upstream of the glottis, then only part of the entrance pressure loss will occur in the region where the sum of the two components exists as one single flow (region I in Fig. 2); the rest will occur in the separate flow channels of region II. When a flow $U$ passes through a converging or diverging channel with inlet area $A_1$ and outlet area $A_2$, the pressure change $\Delta P$ is given by the following equation.

$$\Delta P = 0.5 \rho U^2 \left\{ \frac{1}{A_2^2} - \frac{1}{A_1^2} + \eta \left( \frac{1}{A_2} - \frac{1}{A_1} \right)^2 \right\}$$

Note that the term with the $\eta$-coefficient accounts for energy losses ($\eta = 0$ would correspond to a lossless conversion of potential into kinetic energy; $\eta = 1$ describes the situation where energy is lost but where momentum is preserved like in a strongly diverging duct).

As is clear from Equation (1), the magnitude of the pressure loss term in region I will be determined by the total flow and the area at which the flow can no longer be considered one single flow component (i.e., at $A_i$). Of course, $A_i$ will also affect the pressure drops at both entrances of region II. If one realizes that other pressure changes due to viscous losses, inertia, and tapering of the ducts can also be treated separately for each flow channel, it should be obvious that the transglottal pressure ($P_{tr}$, i.e., the pressure across region II in Fig. 2) can be expressed by three different equations, one for each flow path: (1) the membranous part of the glottis (2) the parallel chink, and (3) the flow path through the subglottal and supraglottal system. Since we have three unknowns (viz. $U_g$, $U_c$, and $P_{tr}$) these three equations are enough to describe our synthesizer.

To derive the first equation for $P_{tr}$ we have assumed that for the inlet region Equation (1) can be applied, and that for the expansion region, analogously to the approach in the original two-mass model, conservation of momentum must be assumed. Thus we find:
Pressure across region I and III ($U_{tot} = U_g + U_c$):

$$P_r(t) = P_{inle}(t) - [h_{sub}(t) + h_{supra}(t)] \otimes U_{tot}(t) +$$

$$- (1 + \eta_i) \frac{\rho U_{tot}^2(t)}{2 A_i^2(t)} +$$

$$- \frac{\rho U_{tot}^2(t)}{A_i^2(t)} + \frac{\rho U_g^2(t)}{A_{g2}(t) A_i(t)} + \frac{\rho U_c^2(t)}{A_{c2}(t) A_i(t)}$$

The meaning of the mathematical symbols is explained in the appendix. For deriving the second and third equations it is important to realize that we assume a uniform velocity profile at the point where the total flow $U_{tot}$ splits. This means that the area occupied by $U_g$ (denoted by $A_{ig}$) and the area occupied by $U_c$ (denoted by $A_{ic}$) are given by

$$\frac{U_{tot}}{A_i} = \frac{U_g}{A_{ig}} = \frac{U_c}{A_{ic}}$$

Using Equation (1) for describing the pressure loss at the entrance while using Equation (3) to rewrite the inlet areas $A_{ig}$ and $A_{ic}$ in terms of $A_i$, $U_g$, $U_c$, and $U_{tot}$, we find:

Pressure across the membranous glottis (including the linked leak):

$$P_r(t) = \frac{\rho(\eta_i - 1) U_{tot}^2(t)}{2 A_i^2(t)} + \frac{\rho(\eta_i + 1) U_g^2(t)}{2 A_{g1}^2(t)} - \frac{\rho \eta_i U_g U_{tot}(t)}{A_{g1}(t) A_i(t)}$$

$$+ R_{g0}(t) U_g(t) + \frac{d[L_g(t) U_g(t)]}{dt}$$

$$+ \frac{\rho}{2} U_g^2(t) \left[ \frac{1}{[A_{g2}(t) + A_{g1}(t)]^2} - \frac{1}{[A_{g1}(t) + A_{g2}(t)]^2} \right]$$

$$+ \eta_{12}(t) \left[ \frac{1}{[A_{g2}(t) + A_{g1}(t)]} - \frac{1}{[A_{g1}(t) + A_{g2}(t)]} \right]^2$$

Across the parallel chink:

$$P_r(t) = \frac{\rho(\eta_i - 1) U_{tot}^2(t)}{2 A_i^2(t)} + \frac{\rho(\eta_i + 1) U_c^2(t)}{2 A_{c1}^2(t)} - \frac{\rho \eta_i U_c U_{tot}(t)}{A_{c1}(t) A_i(t)}$$

$$+ R_{c0}(t) U_c(t) + \frac{d[L_c(t) U_c(t)]}{dt}$$

$$+ \frac{\rho}{2} U_c^2(t) \left[ \frac{1}{A_{c2}(t)} - \frac{1}{A_{c1}(t)} \right]^2$$

$$+ \eta_{12}(t) \left[ \frac{1}{A_{c2}(t)} - \frac{1}{A_{c1}(t)} \right]^2$$

Analogously to the approach of Ishizaka & Flanagan (1972), Equations (2), (4), and (5) can be translated into an electrical equivalent network (Fig. 3). The glottal
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Figure 3. Schematic diagram of a speech synthesizer with a voice source containing a parallel chink.

region from Fig. 2 is represented by a network where the two parallel flow tubes in region II appear as two (flow dependent) impedances in parallel \([Z_g \text{ and } Z_c]\) which are described by Equations (4) and (5) respectively, while the inlet pressure loss and exit pressure recovery of region I and III respectively are depicted by separate flow dependent impedances \([R_i \text{ (term with } A^2)\text{ and } R_e \text{ (the last three terms) in Equation (2)}]\).

In order to investigate the main characteristics of various types of leakage we carried out a synthesis experiment in which we synthesized a number of vowels. To that end, the glottal source was loaded by a static vocal tract using the method described in Sondhi & Schroeter (1987). The subglottal system consisted of a pressure source with an internal impedance of 10 cgs Ohms. Furthermore, three subglottal resonance frequencies were taken into account (cf. Ishizaka, Matsudaira, & Kaneko, 1976; Cranen & Boves, 1987). For controlling the glottal inlet and outlet areas in Equations (2–5) in such a way that abduction was more or less realistically accounted for, we used a parametric model analogous to the one proposed by Titze (1984) that we extended with a parallel chink. We did not use any noise source to simulate glottal noise arising from a possible dc flow.

5. Results

Fig. 4(a) shows glottal waveforms and spectra that were computed using a tract shape representing the vowel /a/ excited by a glottis with linked leak; Fig. 4(b) shows a similar picture for a glottis with a parallel chink. The dashed lines represent the waveforms for a glottis without leakage; the solid lines a glottis with leakage. Both types of leak were chosen such that they gave rise to a dc flow offset of approximately 100 cm³/s (abduction quotient \(Q_a = 0.9\), \(A_c = 0.04\ cm^2\)). In order to make comparison of slopes easier, the spectra are normalized with respect to their maxima. It should be kept in mind though, that for a leaky glottis the absolute levels of the spectra are slightly lower, in particular at the lower frequencies. Note that in the case of a glottis with parallel chink the interaction ripple corresponds to an (harmonic part of the) glottal flow spectrum above the second formant which seems slightly enhanced in comparison to the no leakage case.

In order to allow a deeper understanding of this ripple just after closure, Fig. 5 shows, from top to bottom, how an increase in abduction affects the flow derivatives
Figure 4. Left: glottal flow and its derivative during the simulation of an /a/ for a glottis without (dashed lines) and with (solid lines) leakage. Right: Corresponding (normalized) spectra of the derivative waveforms on the left.
(a) Leakage caused by abduction (b) Leakage caused by a parallel chink. Note that if we disregard the absolute energy level, the source spectrum of the glottis with leakage has higher energy at higher frequencies relatively to the case without parallel chink. In this simulation we have chosen \( A_t = 1.1(A_{c1} + A_{c2}) \).

in the membranous glottis and in the parallel chink as well as their sum and the transglottal pressure. Fig. 6 shows how glottal flow, flow derivative, and translottal pressure waveforms change when we simulate a gradual transition from a glottis with parallel chink (for area of parallel chink cf. second panel from top) to a glottis with linked leak of approximately the same size (cf. dc-offset in top panel).
From Figs 5 and 6 it is clear that during the interval that the membranous glottis is closed, the transglottal pressure induces a flow in the parallel chink which more or less "follows" the ripple behavior in the transglottal pressure waveform: energy which has accumulated in the resonance cavities below and above the glottis is re-injected through the parallel chink just after the membranous glottis has closed. The exact ripple behavior of the chink flow will not only depend on the transglottal pressure but, of course, also on the impedance of the parallel chink. If the impedance were purely resistive, a chink flow would result that is congruent with the transglottal pressure wave; if the impedance were purely inertive, the chink flow would have a waveform that could be obtained by integrating the transglottal pressure wave. In practice, the impedance of the parallel chink will probably behave as a resistance in series with an inductance and the waveform of the flow through the parallel chink will look like a low-pass filtered version of transglottal pressure (the output of a leaky integrator).

The acoustic excitation of the subglottal and supraglottal cavities caused by an abrupt closure of the membranous glottis gives rise to transglottal pressure waveforms consisting of decaying resonance ripples starting at the moment of closure. When the closure is really abrupt, i.e., when the folds are fully adducted, the transglottal pressure waveforms may exhibit a relatively large peak at glottal closure (cf. Fig. 6). If abduction increases, however, opening and closure of the folds will not be as abrupt anymore and the peak in the transglottal pressure will decrease. As a consequence, the flow through a remaining leak opening will show less ripple as well.

In our simulations, we found that the spectral enhancement at the higher frequencies caused by the source/tract interaction in the closed glottis interval
depends quite strongly on the assumptions that are made about the value of \( A_i \). This is understandable because this assumption determines to a great extent how the resistive parts of the impedances are distributed over the glottal impedance network in Fig. 3. Since it is likely that the 3D-shaping of the inlet and outlet regions affect the actual airstream behavior, geometrical details may play a more important role than recognized thus far. In order to obtain any spectral enhancement effects, it appeared to be important to assume that the flow splits at a relatively short distance upstream of the glottis \( (A_i < 1.2(A_{g1} + A_{c1})) \). For larger values of \( A_i \), the spectral slope at higher frequencies becomes the same as for the no leakage case. Whether the spectral enhancement is plausible from a physical/physiological point of view remains a topic to be investigated.

6. Discussion

In the classical linear source-filter model of speech production dc components of the flow are acoustically unimportant. Probably for that reason not much attention has been given to modeling glottal leakage. Most voice-source modeling studies have been done without explicitly taking into account glottal leakage. In particular, there has been a general belief, so it seems, that the glottal impedance during the closed glottis interval is high enough to assume that the amount of non-linear source-tract interaction during that interval is negligible. As a consequence the notion has
grown that if source-tract interaction becomes significant at all, it has primarily to be expected in the open glottis interval. In this study, we have shown simulation results which suggest that the source-tract interaction might be non-negligible in the (first part of the) closed glottis interval as well.

In earlier simulation studies of vowel sounds we found it impossible to explain a dc-offset flow of 100 cm$^3$/s by abduction alone because an increased abduction would cause both the flow derivative at closure and the corresponding transglottal pressure peaks to drop too much below expected standard values. Using the concept of a parallel chink, we have shown in this study that the co-occurrence of a dc-offset flow of 100 cm$^3$/s, a peak-to-peak flow of 400 cm$^3$/s and transglottal pressure peaks of approximately 16–20 cm H$_2$O at an average of 8 cm H$_2$O [which we know from measurements are common values (cf. Cranen & Boves, 1985)] are well attainable provided the membranous part of the glottis closes abruptly and the chink is given a cross-sectional area of 0.04 cm$^2$ (as compared to a maximum area of the membranous glottis of 0.2 cm$^2$).

Our simulation results suggest that a constant leak opening in combination with an abrupt and fully closing second orifice is not much less effective in producing voiced sounds than one single perfectly closing orifice. The introduction of a parallel chink forced us to assume that the glottal air stream splits into two components. This assumption appeared to have an intriguing effect: under some circumstances a spectrum can be observed in which the higher frequencies are slightly enhanced (at the cost of the lower frequencies). However, a perspective relativation is in place. Firstly, the equations used in our simulation are a severe simplification of reality. Secondly, so far, we have no firm experimental confirmation that it is realistic to consider a posterior chink as an impedance which is essentially parallel to the impedance of the membranous glottis. To find a decisive answer to this question more research is needed.

The fact that not the leak area alone, but also the way in which the leak area is formed determines the deterministic part of the glottal flow pulse (i.e., the harmonic component of the source spectrum) sheds an interesting light on voice quality aspects like “breathiness”. For instance, if one takes into account that any noise that is generated by dc flow becomes more perceptible if the slope of the harmonic spectrum shows a steeper roll-off, our observations would imply that dc flow alone is a bad predictor for breathiness. Another related implication would be that “vocal efficiency” measures that use ac/dc ratios (as in Isshiki, 1983) would have to be reconsidered.

Finally, a parallel chink inevitably has to become a linked leak as soon as the folds are abducted. Therefore, since a (not excessively large) parallel chink does not seem to have any negative effects on the spectral slope while a linked leak does, another implication of our findings would be that the presence of a parallel chink provides a way to create relatively large spectral changes by relatively small abduction gestures.

REFERENCES


### Appendix: Mathematical symbols

- \( A_{g_1}(t) \): time-varying part of the inlet area of the membranous glottis
- \( A_{g_2}(t) \): time-varying part of the outlet area of the membranous glottis
- \( A_{l_1}(t) \): area of the linked leak at the glottal inlet
- \( A_{l_2}(t) \): area of the linked leak at the glottal outlet
- \( A_{c_1}(t) \): area of the parallel chink at the glottal inlet
- \( A_{c_2}(t) \): area of the parallel chink at the glottal outlet
- \( A_i(t) \): area at which the entrance flow splits
- \( h_{sub}(t) \): subglottal impedance impulse response
- \( h_{supra}(t) \): vocal tract impedance impulse response
- \( l_g \): length of membranous glottis
- \( l_c \): length of parallel chink
- \( d_g \): depth of membranous glottis and parallel chink
- \( \otimes \): convolution
- \( \eta_i = 0.37 \): entrance loss factor
- \( \eta_{12}(t) = \{ 0.4 \text{ if } [A_{g_1}(t) + A_{l_1}(t)] \geq [A_{g_2}(t) + A_{l_2}(t)] \}
- 1.0 \text{ if } [A_{g_1}(t) + A_{l_1}(t)] < [A_{g_2}(t) + A_{l_2}(t)] \}
- \) loss coefficient to account for tapering of membranous glottis (cf. Ishizaka, 1983)

\[
R_{v,g}(t) = 12 \mu_l r_g^2 \int_0^{d_g} \frac{1}{A_e(y, t) + A_l(y, t)} dy
\]

\[
R_{v,c}(t) = 12 \mu_l r_c^2 \int_0^{d_c} \frac{1}{A_c(y, t)} dy
\]
viscous resistance of membranous glottis and parallel chink, respectively

\[ L_g(t) = \rho \left[ \frac{\Delta d_1}{(A_{g1} + A_{11})} + \int_0^{d_1} \frac{1}{A_g(y, t) + A_l(y, t)} \, dy + \frac{\Delta d_2}{A_{g2} + A_{l2}} \right] \]

\[ L_c(t) = \rho \left[ \frac{\Delta d_{1,c}}{A_{c1}} + \int_0^{d_{1,c}} \frac{1}{A_c(t)} \, dy + \frac{\Delta d_{2,c}}{A_{c2}} \right] \]

Inductance of membranous glottis and parallel chink, respectively

Note: \( \Delta d_{1,R} = 0.61 \sqrt{(A_{g1} + A_{11})/\pi} \), \( \Delta d_{2,R} = 0.61 \sqrt{(A_{g2} + A_{l2})/\pi} \), \( \Delta d_{1,c} = 0.61 \sqrt{A_{c1}/\pi} \), and \( \Delta d_{2,c} = 0.61 \sqrt{A_{c2}/\pi} \) are end correction terms for a small tube ending in a large volume without baffle (cf. Beranek, 1986; p. 133.)