A study of double pomeron exchange in $\pi^+ p$ and $K^+ p$ interactions at 250 GeV/c

EHS/NA22 Collaboration

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Abstract. Quasi-elastic scattering with central dipion
cluster production is studied in $\pi^+ p$ and $K^+ p$ interac-
tions at 250 GeV/c. The cross section of double pomeron
exchange is obtained as $\sigma_{DPE}(\pi^+ p \rightarrow \pi^+ (\pi^- \pi^+)) = 24 \pm 5 \mu b$
and $\sigma_{DPE}(K^+ p \rightarrow K^+ (\pi^- \pi^+)) = 19 \pm 5 \mu b$. The
low energy pomeron-pomeron cross section is estimated
for the first time.

1 Introduction

New direct information on the interaction properties of
the pomeron and on its role in strong interaction physics
can be extracted from the study of the processes in which
it can be exchanged more than once. The simplest one
among these is double pomeron exchange (DPE) [1]-[8].
This process is characterized by two fast leading hadrons
(with Feynman $|x| > 0.9$) and a centrally produced dif-
fractive cluster. Large rapidity gaps are required to
isolate these central particles kinematically from the
background. According to the existing literature (e.g.
[5, 7]), this background is mainly due to single diffraction
dissociation or pomeron-regge exchange.

DPE has recently been studied in $pp$-interactions at
ISR energies (see Refs. in [7], and [9]-[12]). First results
are now also available from the SPS collider [13]. In
meson-nucleon scattering, attempts have been undertak-
en to study DPE in $K^+ p$-interactions at 70 GeV/c [14]
and in $\pi^\pm p$-interactions at 147 GeV/c [15]. However,
due to the small rapidity interval available, only upper
limits for the DPE cross section could be obtained.

A better opportunity to study DPE in meson-nucleon
scattering is given by the European Hybrid Spectrometer
[16-18] at the CERN SPS exposed to a positive meson
enriched beam with momentum 250 GeV/c. This mo-
mentum corresponds to a sufficiently large rapidity gap
(7-8 units) between the colliding hadrons. A first study
of the reactions

$$\pi^+ p \rightarrow \pi^+ (\pi^- \pi^+) p \quad (1)$$

$$K^+ p \rightarrow K^+ (\pi^- \pi^+) p \quad (2)$$

at 250 GeV/c has been performed in [19] where an upper
limit of the DPE process has been obtained.

In this paper we present our final results on the study
of DPE in reactions (1) and (2). The experimental details
are shortly described in Sect. 2. The estimate of the cross
section and the study of the characteristics of the DPE
process are presented in Sect. 3. An attempt to extract
the low energy pomeron-pomeron cross section is under-
taken in Sect. 4 and a summary is given in Sect. 5.
2 Experimental details

The full experimental set-up of the European Hybrid Spectrometer (EHS) with the rapid cycling bubble chamber (RCBC) filled with H$_2$ as vertex detector is described in detail in [16-18]. Tracks of secondary charged particles are reconstructed from hits in the wire and drift chambers of the two lever arm spectrometer and from measurement in RCBC. The momentum resolution varies from (1–2)% for tracks reconstructed in the bubble chamber, to (1–2.5)% for tracks reconstructed in the first lever arm, and to 1.5% for tracks reconstructed in the full spectrometer.

Particle identification in EHS is supplied by RCBC, the Cerenkovs SAD and FC, the ionization sampling device ISIS and the transition radiation detector TRD.

Our results are based on 144220 $\pi^+ p$ events and 49564 $K^+ p$ events. The number of well measured and reconstructed 4-prong events with the proper charge balance and $\Delta p/p < 25\%$ for each track are equal to 9753 for $\pi^+ p$ and 3706 for $K^+ p$-interactions. The corresponding sensitivity, deduced from the results of [17], is 0.25 $\mu$b and 0.58 $\mu$b per event, respectively. The events are processed through the HYDRA kinematics program. We further select events with a $\chi^2$-probability $P(\chi^2)$ for the 4C hypotheses (1) and (2) larger than 0.01% for which the leading particle in the c.m.s. forward hemisphere is a $\pi^+(K^+)$-meson with $x_{\pi^+}(x_{K^+}) > 0.9$ and the leading particle in the backward hemisphere is a proton with $|x_p| > 0.9$. These criteria lead to 83 events for reaction (1) and 35 events for reaction (2) all with, in fact, $P(\chi^2) > 0.2\%$ for reaction (1) and $P(\chi^2) > 0.1\%$ for reaction (2). No event is found to simultaneously satisfy hypotheses (1) or (2) with a leading proton in the forward and with a leading pion (kaon) in the backward hemisphere, or a hypothesis with central ($K^+ K^-$) or ($p\bar{p}$) instead of ($\pi^+ \pi^-$)-pair production.

We use the information on the identification of low-momentum ($p_{lab} < 1.2$ GeV/c) particles in RCBC and of centrally produced particles (with $p_{lab} < 20$ GeV/c) from the identification device ISIS, based on multiple sampling of the ionization [16]. We exclude events for which the leading backward particle is not consistent with the proton hypothesis and events for which at least one of the centrally produced particles cannot be accepted as a pion due to the $\chi^2$ probability for the pion hypothesis being smaller than that for another hypothesis. The leading forward particle ($p_{lab} > 225$ GeV/c) is assigned the pion mass for reaction (1) and the kaon mass for reaction (2) for all events accepted. The number of events excluded is 10 for reaction (1) and 6 for reaction (2).

Events with low squared four momentum transfer to the quasi-elastically scattered meson ($|t_{\pi^+ p}+1.0^{+}\,\text{GeV}^2/c^4| < 0.03$ GeV$^2$/c$^4$) have very low trigger efficiency and are excluded from further consideration. The remaining events are weighted to correct for losses induced by the interaction trigger [20]. The number of events remaining is 69 (or 128.8 weighted events) for reaction (1) and is 26 (or 45.6 weighted events) for reaction (2). Systematic errors due to this procedure are small compared to the statistical errors. The corresponding cross sections are

![Fig. 1. Squared four-momentum transfer distribution $d\sigma/dt_{\pi^+ p}$ for reactions (1) (crosses) and (2) (circles). The results of the exponential fit are also shown](image1)

![Fig. 2. Squared four-momentum transfer distribution $d\sigma/dt_{p^+ p}$ for reactions (1) (crosses) and (2) (circles). The results of the exponential fit are also shown](image2)
\[ \sigma'(\pi^+ p \rightarrow \pi^+ p) = 31 \pm 4 \text{ mb} \]
\[ \sigma(K^+ p \rightarrow K^+ p) = 25 \pm 5 \text{ mb} \]

The correction for the loss at \( \frac{1}{s} < 0.03 \) (GeV/c)^2 is estimated from a fit of the experimentally observed spectra d\sigma/dt to the exponential form \( \sim b \exp[-(t + 0.03)] \). The values of the slope parameter fitted by the maximum likelihood method are \( b_p = (6.85 \pm 0.19) \text{ (GeV/c)^{-2}} \) and \( b_K = (5.98 \pm 0.34) \text{ (GeV/c)^{-2}} \) for the \( \pi^+ \) and \( K^+ \) induced reaction, respectively. For d\sigma/dt (Fig. 2) we obtain \( b_p = (7.29 \pm 0.26) \text{ (GeV/c)^{-2}} \) for reaction (1) and \( b_K = (6.66 \pm 0.40) \text{ (GeV/c)^{-2}} \) for reaction (2), in good agreement with \( b\pi^{-7}(\text{GeV/c)^{-2}} \) obtained for the reaction pp \( \rightarrow p(\pi^+ \pi^-)p \) with quasi-elastically scattered protons and a centrally produced \( (\pi^+ \pi^-) \)-cluster at \( \sqrt{s} = 30.7 \text{ GeV} \) [21].

The corrected cross sections are
\[ \sigma(\pi^+ p \rightarrow \pi^+ p) = 38 \pm 6 \text{ mb} \]
\[ \sigma(K^+ p \rightarrow K^+ p) = 29 \pm 6 \text{ mb} \]

The values (4) are close to those reported previously in [18], where only half of the present \( \pi^+ p \) statistics was used, and can be considered as upper limits for the DPE cross section.

3 Cross section and characteristics of DPE

The rapidity \( y \) and Feynman-x distributions for the DPE candidates are shown in Figs. 3-4. With the \( x_p \)-cut on the leading particles, one obtains good rapidity separation of leading hadrons and centrally produced pions. However, even large rapidity gaps between leading and centrally produced hadrons do not necessarily exclude all non-pomeron exchanges. In particular, there could be a remnant of single diffraction dissociation, characterized by one of the exchanges being reggeon instead of pomeron.

However, it is expected that the background from single pomeron exchange (SPE) is different in the various phase space regions. This follows from a nonsimilar behaviour of the differential cross section for the pomeron-pomeron (PP), pomeron-reggeon (PR) and reggeon-pomeron (RP) exchange processes.

Below, the following variables will be used to describe the differential cross section:
\[ m_{hnn} = \text{the effective mass of the central cluster,} \]
\[ t_1 = t_{p-p}, \]
\[ t_2 = t_{M-M} (M \text{ denotes the leading meson: } \pi^+ \text{ or } K^+). \]
where \( a_P(t) = 1 + a_P t \) is the pomeron trajectory and \( a_P \) is its slope, \( a_R(t) = \frac{1}{2} + a_R t \) is the reggeon trajectory with intercept equal \( \frac{1}{2} \) and slope \( a_R \). The functions \( g_{PP}^2(t) \), \( g_{KR}^2(t) \) describe the pomeron-hadron and reggeon-hadron couplings (see Fig. 5) and are parametrized as

\[
\begin{align*}
g_{PP}^2(t) &= g_{PP}^2(0) e^{a_{PP} t}, \\
g_{KR}^2(t) &= g_{KR}^2(0) e^{a_{KR} t},
\end{align*}
\]

(6a)

(6b).

The functions \( \sigma_{PP}(m_{\pi\pi}^2, t_1, t_2) \) and \( \sigma_{PR(RP)}(m_{\pi\pi}^2, t_1, t_2) \) define the pomeron-pomeron and pomeron-reggeon cross sections. They can depend on the "masses" \( t_1 \) and \( t_2 \) of the pomerons (reggeons); in this case we shall use the parametrization

\[
\begin{align*}
\sigma_{PP}(m_{\pi\pi}^2, t_1, t_2) &= \sigma_{PP}(m_{\pi\pi}^2, 0) e^{a_{PP}(t_1+t_2)}, \\
\sigma_{PR}(m_{\pi\pi}^2, t_1, t_2) &= \sigma_{PR}(m_{\pi\pi}^2, 0) e^{a_{PR}(t_1+t_2)},
\end{align*}
\]

(7a)

(7b)

with \( \sigma_{PP}(m_{\pi\pi}^2, 0) \), \( \sigma_{PR}(m_{\pi\pi}^2, 0) \), \( R_{PP}^2 \) and \( R_{PR}^2 \) as open parameters. This parametrization is the simplest and most convenient one. It allows to integrate (5) and to obtain (8) and (15). The distribution in \( t_1 \) and \( t_2 \) (Figs. 1 and 2) are described well by the exponential form. If the pomeron-pomeron interaction itself proceeds via one-pion exchange, with the usual pion-pomeron coupling \( g_{PP}^2(q^2) = g_{PP}^2(0) \exp(R_{PP}^2 q^2) \), this leads to \( \sigma_{PP}(m_{\pi\pi}^2, t_1, t_2) = \sigma_{PP}(m_{\pi\pi}^2, 0) \exp(R_{PP}^2(t_1+t_2)) \) which is just a special case of (7). Hitherto, no data exist on the parameters of expression (7a) characterizing the pomeron-pomeron interaction at low energies.

Note that in (5) the contribution of double reggeon (RR) exchange is omitted. Due to the small mass \( m_{\pi\pi} \) of the central pion cluster (see below), (RR) is suppressed relative to DPE by a factor \( (m_{\pi\pi}^2/s) \sim 10^{-4}-10^{-3} \). Furthermore, possible interference of DPE and SPE diagrams is ignored.

Integrating (5) over \( t_1 \) and \( t_2 \) one obtains:

\[
\begin{align*}
\frac{d^2 \sigma_{PP}}{dz_1 dz_2} &= \frac{1}{4} g_{MP}^2(0) g_{PP}^2(0) \sigma_{PP}(m_{\pi\pi}^2, 0) \\
& \quad \cdot \frac{1}{(2a_P z_1 + R_{MP}^2 + R_{PP}^2)(2a_P z_2 + R_{PP}^2 + R_{PR}^2)} \\
& = \sigma_{PP}(m_{\pi\pi}^2, 0) f_{PP}(z_1, z_2),
\end{align*}
\]

(8a)

For a limited interval of \( m_{\pi\pi} \), where one can neglect the variation of \( \sigma_{PP} \) and \( \sigma_{PR} \), DPE (8a) predicts only little variation of the density distribution in \( (z_1, z_2) \), while SPE (8b) or (8c) leads to exponentially decreasing distributions in \( z_1 \) or \( z_2 \).

The experimentally observed \( (z_1, z_2) \) distribution and its kinematical boundaries \( (z_1, z_2 > z_{\text{min}} \approx 2.3 \) and \( z_1 + z_2 < \ln(s/4m_{\pi\pi}^2) \approx 8.3 \) are shown in Fig. 6. The region of \( z_1 + z_2 > 7.4 \) corresponds to very low dipion mass (note that \( z_1 + z_2 = \ln(s/m_{\pi\pi}^2) \)), \( m_{\pi\pi} \) is the dipion transverse mass; for \( z_1 + z_2 > 7.4 \), \( m_{\pi\pi}^2 < 0.28 \text{ GeV}/c^2 \) holds for which \( \sigma_{PP} \) and \( \sigma_{PR} \) are suppressed due to small phase space for the \( \pi\pi \) system. In the region \( z_1 + z_2 < 7.4 \), indeed, no strong variation of density is observed. This means, on the one hand, that SPE processes do not play a dominant role, and on the other, that \( \sigma_{PP} \) does not depend significantly on \( m_{\pi\pi}^2 \sim m_{\pi\pi}^2 = s \cdot \exp[-(z_1 + z_2)] \).

In order to estimate the contribution from SPE, we therefore ignore the possible dependence of \( \sigma_{PP} \) and \( \sigma_{PR} \) on \( m_{\pi\pi} \) in the region of \( z_1 + z_2 < 7.4 \), and fit the one-
dimensional distributions in $z_1$ and $z_2$ (Fig. 7) by the expressions
\[
\frac{d\sigma}{dz_1} \sim \int_{z_{min}}^{z_0} \{ f_{PP}(z_1, z_2) + c_R [f_{PR}(z_1, z_2) + f_{RP}(z_1, z_2)] \} \, dz_2, \tag{9a}
\]
\[
\frac{d\sigma}{dz_2} \sim \int_{z_{min}}^{z_0} \{ f_{PP}(z_1, z_2) + c_R [f_{PR}(z_1, z_2) + f_{RP}(z_1, z_2)] \} \, dz_1, \tag{9b}
\]
with $c_R$ characterizing the SPE contribution as a free parameter.

The parameters entering into the functions $f_{PP}$, $f_{PR}$, $f_{RP}$ (see (8)) are taken from [6, 8, 22, 23]. The following values are used (all parameters are in units (GeV/c)$^{-2}$):
\[
\alpha_p = 0.2 \pm 0.4, \quad \alpha_R = 0.7 \pm 1; \quad g_{PP}(0) = 4, \quad g_{PR}(0) = 1.4, \quad g_{RP}(0) = 1.1; \quad g_{PP}^2(0) = 1.3, \quad g_{PR}^2(0) = 0.8 \pm 1.2, \quad g_{RP}^2(0) = 0.2 \pm 0.6; \quad R_{PP}^2 = 3.5 \pm 4, \quad R_{PR}^2 = 1.4 \pm 2, \quad R_{RP}^2 = 0.74 \pm 1.24; \quad R_{PP}^2 = 7.1, \quad R_{PR}^2 = 2, \quad R_{RP}^2 = 3.1. \tag{10}
\]

$R_p^2$ and $R_R^2$ varies from 0 (where $\sigma_{PP}$ and $\sigma_{PR}$ are assumed to have a negligible dependence on $t_1$ and $t_2$) to $R_{PP}^2 = R_{PR}^2$ and $R_{RP}^2$ (where the PP and PR interactions occur via one-pion exchange). We have verified, that the choice of different sets of these parameters leads to different values of $c_R$, but practically has no influence on the integrated contribution of $\text{SPE}$ in (9).

The combined fit of the $\pi^+ p$ and $K^+ p$ data (assuming the same $c_R$ for both) shows that the contribution from SPE to the cross section in the considered region is small (see Fig. 7), but determined with large errors, $(15 \pm 3)%$ for $\pi^+ p$ and $(10 \pm 3)%$ for $K^+ p$-interactions. Subtracting these contributions, one obtains for $|t_1| > 0.03$ GeV/c$^2$:
\[
\sigma'_{\text{DPE}}(\pi^+ p \rightarrow \pi^+ (\pi^+ \pi^-) p) = 26 \pm 10 \mu b, \quad \sigma'_{\text{DPE}}(K^+ p \rightarrow K^+ (\pi^+ \pi^-) p) = 22 \pm 6 \mu b. \tag{11}
\]

Another estimate of the DPE cross section can be obtained with the help of the rapidity gap method [4, 5, 24]. It can be shown that the rapidity gap $\Delta_{1(2)}$ between hadron $h_{1(2)}$ and the nearest centrally produced pion $\pi_{1(2)}$ is of very similar value as $z_{1(2)}$, and the distribution in $\Delta_{1(2)}$ is qualitatively similar to that in $z_{1(2)}$. In the case that a reggeon is exchanged between hadron $h_{1(2)}$ and the nearest pion $\pi_{1(2)}$, the distribution in $\Delta_{1(2)}$ is expected to have a rapidly falling form (see (8c) or (8b), and [5]). One can assume that the largest “arm” of the rapidity gap $\Delta_{max} = \max(\Delta_1, \Delta_2)$ corresponds to pomeron exchange, and the smallest one $\Delta_{min} = \min(\Delta_1, \Delta_2)$ corresponds to pomeron (DPE) or reggeon exchange (SPE). The relative contribution of the latter should decrease as $\Delta_{min}$ increases.

The distribution in $\Delta_{min}$ is shown in Fig. 8 for the combined $\pi^+ p$ and $K^+ p$ data. Under simplifying assumptions (see [5]), the SPE contribution can be represented by $\sim e^{-\Delta_{min}}(z_0 - 2\Delta_{min})$. Assuming that at the lower values of $\Delta_{min}$ the differential cross section $d\sigma/d\Delta_{min}$ is completely determined by SPE, one can estimate an integral contribution in the region of $\Delta_{min} > 0.03$ GeV/c$^2$ is
\[
\sigma'_{\text{DPE}}(\pi^+ p \rightarrow \pi^+ (\pi^+ \pi^-) p) = 19 \pm 4 \mu b, \quad \sigma'_{\text{DPE}}(K^+ p \rightarrow K^+ (\pi^+ \pi^-) p) = 16 \pm 4 \mu b. \tag{12}
\]

Within the large errors these values are consistent with estimates (11).

The correction for loss of events with low momentum transfer ($|t_1| < 0.03$ GeV/c$^2$) is carried out according to

\[
\begin{align*}
0 & \leq D_{min} < 0.03, & 0.5 & \leq D_{min} < 0.1, & 0.03 & \leq D_{min} < 0.5, \\
& & & & & 0.5 & \leq D_{min} < 1.5.
\end{align*}
\]

Fig. 8. Minimal rapidity gap distribution for the combined $\pi^+ p$ and $K^+ p$ data. The curve represents the SPE contribution parameterized as $e^{-\Delta_{min}}(z_0 - 2\Delta_{min})$ and normalized at the point $\Delta_{min} = 1.5$. 357
Table 1. The mean characteristics of leading hadrons in the DPE

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Particle</th>
<th>$\langle x \rangle$</th>
<th>$\langle y \rangle$</th>
<th>$\langle t_d^{-1}\rangle$ (GeV/c)^2</th>
<th>$b$(GeV/c)^{-2} (slope in d$\sigma$/d$dt$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>$p$</td>
<td>-0.963 ± 0.003</td>
<td>-3.06 ± 0.02</td>
<td>0.15 ± 0.02</td>
<td>6.8 ± 0.6</td>
</tr>
<tr>
<td></td>
<td>$\pi^+$</td>
<td>0.986 ± 0.004</td>
<td>4.03 ± 0.05</td>
<td>0.17 ± 0.03</td>
<td>7.3 ± 0.4</td>
</tr>
<tr>
<td>(2)</td>
<td>$p$</td>
<td>-0.976 ± 0.006</td>
<td>-3.09 ± 0.02</td>
<td>0.13 ± 0.04</td>
<td>7.8 ± 0.9</td>
</tr>
<tr>
<td></td>
<td>$K^+$</td>
<td>0.938 ± 0.006</td>
<td>3.49 ± 0.04</td>
<td>0.20 ± 0.05</td>
<td>6.0 ± 0.7</td>
</tr>
</tbody>
</table>

Table 2. The mean characteristics of centrally produced pions in the DPE

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Particle</th>
<th>$\langle x \rangle$</th>
<th>$\langle y \rangle$</th>
<th>$\langle p_T \rangle$ (GeV/c)</th>
<th>$\langle m_{\pi\pi} \rangle$ (GeV/c^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>$\pi^+$</td>
<td>0.005 ± 0.004</td>
<td>0.17 ± 0.11</td>
<td>0.38 ± 0.03</td>
<td>0.986 ± 0.004</td>
</tr>
<tr>
<td></td>
<td>$\pi^-$</td>
<td>0.016 ± 0.004</td>
<td>0.45 ± 0.12</td>
<td>0.33 ± 0.03</td>
<td>4.03 ± 0.05</td>
</tr>
<tr>
<td></td>
<td>$\rho$</td>
<td>0.021 ± 0.006</td>
<td>0.26 ± 0.08</td>
<td>0.52 ± 0.04</td>
<td>0.73 ± 0.4</td>
</tr>
<tr>
<td></td>
<td>$\pi_0$</td>
<td>0.014 ± 0.007</td>
<td>0.29 ± 0.16</td>
<td>0.45 ± 0.04</td>
<td>7.3 ± 0.4</td>
</tr>
<tr>
<td></td>
<td>$\rho^+$</td>
<td>0.029 ± 0.010</td>
<td>0.36 ± 0.12</td>
<td>0.47 ± 0.05</td>
<td>7.8 ± 0.9</td>
</tr>
</tbody>
</table>

Table 3. The combined mean characteristics in DPE

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Combination</th>
<th>$\langle \Delta \rangle$</th>
<th>$\langle m_{\pi\pi} \rangle$ (GeV/c^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>($p\pi_0$)$_{min}$</td>
<td>2.9 ± 0.1</td>
<td>2.98 ± 0.14</td>
</tr>
<tr>
<td></td>
<td>($\pi^+\pi^-$)$_{min}$</td>
<td>3.2 ± 0.1</td>
<td>2.10 ± 0.16</td>
</tr>
<tr>
<td></td>
<td>$p\pi^+$</td>
<td>3.44 ± 0.15</td>
<td>2.37 ± 0.16</td>
</tr>
<tr>
<td></td>
<td>$\pi^-$</td>
<td>2.37 ± 0.16</td>
<td>2.37 ± 0.16</td>
</tr>
<tr>
<td>(2)</td>
<td>($p\pi_0$)$_{min}$</td>
<td>3.1 ± 0.2</td>
<td>3.50 ± 0.15</td>
</tr>
<tr>
<td></td>
<td>($K^-\pi^+$)$_{min}$</td>
<td>2.7 ± 0.1</td>
<td>1.94 ± 0.19</td>
</tr>
<tr>
<td></td>
<td>$p\pi^-$</td>
<td>3.55 ± 0.20</td>
<td>2.88 ± 0.29</td>
</tr>
<tr>
<td></td>
<td>$K^-\pi^+$</td>
<td>2.88 ± 0.29</td>
<td>2.88 ± 0.29</td>
</tr>
</tbody>
</table>

the method described in Sect. 2, with the help of the fitted slope parameters $b_\pi$ and $b_K$ for the DPE events of Table 1. The final values of the DPE cross sections are

$$\sigma_{DPE}(\pi^+ p \rightarrow \pi^+(\pi^+ \pi^-) p) = 24 \pm 5 \text{ mb},$$

$$\sigma_{DPE}(K^- p \rightarrow K^+(\pi^+ \pi^-) p) = 19 \pm 5 \text{ mb}. \quad (13)$$

Average characteristics of the events with $\Delta_{min} > 2$ (the almost pure sample of the DPE events) are shown in Tables 1-3. The characteristics of leading hadrons are presented in Table 1. The average rapidity gap between leading hadrons is 7.1 for reaction (1) and 6.6 for reaction (2). The slope parameters in the differential spectra $d\sigma/d\xi$ for DPE are consistent with those for the whole event sample.

The central pion characteristics are presented in Table 2. As can be seen from the last column, only low mass central clusters are produced ($\langle m_{\pi\pi} \rangle \sim 0.7 \text{ GeV/c^2}$). They have comparatively large average transverse momentum ($\langle p_T \rangle_{exc} \sim 0.5 \text{ GeV/c}$), low $x(\langle x_{exc} \rangle \sim 0.02 = \pm 0.03)$ and rapidity ($\langle y_{exc} \rangle \sim 0.3$). The average transverse momentum of the centrally produced pions is approximately the same as that of pions produced in the central region of 4-prong non-single-diffractive $(\pi^+/K^+) p$-interactions at 250 GeV/c [25].

The average rapidity gap between the leading hadron and the nearest central pion (Table 3) is about 3 units. The average effective masses $\langle m_{\pi\pi} \rangle = 2.4$, $\langle m_{\pi\pi} \rangle = 2.9$ and $\langle m_{\pi\pi} \rangle = 3.5 \text{ GeV/c^2}$ are far from the $\rho^0$, $K^0(890)$ and $\Delta^{++}(1232)$ masses, respectively, and it has been verified that no peaks are observed in the region of these resonances.

4 The pomeron-pomeron cross section

A first attempt is undertaken to extract information on the properties of the pomeron-pomeron interaction, itself. The unknown parameter $R_0$, which determines the dependence of the pomeron-pomeron cross section on the pomeron "masses" $\alpha_i$, $\alpha_2$ (in (7a)), can be estimated from fitting the DPE data by the four-dimensional distribution (5a). A maximum likelihood fit is performed for fixed values of $\alpha_P' = 0.2, 0.3$ and 0.4, with as free parameters $R_1 = R_0 + R_{1P}$, $R_2 = R_0 + R_{2P}$ and $R_3 = R_0 + R_{3P}$. The mass dependence of $\sigma_{PP}(m_{\pi\pi}^2, 0)$ for the massless pomeron cross section (which is expected to be weak) is parametrized by the two-pion phase space dependence

$$\sigma_{PP}(m_{\pi\pi}^2, 0) \sim \sqrt{1 - 4m_{\pi\pi}^2/m_{\pi\pi}^2}. \quad (14)$$

The results of the fit are given in Table 4 for the three values of $\alpha_P'$. At given $\alpha_P'$, three different estimates

<table>
<thead>
<tr>
<th>$\alpha_P'$ (GeV/c)^{-2}</th>
<th>$R_1$ (GeV/c)^{-2}</th>
<th>$R_2$ (GeV/c)^{-2}</th>
<th>$R_3$ (GeV/c)^{-2}</th>
<th>$R_3'$ (GeV/c)^{-2}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>5.7 ± 0.5</td>
<td>4.6 ± 0.4</td>
<td>3.7 ± 0.7</td>
<td>2.4 ± 0.4</td>
</tr>
<tr>
<td>0.3</td>
<td>5.0 ± 0.5</td>
<td>4.0 ± 0.4</td>
<td>3.1 ± 0.7</td>
<td>1.9 ± 0.4</td>
</tr>
<tr>
<td>0.4</td>
<td>4.3 ± 0.5</td>
<td>3.3 ± 0.4</td>
<td>2.4 ± 0.7</td>
<td>1.2 ± 0.4</td>
</tr>
</tbody>
</table>
of \( R^2_0 \) are obtained by subtracting from the fitted values of \( R^2_1, R^2_2 \) and \( R^2_3 \) the known "average" value of \( R^2_{pp} = 3.8 \pm 0.2, R^2_{pp} = 1.7 \pm 0.3 \) and \( R^2_{pp} = 1.0 \pm 0.2 \) (GeV/c)^{-2}, respectively (see Sect. 3). The three estimates of \( R^2_0 \) are consistent within errors and their average is given in the last column of Table 4.

At no value of \( \alpha_p \), the obtained \( R^2_0 \) value is consistent with zero. Thus, the dependence of \( \sigma_{pp}(m^2_{\pi \pi}, t_1, t_2) \) on \( t_1, t_2 \) cannot be ignored. The closeness of \( R^2_0 \) and \( R^2_{pp} = 1.7 \pm 0.3 \) (GeV/c)^{-2} may indicate that the one-pion exchange (OPE) between the two pomeron may play a significant rôle within the pomeron-pomeron interaction. However, this closeness is only a necessary, but not a sufficient condition. For additional information from the angular distribution see below.

The differential cross section \( d\sigma_{DPE}/dm_{\pi \pi} \) for reactions (1) and (2) is shown in Fig. 9. The theoretical expression for \( d\sigma_{DPE}/dm_{\pi \pi} \) can be deduced from (8a) and has the form [8]:

\[
\frac{d\sigma_{DPE}}{dm_{\pi \pi}} = g^2_{pp}(0)g^2_{pp}(0)\frac{\sigma_{pp}(m^2_{\pi \pi}, 0)}{8\alpha_P m_{\pi \pi}} \alpha_P \ln \frac{m^2_{\pi \pi} + R^2_M + R^2_F + 2R^2_0}{2} \times \frac{2\alpha_P \ln \left( \frac{1 - x_m}{m^2_{\pi \pi} + R^2_M + R^2_0} \right)}{2\alpha_P \ln \left( \frac{1}{1 - x_m} + R^2_M + R^2_0 \right)} \frac{2\alpha_P \ln \left( \frac{1 - x_m}{m^2_{\pi \pi} + R^2_F + R^2_0} \right)}{2\alpha_P \ln \left( \frac{1}{1 - x_m} + R^2_F + R^2_0 \right)},
\]

where \( x_m = 0.9 \) is the Feynman x boundary of the leading hadron. From a comparison of the data and (15), the massless pomeron-pomeron cross section \( \sigma_{pp}(m^2_{\pi \pi}, 0) \) is extracted. The results are found to be practically independent of the choice of parameter sets in Table 4 and coincide within errors for the \( \pi^+p \) and \( K^+p \) data. The dependence of \( \sigma_{pp}(m^2_{\pi \pi}, 0) \) on \( \sqrt{s_{pp}} = m_{\pi \pi} \) averaged over the \( \pi^+p \) and \( K^+p \) data is presented in Fig. 10. Above the rise at the two-pion threshold, \( \sigma_{pp}(s_{pp}, 0) \) is consistent with being constant in the interval \( 0.45 < \sqrt{s_{pp}} < 1.5 \) GeV. The cross section averaged over this interval is \( \sigma_0 = 0.11 \pm 0.02 \) mb. Note, that in the quoted errors the possible uncertainty of the pomeron-hadron vertex parameters \( g^2_{pp}(0), g^2_{pp}(0), g^2_{pp}(0) \) (which can reach 20%) is not included.

As the pomeron isospin is \( T = 0, \sigma_{pp}^{0+*} = 2\sigma_{pp}^{p2} \) is expected. So the massless pomeron-pomeron total cross section (to two pions) is \( \sigma_{pp}^{p2} = 0.17 \pm 0.03 \) mb at \( \sqrt{s_{pp}} = 0.45 \pm 1.5 \) GeV and \( \sigma_{pp}^{p2} = 0.08 \pm 0.02 \) mb at \( \sqrt{s_{pp}} < 0.45 \) GeV. These are first estimates of the low energy pomeron-pomeron cross section.

Our value for the massless pomeron-pomeron total cross section \( \sigma_{pp}^{p2} = 0.17 \pm 0.03 \) mb for \( 0.45 < m_{\pi \pi} < 1.5 \) GeV/c is in agreement with the theoretical prediction [8] for the high-energy limit \( \sigma_{pp}^{p2}(s_{pp} \to \infty) \approx 0.14 \) mb. Furthermore, some estimate for \( \sigma_{pp}^{p2} \) can be extracted from [26], where the DPE contribution \( \sigma_{DPE}^{p2} \) to the invariant cross section for the reaction \( pp \to ppX \) is given. We extract \( \sigma_{pp}^{p2}(m^2_{\pi \pi}, 0) = 0.48 \pm 0.07 \) mb at \( m_{\pi \pi} = 3.6 \) GeV/c, noticeably larger than our estimate. This is probably due to the contribution from other channels, as \( pp \to 4\pi \to 2K \) etc.

Using the parametrizations (7) and (14) one can express the dependence of the total pomeron-pomeron cross section on their four-momenta \( q_1 \) and \( q_2 \) \( (q_1^2 = t_1, q_2^2 = t_2) \) as

\[
\sigma_{pp}^{p2}(q_1, q_2) = a\sqrt{1 - 4m^2_{\pi \pi}/(q_1 + q_2)^2} \exp \left[ R_0^2(q_1^2 + q_2^2) \right]
\]

with \( a = (0.16 \pm 0.02) \) mb and \( R_0^2 = (1.8 \pm 0.4 \pm 0.6) \) (GeV/c)^{-2}, where the second error in \( R_0^2 \) is due to uncertainty in the pomeron slope \( \alpha_p \).

The angular distribution of the centrally produced pions in the pomeron-pomeron frame is shown in Fig. 11. One can see that the angular distribution is con-
consistent with being isotropic, except for the low angle region. This means that the S-wave pomeron-pomeron interaction dominates. However, some contribution from the OPE mechanism (peaked at lower angles) cannot be excluded.

5 Summary

The reactions $\pi^+ p \rightarrow \pi^+ (\pi^+ \pi^-) p$ and $K^+ p \rightarrow K^+ (\pi^+ \pi^-) p$ with quasi-elastically scattered leading hadrons ($|x| > 0.9$) and centrally produced pion pairs are studied at 250 GeV/c. It is shown that a dominant part of these reactions is consistent with the DPE process, and its cross section is obtained as

$$\sigma_{\text{DPE}}(\pi^+ p \rightarrow \pi^+ (\pi^+ \pi^-) p) = 24 \pm 5 \mu b,$$

$$\sigma_{\text{DPE}}(K^+ p \rightarrow K^+ (\pi^+ \pi^-) p) = 19 \pm 5 \mu b.$$

For the first time estimates are obtained for the cross section of the massless pomeron interaction at low energies, $\sigma_0^{\text{int}} = 0.17 \pm 0.03 \text{ mb}$ at $\sqrt{s_{pp}} = 0.45 - 1.5 \text{ GeV}$ and $\sigma_0^{\text{int}} = 0.08 \pm 0.02 \text{ mb}$ at $\sqrt{s_{pp}} < 0.45 \text{ GeV}$.

An approximate expression for the dependence of the pomeron-pomeron cross section on the four-momenta $q_1$ and $q_2$ is obtained as

$$\sigma_{\text{pp}}^{\text{int}}(q_1, q_2) = a \sqrt{1 - 4m^2/(q_1 + q_2)^2} \exp[R_0^2(q_1^2 + q_2^2)]$$

with $a = (0.16 \pm 0.02) \text{ mb}$ and $R_0^2 = (1.8 \pm 0.4 \pm 0.6) (\text{GeV/c})^{-2}$.

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