Combined Forward-Backward Asymmetry Measurements in Top-Antitop Quark Production at the Tevatron

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The CDF and D0 experiments at the Fermilab Tevatron have measured the asymmetry between yields of forward- and backward-produced top and antitop quarks based on their rapidity difference and the asymmetry between their decay leptons. These measurements use the full data sets collected in proton-antiproton collisions at a center-of-mass energy of \( \sqrt{s} = 1.96 \) TeV. We report the results of combinations of the inclusive asymmetries and their differential dependencies on relevant kinematic quantities. The combined inclusive asymmetry is \( A_{FB}^{\tilde{t}} = 0.128 \pm 0.025 \). The combined inclusive and differential asymmetries are consistent with recent standard model predictions.

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The production of top and antitop quark (\( t\tilde{t} \)) pairs at the Tevatron proton-antiproton (\( p\bar{p} \)) collider at Fermilab is dominated by the \( q\bar{q} \) annihilation process, which can lead to asymmetries, \( A_{FB} \), in the number of top quarks produced within the hemisphere centered on the beam proton (forward) relative to those that are produced within the antiproton hemisphere (backward). In the standard model (SM), no forward-backward asymmetries are expected at leading order in perturbative quantum chromodynamics (QCD). However, contributions to the asymmetry from interference of leading order and higher-order amplitudes, and smaller offsetting contributions from the interference of initial- and final-state radiation, combine to yield a nonzero asymmetry [1–5]. Compared to older predictions [6] of the inclusive asymmetry at next-to-leading order (NLO) QCD, the latest higher-order corrections in QCD and electroweak (EW) theory are almost of the same size as the inclusive prediction at NLO QCD. Measurements of the inclusive asymmetries and their dependence on kinematic quantities of top quarks and their decay leptons are used to probe the \( t\tilde{t} \) production mechanism. Beyond-the-SM (BSM) interactions [7] can significantly alter the dynamics, even such that differential asymmetries can be strikingly changed while inclusive asymmetries are only marginally affected.

Inclusive and differential measurements [8,9] by the CDF [10] and D0 [11] Collaborations in 2011 were only marginally consistent with each other, and with then-existing SM predictions [6]. Both collaborations have since completed measurements using the full Tevatron Run II \( p\bar{p} \) collision data, corresponding to integrated luminosities between 9 and 10 fb\(^{-1}\). Assuming SM \( t \) and \( \bar{t} \) decays, they have measured asymmetries using events containing a single charged lepton (\( \ell + \) jets), where one \( W \) boson from a top quark decays to a charged lepton and a neutrino and the other decays to a quark and an antiquark that evolve into jets and in events containing two charged leptons (\( \ell\ell \)) where both \( W \) bosons decay leptonically. Both collaborations have measured inclusive and differential asymmetries as functions of kinematic quantities of the top quarks and their decay leptons. More refined analysis techniques have been employed since the initial measurements. In the \( \ell + \) jets channel, CDF performed a detailed investigation of the inclusive and differential \( t\tilde{t} \) asymmetries [12], and D0 used a novel partial event reconstruction for the inclusive and differential measurement of \( A_{FB}^{\tilde{t}} \) [13]. In the \( \ell\ell \) channel, CDF used several kinematic distributions to minimize the expected total uncertainty [14], while D0 carried out a simultaneous measurement of \( A_{FB}^{\tilde{t}} \) and the top quark polarization [15].

We present the combinations of the final CDF and D0 measurements and compare them with current SM calculations [16]. Careful assessment of the correlations of systematic uncertainties between analysis channels and experiments is required for comparing the data with predictions.

For reconstructed top and antitop quarks, \( A_{FB}^{\tilde{t}} \) is defined by

\[
A_{FB}^{\tilde{t}} = \frac{N(\Delta y_{\tilde{t}} > 0) - N(\Delta y_{\tilde{t}} < 0)}{N(\Delta y_{\tilde{t}} > 0) + N(\Delta y_{\tilde{t}} < 0)},
\]

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where $\Delta y_{\ell} = y_t - y_{\bar{t}}$ is the rapidity difference [17] between the $t$ and $\bar{t}$ quark, and $N$ is the signal yield in a particular configuration. Typically, measurements of $t\bar{t}$ forward-backward asymmetries require reconstruction of top and antitop quarks using all available information associated with the final-state particles [18]. Background contributions are subtracted from the yield of $t\bar{t}$ candidates, thereby providing the $t\bar{t}$ signal. The latter is corrected for detector effects, so as to unfold from the reconstructed $t$ and $\bar{t}$ quarks to the parton level.

The asymmetry in $t$ and $\bar{t}$ quark production also leads to asymmetries in their decay leptons which, while smaller in magnitude, do not need unfolding, but must be corrected for acceptance effects. The single-lepton asymmetry is defined by

$$A_{FB}^l = \frac{N(q_{+}\eta_\ell > 0) - N(q_{-}\eta_\ell < 0)}{N(q_{+}\eta_\ell > 0) + N(q_{-}\eta_\ell < 0)},$$

where $q_\ell$ is the sign of the electric charge and $\eta_\ell$ the pseudorapidity of the lepton in the laboratory frame. For the $\ell\bar{\ell}$ channel, the dilepton asymmetry is defined as

$$A_{FB}^{l\bar{l}} = \frac{N(\Delta \eta > 0) - N(\Delta \eta < 0)}{N(\Delta \eta > 0) + N(\Delta \eta < 0)},$$

where $\Delta \eta = \eta_{\ell} - \eta_{\bar{\ell}}$ is the pseudorapidity difference between the positive- and negative-charge lepton. The asymmetries obtained using top quarks and leptons are correlated, as a positive rapidity difference between a $t$ and a $\bar{t}$ quark is likely to produce a positive pseudorapidity difference between a positive- and negative-charge decay lepton.

Inclusive and differential measurements of $A_{FB}^l$ at the Tevatron were reported in Refs. [12,13] for the $\ell +$ jets channel and in Refs. [14,15] for the $\ell\bar{\ell}$ channel. Measurements of $A_{FB}^{l\bar{l}}$ for the $\ell +$ jets channel are given in Refs. [19,20] and in Refs. [21,22] for the $\ell\bar{\ell}$ channel. Measurements of $A_{FB}^l$ are reported in Refs. [21,22].

We combine the following CDF and D0 results using the best linear unbiased estimator (BLUE) [23–25]: the inclusive asymmetries $A_{FB}^l$, $A_{FB}^{l\bar{l}}$, and $A_{FB}^{l\bar{\ell}}$, each extrapolated to the full phase space relying on corresponding Monte Carlo simulations, and the differential asymmetry of $A_{FB}^l$ as a function of the invariant mass of the $t\bar{t}$ system ($m_{t\bar{t}}$). For combinations of inclusive asymmetries, the input uncertainties are symmetrized, while they are treated as asymmetric in the case of the combination of the asymmetry as a function of $m_{t\bar{t}}$. A mutually compatible classification of all systematic uncertainties is not available for $A_{FB}^l$ as a function of $|\Delta y_{\ell}|$. Hence, we provide results of a simultaneous least-squares fit to determine the slope parameter of the asymmetry in the CDF and D0 data, assuming a linear dependence. A similar fit is also provided for $A_{FB}^{l\bar{\ell}}$ as a function of $m_{t\bar{t}}$. The CDF and D0 differential asymmetries, $A_{FB}^l$ as a function of $q_{\ell}\eta_\ell$ and $A_{FB}^{l\bar{l}}$ as a function of $\Delta \eta$ are not combined, but are displayed together for ease of comparison.

Predictions of inclusive and differential $A_{FB}^l$ distributions at next-to-next-to-leading order (NNLO) QCD calculations are available from Ref. [1]. The contribution from EW NLO corrections to the NLO QCD asymmetries are not negligible [3]. Hence, we compare the measurements to the latest NNLO QCD + NLO EW inclusive and differential $A_{FB}^l$ calculations [1,26]. The combined inclusive-lepton asymmetries $A_{FB}^l$ and $A_{FB}^{l\bar{l}}$ are compared to the NLO QCD + NLO EW predictions of Ref. [3].

To accommodate correlations among analysis channels and between experiments, we classify systematic uncertainties into the following categories.

(i) Background modeling. The uncertainties in the distribution and normalization of the background are assumed to be uncorrelated since the backgrounds are estimated differently in different analyses, and in the two experiments.

(ii) Signal modeling. The uncertainties in modeling the signal, parton showering [27], initial- and final-state radiation [28], and color connections [29] are taken to be fully correlated among analysis channels and experiments because they all rely on the same assumptions.

(iii) Detector modeling. The uncertainties in jet-energy scale [30] and the modeling of the detector are fully correlated within each experiment and uncorrelated between the two experiments.

(iv) Method. The uncertainties in the methods used to correct for detector acceptance, efficiency, and potential biases in the reconstruction of top quark kinematic properties are mostly taken to be uncorrelated between experiments and analysis channels. However, the uncertainties on the phase-space correction procedures for the leptonic asymmetry in the D0/$\ell +$ jets [13] and $\ell\bar{\ell}$ [15] analyses are estimated using the same methods and are, therefore, correlated with each other but are uncorrelated with the CDF results.

(v) Parton-density distribution functions. The uncertainties in parton-density distribution functions (PDF) and the pileup in energy from overlapping $p\bar{p}$ interactions are treated as fully correlated between the analysis channels and the two experiments, because they characterize the same potential systematic biases.

The combined inclusive asymmetry is $A_{FB}^l = 0.128 \pm 0.021$ (stat) $\pm 0.014$ (syst), consistent with the NNLO QCD + NLO EW prediction of 0.095 $\pm$ 0.007 [2] within 1.3 standard deviations (SD). The combination has a $\chi^2$ of 1.7 for 3 degrees of freedom (DOF). BLUE also provides the weights in the combination for the CDF $\ell +$ jets, D0/$\ell +$ jets, CDF $\ell\bar{\ell}$, and D0$\ell\bar{\ell}$ results, which are 0.25, 0.64, 0.01, and 0.11, respectively.

The CDF and D0 differential $A_{FB}^l$ asymmetries as a function of $m_{t\bar{t}}$ are measured only for the $\ell +$ jets channel. We combine the D0 bins in the range of $350 < m_{t\bar{t}} < 550$ GeV/$c^2$ to provide uniform, 100-GeV/$c^2$-wide, bins...
The combined differential $A_t^\text{FB}$ values in bins of $m_t$, with the probability (Prob.) for the CDF and D0 inputs to agree with each other, with statistical (Stat.), systematic (Tot. syst.), and total uncertainties. The systematic uncertainties are broken down into uncertainties in the distribution of the background (Bkd. distr.), background normalization (Bkd. norm.), signal modeling (Signal), detector modeling (Det.), measurement method (Meth.), and parton distribution function (PDF).

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<tr>
<td>350–450</td>
<td>0.081</td>
<td>95%</td>
<td>0.037</td>
<td>0.031</td>
<td>0.009</td>
<td>0.012</td>
<td>0.004</td>
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<td>450–550</td>
<td>0.193</td>
<td>22%</td>
<td>0.048</td>
<td>0.042</td>
<td>0.010</td>
<td>0.016</td>
<td>0.007</td>
<td>0.006</td>
<td>0.007</td>
<td>0.006</td>
<td>0.023</td>
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<tr>
<td>550–650</td>
<td>0.258</td>
<td>98%</td>
<td>0.093</td>
<td>0.063</td>
<td>0.008</td>
<td>0.062</td>
<td>0.017</td>
<td>0.017</td>
<td>0.006</td>
<td>0.008</td>
<td>0.068</td>
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<tr>
<td>&gt; 650</td>
<td>0.319</td>
<td>8%</td>
<td>0.147</td>
<td>0.123</td>
<td>0.018</td>
<td>0.065</td>
<td>0.021</td>
<td>0.026</td>
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<td>0.019</td>
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for the combination. For the two measurements, we use covariance matrices [31] that take into account the bin-to-bin correlations from the unfolding of differential distributions. The correlations in systematic uncertainties among channels and experiments for each $m_t$ bin are assumed to be equal to those in the inclusive measurements. However, the uncorrelated background uncertainties for the differential asymmetries are subdivided into two separate components, one for the overall normalization and one for the differential distribution (shape) of the background. According to the different experimental methodologies, these are treated as correlated between bins for the CDF measurement and as uncorrelated for the D0 measurement. We verify that changing the correlations of systematic uncertainties between $-1$ and $+1$ has negligible impact on the combined result because the statistical uncertainties dominate.

The combined $A_t^\text{FB}$ values, and their statistical and systematic uncertainties for each category, are given in Table I, which also reports the probabilities for the CDF and D0 inputs to agree with each other in each mass bin. Overall, the differential combination has a χ² of 5.2 for 4 DOF. The correlations in the total uncertainties between $m_t$ bins are given in Ref. [31]. The values of $A_t^\text{FB}$ as a function of $m_t$ for each experiment and their combination are shown in Fig. 1, together with the NNLO QCD + NLO EW predictions [26].

The counter-intuitive value of the combined asymmetry in the 550–650 GeV/c² mass bin is due to the specific pattern of the CDF and D0 bin-to-bin correlations stemming from different choices in the regularized matrix unfolding. The opposite correlations observed between the 550–650 GeV/c² and the > 650 GeV/c² mass bins in the CDF (large and positive) and D0 (small and negative) measurements give rise to a combined asymmetry in the 550–650 GeV/c² mass bin that is smaller than that found in either measurement [31].

To reduce the correlations between the slope and the intercept, we use a linear fit of the form $A_t^\text{FB}(m_t) = \alpha_{m_t}(m_t - 450 \text{ GeV/c}^2) + \beta_{m_t}$ taking into account the correlations (see Table IV in Ref. [31]). The linear fit yields a slope of $\alpha_{m_t} = (9.71 \pm 3.28) \times 10^{-4} \text{ GeV}^{-1} \text{c}^2$ with an intercept at a $m_t$ value of 450 GeV/c² of $\beta_{m_t} = 0.131 \pm 0.034$. The fit has a χ² of 0.3 for 2 DOF. The values predicted at NNLO QCD + NLO EW are $\alpha_{m_t}^{\text{SM}} = (5.11^{+0.42}_{-0.64}) \times 10^{-4} \text{ GeV}^{-1} \text{c}^2$ and an intercept of $\beta_{m_t}^{\text{SM}} = 0.087^{+0.005}_{-0.006}$. The predicted dependence is determined by a linear fit to the binned prediction from Ref. [26]. The NNLO QCD + NLO EW binned predictions of the differential $A_t^\text{FB}$ and of the corresponding slope parameters agree with the combined experimental results to within 1.3 SD.

The differential $t\bar{t}$ asymmetry as a function of $|\Delta y_{t\bar{t}}|$ is available from CDF for both the $\ell +$ jets and $\ell \ell \ell$ channels, and from D0 for the $\ell +$ jets channel. The choice of binning differs for these measurements. We perform

![Figure 1](https://example.com/figure1.png)

**FIG. 1.** Results for $A_t^\text{FB}$ vs $m_t$ for the individual CDF and D0 measurements and for their combination. The inputs to the combination are displaced at different abscissa values within each $m_t$ bin for ease of visibility. The inner error bar indicates the statistical uncertainty, while the outer error bar corresponds to the total uncertainty including the systematic uncertainty added in quadrature. The value of the combined data point for the mass region of 550–650 GeV/c² is discussed in Ref. [31] in more detail. The linear dependence of the combined result is given by the solid black line together with the 1 SD total uncertainty of the two-parameter fit given by the shaded gray area. The dashed orange line shows the NNLO QCD + NLO EW prediction of Refs. [1,2,26], while the shaded orange area reflects its 1 SD uncertainty.
lepton asymmetries gives a prediction and the combined result differ by 1.5 SD. A fit to the binned NNLO QCD results is given by the solid black line, with the 1 SD total uncertainty on the one-parameter fit given by the shaded gray area. The dashed orange line shows the NNLO QCD + NLO EW prediction [1,2,26], while the shaded orange area reflects its 1 SD uncertainty.

a simultaneous least-squares fit to a linear function \( A_{FB}^\eta(\Delta y_{\eta}) = \alpha_{\Delta y_{\eta}} |\Delta y_{\eta}| \) for all available measurements, employing a combined 10 \( \times \) 10 covariance matrix \( C_{ij} \). We define \( \chi^2(\Delta y_{\eta}) = \sum_{ij} [y_i - f_i(\Delta y_{\eta})] C_{ij}^{-1} [y_j - f_j(\Delta y_{\eta})] \), with \( y_i \) and \( y_j \) representing the bin \( i \) and \( j \) of each of the three measurements, and \( f_i(\Delta y_{\eta}) \) and \( f_j(\Delta y_{\eta}) \) representing the expectations from a linear function. The definition of the asymmetry ensures that \( A_{FB}^\eta = 0 \) at \( \Delta y_{\eta} = 0 \). The correlations of the systematic uncertainties among analysis channels and experiments are assumed to be equal to those in the \( A_{FB}^\eta \) vs \( m_t \) measurements. Figure 2 shows the individual measurements and the result of the linear fit. The linear dependence for the combination is measured to be \( \alpha_{\Delta y_{\eta}} = 0.187 \pm 0.038 \) with a \( \chi^2 \) of 10.9 for 9 DOF. A fit to the binned NNLO QCD + NLO EW predictions of Ref. [1,2,26] gives the slope \( \alpha_{\Delta y_{\eta}}^{SM} = 0.129_{-0.012}^{+0.006} \). The prediction and the combined result differ by 1.5 SD.

The combined fit to the CDF and D0 inclusive single-lepton asymmetries gives \( A_{FB}^\eta = 0.073 \pm 0.016 \) (stat) \( \pm 0.012 \) (syst). The fit has a \( \chi^2 \) of 2.2 for 3 DOF, and the result is consistent with the NLO QCD+ prediction of \( 0.038 \pm 0.003 \) [3] to within 1.6 SD. The weights of the CDF \( \ell^+ + \text{jets} \), D0 \( \ell^+ + \text{jets} \), CDF \( \ell \ell \) and D0 \( \ell \ell \) results in the fit are 0.40, 0.27, 0.11, and 0.23, respectively. The individual CDF and D0 measurements of \( A_{FB}^\eta \) as a function of \( q_\ell \ell_\ell \) are shown in Fig. 3.

The combined fit to the CDF and D0 inclusive \( A_{FB}^\eta \) measurements yields \( A_{FB}^\eta = 0.108 \pm 0.043 \) (stat) \( \pm 0.016 \) (syst). The fit has a \( \chi^2 \) of 0.2 for 1 DOF, and the result is consistent with the NLO QCD + NLO EW prediction of \( 0.048 \pm 0.004 \) [3] to within 1.3 SD. The weights of the CDF and D0 \( \ell \ell \) results in the fit are 0.32 and 0.68, respectively. The individual CDF and D0 measurements of \( A_{FB}^\eta \) as a function of \( \Delta \eta \) are shown in Fig. 4.

In summary, we report combinations of the measurements of top-antitop quark forward-backward asymmetries performed in a \( p\bar{p} \) collision sample corresponding to 9–10 fb\(^{-1} \) collected by the CDF and D0 experiments at the Tevatron. The resulting combined inclusive asymmetry is \( A_{FB}^\eta = 0.128 \pm 0.025 \) compared to the prediction at NNLO QCD + NLO EW of 0.095 \( \pm 0.007 \). All three inclusive observables agree with the existing SM predictions.
predictions to within 1.6 standard deviations. The differential asymmetries as a function of $m_{\bar{t}t}$ and $\Delta y_{\bar{t}t}$ agree to within 1.5 standard deviations. All measurements favor somewhat larger positive asymmetries than the predictions, but none of the observed differences are larger than 2 standard deviations. Hence, we conclude that the measurements and their combinations, shown in Fig. 5, are consistent with each other and with the SM predictions.

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The rapidity $y$ is defined as $y(\beta, \theta) = 1/2 \ln \left( \frac{1 + \beta \cos(\theta)}{1 - \beta \cos(\theta)} \right)$, where $\theta$ is the polar angle relative to the proton beam and $\beta$ is the ratio of a particle’s momentum to its energy. The pseudorapidity is defined as $\eta = y(\theta, 1)$. 


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[17] The rapidity $y$ is defined as $y(\beta, \theta) = 1/2 \ln \left( \frac{1 + \beta \cos(\theta)}{1 - \beta \cos(\theta)} \right)$, where $\theta$ is the polar angle relative to the proton beam and $\beta$ is the ratio of a particle’s momentum to its energy. The pseudorapidity is defined as $\eta = y(\theta, 1)$. 


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