Specify Urgency in Timed I/O Automata*

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Abstract

Tools and techniques based on timed automata (such as Uppaal and the timed I/O automata framework) have proven to be extremely useful for the analysis of protocols and control software for real-time systems. However, a significant limitation of these approaches is that, due to the expressiveness of the modeling languages, timelocks — degenerate states in which time is unable to pass — can freely arise and cannot, in the general case, be detected. As a remedy to this problem Sifakis et al. advocate the use of deadline predicates for the specification of progress properties of Alur-Dill style timed automata. In this article, we extend these ideas to a more general setting, which may serve as a basis for deductive verification techniques. More specifically, we extend the TIOA framework of Lynch et al with urgency predicates. We identify a suitable language to describe the resulting timed I/O automata with urgency and show that for this language time reactivity holds by construction. We also establish that the class of timed I/O automata with urgency is closed under composition. The use of urgency predicates is compared with three alternative approaches to specifying progress properties that have been advocated in the literature: invariants, stopping conditions and deadline predicates. We argue that in practice the use of urgency predicates leads to shorter and more natural specifications than any of the other approaches. Some preliminary results on proving invariant properties of timed (I/O) automata with urgency are presented.

1. Introduction

In the literature on real-time systems there appears to be broad consensus on how to express quantitative timing constraints in state based modeling formalisms. Following the

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of Alur-Dill style timed automata, a system model that has limited expressivity in order to enable automatic state space exploration and model checking. In this article, we study the specification of progress properties in the much more general model of timed I/O automata (TIOA) of Lynch et al [16]. Even though fragments of the TIOA framework can be translated into timed automata [23], analysis of general TIOA models requires the use of deductive verification techniques and theorem provers such as PVS [15]. Inspired by the work of Sifakis et al, we introduce a similar notion of urgency predicates within the TIOA framework, both at the semantic level where we have infinite sets of states, transitions and trajectories, and at the syntactic level where system behavior is described finitely in terms of a logical language.

In the I/O automaton framework, transitions are typically specified using precondition/effect notation, that is, some type of guarded commands. This means that, for a given action name \( b \) with parameters \( h \), a precondition predicate \( \text{pre}(\vec{v}, h) \) is given that defines from which states \( \vec{v} \) action \( b(h) \) is enabled, and an effect predicate \( \text{eff}(\vec{v}, h, \vec{v}') \) that defines to which states \( \vec{v}' \) one may jump after doing action \( b(h) \) in state \( \vec{v} \). For the specification of timed systems we add a third predicate, the urgency predicate \( \text{urg}(\vec{v}, h) \), to every transition definition. The meaning of the urgency predicate is that if, for some \( h \), the state predicate

\[
\text{pre}(\vec{v}, h) \land \text{urg}(\vec{v}, h)
\]

becomes true at a time point \( t \) in a trajectory, then \( t \) must be the limit time of that trajectory. Intuitively, the precondition specifies when a transition may occur, and the urgency predicate specifies when the transition becomes urgent, that is, either this or some other enabled discrete transition must occur immediately. A small but significant difference between our approach and the one of Sifakis et al [24, 4] is that Sifakis et al require that a deadline predicate implies the precondition predicate, whereas we achieve a similar effect by conjoining the urgency predicate with the precondition.

The main contributions of this article are:

1. Extension of the work of [24, 4, 3] on deadline predicates to a much more expressive setting, which may serve as a basis for deductive verification techniques. More specifically, we extend the TIOA framework of [16] with urgency predicates at the semantic level, and define a suitable language to describe the resulting timed I/O automata with urgency. For this language, time reactivity holds by construction. We also establish that the class of TIOAs with urgency is closed under composition. In general, under the usual semantics, timed automata with urgency and timed automata with deadlines are not closed under composition, this problem has been studied in [7], where an alternative semantics is given that preserves compositionality.

2. A comparison of urgency predicates with three alternative ways to specifying progress properties: invariants [14, 2], stopping conditions [16] and deadlines [24, 4, 3]. Deadlines, stopping conditions and urgency predicates are shown to be (essentially) equally expressive. Invariants are slightly more expressive since they allow to bound the time at which an action occurs by a right open interval. Only use of urgency and deadline predicates gives time reactivity by construction. We argue that in practice the use of urgency predicates leads to shorter and more natural specifications than any of the other methods.

3. Some preliminary results on proving invariant properties of timed (I/O) automata with urgency. A similar approach for discrete time can also be found in [11].

The full version of the present paper appears as [10]. The proofs which have been omitted here, due to space limitation, are available in this technical report.

2. Timed (I/O) Automata with Urgency

In this section, we describe our extension of the timed I/O automata framework of Lynch et al [16, 17] with urgency. In Subsections 2.1 and 2.2 we begin with recalling some definitions from [16, 17]: we introduce a basic vocabulary for describing timed behaviors and recall the notion of a timed automaton. In Subsection 2.3, we add a notion of urgent transitions to timed automata, both at the semantic and at the syntactic level. The class of timed automata with urgency is not closed under composition, in general. In order to obtain compositionality, we add, in Subsection 2.4, an input/output distinction. Subsection 2.5, finally, defines a parallel composition operator and establishes that both the class of timed automata and the class of timed I/O automata with urgency are closed under composition.

2.1. Describing Timed System Behavior

In this section, we list the basic notions that are used in describing the behavior of a timed system, including both discrete and continuous changes. We simply sketch this material, leaving the reader to consult [16, 17] for the details.

The time domains we use is the set \( \mathbb{R} \) of real numbers (in [16, 17] also other time domains are considered). States of automata will consist of valuations of variables. Each variable has both a static type, which defines the set of values it may assume, and a dynamic type, which gives the set of trajectories it may follow. We assume that dynamic types are closed under some simple operations: shifting the time domain, taking subintervals and pasting together intervals. We call a variable discrete if its dynamic type equals the pasting-closure of a set of constant-valued functions (i.e.,
the step-functions), and analog if its dynamic type equals the pasting-closure of a set of continuous functions (i.e., the piecewise-continuous functions).

A valuation for a set \( V \) of variables is a function that associates with each variable \( v \in V \) a value in its static type. We write \( \text{val}(V) \) for the set of all valuations for \( V \). A trajectory for a set \( V \) of variables describes the evolution of the variables in \( V \) over time; formally, it is a function from \( 0 \) to valuations of \( V \), that is, a trajectory defines a value for each variable at each time in the interval. We write \( \text{dom}(\tau) \) for the domain of trajectory \( \tau \). A point trajectory is one with the trivial domain \( \{0\} \). We write \( \varphi(x) \) for the point trajectory for valuation \( x \). The limit time of a trajectory \( \tau, \tau.\text{lim} \), is the supremum of the times in its domain. \( \tau.\text{val} \) is defined to be the first valuation of \( \tau \), and if \( \tau \) is right-closed, \( \tau.\text{load} \) is the last valuation. Suppose \( \tau \) and \( \tau' \) are trajectories for \( V \), with \( \tau \) closed. The concatenation of \( \tau \) and \( \tau' \), denoted by \( \tau \circ \tau' \), is the trajectory obtained by taking the union of the first trajectory and the function obtained by shifting the domain of the second trajectory until the start time agrees with the limit time of the first trajectory; the last valuation of the first trajectory, which may not be the same as the first valuation of the second trajectory, is the one that appears in the concatenation. Trajectory \( \tau \) is a prefix of trajectory \( \tau' \), denoted by \( \tau \prec \tau' \), if \( \tau \) can be obtained by restricting \( \tau' \) to a subset of its domain. For every \( t \in \text{dom}(\tau) \), we define \( \tau \geq t \) to be the trajectory obtained by taking the part of \( \tau \) from \( t \) onwards, and then shifting the domain so that it starts with \( 0 \) again. Formally, \( \text{dom}(\tau \geq t) = \{u \in \mathbb{R} \mid u + t \in \text{dom}(\tau)\} \) and for all \( u \) in the domain, \( \tau \geq t(u) = \tau(u + t) \).

2.2. Timed Automata

A timed automaton in the sense of \([16, 17]\) is a state machine whose states are divided into variables and that has a set of discrete actions. The state of a timed automaton may change in two ways: by discrete transitions, which change the state atomically, and by trajectories, which describe the evolution of the state over intervals of time. Discrete transitions are labeled with actions, which are classified as either external or internal. The external actions are used to synchronize with the automaton’s environment, while the internal actions are only visible to the automaton itself.

Formally, a timed automaton is a tuple \( \mathcal{A} = (X, Q, \Theta, E, H, D, T) \) with

- A set \( X \) of internal variables.
- A set \( Q \subseteq \text{val}(X) \) of states.
- A nonempty set \( \Theta \subseteq Q \) of start states.
- A set \( E \) of external actions and a set \( H \) of internal actions, disjoint from each other. We write \( A = E \cup H \).
- A set \( D \subseteq Q \times A \times Q \) of discrete transitions. An edge \( e = (x, a, x') \in D \), also written as \( x \xrightarrow{a} x' \), represent a transition from state \( x \) to \( x' \) labeled with action \( a \). We say that \( a \) is enabled in \( x \) if \( x \xrightarrow{a} x' \) for some \( x' \).
- A set \( T \) of trajectories for \( X \) such that \( \tau(t) \in Q \) for each \( \tau \in T \) and \( t \in \text{dom}(\tau) \). We require that the following axioms hold:

\begin{enumerate}
  \item \( T_0 \) (Existence of point trajectories) If \( x \in Q \) then \( \varphi(x) \in T \).
  \item \( T_1 \) (Prefix closure) For every \( \tau \in T \) and every \( \tau' \leq \tau, \tau' \in T \).
  \item \( T_2 \) (Suffix closure) For every \( \tau \in T \) and every \( t \in \text{dom}(\tau), \tau \geq t \in T \).
  \item \( T_3 \) (Concatenation closure) Let \( \tau_0 \tau_1 \tau_2 \ldots \) be a sequence of trajectories in \( T \) such that, for each non-final index, \( i, \tau_i \) is closed and \( \tau_i.\text{val} = \tau_{i+1}.\text{val} \). Then \( \tau_0 \cap \tau_1 \cap \tau_2 \ldots \in T \).
\end{enumerate}

A trajectory \( \tau \) is maximal in \( T \) if there exists no \( \tau' \in T \) with \( \tau < \tau' \). The following lemma (which as far as we know is new) states that each trajectory can be extended into a maximal one. Intuitively this is an obvious property, but the proof requires some work due to the fact that we know so little about \( T \).

**Lemma 1** Let \( \mathcal{A} \) be a timed automaton and let \( T \) be its set of trajectories. Then each trajectory in \( T \) is a prefix of a trajectory that is maximal in \( T \).

2.3. Adding Urgency

We now extend timed automata with extra state predicates, urgency predicates, one for each action.

A timed automaton with urgency is a pair \( \langle \mathcal{A}, U \rangle \) of a timed automaton \( \mathcal{A} = (X, Q, \Theta, E, H, D, T) \) and an urgency predicate \( U : Q \times A \rightarrow \text{Bool} \). If \( U(x, a) = \text{true} \) then we say that action \( a \) is potentially urgent in a state \( x \). Action \( a \) is urgent in state \( x \) if it is potentially urgent and in addition enabled. We require that the following two axioms hold:

\begin{enumerate}
  \item \( T_4 \) (Urgency) For every \( \tau \in T, t \in \text{dom}(\tau) \) and \( a \in A \): if \( a \) is urgent in \( \tau(t) \) then \( t = \tau.\lim \).
  \item \( T_5 \) (Maximality) For every \( \tau \in T \), if \( \tau \) is maximal and finite then \( \tau \) is right-closed and some \( a \in A \) is urgent in \( \tau.\text{load} \).
\end{enumerate}

Axiom \( T_4 \) states that as soon as an action becomes urgent, this action or some other action that is enabled has to occur immediately. Axiom \( T_5 \) states that each maximal and finite trajectory enables an urgent action at the end.

\footnote{The reader may wonder why we do not impose a stronger axiom stating that if an action becomes urgent this action or some other urgent action has to occur immediately. Such an approach, which involves the use of
Timed automata with urgency can be conveniently specified in a slight variation of the TIOA language [15].

Example 1 To illustrate this language, we consider the simple model of a train displayed in Figure 1. The automaton runs cyclically through states start, light and gate. After spending between (2, 5] time units in start the automaton jumps to light, then within (5, 10] time units after arrival in light the automaton jumps to gate, and after exactly 2 time units in gate the automaton returns to the initial state start.

Example 2 Figure 2 gives another example of a specification in our language. It is a model of a reliable FIFO channel that delivers its messages within a certain time bound, represented by the automaton parameter b, which is a positive real number.

The other automaton parameter M represents the type of messages communicated by the channel. The states of the automaton are valuations of the state variables queue and now. The discrete variable queue holds a finite sequence of pairs consisting of a message that has been sent and its delivery deadline. The clock variable now records the current real time. A send(m) action, which is always enabled and never becomes urgent, adds to the queue a new pair whose first component is m and whose second component is the deadline now + b. A receive(m) action can occur when m is the first message in the queue and it results in the removal of the first message from the queue. The receive(m) action becomes urgent when the delivery deadline u of the first message equals the current time now.
By construction the set of trajectories denoted by a specification in our language satisfies axioms T0-T4. However, the example below shows that axiom T5 does not need to hold in general.

**Example 3** The timed automaton specified in Figure 3 has a transition with precondition \( x > 4 \land b = \text{false} \) and urgency predicate \( \text{true} \). Axiom T5 does not hold, since time can only advance up to \( x = 4 \) but at that time the transition is not (yet) enabled.

**Theorem 1** If the predicate \( \bigvee Q \text{Urg}(\tau) \) is left-closed then axiom T5 holds.

**Proof:** Suppose that \( \tau \) is a maximal and finite trajectory. Assume that the domain of \( \tau \) is right-open. Then, by T4, nowhere on \( \tau \) an action becomes urgent. But this means that the extension of \( \tau \) with a single state at the end gives a legal trajectory, thus contradicting the assumption that \( \tau \) is maximal. Hence, without loss of generality, we may assume that the domain of \( \tau \) is right-closed. Assume that no action \( a \) is urgent in \( \tau. \) This means that the disjunction \( \bigvee Q \text{Urg}(\tau) \) does not hold in \( x \). But this means that there exists a small \( \varepsilon \)-extension of \( \tau \) in which no action \( a \) is urgent. This extension is then a legal trajectory, which contradicts with the (assumed) maximality of \( \tau \). □

### 2.4. Adding an I/O Distinction

In this section, we further refine the model of timed automata by distinguishing between input and output actions as in [16].

A *timed I/O automaton with urgency* is a quadruple \( (A, U, I, O) \) where \( (A, U) \) is a timed automaton with urgency, with \( A = (X, Q, \Theta, E, H, D, T) \). I and O partition \( E \) into input and output actions, that is \( E = I \cup O \) and \( I \cap O = \emptyset \). Actions in \( H \cup O \) are called locally controlled. We write \( L = H \cup O \) and \( A = E \cup H \). We require that the following axiom holds:

**E0 (Inputs not urgent)** For every \( x \in Q \) and every \( a \in I \), \( U(x, a) = \text{false} \).

**E1 (Input action enabling)** For every \( x \in Q \) and every \( a \in I \), there exists \( x' \in Q \) such that \( (x, a, x') \in D \).

The input actions are assumed not to be under the automaton’s control—they just arrive from the outside—while the automaton itself specifies what output and internal actions should be performed. In line with these intuitions, axiom E0 states that input actions never become urgent. Axiom E1 is the usual input enabling condition of ordinary I/O automata [20]; it says that a TIOA with urgency is able to accommodate an input action whenever it arrives. At the syntactic level, a sufficient condition for axioms E0 and E1 to hold is, in each transition definition for an input action, the precondition is true and the urgency predicate is false.

A desirable property for models of real-time systems is **time reactivity**. This means that in each state, either time is allowed to advance forever, or time may advance for a while up to a point where the system is prepared to react with some locally controlled action. In [16], an axiom E2 is required for timed I/O automata which captures this property:
For a TIOA with urgency, time reactivity is implied by the other axioms.

**Theorem 2** Each timed I/O automaton with urgency satisfies axiom E2.

### 2.5. Composition

We say that timed automata \(A_1\) and \(A_2\) are *compatible* if they have no state variables in common, and if neither automaton has an internal action that is an action of the other automaton. If \(A_1\) and \(A_2\) are compatible then their composition \(A_1 \| A_2\) is defined formally to be the timed automaton \(A = (X, Q, \Theta, E, H, D, T)\) where

- \(X = X_1 \cup X_2\),
- \(Q = \{x \in val(X) | x[X_1 \in Q_1, i \in \{1,2\}\}\}\),
- \(\Theta = \{x \in Q | x[X_i \in \Theta_i, i \in \{1,2\}\}\}\),
- \(E = E_1 \cup E_2\) and \(H = H_1 \cup H_2\),
- For each \(x, x' \in Q\) and each \(a \in A\), \(x \xrightarrow{a} A x'\) iff for \(i \in \{1,2\}\), either (1) \(a \in A_i\) and \(x[X_i \xrightarrow{a} A x' X_i],\) or
  (2) \(a \notin A_i\) and \(x[X_i = x'[X_i]\).
- \(\tau \in T \iff \tau \downarrow X_i \in T_i, i \in \{1,2\}\).

We refer to [17] for a proof that \(A_1 \| A_2\) is a timed automaton, that is, the above structure satisfies axioms T0-T3.

Two timed automata with urgency, \((A_1, U_1)\) and \((A_2, U_2)\), are compatible if the underlying timed automata \(A_1\) and \(A_2\) are compatible. In this case, the composition is defined to be the structure \((A, U)\), where \(A = A_1 \| A_2\) and \(U\) is given by

\[
U((x_1, x_2), a) = U_1(x_1, a) \lor U_2(x_2, a),
\]

where by convention \(U_i(x_i, a) = false\) if \(a\) is not in the signature of \(A_i\). So an action is urgent in a state of the composed system iff it is urgent in one of the component states. In general, the composition is not a timed automaton with urgency. The problem is due to axiom T5: if, for instance, we compose a system in which action \(a\) becomes urgent at time 1 with a system that has \(a\) in its signature but without any \(a\)-transition, then the composed system has a maximal trajectory of length 1 in which no transition is enabled. Several papers address the issue of how timelock freedom (or more generally, liveness) can be preserved by composition, see for instance [5, 4, 3]. In this article, we present one simple but useful result along these lines: the class of timed I/O automata with urgency is closed under composition.

We say that two timed I/O automata with urgency, \((A_1, U_1, I_1, O_1)\) and \((A_2, U_2, I_2, O_2)\), are compatible if the underlying timed automata \(A_1\) and \(A_2\) are compatible, and also they have no output actions in common. A consequence of these conditions is that each action is controlled by at most one component. In this case, the composition is defined to be the structure \((A, U, I, O)\), where \((A, U)\) is the composition of \((A_1, U_1)\) and \((A_2, U_2)\), \(I = (I_1 \cup I_2) - (O_1 \cup O_2)\), and \(O = O_1 \cup O_2\). That is, an external action of the composition is classified as an output if it is an output of one of the component automata, otherwise it is classified as an input.

**Theorem 3** The composition of two compatible timed I/O automata with urgency is again a timed I/O automaton with urgency.

### 3. Expressivity

In this section, we compare the expressivity of urgency predicates with that of the deadline predicates of [24, 4], the stopping conditions of [16], and the invariants as used e.g. in [14, 2, 18].

#### 3.1. Deadline Predicates

Instead of using urgency predicates, we could follow the approach of Sifakis et al. [24, 4] even more closely by using deadline predicates. This would mean that, for a given action name \(b\) with parameters \(h\), besides the precondition \(pre(v, h)\) and the effect \(eff(v, v', h)\), also a deadline predicate \(dl(v, h)\) is specified such that \(dl(v, h) \iff pre(v, h)\) holds. The semantics of a deadline predicate is that if, for some \(h\), the state predicate

\[
dl(v, h) = \implies pre(v, h)
\]

becomes true at a time point \(t\) in a trajectory, then \(t\) must be the limit time of that trajectory.

Clearly, any definition of a timed automaton with urgency predicates can be transformed into an equivalent (in the sense that the defined automata are semantically equal) definition with deadline predicates by replacing each urgency predicate \(urg(v, h)\) by a deadline predicate \(pre(v, h) \land \neg urg(v, h)\). Conversely, any definition with deadline predicates can be transformed into an equivalent definition with urgency predicates by replacing each deadline predicate \(dl(v, h)\) by an identical urgency predicate \(urg(v, h)\). Studying the examples in Figure 1 and Figure 2, and the examples in [17] indicates that the use of urgency predicates leads to slightly shorter specifications than the use of deadlines.

#### 3.2. Stopping Conditions

Another alternative for urgency predicates are the stopping conditions as used in [16]. A stopping condition is a
state predicate \( sc(v) \) such that if \( sc(v) \) becomes true at a time point \( t \) in a trajectory, then \( t \) must be the limit time of that trajectory.

We again checked the examples from Figure 1, Figure 2 and [17], and in each case urgency predicates lead to shorter and (in our view) more natural specifications than stopping conditions. Figure 4, for instance, shows how the transitions and trajectories of the example of Figure 1 can be rewritten using a stopping condition. The disadvantages should be clear: upper bounds are no longer specified next to the corresponding lower bounds, and parts of the preconditions have to be repeated in the stopping condition.

\[
\begin{align*}
\text{transitions} & : \text{external} \text{ coming} \\
& \quad \text{pre} \ x > 2 \land \text{control} = \text{start} \\
& \quad \text{eff} \ \text{control} := \text{light}; x := 0 \\
& \text{external} \text{ approaching} \\
& \quad \text{pre} \ x > 5 \land \text{control} = \text{light} \\
& \quad \text{eff} \ \text{control} := \text{gate}; x := 0 \\
& \text{external} \text{ passing} \\
& \quad \text{pre} \ x = 2 \land \text{control} = \text{gate} \\
& \quad \text{eff} \ \text{control} := \text{start}; x := 0 \\
\text{trajectories} & : \text{stops when} \\
& \quad (\text{control} = \text{start} \land x \geq 5) \lor \\
& \quad (\text{control} = \text{light} \land x \geq 10) \lor \\
& \quad (\text{control} = \text{gate} \land x = 2)
\end{align*}
\]

Figure 4. The train model defined using a stopping condition.

Any definition of a timed automaton with urgency predicates can be transformed into an equivalent definition with stopping conditions by replacing the urgency predicates by a single stopping condition that is the disjunction of the formula \( \text{pre}(v, h) \land \text{urg}(v, h) \), for all transition definitions.

Stopping conditions are more expressive than urgency predicates since they allow one to define timed automata that are not time reactive, a clear disadvantage of these specification styles. The example of Figure 5, for instance, can easily be encoded using invariants (replace the stopping condition by an invariant \( x \leq 1 \)).

Invariants also allow one to specify strict upper bounds on the timing of events, as illustrated in Figure 6. The same timed automaton can not be specified using urgency predicates, for the simple reason that it has a maximal trajectory that is right-open, which is in violation with axiom T5. If we are willing to consider timed automata up to some suitable equivalence (for instance, the trace equivalence defined in [16]) then it is possible to specify strict upper bounds with urgency predicates, but this requires the use of auxiliary variables and unbounded nondeterminism. Figure 7 illustrates the specification of a strict upper bound with an urgency predicate. The idea is to choose nondeterministically a value in the interval \( [0, 1) \) and then make \( a \) urgent when time has reached this value. Apart from the fact that the second specification is less intuitive, the use of unbounded nondeterminism will constitute a serious obstacle for automatic verification methods. In all practical applications of timed automata that we are aware of, the use of only non
strict upper bounds on timing is not a restriction. For applications where use of strict upper bounds is essential, use of invariants is probably more appropriate than use of urgency predicates.

Timed automata with urgency predicates can (in many cases) be translated to equivalent timed automata with invariants. Robson [23] describes how a fragment of TIOA with urgency predicates can be translated to the input language of UPPAAL. Below we discuss a more general translation scheme. We say that a state predicate \( p(v) \) is stable (under time progress) if \( \varphi(v) \implies \forall d > 0 : \varphi(v + d) \).

Typically, a predicate will be stable if it only involves lower bounds on clocks and no upper bounds. The lower hull of a state predicate \( \varphi(v) \) is the set of valuations given by

\[
LH(\varphi) = \{ x | \varphi(x) \land \exists e > 0 \forall 0 < e' < e : \neg \varphi(x \oplus e') \}
\]

The upper hull of a state predicate can be defined similarly, just replace \( \ominus \) by \( \oplus \) in the above definition. If \( \varphi \) only involves (non-strict) lower bounds on clocks then the lower hull can easily be expressed again as a predicate by replacing the \( \geq \) signs with \( \leq \).

Now consider a definition of a timed automaton with urgency predicates such that all predicates \( \text{Urg}(\text{tr}) \) are left-closed and stable. An equivalent timed automaton with invariants can be obtained by replacing the urgency predicates with the invariant

\[
in = \neg \left( \bigvee_{\text{tr}} \text{Urg}(\text{tr}) \right) \lor LH \left( \bigvee_{\text{tr}} \text{Urg}(\text{tr}) \right),
\]

provided that the state predicate \( \text{inv} \) holds initially and after each discrete transition, i.e.,

\[
\text{pre}((v, \bar{h}), (v', \bar{h}')) \land \text{eff}((v, \bar{h}, v')) \implies \text{inv}(v').
\]

The proof of the equivalence is straightforward and left to the reader.

Any timed automaton definition with right-closed invariants can be easily translated to a timed automaton with stopping conditions: the stopping condition is defined to be (a predicate denoting) the upper hull of the invariant. The translation scheme of Section 3.2 can then be used (provided axiom \( T5' \) holds) to translate the resulting timed automaton with stopping conditions to a timed automaton with urgency.

4. Proving Invariant Properties

In this section, we discuss how to establish invariant properties for specifications that involve urgency predicates. It is important to distinguish invariant properties from the invariant assertions that were discussed in the previous section as a construct to specify progress. An invariant property is a state predicate that holds for all reachable states of a given system. An invariant in the sense of the previous section is actually an invariant property of the system that it helps to define. The converse implication typically does not hold.

An execution fragment of a timed automaton \( \mathcal{A} \) is a sequence \( \alpha = \tau_0 a_1 \tau_1 a_2 \tau_2 \ldots \), where each \( a_i \) is an action of \( \mathcal{A} \), each \( \tau_i \) is a trajectory of \( \mathcal{A} \), and for every \( i \), \( \tau_i.\text{load} \stackrel{a_{i+1}}{\rightarrow} \tau_{i+1}.\text{val} \). An execution fragment records what happens during a particular run of a system, including all the discrete state changes and all the changes that occur while time advances. An execution is an execution fragment whose first state is a start state of \( \mathcal{A} \). A state is reachable in \( \mathcal{A} \) if it is the last state of the last trajectory of a finite execution of \( \mathcal{A} \). A state predicate \( \varphi \) is an invariant of \( \mathcal{A} \) if it holds for all reachable states of \( \mathcal{A} \).

In order to prove that an assertion \( \varphi \) is an invariant of \( \mathcal{A} \), it suffices to prove that it holds initially and is preserved by all discrete transitions as well as by all time transitions \( \tau.\text{time} = d \land \tau.\text{load} = x' \).

If we manage to give a simple and tractable characterization of the \( \tau.\text{time} \) predicate, then all the invariant proof techniques which are presented (for instance) in [21] become available in our setting.

Let \( \text{tr} \) be a transition definition for an action name \( b \) with parameter \( \bar{h} \) with precondition \( \text{pre}(v, \bar{h}, v') \), urgency predicate \( \text{Urg}(v, \bar{h}, v') \), and effect predicate \( \text{eff}(v, \bar{h}, v') \). For \( d \geq 0 \), the time progress predicate \( \text{tp}(v, \text{tr}, d) \) expresses that transition \( \text{tr} \) permits time to advance with an amount \( d \) from state \( v'. \)
The predicate is formally defined in terms of the $\text{Urg}(tr)$ predicate of (2):

$$tp(v, tr, d) \triangleq \forall 0 \leq e < d : -\text{Urg}(tr)(v \oplus e, \tilde{h})$$

Similarly, the time progress predicates for the approaching and passing transitions can be written resp. as $\text{control} = \text{light} \quad \Rightarrow \quad x + d \leq 10$ and $\text{control} = \text{gate} \quad \Rightarrow \quad x + d \leq 2$ respectively. With these characterizations it is trivial to prove, for example, that $\text{control} = \text{start} \quad \Rightarrow \quad x + d \leq 5$ is inductive (and hence an invariant): it holds initially, and it is preserved by all discrete transitions and all $\text{time}(d)$ steps.

The above quantifier elimination can be generalized under some reasonable assumptions. If $\phi(v)$ is a state predicate then we write $\text{post}(\phi)(v)$ for the state predicate that holds for states that have a time predecessor satisfying $\phi$:

$$\text{post}(\phi)(v) \triangleq \exists w \exists e > 0 : (v = w \oplus e) \land \phi(w)$$

If $\phi$ only involves lower bounds on clocks, then $\text{post}(\phi)$ can typically be obtained from $\phi$ by making these lower bounds strict, so quantifier elimination from $\text{post}(\phi)$ is easy. Hence, if preconditions and urgency predicates only involve lower bounds on clocks (which appears to be a good specification style anyway), then their conjunction is stable one may use the following lemma to eliminate the quantification over $e$ from the time progress predicate (5).

**Lemma 2** Let $\phi$ be a state predicate that is stable under time progress. Then

$$(\forall 0 \leq e < d : -\phi(v \oplus e)) \iff \neg \text{post}(\phi)(v \oplus d)$$

**Proof:** Equivalence (7) can be rewritten into

$$(\exists 0 \leq e < d : \phi(v \oplus e)) \iff \text{post}(\phi)(v \oplus d)$$

We prove both implications:

$\Rightarrow$ Assume $\phi(v \oplus e)$, for certain $e \in [0, d)$. Then $\text{post}(\phi)(v \oplus d)$ holds since there is a state, namely $v \oplus e$, that is a time predecessor of $v \oplus e$ and in which $\phi$ holds.

$\Leftarrow$ Assume $\text{post}(\phi)(v \oplus d)$. Then by (6) there exists an $e' > 0$ and a state $w'$ such that $\phi(w')$ holds and $v \oplus e' \in [w \oplus e']$. Depending on the relationship between $d$ and $e'$ we have two cases:

**Case $e' \leq d$:** Then $v \oplus (d - e') = w$. Choose $e = d - e'$. Then $d \in [0, d)$ and $\phi(v \oplus e)$.

**Case $d < e'$:** Then $v \oplus e' - d = w'$. Since $\phi(w')$ holds and $\phi$ is stable under time progress, also $\phi(v \oplus e)$ holds. So we may choose $e = 0$ to obtain $\phi(v \oplus e)$, as required.

5. Concluding Remarks

In this article, we introduced a notion of urgency predicates and compared it with three other constructs for specifying progress properties that have been proposed in the literature: invariants, stopping conditions and deadlines. We showed that under some rather realistic assumptions (use of clock variables, no strict upper bounds on progress, absence of time deadlocks, etc.) the four notions are equally expressive. Nevertheless, a clear advantage of deadlines and urgency predicates in practice is that one gets absence of time deadlocks (time reactivity) for free. A potential advantage of invariants is that they allow one to bound the time at which a (locally controlled) action occurs by a right-open interval. However, we are not aware of practical applications in which this feature is really needed. We argued that if one uses a preconditions/effects style specification language, urgency predicates lead to shorter and more natural specifications than any of the other constructs, in particular invariant. In the graphical syntax used by e.g. UPYAP, the use of urgency/deadline predicates would not lead to shorter specifications than the use of invariants. Typically, in first case one will decorate an edge of the graph (i.e., a transition) with a label $x \geq 4$, and in the second case a label $x \leq 4$ will be attached to a vertex of the graph (i.e., a location). But whereas the use of invariants may easily lead to time deadlocks, urgency/deadline predicates only stop time if there is a good reason for it, that is a specific transition that must be taken, and in this manner time deadlocks are avoided.

Folklore has it that urgency/deadline predicates are more difficult to implement in model checkers than invariants because they easily lead to non-convex zones. Non-convex zones indeed arise in the implementation of timed automata with deadlines in the IF toolset [6]. In particular, time transitions may lead from one convex zone to several convex
zones (not only one, as in standard timed automata with invariants). When such a situation arises in IF, the non-convex zone is automatically split in several, possibly overlapping, convex zones. The main reason why non-convex zones do not arise in standard timed automata with invariants (such as those implemented in UPPAAL) is that rather strong restrictions are imposed on the syntax of invariants. Only conjunctions of upper bounds on clocks—where the bounds are given by integer expressions—are allowed. If similar restrictions would be imposed in a syntax for urgency predicates, then no non-convex zones would arise in that setting either! More specifically, one would have to require that each urgency predicate is the conjunction of lower bounds on clocks, where the (non-strict) bounds are given by integer expressions. In addition, the urgency predicate of an input action a? should always be false. We think it would be a clear improvement to the current version of UPPAAL (3.4.7) to add such a restricted notion of urgency predicates to the syntax, replacing the notion of an urgent channel. Adding general urgency predicates to UPPAAL would of course also be a possibility, but this would require splitting of zones as in the IF toolset.

In the setting that we studied, urgency predicates appear to be a very nice way to specify progress properties, with clear advantages over some other constructs that have been advocated in the literature. Some remaining questions for future research are: (1) Exploration of proof rules to reason with urgency predicates in simulations and liveness proofs. (2) Establish versions of the compositionality results of [5, 4, 3] in the setting of this paper. (3) Extension of our specification language and expressiveness results to a hybrid setting in which besides clocks also other continuously evolving variables are allowed.

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References