Angular dependence of factorial moments in $\pi^+ / K^+ p$ interactions at 250 GeV/c

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Abstract

A sample of $\pi^+ p$ and $K^+ p$ non-diffractive interactions at $\sqrt{s} = 22$ GeV is used to investigate factorial multiplicity moments as a function of the CMS opening angle $\theta$ between particles. The angular dependence is very different for unlike-charge and like-charge particle combinations. For the latter, factorial moments increase with decreasing opening angle approximately as a power law. The $\theta$ dependence is stronger for central production-angle intervals than in the forward and backward regions. The predictions of the standard version of the FRITIOF model deviate strongly from the data, but including Bose-Einstein correlations leads to qualitative agreement.

1. Introduction

In recent years, much attention has been paid to the possible existence of dynamical density fluctuations of
particles produced in high energy collisions [1]. Such fluctuations have indeed been observed experimentally (see [2] and references therein). The effect has become known under the term intermittency, defined [3] as a power-law rise of normalized factorial moments $F_q$ of order $q$ of the (charged) particle multiplicity $n$,

$$F_q = \frac{(n(n-1)\ldots(n-q+1))}{\langle n \rangle^q}$$

(1)

as a function of decreasing size $\delta$ of phase space cells:

$$F_q(\delta) \sim \delta^{-\phi_q}.$$  

(2)

The positive powers $\phi_q$ are known as intermittency indices.

The primary reasons for the interest in intermittency are, on the one hand, that current models of hadron production in hadronic collisions cannot reproduce the strength of the observed fluctuations and, on the other, that perturbative QCD itself turns out to be inherently intermittent.

The first studies were performed in the kinematical variables rapidity $y$ or pseudo-rapidity $\eta$, azimuthal angle $\phi$ and transverse momentum $p_t$. Later analyses [4,5], have indicated, that the squared four-momentum difference $Q^2 = -(q_1 - q_2)^2$ or the invariant mass $M_{inv} = (Q^2 + 4m^2)^{1/2}$ of the particles are more suitable to isolate the effect.

Studies of factorial moments in terms of $Q^2$ and $M_{inv}$ [4-7] show, that correlations between like-charge particles and, consequently, Bose–Einstein (BE) correlations play a predominant role in the observed intermittency effect at small $Q^2$ of $M_{inv}$, while indications exist that QCD is responsible at larger $Q^2$. However, detailed understanding of the intermittency phenomena is still lacking and experimental data for the $F_q$ dependence are needed in variables sensitive to these details.

In perturbative QCD, the intermittency indices $\phi_q$ are directly related to the anomalous dimension $\gamma_0 = (3\alpha_s/2\pi)^{1/2}$ [8-10] and, therefore, to the running coupling constant $\alpha_s$. In the same theoretical context, it has been argued [9,10] that the opening angle $\theta$ between particles is a suitable and sensitive variable to analyse. It is, of course, closely related to $Q$ (or $M_{inv}$).

The theoretical analysis in [9,10] is particularly applicable to hadron production in $e^+ e^-$ collisions. A proper interpretation of future $e^+ e^-$ data in a perturbative QCD framework requires, however, a good understanding of similar data from interactions which are not dominated by hard QCD effects.

In the present analysis we, therefore, investigate the opening-angle dependence of factorial moments in (soft) hadronic collisions. Use is made of the correlation (or density) strip integral method [11], a recent methodological improvement in the study of intermittency.

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The full experimental set-up of EHS, exposed to a positive meson enriched beam with momentum 250 GeV/c, is described in detail in [12,13]. It consists of an active vertex detector (Rapid Cycling Bubble Chamber filled with H2) and a down-stream two-lever-arm spectrometer. Tracks of secondary charged particles are reconstructed from hits in the wire and drift chambers of the spectrometer and from measurement in RCBC. The momentum resolution varies from (1–2)% for tracks reconstructed in RCBC, to (1–2.5)% for tracks reconstructed in the first lever arm and to 1.5% for tracks reconstructed in the full spectrometer.

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with $p_i$ and $p_j$ being the three-momenta of particles $i$ and $j$. An angular distance measure for more than two particles is defined as:

$$\text{dist}(p_{i_1}, \ldots, p_{i_q}) = \max_{k_1, k_2} \theta_{i_k, i_l}.$$

In terms of the density strip integral, the numerator of $F_q$ can be determined by counting, for each event, the number of $q$-tuples that have a pairwise angular opening smaller than a given value $\theta$ and then averaging.

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Fig. 1. Dependence of factorial moments of order $q = 2$ to $q = 5$ on the opening angle $\theta$ for various charge configurations.
over all events. Using the Heaviside unit step function \( \Theta \), this can mathematically be expressed as

\[
F_\gamma(\theta) = \frac{1}{\text{norm}} \left( q! \sum_{i_1 < \ldots < i_q} \prod_{k_i,k_2} \Theta(\theta - \theta_{k_1,k_2}) \right),
\]

(5)

where the factor \( q! \) takes into account the number of permutations within a \( q \)-tuple.

The normalization is obtained from "mixed" events constructed by random selection of tracks from different events in a track pool [5]. The multiplicity of a mixed event is taken to be a Poissonian random variable, thereby ensuring that no extra correlations are introduced. The mixed events are treated in the same way as real events. A correction factor is applied for the difference in average multiplicity of the Poissonian in the experimental distribution.

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In Fig. 1, the data for \( \ln F_q \) are plotted as a function of \( -\ln \theta \) for all charges combined and, separately, for positive, negative and unlike-charge particle combinations. Factorial moments are plotted for order \( q = 2-5 \) in the four sub-figures, respectively. On the double logarithmic plot used, the increase should be linear if (2) was strictly applicable. For our data, this holds at most in limited \( \theta \) regions. Nevertheless, as a rough indication for the \( \theta \) dependence, intermittency indices \( \phi_q \) are given in column 4 of Table 1, as obtained from a fit by

\[
\ln F_q = a - \phi_q \ln \theta \tag{6}
\]

in the \( \theta \) range given in column 2 and indicated by arrows in Fig. 1.

From Fig. 1 and Table 1 it can be seen that:

(i) for all-negative particle combinations, the \( \ln F_q \) (for all orders) rise for all values of \( -\ln \theta \);

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<table>
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<th>( q )</th>
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<td>4° &lt; ( \theta &lt; 47^\circ )</td>
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<td>cc</td>
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<td>-0.31 ± 0.04</td>
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<tr>
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<td></td>
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<td>-0.68 ± 0.21</td>
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<td>+ + + -</td>
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<td>-0.3 ± 0.3</td>
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<tr>
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<td>-1.0 ± 0.4</td>
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<tr>
<td></td>
<td></td>
<td>cccc</td>
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(ii) for all-positive particle combinations, the $F_q$ first decrease when $-\ln \theta$ increases, have a minimum and then increase for high $-\ln \theta$ values;

(iii) for mixed-charge combinations $F_2$ increases at high $-\ln \theta$, but no strong $\theta$ dependence is observed in the higher orders.

As in our previous analyses [4,5], we conclude that the rise of the factorial moments at small phase-space distances is caused by like-charge combinations, not only for $q=2$, but also for orders up to $5$. The increase of $F_2$ at small distances for the $(+ -)$ combination is difficult to interpret because of contamination from Dalitz decay, $\gamma$ conversions and $\eta$ or $\eta'$ decays.

A comparison of the data has been performed with the FRITIOF Monte Carlo model versions 2.0 [14] and 7.0 [15]. Events generated with these models are subject to the same selection criteria as the real data. Since version 2.0 is better tuned to our data [5] and the results obtained from both versions are very similar, we show the comparison for version 2.0. To include BE correlations, we use the algorithm developed for

**FRITIOF 2.0**

![Graphs showing FRITIOF 2.0 results for $q=2, 3, 4, 5$.](image)

Fig. 2. Same as in Fig. 1, but for FRITIOF 2.0 without Bose–Einstein effect.
JETSET 7.3 [16] with exponential parametrization in \( Q \) and measured parameters \( r \) and \( \lambda \) [5].

Fig. 2 and Fig. 3 show, respectively, the FRITIOF 2.0 predictions without and with BE correlations. The intermittency indices are given in columns 5 and 6 of Table 1.

FRITIOF without BE correlations (Fig. 2) strongly deviates from the data. FRITIOF with BE correlations (Fig. 3) can reproduce the rise of \( F_q \) but, as already noted in [16], the effect is overestimated, particularly for high-order factorial moments.

In a more differential analysis, we study the angular dependence of factorial moments in intervals of the CMS production angle \( \Theta_0 \) defined as

\[
\Theta_0 = \arccos\left( \frac{p \cdot p_{\text{beam}}}{|p||p_{\text{beam}}|} \right).
\]

Fig. 4 shows a strong \( \Theta_0 \) dependence of the intermittency indices \( \phi_2 \) and \( \phi_3 \), with the largest values obtained in \( \Theta_0 \) intervals close to 90°. For this \( \Theta_0 \) region, the effect is even stronger in \( \Theta_0 \) than in \( Q^2 \) (not shown).
For like-charge combinations, FRITIOF without BE correlations does not reproduce the data (dashed lines in Figs. 4a, 4b). Including BE effects in the model leads to reasonable agreement with the data (full lines in Fig. 4a, 4b).

5. Conclusions

In this paper we present the first results on the opening-angle dependence of factorial multiplicity moments \( F_q \) for various charge combinations. For like-charge particle combinations, the \( F_q \) rise with decreasing cms opening angle \( \theta \) between particles, for \( q = 2-5 \). Only a weak dependence is observed for unlike-charge combinations. The intermittency indices \( \phi_q \) depend strongly on the production angle \( \theta_0 \) of the particles. For the region close to \( \theta_0 = 90^\circ \), the dependence of factorial moments is even stronger in \( \theta \) than in \( Q^2 \). The standard version of the Monte Carlo program FRITIOF is unable to reproduce the data. When Bose-Einstein correlations are included, the model reproduces the rise of the factorial moments, but the effect is overestimated in particular for higher orders. Because of its relevance in a perturbative-QCD treatment of intermittency, the angular variable would be particularly well suited for an analysis of \( e^+e^- \) data at LEP. A comparison to the present (soft) NA22 results might help in clarifying the importance of perturbative and non-perturbative contributions to hadron production in \( e^+e^- \) annihilation at high energies.

6. Acknowledgement

It is a pleasure to thank the EHS coordinator L. Montanet and the operating crews and staffs of the EHS, SPS and H2 beam, as well as the scanning and processing teams of our laboratories of their invaluable help with this experiment. We are grateful to the III. Physikalisches Institut B, RWTH Aachen, Germany, the DESY-Institut für Hochenergiephysik, Berlin-Zeuthen, Germany, the Department of High Energy Physics, Helsinki University, Finland, and the University of Warsaw and Institute of Nuclear Problems, Poland for early contributions to this experiment. We, furthermore, would like to thank R. Peschanski and J.-
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<td>-0.19 ± 0.01</td>
</tr>
<tr>
<td>4</td>
<td>( 18^\circ &lt; \theta &lt; 51^\circ )</td>
<td>+ + + +</td>
<td>0.32 ± 0.10</td>
<td>-0.44 ± 0.11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+ + + -</td>
<td>-0.02 ± 0.06</td>
<td>-0.35 ± 0.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+ + - +</td>
<td>-0.26 ± 0.06</td>
<td>-0.31 ± 0.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+ - + -</td>
<td>0.02 ± 0.08</td>
<td>-0.26 ± 0.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- - - -</td>
<td>0.95 ± 0.18</td>
<td>-0.68 ± 0.21</td>
</tr>
<tr>
<td></td>
<td></td>
<td>cccc</td>
<td>-0.05 ± 0.05</td>
<td>-0.31 ± 0.04</td>
</tr>
<tr>
<td>5</td>
<td>( 27^\circ &lt; \theta &lt; 51^\circ )</td>
<td>+ + + + +</td>
<td>0.6 ± 0.4</td>
<td>0.0 ± 0.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+ + + + -</td>
<td>-0.2 ± 0.2</td>
<td>-0.3 ± 0.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+ + + - +</td>
<td>-0.5 ± 0.2</td>
<td>-0.5 ± 0.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+ + - + -</td>
<td>-0.4 ± 0.2</td>
<td>-0.3 ± 0.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+ - + - -</td>
<td>0.6 ± 0.4</td>
<td>-1.0 ± 0.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- - - - -</td>
<td>1.8 ± 0.6</td>
<td>-1.9 ± 2.0</td>
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<tr>
<td></td>
<td></td>
<td>cccccc</td>
<td>0.6 ± 0.4</td>
<td>-0.4 ± 0.2</td>
</tr>
</tbody>
</table>
(ii) for all-positive particle combinations, the $F_q$ first decrease when $-\ln \theta$ increases, have a minimum and then increase for high $-\ln \theta$ values;

(iii) for mixed-charge combinations $F_2$ increases at high $-\ln \theta$, but no strong $\theta$ dependence is observed in the higher orders.

As in our previous analyses [4,5], we conclude that the rise of the factorial moments at small phase-space distances is caused by like-charge combinations, not only for $q=2$, but also for orders up to 5. The increase of $F_2$ at small distances for the $(+ -)$ combination is difficult to interpret because of contamination from Dalitz decay, $\gamma$ conversions and $\eta$ or $\eta'$ decays.

A comparison of the data has been performed with the FRITIOF Monte Carlo model versions 2.0 [14] and 7.0 [15]. Events generated with these models are subject to the same selection criteria as the real data. Since version 2.0 is better tuned to our data [5] and the results obtained from both versions are very similar, we show the comparison for version 2.0. To include BE correlations, we use the algorithm developed for

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![FRITIOF 2.0](image)

Fig. 2. Same as in Fig. 1, but for FRITIOF 2.0 without Bose–Einstein effect.
JETSET 7.3 [16] with exponential parametrization in $Q$ and measured parameters $r$ and $\lambda$ [5].

Fig. 2 and Fig. 3 show, respectively, the FRITIOF 2.0 predictions without and with BE correlations. The intermittency indices are given in columns 5 and 6 of Table 1.

FRITIOF without BE correlations (Fig. 2) strongly deviates from the data. FRITIOF with BE correlations (Fig. 3) can reproduce the rise of $F_n$, but, as already noted in [16], the effect is overestimated, particularly for high-order factorial moments.

In a more differential analysis, we study the angular dependence of factorial moments in intervals of the CMS production angle $\Theta_0$ defined as

$$\Theta_0 = \arccos\left(\frac{P \cdot P_{\text{beam}}}{|P| |P_{\text{beam}}|}\right). \quad (7)$$

Fig. 4 shows a strong $\Theta_0$-dependence of the intermittency indices $\phi_2$ and $\phi_3$, with the largest values obtained in $\Theta_0$ intervals close to $90^\circ$. For this $\Theta_0$ region, the effect is even stronger in $\Theta$ than in $Q^2$ (not shown).

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Fig. 3. Same as in Fig. 1, but for FRITIOF 2.0 with Bose–Einstein effect in exponential parametrization.
5. Conclusions

In this paper we present the first results on the opening-angle dependence of factorial multiplicity moments $F_q$ for various charge combinations. For like-charge particle combinations, the $F_q$ rise with decreasing cms opening angle $\theta$ between particles, for $q=2$–5. Only a weak dependence is observed for unlike-charge combinations. The intermittency indices $\phi_q$ depend strongly on the production angle $\theta_0$ of the particles. For the region close to $\theta_0 = 90^\circ$, the dependence of factorial moments is even stronger in $\theta$ than in $Q^2$. The standard version of the Monte Carlo program FRITIOF is unable to reproduce the data. When Bose–Einstein correlations are included, the model reproduces the rise of the factorial moments, but the effect is overestimated in particular for higher orders. Because of its relevance in a perturbative-QCD treatment of intermittency, the angular variable would be particularly well suited for an analysis of $e^+e^-$ data at LEP. A comparison to the present (soft) NA22 results might help in clarifying the importance of perturbative and non-perturbative contributions to hadron production in $e^+e^-$ annihilation at high energies.

6. Acknowledgement

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References


