Angular dependence of Bose-Einstein correlations in interactions of $\pi^+$ and $K^+$ mesons with protons and nuclei at 250 GeV/c

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Abstract. The angular dependence of Bose-Einstein correlations is studied in interactions of $\pi^+$ and $K^+$ mesons with protons and nuclei at 250 GeV/c. The pion source is found to be elongated along the interaction axis in the c.m.s., with the ratio $\alpha$ between transverse and longitudinal dimension equal to 0.55 ±0.06 for the case of proton, 0.53 ±0.13 for the case of $\Delta$ target. For meson-proton interactions, indication for an increase of the elongation is found with increasing event multiplicity, pair momentum and particle transverse momentum.

1 Introduction

Pion interferometry not only allows to measure the average radius of a pion source, but also to determine its shape [1]. The latter can be obtained from the dependence of the size on a direction given by the angle $\theta$ of the c.m.s. momentum difference $\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_2$ with respect to the collision axis. For hadronic collisions, such shape measurements have been reported in [2-6], where transverse and longitudinal radii $r_T$ and $r_L$ of the pion source have been estimated in the framework of the surface emission model proposed by Kopilov and Podgoretski [7]. The pion emission region is found to be oblate ($r_L < r_T$) at the lower energies [2-4], but prolate ($r_L > r_T$) at higher energies [5,6].

Recently, a method for direct determination of the ratio $a = r_T/r_L$ from the angular distribution of the vector $\mathbf{q}$ itself has been proposed by Podgoretski [8]. As a minimal assumption on the form of the pion source, rotational symmetry is used around the interaction axis.

In general, the angular distribution of $\mathbf{q}$ for pion pairs with $|\mathbf{q}| < q_{cut}$ and very small c.m.s. energy difference $q_0 = |E_1 - E_2|$ is given [8] by

$$\varphi(\cos \theta) \propto \int_0^{q_{cut}} f(q^2 r_T^2 + q^2 (r_L^2 - r_T^2) \cos^2 \theta) \, dq^2$$

$$\approx [r_T^2 + (r_L^2 - r_T^2) \cos^2 \theta]^{-1.5} \int_0^{q_{cut}} f(x^2) \, dx^2$$

where $x^2 = q^2[r_T^2 + (r_L^2 - r_T^2) \cos^2 \theta]$ and the function $f(x^2)$ is the Fourier transform of the spatial distribution of the source, normalized to unity as $q \to 0$. One can show from (1) that, independently of the particular form of $f(x^2)$, the function $\varphi(\cos \theta)$ becomes constant at sufficiently small $q_{cut}$, $q_{cut} \ll 1/r_L$. At sufficiently large $q_{cut}$ (i.e., above the correlation region) the integral in (1) is practically independent of $q_{cut}$, and the angular distribution $\varphi(\cos \theta)$ (normalized to unity in the interval $-1 \leq \cos \theta \leq 1$) turns out to be

$$\varphi(\cos \theta) = \frac{a^2}{2[a^2 + (1 - a^2) \cos^2 \theta]^{1.5}}$$

The ratio $a = r_T/r_L$ can be determined by fitting distribution (3) to the experimental angular distribution obtained after subtraction of a background (reference) distribution for which like-pion interference effects are absent. More simply, it can be determined from the asymmetry parameter $\Delta = (N_1 - N_2)/(N_1 + N_2)$, where $N_1$ and $N_2$ are the numbers
of correlated pion pairs (i.e. pairs after subtraction of the reference distribution) with $|\cos \theta| < 1/2$ and $|\cos \theta| > 1/2$, respectively, as

$$a^2 = \frac{1}{3} \left( \frac{4}{(1 + \Delta)^2} - 1 \right). \tag{4}$$

The (large) advantages of the method described above are that it does not require a fit to any particular form of the spatial distribution of the source, it is insensitive to the strength of the correlation, and it can be based on a smaller statistics than that required for separate measurement of $r_T$ and $r_L$. The method has been successfully applied to $p(p)p$ - collisions at $\sqrt{s} = 53$ and 63 GeV [9].

In this work, we report on an investigation of the angular dependence of Bose-Einstein correlations in collisions of $\pi^+$ and $K^+$ mesons with protons, as well as with Al and Au nuclei at 250 GeV/c. The data have been obtained in the NA22 experiment, performed at the CERN SPS with the help of the European Hybrid Spectrometer EHS. Earlier results concerning Bose-Einstein correlations in $\pi^+p$ and $K^+p$ collisions in the same experiment are published in [6,10], those in $\pi^+p$ and $K^+p$ collisions with Al and Au in [11].

The experimental procedure is described in the following section. In Sect. 3 we present the results for the ratio $a$ in meson-proton collisions, as well as for its dependence on charge multiplicity and on a number of kinematical variables of the pions. The results for meson-nucleus interactions are presented in Sect. 4. The conclusions are summarized in Sect. 5.

### 2 Data sample and reconstruction procedure

The experimental set-up of EHS, exposed to a positive meson enriched beam of momentum 250 GeV/c, is described in detail in [12], the data reduction procedures in [13,14]. A rapid cycling bubble chamber RCBC filled with hydrogen was used as an active vertex detector. The RCBC was equipped with two nuclear targets consisting of an aluminium and a gold foil of thickness 2.5 mm and 0.64 mm, respectively, corresponding to 0.5% of an interaction length. The foils were placed side by side, orthogonally to the beam, 15.5 cm behind the entrance window of the chamber. Tracks of secondary charged particles were reconstructed from hits in the wire and drift chambers of the spectrometer and from measurement in RCBC. Depending on the momentum, the average momentum resolution varies from 1 to 2% for tracks reconstructed in RCBC, from 1 to 2.5% for tracks reconstructed in the first lever arm, and to 1.5% for those reconstructed in the full spectrometer.

The selection criteria for events in hydrogen and in foils are described in detail in [6] and [15]. The accepted events are satisfactorily measured and reconstructed and contain at least two negatively charged tracks with momentum error less than 4% for hydrogen events and less than 25% for foil events. Additionally, for foil events, we exclude negative tracks with momentum less than 100 MeV/c (in order to minimize the influence from ionization losses in the foils) and stopping tracks identified as antiproton. The number of accepted events is 102,568 for meson-proton, 3,764 for meson-aluminium and 3,004 for meson-gold interactions.

![Fig. 1. The correlation function for pairs of negative pions at $q_0 < (q_0)_{\text{cut}}$ in $(\pi^+K^+)$-interactions](image)

The resolution in $|q|$ is estimated to be $\sigma_q = 3$ MeV/c at $|q| < 0.05$ GeV/c. All negative particles are assumed to be pions.

The reference sample is formed by the commonly applied multiplicity-dependent mixed-event technique, i.e. a pion from one event is combined with pions randomly chosen from different events of the same multiplicity class ($n = 6, n = 8, n = 10, n = 12, n \geq 14$).

### 3 Results for meson-proton interactions

Typical $(\pi^-\pi^-)$ correlation functions $R(|q|)$, defined as the ratio between the experimentally observed distribution and an uncorrelated reference distribution in $|q|$, are presented in Fig. 1 for $(q_0)_{\text{cut}} = 0.03, 0.05$ and 0.1 GeV, respectively. For all three $(q_0)_{\text{cut}}$ values, the correlation region (peak close to $|q| = 0$) reaches up to $|q| \sim 0.4 \pm 0.5$ GeV/c. The function $R(|q|)$ is normalized to unity in the region $0.4 < |q| < 2.0$ GeV/c, i.e. in the plateau for $|q|$ values above the correlation region.

In [6] it has been shown for the case of $(q_0)_{\text{cut}} = 0.05$ GeV/c that the sharp peak at small $|q|$ cannot be fitted by the usual single Gaussian shape. Furthermore, in [16, 17] evidence is given even for a power-like shape of the peak. In the following, we therefore take full advantage of the shape independence of the method described in Sect. 1.

Fig. 2 shows the angular distribution of the vector $q$ for different values of the cuts $(q_0)_{\text{cut}}$ and $q_{\text{cut}}$, for $(\pi^-\pi^-)$ pairs from the same event (points) and for mixed pairs from different events normalized as in Fig. 1 (histogram). The result of the subtraction of these distributions, i.e. the angular distribution of the correlated pion pair, is presented in the corresponding sub-figures of Fig. 3. The ratio $a$ is obtained
Fig. 3. The angular distribution of the vector $q$ at different $q_{\text{cut}}$ and $(q_0)_{\text{cut}}$ values in $(\pi^-/K^+)$ interactions. Points: experimentally observed distribution; histograms: reference distribution.

from fits by (3) to the data of Fig. 3. The results are given as solid lines, the obtained parameter values $a_{fit}$ are indicated in the figure. We have verified that, within errors, the values of $a$ extracted from the asymmetry parameter according to (4) are the same as those obtained from the fit.

The shift towards small values of $|\cos \theta|$ in Fig. 2b,c,d has a "natural" origin. The di-pion momentum $p = p_1 + p_2$ has predominantly small angles with respect to the collision axis. Therefore, at small $q_0$, the vector $q$ is concentrated in the plane perpendicular to the interaction axis (i.e. at small $|\cos \theta|$). This kinematical effect, inherent for both experimental and reference distributions, is cancelled in their difference, which is attributed then to BE correlations.

In [18] it has been observed that the mixing technique generates 'methodological' correlations in the small $|q|$ region with a shape similar to that of BE correlations. These could, in principle, give some additional effect in the $|\cos \theta|$ distribution. For a verification of our results, the FRITIOF model [19] is used without and with BE correlations. Events generated according to this model are subject to the same selection criteria as the real data. The number of FRITIOF events passing these criteria is 73.953 without and 73.990 with BE correlations. The model analysis is performed in the region of $q_0 < 0.05$ GeV, $|q| < 0.5$ GeV/c.

In Figs. 4a and 4c the $|\cos \theta|$ distributions are shown for FRITIOF without BE correlations. Indeed, small positive correlations are seen in Fig. 4c at $|\cos \theta| > 0.2$, although no correlations are included in the model. However, the $|\cos \theta|$ dependence of these correlations is different from that observed in the experimental data (Fig. 2c and Fig. 3c, respectively). That means that the 'methodological' correlations cannot reproduce the experimentally observed character of the $|\cos \theta|$ distribution.

Figs. 4b and 4d show the same distributions for the case that the BE effect is included in FRITIOF. An exponential parametrization is used in the four-momentum difference $Q = -(p_1 - p_2)^2/2$, as $R(Q) = 1 + \lambda \exp(-rQ)$ with $\lambda = 0.5$ and $r = 0.8$ fm. Correlation is assumed to be

![Fig. 2](image2.png)

![Fig. 3](image3.png)

![Fig. 4](image4.png)
present for all pions \(^1\). Fig. 4b shows a clear signal (points) above the background curve (histogram) for FRITIOF with BE correlations. However, as expected from the spherical parametrization, Fig. 4d, indeed, contains no \(\cos \theta\) dependence \((a = 1.02 \pm 0.04)\).

The stability of the results has also been checked with the alternative overall mixing technique used in \([16]\). The result is the same \((a = 1.02 \pm 0.03)\).

As expected from (1), the angular distribution of \(q\) is isotropic for the smallest value of \(q_{\text{cut}}\), \(q_{\text{cut}} = 0.10\) GeV/c (Fig. 3a) (so the extracted parameter \(a\) is near unity). With increasing \(q_{\text{cut}}\) it becomes more and more anisotropic, but practically stops to vary at the "asymptotic" limit of \(0.5\) GeV/c, i.e. just at the high \(|q|\) end of the correlation peak. This saturation effect is shown in more detail in Fig. 5 for three \((q_0)_{\text{cut}}\) values.

In the saturation region, the error on the parameter \(a\) is rising slightly with increasing \(q_{\text{cut}}\). This is expected \([8]\) from the procedure of background subtraction. So, the value of \(q_{\text{cut}}\) should be chosen at the left edge of the "plateau", i.e. near \(q_{\text{cut}} = 0.5\) GeV/c (see Fig. 1).

As far as \((q_0)_{\text{cut}}\) is concerned, this has to be chosen as small as possible: for our meson-proton data the minimal \((q_0)_{\text{cut}}\) providing reasonable statistics is 0.03 GeV/c. As seen from Fig. 5, at fixed \(q_{\text{cut}}\) the parameter \(a\) decreases with decreasing \((q_0)_{\text{cut}}\). This behaviour has the following origin. Equation (1) is derived, strongly speaking, at \(q_0 = 0\). If \(q_0\) is not very small, equation (1) has to be modified. For example, for the case of a Gaussian density both for the spatial and time distributions, the argument of the subintegral function in (1) includes an additional term \(q_0^2/<q^2>\) \((<q^2>\) being the mean radiation time). One can show that due to this term the parameter \(a\) is closer to unity than the true ratio \(r_T/r_L\); therefore, the larger the average value \(<q_0^2>/<q^2>\), the larger the difference between \(a\) and \(r_T/r_L\). This expected behaviour is clearly confirmed by our data. We, therefore, extract the ratio \(r_T/r_L\) by extrapolation of the \((q_0)_{\text{cut}}\)-dependence of the parameter \(a\) to \((q_0)_{\text{cut}} = 0\) (at fixed \(q_{\text{cut}} = 0.5\) GeV/c). The result of a linear extrapolation from the range of \((q_0)_{\text{cut}} = 0.03 \pm 0.10\) GeV to \((q_0)_{\text{cut}} = 0\) is \(r_T/r_L = a (q_0)_{\text{cut}} = 0) = 0.55 \pm 0.06\), while it is \(a = 0.67 \pm 0.04\) at \(q_{\text{cut}} = 0.5\) GeV/c and \((q_0)_{\text{cut}} = 0.05\) GeV.

The quoted value of the ratio \(a\) agrees with the ratio \(a = 0.56 \pm 0.08\) of the transverse and longitudinal radii, \(r_T = 1.04 \pm 0.12\) fm and \(r_L = 1.85 \pm 0.13\) fm, measured separately in the same experiment \([6]\), but due to the method of direct measurement the error is reduced. A value of \(a = 0.60 \pm 0.02\) is obtained in \([9]\) for \(p(p)p\)-interactions at \(\sqrt{s} = 53\) and 63 GeV for the cuts \(q_{\text{cut}} = 0.4\) GeV/c and \((q_0)_{\text{cut}} = 0.1\) GeV, while a separate measurement of \(r_T\) and \(r_L\) had given \(a = 0.55 \pm 0.07\) \([5]\).

As shown in \([20-22]\), where a general case of non-static pion source is considered, the observed shape of the source is elongated along the direction of its movement, even if the source is spherically symmetric in its rest frame. The magnitude of this elongation depends on the source velocity and on the mean radiation time. In our recent study of Bose-Einstein correlations in different reference frames of \(\pi^+ p\) interactions \([6]\), no evidence is found for the existence of a unique frame in which the pion source is motionless for each \(\pi^+ p\) collision. If the source velocity-vector is predominantly directed along the interaction axis, the observed

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\(^1\) Alternatively, assuming Bose-Einstein correlation (with a larger value of \(\lambda\)) only for pions produced directly or from decay of short-lived resonances, gives similar results for the present analysis.
shape is expected to be prolate. Therefore, our results on the angular dependence of Bose-Einstein correlations may, at least partially, reflect these non-static properties of the source.

The multiplicity dependence of the shape is studied in Fig. 6a, again extrapolated to \((q_0)_{\text{cut}} = 0\) at \(q_{\text{cut}} = 0.5\) GeV/c. An indication is obtained that the source may become more elongated for larger multiplicities. The rapidity dependence is not clear (Fig. 6b). The elongation increases with increasing momentum \(p = |p_1 + p_2|\) of the pion pair in the c.m.s. (Fig. 6c). For pions with small transverse momentum \(p_T < 0.2\) GeV/c the source is less elongated than for the unbiased sample (Fig. 6d). Using the FRITIOF model with BE correlations (for a spherically symmetric source), we have verified that the experimentally observed \(p\) and \(p_T\) dependence is not an artefact due to the kinematical cuts.

4 Results for meson-nucleus interactions

The correlation function \(R(|q|)\) for pairs of negative pions produced with small energy difference \((q_0)_{\text{cut}} = 0.05\) and 0.1 GeV in \((\pi^+ / K^+)\)Al and \((\pi^+ / K^+)\)Au interactions is presented in Fig. 7. Here, \(q\) is defined as the momentum difference in the meson-nucleon c.m.s. The function \(R(|q|)\) is normalized to unity over the region \(0.5 < |q| < 1.0\) GeV/c (except for interactions on Au for \(q_0 < 0.05\), where it is normalized for \(0.4 < |q| < 1.0\) GeV/c) and is sharply peaked at \(|q| \sim 0\).

Figs. (8-10)a,b show the angular distribution of the vector \(q\) at different \((q_0)_{\text{cut}}\) and \(q_{\text{cut}}\) values, for \((\pi^- / \pi^-)\) pairs from the same events (points) and for mixed pairs from different events (histogram). The result of subtraction of these distributions is presented in Figs. (8-10)c,d. As is the case for meson-proton interactions, the resulting distributions are consistent with being isotropic at \(q_{\text{cut}} = 0.1\) GeV/c (Fig. 8), but not at larger \(q_{\text{cut}}\) values (Figs. 9, 10). At sufficiently large values, \(q_{\text{cut}} \geq 0.4\) GeV/c, the parameter \(a\) does not vary outside errors with increasing \(q_{\text{cut}}\) (see Fig. 11).

At \(q_{\text{cut}} = 0.5\) GeV/c and \((q_0)_{\text{cut}} = 0.05\) GeV, the ratio \(a\) obtained from (4) is equal to 0.65±0.13 for meson-aluminium and 0.50±0.21 for meson-gold interactions, within errors equal to the numbers obtained from the fit according to (3) in Fig. 9. The extrapolation of the \((q_0)_{\text{cut}}\)-dependence of \(a\) from the region \((q_0)_{\text{cut}} = 0.03 \pm 0.15\) GeV to \((q_0)_{\text{cut}} = 0\) gives 0.53±0.15 for meson-aluminium and 0.33±0.21 for meson-gold interactions. The quoted values of the ratio \(r_T / r_L\) show that also the nuclear source is elongated along the interaction axis in the c.m.s. of meson-nucleon collision.

The ratio averaged over Al and Au is \(a = 0.46 \pm 0.12\). The large error does not allow to conclude that the nuclear source is more elongated than the proton one, as could be expected from multiple inelastic interactions of the leading hadron (hadron cluster) along its path within the nucleus.

5 Summary

A study of the angular dependence of Bose-Einstein correlations has been performed in interactions of \(\pi^+\) and \(K^+\) mesons with hydrogen, aluminium and gold nuclei at 250 GeV/c, with the help of the EHS spectrometer. Within the framework of static models, the ratio \(a = r_T / r_L\) of transverse and longitudinal radii of the pion source can be determined from the angular distribution of the meson-nucleon c.m.s. momentum difference of negative pion pairs.
The method has been checked by Monte Carlo simulation. The ratio \( a \) extrapolated to \( q_{\text{cut}} = 0 \) at \( q_{\text{cut}} = 0.5 \text{ GeV/c} \) has a value of 0.55±0.06 for \((\pi^+/K^+)p\), 0.53±0.15 for \((\pi^+/K^+)\text{Al}\), and 0.33±0.21 for \((\pi^+/K^+)\text{Au}\) interactions, i.e., the pion source is elongated along the collision axis, both for the meson-proton and meson-nucleus interactions. While no dependence of the elongation is found on the rapidity of the pair, indication for an increase is found with increasing event multiplicity, pair momentum and particle transverse momentum.

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