A Precision Measurement of the Mass of the Top Quark

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The Standard Model (SM) of particle physics contains about two dozen parameters – such as particle masses – whose origins are still unknown and cannot be predicted, but whose values are constrained through their interactions. In particular, the masses of the top (t) quark \(M_t\) and W boson \(M_W\)\(^\dagger\) constrain the mass of the long-hypothesized Higgs boson, but thus far not observed, Higgs boson. A precise measurement of the top-quark mass can therefore point to where to look for the Higgs, and indeed whether the hypothesis of a SM Higgs is consistent with experimental data. Since top quarks are produced in pairs and decay in only \(\approx 10^{-24}\) s into various final states, reconstructing their mass from their decay products is very challenging. Here we report a technique that extracts far more information from each top-quark event and yields a greatly improved precision on the top mass of \(\pm 5.3\ \text{GeV}/c^2\), compared to previous measurements \(^2\). When our new result is combined with our published measurement in a complementary decay mode \(^3\) and with the only other measurements available \(^2\), the new world average for \(M_t\) becomes \(178.0 \pm 4.3\ \text{GeV}/c^2\) \(^4\). As a result, the most likely Higgs mass increases from the experimentally excluded \(^5\) value of \(96\ \text{GeV}/c^2\) \(^6\) to \(117\ \text{GeV}/c^2\), which is beyond current experimental sensitivity. The upper limit on the Higgs mass at 95% confidence level is raised from \(219\ \text{GeV}/c^2\) to \(251\ \text{GeV}/c^2\).

The discovery of the top quark in 1995 served as one of the major confirmations of the validity of the SM \(\dagger\dagger\). Of its many parameters, the mass of the top quark, in particular, reflects some of the most crucial aspects of the Model. This is because, in principle, the top quark is point-like and should be massless; yet, through its interactions with the hypothesized Higgs field, the physical mass of the top quark appears to be about the mass of a gold nucleus. Because it is so heavy, the top quark (along with the W boson) provides an unusually sensitive tool for investigating the Higgs field. \(M_W\) is known to a precision of 0.05%, while the uncertainty on \(M_t\) is at the 3% level \(^1\). Improvements in both measurements are required to restrict further the allowed range of mass for the Higgs; however, given the large uncertainty in the top mass, an improvement in its precision is particularly important. As has been pointed out recently, e.g., in Refs. \(^11\) \(^10\), a potential problem for the SM is that, based on the presently accepted mass of the top quark, the most likely value of the Higgs mass \(^6\) lies in a range that has already been excluded by experiment \(^2\). Precise knowledge of the Higgs mass is crucial for our understanding of the SM and any possible new physics beyond it. For example, in a large class of supersymmetric models (theoretically preferred solutions to the deficiencies of the SM), the Higgs mass has to be less than \(\approx 135\ \text{GeV}/c^2\). While, unlike the SM, supersymmetry predicts more than one Higgs boson, the properties of the lightest one are expected to be essentially the same as those for the SM Higgs boson. Thus, if the SM-like Higgs is heavier than \(\approx 135\ \text{GeV}/c^2\), it would disfavor a large class of supersymmetric models. In addition, the current limits on supersymmetric particles from LEP \(^11\) are extremely sensitive to the mass of the top quark. In fact, for a top-quark mass greater than \(179\ \text{GeV}/c^2\), the bounds on one of the major supersymmetry parameters, \(\tan \beta\), which relates the properties of the SM-like Higgs boson and its heavier partners, would disappear completely \(^12\). Hence, in addition to the impact on searches for the Higgs boson, other important consequences call for improved precision on the top mass, and this goal is the main subject of this paper.

The DØ experiment at the Fermilab Tevatron has studied a sample of \(t\bar{t}\) events produced in proton-antiproton (\(p\bar{p}\)) interactions \(^13\). The total energy of 1.8 TeV released in a head-on collision of a 900 GeV \(p\) and 900 GeV \(\bar{p}\) is almost as large as the rest energy of ten gold nuclei. Each top (antitop) quark decays almost immediately into a bottom \(b\) (\(\bar{b}\)) quark and a \(W^+\) (\(W^-\)) boson, and we have reexamined those events in which one of the \(W\) bosons decays into a charged lepton (electron or muon) and a neutrino, and the other \(W\) into a quark and an antiquark (see Figure \(4\)). These events and their selection criteria are identical to those used to extract the mass of the top quark in our previous publication, and correspond to an integrated luminosity of 125 events/pb. (That is, given \(\approx 10^{-24}\) s into various final states, reconstructing their mass from their decay products is very challenging. Here we report a technique that extracts far more information from each top-quark event and yields a greatly improved precision on the top mass of \(\pm 5.3\ \text{GeV}/c^2\), compared to previous measurements \(^2\). When our new result is combined with our published measurement in a complementary decay mode \(^3\) and with the only other measurements available \(^2\), the new world average for \(M_t\) becomes \(178.0 \pm 4.3\ \text{GeV}/c^2\) \(^4\). As a result, the most likely Higgs mass increases from the experimentally excluded \(^5\) value of \(96\ \text{GeV}/c^2\) \(^6\) to \(117\ \text{GeV}/c^2\), which is beyond current experimental sensitivity. The upper limit on the Higgs mass at 95% confidence level is raised from \(219\ \text{GeV}/c^2\) to \(251\ \text{GeV}/c^2\).

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The previous DØ top-quark mass measurement in this lepton + jets channel is \(M_t = 173.3 \pm 5.6\ \text{(stat)} \pm 5.5\ \text{(syst)}\ \text{GeV}/c^2\), and is based on 91 candidate events. Information pertaining to the detector and to the older analysis can be found in Refs. \(^12\) and \(^13\), respectively.
The new method of the top-quark mass measurement is similar to one suggested previously for $t\bar{t}$ dilepton decay channels (where both $W$ bosons decay leptonically), and used in previous mass analyses of dilepton events, and akin to an approach suggested for the measurement of the mass of the $W$ boson at LEP. The critical differences from previous analyses in the lepton + jets decay channel lie in: (i) the assignment of more weight to events that are well measured or more likely to correspond to $t\bar{t}$ signal, and (ii) the handling of the combinations of final-state objects (lepton, jets, and imbalance in transverse momentum, the latter being a signature for an undetected neutrino) and their identification with top-quark decay products in an event (e.g., from ambiguity in choosing jets that correspond to $b$ or $\bar{b}$ quarks from the decays of the $t$ and $\bar{t}$ quarks). Also, since leading-order matrix elements were used to calculate the event weights, only events with exactly four jets are kept in this analysis, resulting in a candidate sample of 71 events. Although we are left with fewer events, the new method for extracting the mass of the top quark provides substantial improvement in both statistical and systematic uncertainties.

We calculate as a function of top mass the differential probability that the measured variables in any event correspond to signal. The maximum in the product of these individual event probabilities provides the best estimate of the mass of the top quark in the data sample. The impact of biases from imperfections in the detector and event reconstruction algorithms is taken into account in two ways. Geometric acceptance, trigger efficiencies, event selection, etc., enter through a multiplicative acceptance function that is independent of $M_t$. Because the angular directions of all the objects in the event, as well as the electron momentum, are measured with high precision, their measured values are used directly in the calculation of the probability that any event corresponds to $t\bar{t}$ or background production. The known momentum resolution is used to account for uncertainties in measurements of jet energies and muon momenta.

As in the previous analysis, momentum conservation in $\gamma\gamma$-jet events is used to check that the energies of jets in the experiment agree with Monte Carlo (MC) simulation. This calibration has an uncertainty $\delta E = (0.025E + 0.5$ GeV). Consequently, all jet energies in our sample are rescaled by $\pm \delta E$, the analysis redone, and half of the difference in the two rescaled results for $M_t (\delta M_t = 3.3$ GeV/$c^2$) is taken as a systematic uncertainty from this source. All other contributions to systematic uncertainty: Monte Carlo modeling of signal ($\delta M_t = 1.1$ GeV/$c^2$) and background ($\delta M_t = 1.0$ GeV/$c^2$), effect of calorimeter noise and event pile-up ($\delta M_t = 1.3$ GeV/$c^2$), and other corrections from top-quark mass extraction ($\delta M_t = 0.6$ GeV/$c^2$) are much smaller, and discussed in detail in Refs. This should be noted that the new mass measurement method provides a significant ($\approx 40\%$, from $\pm 5.5$ to $\pm 3.9$ GeV/$c^2$) reduction in systematic uncertainty, which is ultimately dominated by the measurement of jet energies. For details on the new analysis, see the Methods section.

The final result is $M_t = 180.1 \pm 3.6$ (stat) $\pm 3.9$ (syst) GeV/$c^2$. The improvement in statistical uncertainty over our previous measurement is equivalent to collecting a factor of 2.4 as much data. Combining the statistical and systematic uncertainties in quadrature, we obtain $M_t = 180.1 \pm 5.3$ GeV/$c^2$, which is consistent with our previous measurement in the same channel (at $\approx 1.4$ standard deviations), and has a precision comparable to all previous top-quark mass measurements combined.

The new measurement can be combined with that obtained for the dilepton sample that was also collected at DØ during Run I (3) ($M_t = 168.4 \pm 12.3$ (stat) $\pm 3.6$ (syst) GeV/$c^2$), to yield the new DØ average for the mass of the top quark:

$$M_t = 179.0 \pm 5.1 \text{ GeV}/c^2 \text{ (DØ).}$$  

(1)

Combining this with measurements from the CDF experiment (2), provides a new “world average” (based on all measurements available) for the top-quark mass (4):

$$M_t = 178.0 \pm 4.3 \text{ GeV}/c^2 \text{ (All available data),}$$  

(2)

dominated by our new measurement. This new world average shifts the best-fit value of the expected Higgs mass from 96 GeV/$c^2$ to 117 GeV/$c^2$ (see Figure 3), which is now outside of the experimentally-excluded region, yet accessible in the current run of the Tevatron and at future runs at the Large Hadron Collider (LHC), currently under construction at CERN. (The upper limit on the Higgs mass at 95% confidence level changes from 219 GeV/$c^2$ to 251 GeV/$c^2$.)

Figure 3 shows the effect of using only the new DØ top mass for fits to the Higgs mass, and indicates a best value of 123 GeV/$c^2$ and the upper limit of 277 GeV/$c^2$ at 95% confidence level. It should be noted that the horizontal scale in Figure 3 is logarithmic, and the limits on the Higgs boson mass are therefore asymmetric.

The new method is already being applied to data being collected by the CDF and DØ experiments at the new run of the Fermilab Tevatron and should provide even higher precision on the determination of the top-quark mass, equivalent to more than a doubling of the data sample, relative to using the conventional method. An ultimate precision of at $\approx 2$ GeV/$c^2$ on the top-quark mass is expected to be reached in several years of Tevatron operation. Further improvement may eventually come from the LHC.
## Methods

The probability density as a function of $M_t$ can be written as a convolution of the calculable cross section and any effects from measurement resolution:

$$P(x, M_t) = \frac{1}{\sigma(M_t)} \int d\sigma(y, M_t) dq_1 dq_2 f(q_1) f(q_2) W(y, x)$$  \hspace{1cm} (3)

where $W(y, x)$, our general transfer function, is the normalized probability for the measured set of variables $x$ to arise from a set of nascent (partonic) variables $y$, $d\sigma(y, M_t)$ is the partonic theoretical differential cross section, $f(q)$ are parton distribution functions that reflect the probability of finding any specific interacting quark (antiquark) with momentum $q$ within the proton (antiproton), and $\sigma(M_t)$ is the total cross section for producing $t\bar{t}$. The integral in Eq. (3) sums over all possible parton states leading to what is observed in the detector.

The acceptance of the detector is given in terms of a function $A(x)$ that relates the probability $P_m(x, M_t)$ of measuring the observed variables $x$ to their production probability $P(x, M_t)$: $P_m(x, M_t) = A(x) P(x, M_t)$. Effects from energy resolution, etc., are taken into account in the transfer function $W(y, x)$. The integrations in Eq. (4) over the eleven well-measured variables (three components of charged-lepton momentum and eight jet angles) and the four equations of energy-momentum conservation, leave five integrals that must be performed to obtain the probability that any event represents $t\bar{t}$ (or background) production for some specified value of top mass $M_t$.

The probability for a $t\bar{t}$ interpretation can be written as:

$$P_{t\bar{t}} = \frac{1}{12\sigma_{t\bar{t}}} \int d^5\Omega \sum_{perm,\nu} |M_{t\bar{t}}|^2 \frac{f(q_1)f(q_2)}{|q_1||q_2|} \Phi_\nu W_{\text{jets}}(E_{\text{part}}, E_{\text{jet}}),$$

where $\Omega$ represents a set of five integration variables, $M_{t\bar{t}}$ is the leading-order matrix element for $t\bar{t}$ production, $f(q_1)$ and $f(q_2)$ are the CTEQ4M parton distribution functions for the incident quarks, $\Phi_\nu$ is the phase-space factor for the 6-object final state, and the sum is over all 12 permutations of the jets and all possible neutrino solutions. $W_{\text{jets}}(E_{\text{part}}, E_{\text{jet}})$ corresponds to a function that maps parton-level energies $E_{\text{part}}$ to energies measured in the detector $E_{\text{jet}}$, and is based on MC studies. A similar expression, using a matrix element for $W^+ \text{ jets}$ production (the dominant background source) that is independent of $M_t$, is used to calculate the probability for a background interpretation, $P_{\text{bkg}}$.

Studies of samples of HERWIG MC events indicate that the new method is capable of providing almost a factor of two reduction in the statistical uncertainty on the extracted $M_t$. These studies also reveal that there is a systematic shift in the extracted $M_t$ that depends on the amount of background there is in the data. To minimize this effect, a selection is introduced based on the probability that an event represents background. The specific value of the $P_{\text{bkg}}$ cutoff is based on MC studies carried out before applying the method to data, and, for a top mass of 175 GeV/c$^2$, retains 71% of the signal and 30% of the background. A total of 22 data events out of our 71 candidates pass this selection.

The final likelihood as a function of $M_t$ is written as:

$$\ln L(M_t) = \sum_{i=1}^{N} \ln[c_1 P_{t\bar{t}}(x_i, M_t) + c_2 P_{\text{bkg}}(x_i)] - N \int A(x) [c_1 P_{t\bar{t}}(x, M_t) + c_2 P_{\text{bkg}}(x)] dx,$$

The integration is performed using MC methods. The best value of $M_t$ (when $L$ is at its maximum $L_{\text{max}}$) represents the most likely mass of top in the final $N$-event sample, and the parameters $c_i$ reflect the amounts of signal and background. MC studies show that there is a downward shift of 0.5 GeV/c$^2$ in the extracted mass, and this correction is applied to the result. Reasonable changes in the cutoff on $P_{\text{bkg}}$ do not have significant impact on $M_t$.

Figure 4 shows the value of $L(M_t)/L_{\text{max}}$ as a function of $M_t$ for the 22 events that pass all selection criteria, after correction for the 0.5 GeV/c$^2$ bias in mass. The likelihood is maximized with respect to the parameters $c_i$ at each mass point. The Gaussian fit in the figure yields $M_t = 180.1$ GeV/c$^2$, with a statistical uncertainty of $\delta M_t = 3.6$ GeV/c$^2$. The systematic uncertainty, dominated by the measurement of jet energies, as discussed above, amounts to $\delta M_t = 3.9$ GeV/c$^2$. When added in quadrature to the statistical uncertainty from the fit, it yields the overall uncertainty on the new top-quark mass measurement of $\pm 5.3$ GeV/c$^2$.

We wish to note the great number of contributions made by the late Harry Melanson to the DØ experiment, through his steady and inspirational leadership of the physics, reconstruction and algorithm efforts.

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Figure 1: Feynman diagrams for $t\bar{t}$ production in $p\bar{p}$ collisions, with subsequent decays into an electron, neutrino, and quarks. Diagram (a) (quark-antiquark production) is dominant, but diagram (b) (gluon fusion) contributes $\approx 10\%$ to the cross section. This particular final state ($e\nu u\bar{d}\bar{b}$) is one of the channels used in the analysis.

$t\bar{t}$ Decay Channels

Figure 2: Relative importance of various $t\bar{t}$ decay modes. The “lepton + jets” channel used in this analysis corresponds to the two offset slices of the pie-chart and amounts to 30% of all the $t\bar{t}$ decays.
Figure 3: Current experimental constraints on the mass of the Higgs boson. The $\chi^2$ for a global fit to electroweak data using the procedure of Ref. [6], is shown as a function of the Higgs mass. The solid line corresponds to the result for the previous world average for the top-quark mass of $174.3 \pm 5.1$ GeV/$c^2$, with the blue band indicating the impact of theoretical uncertainty. The dotted line shows the result for the new world-averaged $M_t$ of $178.0 \pm 4.3$ GeV/$c^2$, while the dashed line corresponds to using just the new DØ average of $179.0 \pm 5.1$ GeV/$c^2$. The yellow shaded area on the left indicates the region of Higgs masses excluded by experiment ($> 114.4$ GeV/$c^2$ at the 95% confidence level [5]). The improved top mass measurement shifts the most likely value of the Higgs mass above the experimentally excluded range.

Figure 4: Determination of the top-quark mass using the maximum likelihood method. The points represent the likelihood of the fit used to extract the top mass, divided by its maximum value, as a function of the mass of the top quark (after a correction for a $-0.5$ GeV/$c^2$ mass bias, see text). The solid line shows a Gaussian fit to the points. The maximum likelihood corresponds to a mass of 180.1 GeV/$c^2$, which is the new DØ measurement of the top mass in the lepton + jets channel. The hatched band corresponds to the range of $\pm 1$ standard deviation, and indicates the $\pm 3.6$ GeV/$c^2$ statistical uncertainty of the fit.