Towards A Unified Semantic Model For Interactive Applications Using Arrows And Generic Editors

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Abstract

In slightly less than two decades Graphical User Interfaces (GUI) have become standard in presenting the user a pleasant and intuitive interface to software applications. Two major paradigms have contributed to this success: the desktop GUI for widget based programming on a single computer, and the World Wide Web for web applications with a multitude of client computers. Applications that are created within these paradigms are constructed in entirely different ways. Desktop applications can use the vast platform dependent API, or rely on platform independent toolkits. Web applications are confined within web browsers, but they can stretch these limits by relying on a wide variety of scripting languages. This has resulted in ‘typical’ desktop and web applications. However, one can observe the trend that desktop and web applications are taking over each others functionality: former desktop GUI applications now also offer back/forward buttons, hyperlinks from web applications, and web applications use multiple windows/dialogues, disallow browser window cloning, and so on. Hence, the clear separation between desktop and web applications is blurring. This implies that in order to reason rigidly about interactive applications in general, we need a semantic model that is capable of handling these two, formerly separated, paradigms. In this paper we present a first step towards such a framework. This work is based on our earlier work on the iData and GEC toolkits.

1 INTRODUCTION

In slightly less than two decades Graphical User Interfaces (GUI) have become standard in presenting the user a pleasant and intuitive interface to software applications. Two major paradigms have contributed to this success: the desktop GUI for widget based programming on a single computer, and the World Wide Web for web applications with a multitude of client computers. At a cursory glance, these two paradigms are closely related as they use the same Windows, Icons, Menus and Pointing device (WIMP) interface style. However, they are completely different. Semantically speaking, desktop GUI applications are essentially a state-based iterative event-handling process (callback semantics), whereas web applications are essentially a stateless, single-step procedure. Desktop applications can use the vast platform dependent APIs that are available, or rely on platform independent libraries, such as wxWidgets. Web applications are confined within web browsers, but they can stretch these limits by relying on a wide variety of scripting languages.
Until recently, one could immediately tell whether an application was a desktop or a web application. However, we can observe the trend that desktop and web applications are taking over each others functionality: formerly ‘pure’ desktop GUI applications now also offer back/forward buttons, and hyperlinks from the web interface, whereas formerly ‘pure’ web applications are using multiple windows and dialogues, and disallow browser window cloning. Hence, the clear-cut separation between desktop and web applications is blurring. This implies that in order to reason rigidly about interactive applications in general, we need a semantic model that is capable of handling these two, formerly separated, paradigms. In this paper we present a first step to a unified semantic model for interactive applications.

The unified semantic model that we propose needs to strike a balance between abstraction and practicality. We obtain an abstract semantic model by postponing reasoning about GUI aspects such as the exact set of interactive elements and their layout and look and feel. This can be done by adopting the concept of generic editor that has been used in our earlier work on desktop GUI applications on the one hand, the GEC Toolkit [1, 2, 3, 4], and web applications on the other hand, the iData Toolkit [15, 16, 17]. In these toolkits interactive elements are modelled by means of data types and functions. This allows us to concentrate on the data that is maintained by the interactive elements of an application. We obtain a practical semantic model by building it on Arrows [12, 14]. With Arrows we get a disciplined way of modelling the information flow between interactive elements, and still handle arbitrarily complex computations.

As an example of an application within this semantic model, Fig. 1 displays the key fragment of a money-converting application. Function \( c_0 \) introduces two interactive elements, labelled with \( \text{euroId} \) and \( \text{poundId} \), that are mutually interconnected in such a way that if either of them is altered by the user, that the other element responds with the amount of money expressed in the local currency. This is not altogether obvious when considering the mutual dependency of this simple program. We prove this property using the unified semantic model (Sect. 5).

Contributions presented in this paper are:

- We present a uniform semantic framework that captures the essence of desktop and web GUI applications.
- We demonstrate that the framework is practical: real desktop and web applications can be created with it.
- We demonstrate that the framework is accurate: we can reason thoroughly about the framework and applications that are created with it.

The layout of this paper is as follows. We first present the major design decisions that our unified framework needs to satisfy in Sect. 2. We define the unified semantic model in Sect. 3 and present a number of its properties in Sect. 4. We illustrate the use of the model by means of a case study in Sect. 5. We present related work in Sect. 6, and conclusions in Sect. 7.
:: Pounds = { pounds :: Real }
:: Euros = { euros :: Real }

c₀ :: A Euros Pounds
c₀ = feedback (edit euroId >>> arr toPound)
    (edit poundId >>> arr toEuro)

where
toPound { euros } = { pounds = euros * exchangerateEuroPound }
toEuro { pounds } = { euros = pounds * exchangeratePoundEuro }
euroId   = ...
poundId  = ...

![Image of iData money converter program]

FIGURE 1. The iData money converter program.

2 DESIGN RATIONALE

The unified semantic model has been inspired mainly by our earlier work on the GEC Toolkit and iData Toolkit. The model consists of two levels: at the bottom level we have the main building blocks, the interactive elements, also known as objects. These objects represent the interactive elements of an interactive application. At the second level, we have programs that create collections of objects and interconnect them: the state of objects can depend on the state of other objects. Briefly, we have made the following design decisions for the unified semantic model:

- Objects correspond with primitive interactive elements such as text input boxes, buttons, scroll bars, and so on but also with arbitrary compositions of interactive elements. In this paper we abstract from all issues related with interactive elements, such as their number, layout, and so on. Instead, we will model them by a (composite) data type, that represents the state of the object. From our earlier work on the GEC Toolkit and iData Toolkit, we know that we can generate arbitrarily complex user interfaces from values with generic programming techniques.

- User manipulations of objects are modelled as editing operations on the state of these objects. This provides us with an object style semantics, in which user actions update the state of an existing object rather than creating a new object (a functional style semantics).
• Objects can be shared within our model. Shared objects have multiple occurrences within an interactive application, but they share the same state. Sharing of objects turns out to provide us with concise and intuitive ways of handling feedback (Sect. 3).

• Programs create and interconnect objects. The expressive power of functions provides programmers with a great deal of freedom to manipulate and create intricate interfaces. For the unified semantic model, we will not yet tackle functional expressions in general. Currently, we restrict ourselves to the Arrow framework. The Arrow framework imposes a typed discipline on top of the untyped unified semantic model.

3 THE UNIFIED SEMANTIC MODEL

In this section we present the unified semantic model. We express the semantic model in Clean. This has the following advantages:

• It helps us to detect and remove shallow errors in the semantic definition as we can rely on the language compiler to detect these mistakes at compile time.

• We can test the semantic model with concrete examples by defining a meaning function that can actually compute the output of an interactive program, given a scenario.

• We can use the proof tool Sparkle [7] to guide us in constructing a complete proof of the desired properties. We intend to do this in a similar style as done in [8]. At this stage we have not yet completed the proofs, but already the use of Sparkle has proven productive in structuring the semantic definitions.

The main disadvantage of this approach is that the semantics needs to deal with collections of elements of arbitrary types. In order to escape typing problems, we have chosen to use a flat type to represent types with:

```
:: STATE = INT !Int | REAL !Real | TUPLE !STATE !STATE
```

This does not cause loss of generality of the results, but it does require extra attention by ourselves to make sure that no intentional type errors are introduced that would otherwise be intercepted by the type checker.

The structure of the semantic model is as follows. We first present the arrows in which programs are to be represented (Sect. 3.1). Based on these arrows, we first construct a rather straightforward functional interpretation semantics of these arrows (Sect. 3.2). This interpretation is a natural semantics of pure server side web applications. Next, we construct a more complex, object oriented, semantics of these arrows (Sect. 3.3). This semantics fits closely to desktop applications. Finally, we demonstrate that these two semantics are equivalent in the traditional functional sense (Sect. 3.4).
3.1 Programs as Arrows

We start with a representation of arrows in which programs will be constructed.

```markdown
:: A = Arr !(STATE → STATE)
    (>>>>) infixr 1 !A !A
    First !A
    Loop !A
    Edit !UID
```

The first four data constructors correspond with the standard arrow combinator functions arr, >>>, first, and loop. The Edit data constructor is specific for interactive applications: it should create an interactive element that presents to the user a state and that can be edited by the user. We arrange sharing of these editors by tagging them with values of type UID:

```markdown
:: UID ::= Int.
```

Whenever the same integer is used, the corresponding editors are shared. We assume that the UID parameter of an Edit data constructor is a positive integer, and that the intended types of editors with the same UID value are equal.

As an example, the program in Fig.1 is represented as:

```markdown
c₀ = feedback (Edit euroId >>>> Arr toPound)
    (Edit poundId >>>> Arr toEuro)
where euroId = 1
    poundId = 2
    toPound (REAL euros) = REAL (euros * exchangerateEuroPound)
    toEuro (REAL pounds) = REAL (pounds * exchangeratePoundEuro)
```

Within the semantic model, the need arises to generate UID values itself. In order to prevent clashes with the values from the programs, these will always be negative. These UID values are generated with the trivial functions:

```markdown
initFreshUID :: UID
initFreshUID = -1

nextFreshUID :: !UID → UID
nextFreshUID uid = uid-1
```

Both web and desktop applications need to maintain state that persists between distinct web page creations and handling of events. For web applications this can be done by serializing the required states in the web page, and for desktop applications this is usually done by storing the states within the interactive elements themselves. In any case, the semantic model needs to have these states. In the model, the collection of states is modelled by means of a straightforward association set:

```markdown
:: ASet k v = ASet ![AElt k v]
:: AElt k v = AElt !k !v

key :: !(AElt k v) → k
val :: !(AElt k v) → v
```
instance zero (ASet k v)
instance (=) (ASet k v) | (= k

isEmptyASet :: ! (ASet k v) → Bool
isMemberAElt :: !k ! (ASet k v) → Bool | (= k
findAElt :: !k ! (ASet k v) → AElt k v | (= k
replaceAElt :: ! (AElt k v) ! (ASet k v) → ASet k v | (= k

replaceOrAddAElt :: ! (AElt k v) ! (ASet k v) → ASet k v | (= k

The definitions of these access functions are such that a set never contains two elements with the same key value.

Within an arrow program, events can be created for only the editors that are defined by the Edit data constructor. Because we consider interactive elements to be editors of values of their state, an event can be modelled by a pair of the UID value of the editor, and a new value for its STATE:

:: Event ::= ( !UID, !STATE )

Obviously, we assume that the intended type of the state is equal to the actual type of the state of the indicated editor.

Finally, given a program that is represented by an arrow expression of type A, we can extract the initial states of this program:

arrowSTATES :: !A !STATE → (!STATE,!ASet UID STATE)
arrowSTATES f a = (b,inits)
where (b,inits,_) = arrowSTATES’ f (a,zero,initFreshUID)

arrowSTATES’ :: !A (!STATE,!ASet UID STATE,!UID)
→ (!STATE,!ASet UID STATE,!UID)
arrowSTATES’ (Arr f) (a,states,uid)
  # states = replaceOrAddAElt (AElt uid (f a)) states = (f a,states,nextFreshUID uid)
arrowSTATES’ (f>>>g) (a,states,uid)
  = ((arrowSTATES’ g) o (arrowSTATES’ f)) (a,states,uid)
arrowSTATES’ (First f) (v:(TUPLE a c),states,uid)
  # states = replaceOrAddAElt (AElt uid v) states
  # (b,states,uid_1) = arrowSTATES’ f (a,states,nextFreshUID uid)
  = (TUPLE b c,states,uid_1)
arrowSTATES’ (Loop f) (b,states,uid)
  # states = replaceOrAddAElt (AElt uid b) states
  # (TUPLE c d,states,uid_1) = arrowSTATES’ f
    (TUPLE b undefined,states,nextFreshUID uid)
  = (c,states,uid_1)
arrowSTATES’ (Edit id) (a,states,uid)
  = (a,replaceOrAddAElt (AElt id a) states,uid)
3.2 Interpretation Semantics

In this section we present a more or less straightforward interpretation semantics for arrow expressions of type \( A \) that have been introduced in Sect. 3.1. This semantics is well suited to capture the nature of pure web server based applications. The interpretation is a state based functional version:

\[
\begin{align*}
\text{CircF } & e ~ a ~ b \quad ::= \quad (\text{CircSt } e ~ a) \rightarrow (\text{CircSt } e ~ b) \\
\text{CircSt } & e ~ a \quad ::= \quad (a, e)
\end{align*}
\]

\( \text{infixr 1} \) \( :: (\text{CircF } e ~ a ~ b) \ (\text{CircF } e ~ b ~ c) \rightarrow (\text{CircF } e ~ a ~ c) \)

(\( \text{infixr 1} \)) \( f ~ g = g \circ f \)

The environment that these functions will manipulate consists of the states of the objects, as discussed above, as well as the current event that is inspected:

\[
\begin{align*}
\text{Env} \quad ::= \quad ( \text{!ASet UID STATE} , \text{!Maybe Event} )
\end{align*}
\]

Initially, the \( \text{(Maybe Event)} \) component is set to \( \text{(Just e)} \) for an event \( e \). It is set to \( \text{Nothing} \) by the concrete object for which the event was intended (see interpretation of object below). For each of the components of the arrow program, we define a functional interpretation:

\[
\text{arrowFunctional} :: \ !A \rightarrow \text{CircF Env} \ (\text{Maybe STATE}) \ (\text{Maybe STATE})
\]

Please note that one might expect \( \text{(CircF Env STATE STATE)} \) as the type of functionals. However, this presumes that values are always sent through the complete program. This is in general not the case for interactive programs. Any of the interactive elements that are ‘in front’ of the interactive element that has been manipulated should not be changed. For this reason, when handling an event, the system starts with a \( \text{Nothing} \) value.

Here is the functional interpretation of an arrow program:

\[
\begin{align*}
\text{arrowFunctional} :: \ !A & \rightarrow \text{CircF Env} \ (\text{Maybe STATE}) \ (\text{Maybe STATE}) \\
\text{arrowFunctional} \ (\text{Arr } f) & = \text{arr}_W f \\
\text{arrowFunctional} \ (f \gggg g) & = \text{arrowFunctional} \ f \gggg \text{arrowFunctional} \ g \\
\text{arrowFunctional} \ (\text{First } f) & = \text{first}_W (\text{arrowFunctional} \ f) \\
\text{arrowFunctional} \ (\text{Loop } f) & = \text{loop}_W (\text{arrowFunctional} \ f) \\
\text{arrowFunctional} \ (\text{Edit } uid) & = \text{object} \ (uid, \text{const id})
\end{align*}
\]

The functions \( \text{arr}_W, \text{first}_W, \) and \( \text{loop}_W \) are the standard functional definitions, but need to handle these awkward \( \text{STATE} \) values instead of straight values and tuples:

\[
\begin{align*}
\text{arr}_W :: \ (!\text{(STATE} \rightarrow \text{STATE}) & \rightarrow \text{CircF Env} \ (\text{Maybe STATE}) \ (\text{Maybe STATE}) \\
\text{arr}_W \ \text{fun} & = \lambda (ma, \text{env}) \rightarrow \text{case } ma \ \text{of} \\
& \quad \quad \text{Just } a \rightarrow (\text{Just } (\text{fun} \ a), \text{env}) \\
& \quad \quad \text{nothing} \rightarrow (\text{Nothing}, \text{env})
\end{align*}
\]

\[
\begin{align*}
\text{first}_W :: \ !\text{(CircF Env} \ (\text{Maybe STATE}) \ (\text{Maybe STATE})) & \rightarrow \text{CircF Env} \ (\text{Maybe STATE}) \ (\text{Maybe STATE}) \\
\text{first}_W \ f &
\end{align*}
\]
\[ \lambda (m_{ac}, env) \rightarrow \textbf{case } m_{ac} \text{ of} \]
\[ \text{Just } (\text{TUPLE } a \ c) \rightarrow \textbf{let } (mb, env') = f (\text{Just } a, env) \]
\[ \text{in } (\text{Just } (\text{TUPLE } (\text{fromJust } mb) \ c), env') \]
\[ \text{nothing } \rightarrow (\text{Nothing}, env) \]

\text{loop}_W :: ! (\text{CircF Env (Maybe STATE)} \rightarrow (\text{Maybe STATE})) \rightarrow \text{CircF Env (Maybe STATE)} \\
\text{loop}_W f = \text{loop'} f \]
\text{where} \\
\text{loop'} f (mb, env) = \textbf{case } mb \text{ of} \\
\text{Just } b \rightarrow \textbf{let } (mt, env_1) = f (\text{Just } (\text{TUPLE } b \ d), env) \]
\[ (c, d) = \text{toTuple } (\text{fromJust } mt) \]
\[ \text{in } (\text{Just } c, env_1) \]
\[ \text{nothing } \rightarrow (\text{Nothing}, env) \]

Of more interest is the functional interpretation of the \textit{Edit} alternative.

\text{:: } F ::= (\text{ASet UID STATE}) \rightarrow \text{STATE} \\
\text{object ::= } (!\text{UID}!, !\text{F}) \rightarrow (!\text{ Maybe STATE}, (!\text{ASet UID STATE}, \text{Maybe Event})) \\
\rightarrow (!\text{Maybe STATE}, (!\text{ASet UID STATE}, \text{Maybe Event})) \\
\text{object (uid,f) (ma,(states,event)) = case } (\text{ma}, \text{event}) \text{ of} \\
\[ (_-, \text{Just } (\text{id}, v)) \]
\[ \mid \text{uid} = \text{id} = \textbf{let } b = f \text{ states } v \]
\[ \text{in } (\text{Just } b, (\text{replaceAElt } \text{AElt } \text{uid } v \text{ states}, \text{Nothing})) \]
\[ \text{Just a,Nothing}= \textbf{let } b = f \text{ states } a \]
\[ \text{in } (\text{Just } b, (\text{replaceAElt } \text{AElt } \text{uid } a \text{ states}, \text{Nothing})) \]
\[ \text{otherwise } = (\text{Nothing}, (\text{states}, \text{event})) \]

This definition shows that the object to which an event is directed (first alternative of the case distinction), takes over the value of the event and updates its state in the environment. In addition, it signals the fact that it has handled the event by setting the event to \text{Nothing}. The new value that should be propagated through the program depends on this new value. The second case states that whenever such a value has been set, that the interactive elements that depend on this value use this value. Finally, the third case states that elements that occur before the modified element are not affected.

The meaning of a functional style program is to apply the functional interpretation to each and every event that is directed to the program. Of course, first the program needs to be initialized. This is expressed by the following, general, \text{meaning} function:

\text{meaning ::= } ! (\text{A } (!\text{Event}!, \text{Env}) \rightarrow (\text{!STATE}!, \text{Env})) \\
\text{!A } !\text{STATE}! [\text{Event}] \rightarrow (\text{![STATE]!}, \text{Env}) \\
\text{meaning } mf \ f \ a \ es = (\text{bs}, \text{env}) \\
\text{where } (\text{bs,states}) = \text{arrowSTATES } f \ a \\
(\text{bs,env}) = \text{mapSt } (mf \ f) (\text{es}, (\text{states}, \text{Nothing})) \\

A functional style interpretation therefore has the following meaning:
meaning_W :: (A STATE [Event] → ([STATE], Env))
meaning_W = meaning handle_event_W

handle_event_W :: !A !([Event], Env) → ([STATE], Env)

let (mb, env_1) = arrowFunctional f (Nothing, (states, Just e))
in (fromJust mb, env_1)

3.3 Object Semantics

In this section we present an object based semantics of arrow expressions of type A. It turns out that this is a much more complicated semantic model than the functional interpretation semantics as given above in Sect. 3.2. This semantics is well suited for desktop GUI applications that distributes the interactive behavior over a collection of persistent widgets with state and callback functions.

It is useful break up the concept of a widget with state and callback functions into its state and its callback functions. The environment Env already contains all states of all elements, so what remains are the callback functions. The key idea is to present programs of type A in a different way, namely as the sequence of callback functions that they would call in case of occurring events. We call this a wiring:

```
:: Wiring ::= [Wire]
:: Wire ::= (.UID, !F)
```

Every program of type A can be transformed into a linearized form of type Wiring:

```
arrowWiring :: !A → Wiring
arrowWiring f = fst (arrowWiring' f ([], initFreshUID ))
where
```

```
arrowWiring' :: !A !([Wiring], UID) → ([Wiring], UID)
arrowWiring' (Arr fun) (wiring, uid) = (appendWire (uid, const fun) wiring, nextFreshUID uid)
arrowWiring' (g >>= g) (wiring, uid) = ((arrowWiring' g) o (arrowWiring' f)) (wiring, uid)
arrowWiring' (First f) (wiring, uid)
  δ wiring = appendWire (uid,F_1) wiring
  δ (wiring,uid_1) = arrowWiring' f (wiring, nextFreshUID uid)
  δ wiring = appendWire (uid,F_2) wiring
  = (wiring,uid_1)
where
```

```
F_1 _ (TUPLE a c) = a
F_2 states b = let (TUPLE a c) = val (findAElt uid states) in TUPLE b c
```

```
arrowWiring' (Loop f) (wiring, uid)
  δ wiring = appendWire (uid,F_1) wiring
  δ (wiring,uid_1) = arrowWiring' f (wiring, nextFreshUID uid)
  δ wiring = appendWire (uid,F_2) wiring
  = (wiring,uid_1)
where
```
\[
F_1 \ b = \text{TUPLE} \ b \ \text{undef}
\]
\[
F_2 \ b \ (\text{TUPLE} \ c \ d) = c
\]
\[
\text{arrowWiring}^* \ (\text{Edit} \ eid) \ (\text{wiring}, \text{uid})
= (\text{appendWire} \ (\text{eid}, \text{const} \ id) \ \text{wiring}, \text{uid})
\]

This definition states that every element of an arrow is mapped to an object with state and callback function. We only have objects in this semantics. For this reason, the meaning of an object based semantics is to let the objects that can be derived from an arrow based program of type \(A\) (done by \text{arrowWiring}) handle each and every event. For this we can reuse the \text{object} function that we have already defined:

\[
\text{meaning}_D :: (\text{A} \ \text{STATE} \ [\text{Event}] \rightarrow ([\text{STATE}], \text{Env}))
\]
\[
\text{meaning}_D = \text{meaning} \ (\text{handle_event}_D \ o \ \text{arrowWiring})
\]

\[
\text{handle_event_from_start}_D :: \text{!Wiring} \ !(\text{!Event}, \text{!Env}) \rightarrow ([\text{STATE}], \text{!Env})
\]
\[
\text{foldl} \ (\text{flip} \ \text{object}) \ (\text{Nothing}, \text{env}) \ \text{wiring}
= (\text{fromJust} \ \text{mb}, \text{env})
\]

### 3.4 Equivalence of Interpretation vs Object Semantics

In the above two sections, we have introduced two meanings of arrow programs of type \(A\):

\[
\text{meaning}_W :: (\text{A} \ \text{STATE} \ [\text{Event}] \rightarrow ([\text{STATE}], \text{!Env}))
\]
\[
\text{meaning}_W = \text{meaning} \ \text{handle_event}_W
\]

\[
\text{meaning}_D :: (\text{A} \ \text{STATE} \ [\text{Event}] \rightarrow ([\text{STATE}], \text{!Env}))
\]
\[
\text{meaning}_D = \text{meaning} \ (\text{handle_event}_D \ o \ \text{arrowWiring})
\]

We need to prove that \(\text{meaning}_W = \text{meaning}_D\). As stated earlier, we want to do the proof in the proof assistant \text{Sparkle}. Unfortunately, at this stage we have not yet finished the proof. The proof proceeds by case distinction on the structure of \(A\).

### 4 UNIFIED MODEL PROPERTIES

We define a number of semantic equivalence relations:

**Definition 1 (Initial I/O Equivalence)**

Two systems are said to be **initially I/O equivalent** if for the initial event the corresponding output is the same.

**Definition 2 (Behavioural I/O Equivalence)**

Two systems are said to be **behaviorally I/O equivalent** if for each list of input events the list of corresponding outputs is the same.
**Definition 3 (Event Identical)**

Two systems are said to be event identical w.r.t. to a set of UntypedId if for each list of input events the collection of the Objects that correspond with the ids are identical.

**Definition 4 (Identical)**

Two systems are said to be identical if the collection of their Objects is identical up to a global renaming of their id_self’s.

In the unified model it is straightforward to prove that the interpretation semantics as given in Sect. 3.2 satisfies the usual collection of Arrow laws:

**Definition 5 (Arrow Laws)**

\[
\begin{align*}
\text{arr id} & \triangleright> f = f = f \triangleright> \text{arr id} \\
f \triangleright> (g \triangleright> h) &= (f \triangleright> g) \triangleright> h \\
\text{arr} (f \triangleright> g) &= \text{arr } f \triangleright> \text{arr } g \\
\text{arr} (g \circ f) &= \text{arr } f \triangleright> \text{arr } g
\end{align*}
\]

\[
\begin{align*}
\text{first } (f \triangleright> g) &= \text{first } f \triangleright> \text{first } g \\
\text{first } f \triangleright> \text{arr } \text{fst} &= \text{arr } \text{fst} \triangleright> f \\
\text{first } (\text{arr } f) &= \text{arr } (\text{first } f) \\
\text{first } f \triangleright> \text{arr } (\text{id} \times g) &= \text{arr } (\text{id} \times g) \triangleright> \text{first } f \\
\text{first } (\text{first } f) \triangleright> \text{arr } \text{assoc} &= \text{arr } \text{assoc} \triangleright> \text{first } f
\end{align*}
\]

\[
\begin{align*}
\text{loop } (\text{first } h \triangleright> f) &= h \triangleright> \text{loop } f \\
\text{loop } (f \triangleright> \text{first } h) &= \text{loop } f \triangleright> h \\
\text{loop } (f \triangleright> \text{arr } (\text{id} \times k)) &= \text{loop } (\text{arr } (\text{id} \times k) \triangleright> f) \\
\text{loop } (\text{loop } f) &= \text{loop } (\text{arr } \text{assoc}^{-1} \triangleright> f \triangleright> \text{arr } \text{assoc}) \\
\text{second } (\text{loop } f) &= \text{loop } (\text{arr } \text{assoc} \triangleright> \text{second } f \triangleright> \text{arr } \text{assoc}^{-1}) \\
\text{loop } (\text{arr } f) &= \text{arr } (\text{simple_loop } f)
\end{align*}
\]

where simple_loop is the ‘stateless’ variant of our loop combinator:

\[
\text{simple_loop} :: ((b,d) \rightarrow (c,d)) \rightarrow b \rightarrow c
\]

\[
\text{simple_loop } f \ b = \text{let } (c,d) = f \ (b,d) \ \text{in } c
\]

Proving the Arrow laws for the object semantics directly is very hard. Even if this can be done, then the result is weaker than that for the interpretative semantics, because the laws must necessarily be behavioural I/O equivalence instead of equality (because \((\text{Arr } \varepsilon \triangleright> \triangleright> \varepsilon))\) will always create one more state in the environment than \(\varepsilon\) by itself). However, using the equivalence result of Sect. 3.4 we need not go through the trouble of proving the Arrow laws directly.

Arrow-structured programs have the further property that every user manipulation ends in a stable state of the application; i.e. events always arrive at the sink element. This is an important property for both desktop GUI applications (the
program ‘freezes’ if this property does not hold) as well as web applications (no new page is computed, causing the browser to fail). This depends solely on the functions that are provided by the programmer when using the arr combinator.

5 CASE STUDY

In Sect. 1 we have presented a small money exchange program that has two intricately mutually interconnected objects (Fig. 1). Consider the following variations of this little program:

\[
\begin{align*}
c_0 &= \text{feedback} \ (\text{edit euroId} \ >>> \ arr \ toPound) \\
&\quad \ (\text{edit poundId} >>> arr \ toEuro) \\
c_1 &= \text{feedback} \ (\text{edit euroId}) \\
&\quad \ (arr \ toPound >>> \text{edit poundId} >>> arr \ toEuro) \\
c_2 &= c_0 >>> arr \ toEuro \\
c_3 &= c_1 >>> arr \ toPound
\end{align*}
\]

For this program, it is reasonable to assume \(\text{toPound} = \text{toEuro}^{-1}\). All programs are event identical w.r.t. \(\{\text{euroId}, \text{poundId}\}\). These are the objects that are visible to the user, so from a user’s perception it does not matter which one to use. Programs \(c_0\) and \(c_3\) are identical, hence they are also initial I/O and behavioral I/O equivalent. Programs \(c_1\) and \(c_2\) are Arrow equivalent, hence also initial and behavioral I/O equivalent.

We can use the framework to explain what happens if we drop the assumption that the conversion functions are each other’s inverse, so \(\text{toPound} \neq \text{toEuro}^{-1}\). Editing the pound editor behaves as before, but every event \((\text{euroId}, x)\) results in displaying the value \((\text{toPound} \ x)\) in the poundId object, but also modifies the entered value in the euroId object into \((\text{toEuro} \ (\text{toPound} \ x))\).

6 RELATED WORK

This work is about providing a semantic framework that can handle both interactive applications for the desktop as well as web applications, or even a mixture of these systems. Due to the radically different nature of these two paradigms, such a semantic framework needs to be sufficiently abstract without losing the opportunity to reason about the application logic.

In this project we have chosen to build the semantic model on Arrows. The advantage of using a functional style formalism is that integration of computation can be done within the framework, using functions. Other projects, such as Fruit [6] and Fran [11] have taken this route as well. In these systems Arrows were also necessary to eliminate subtle performance problems. In our case, we use them chiefly to structure our programs in order to facilitate reasoning.

Another way of modelling interactive programs is to regard them as collections of communicating processes. From this point of view, it seems to be natural to provide a model in terms of a process algebra. There is a wide variety of process algebras available, such as CCS (Calculus of Communicating Systems) [13],
CSP (Communicating Sequential Processes) [10], ACP (Algebra of Communicating Processes) [5], and \( \mu \text{GRL} \) (micro Common Representation Language) [9]. Especially the latter might be interesting in this context because it augments ACP with algebraic data types in a spirit that is very similar to functional programming. In general, the fine grained control over concurrency that is usually provided by process algebraic models is not necessary when dealing with interactive applications. It is our opinion that the dependency between interactive elements can be dealt with more suitably in a functional style than an inherent concurrent style.

7 CONCLUSIONS AND FUTURE WORK

In this paper we have identified the trend that interactive applications that are developed for either the desktop or the web tend to take over functionality that used to be the exclusive domain of either paradigm. This implies that in order to reason about interactive programs, we need a suitable model that can capture aspects of both paradigms. In this paper we have presented a first step to such a unified semantic model.

We have restricted ourselves deliberately to programs that are expressed in the Arrow framework, because we expected that this would facilitate reasoning over programs. In our experience with the two toolkits, it is very natural to write non-Arrow programs that use the full expressiveness of the functional host language. We want to investigate if reasoning in a much less restricted framework still allows feasible reasoning.

REFERENCES


