Combination of D0 measurements of the top quark mass

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I. INTRODUCTION

The top quark is the heaviest known elementary particle with a mass approximately twice that of the electroweak vector bosons, and factor of 1.4 larger than that of the more recently discovered Higgs boson [1]. Within the standard model (SM), this large mass arises from a large Yukawa coupling (\( \approx 0.9 \)) to the Higgs field. Consequently, loops involving the top quark contribute significantly to electroweak quantum corrections, and therefore a precise measurement of the top quark mass, \( m_t \), provides a means to test the consistency of the SM. Furthermore, the precise values of both the mass of the Higgs boson and the Yukawa coupling of the top quark may play a critical role in the history and stability of the universe (see e.g., Ref. [2]).

The top quark was discovered in 1995 by the CDF and D0 experiments during Run I (1992–1996) of the Fermilab Tevatron \( p\bar{p} \) collider at \( \sqrt{s} = 1.8 \text{ TeV} \) [3, 4]. Run II (2001–2011) at \( \sqrt{s} = 1.96 \text{ TeV} \) followed, providing a factor of \( \approx 150 \) more top-antitop quark pairs than Run I, and far more precise measurements of \( m_t \). Using \( t\bar{t} \) events produced in the D0 detector [5–8], we have measured \( m_t \) in different decay channels [9–15], using the full integrated luminosity of Run I (\( \int L \, dt = 0.1 \text{ fb}^{-1} \)) and Run II (\( \int L \, dt = 9.7 \text{ fb}^{-1} \)). This article reports the combination of these direct top quark mass measurements.

Direct measurements of the top quark mass have also been performed by the CDF experiment (see e.g., Ref. [16]) at the Tevatron, and by the ATLAS (see e.g., Ref. [17]) and CMS (see e.g., Ref. [18]) experiments at the CERN LHC. In 2012, the Tevatron experiments combined their measurements in Ref. [19] with the result \( m_t = 173.18 \pm 0.94 \text{ GeV} \). In 2014, a preliminary combination of ATLAS, CDF, CMS, and D0 measurements yielded \( m_t = 173.34 \pm 0.76 \text{ GeV} \). Both combinations are by now outdated as they do not include the latest and more precise measurements, in particular, the final D0 Run II measurements discussed in this article.

The top quark mass is a fundamental free parameter of the SM. However, its definition depends on the scheme of theoretical calculations used for the perturbative expansion in quantum chromodynamics (QCD). The inputs to the combination presented in this article are the direct measurements calibrated using Monte Carlo (MC) simulations. Hence, the measured mass corresponds to the MC mass parameter. However, because of the presence...
II. DECAY CHANNELS AND INPUT MEASUREMENTS

To measure the top quark mass, we use $p\bar{p} \rightarrow t\bar{t}$ events and assume that the top and antitop quark masses are equal [24,28]. Within the SM, the top quark decays into a $W$ boson and a $b$ quark almost 100% of the time. Different channels arise from the possible decays of the pair of $W$ bosons:

i. The “dilepton” channel ($\ell\ell'$) corresponds to events ($\approx 4.5\%$ of the total) where both $W$ bosons decay into electrons or muons. This channel is quite free from background but has a small yield. The background is mainly due to $Z$+jets production, but also receives contributions from diboson ($WW$, $WZ$, $ZZ$), $W$+jets, and multijet production.

ii. The “lepton+jets” channel ($\ell + \text{jets}$) corresponds to events ($\approx 30\%$ of the total) where one $W$ boson decays into $q\bar{q}'$ and the other into an electron or a muon and a neutrino. This channel has a moderate yield and a background arising from $W$+jets production, $Z$+jets production, and multijet processes.

iii. The “all jets” channel ($\approx 46\%$ of the total) has events in which both $W$ bosons decay to $q\bar{q}'$ that evolve into jets. The yield is high, but the background from multijet production is very large.

iv. The “tau channel” ($\approx 20\%$ of the total) arises from events in which at least one of the $W$ bosons decays into $\tau\nu_\tau$. As the decays $\tau \rightarrow \text{hadrons} + \nu_\tau$ are difficult to distinguish from QCD jets, it is not exploited for the top quark mass measurement. However, the $\tau \rightarrow \ell\nu_\ell\nu_\ell$ decays provide contributions to the $\ell\ell'$ and $\ell + \text{jets}$ channels.

The high mass of the top quark means that the decay products tend to have high transverse momenta ($p_T$) relative to the beam axis and large angular separations. Reconstructing and identifying $t\bar{t}$ events requires reconstruction and identification of high $p_T$ electrons, muons, and jets, and the measurement of the imbalance in transverse momentum in each event ($p_T^{\text{miss}}$) due to escaping neutrinos. In addition, identifying $b$ jets is an effective way of improving the purity of the selections. Good momentum resolution is required for all these objects, and the jet energy scale (JES) has to be known with high precision.

In the Run II $\ell + \text{jets}$ measurements, the uncertainty in the JES is reduced by performing an in situ calibration, which exploits the $W \rightarrow qq'$ decay by requiring the mass of the corresponding dijet system to be consistent with the mass of the $W$ boson (80.4 GeV). This calibration, determined using light-quark jets (including charm jets), is applied to jets of all flavors associated with $t\bar{t}$ decay. It is then propagated to the Run II $\ell\ell'$ measurements.

The input measurements of $m_t$ for the presented combination are shown in Table I and consist of measurements performed during Run I and Run II in the $\ell\ell'$ and $\ell + \text{jets}$ channels using the full data sets. D0 also measured the top quark mass using the “all jets” channel in Run I [29]; however, this measurement is not considered in the combination because its uncertainty is large and some subcomponents of the systematic uncertainty are not available. Just as in Run I, two $\ell\ell'$ mass measurements were performed in Run II using a neutrino weighting (NW) technique and a matrix element method (ME) [13]. We discuss their combination in the following section.

To combine the $m_t$ measurements, we use the Best Linear Unbiased Estimate (BLUE) [30], assuming Gaussian uncertainties, both for the $\ell\ell'$ Run II and the final D0 combinations.

III. COMBINATION OF RUN II DILEPTON MEASUREMENTS

In the $\ell\ell'$ channel, the presence of two undetected neutrinos with high $p_T$ makes it impossible to fully reconstruct the kinematics of the final state. To overcome this problem, we use two methods in Run II. The NW measurement [12] is based on a weight function for each event which is computed by comparing the $x$- and $y$- components of the observed $p_T$ and the hypothesized $p_T$ components of the neutrinos, integrating over the neutrino pseudorapidities [31]. The maximum weight value indicates the most likely value of $m_t$ in that event. The first and second moments of this function are retained as the event-by-event variables sensitive to $m_t$. Their distributions in MC events are used to form two-dimensional tem-
TABLE I: Summary of the input measurements to the combination. We indicate the method used to extract the mass of the top quark from the data (see the corresponding references for further details).

<table>
<thead>
<tr>
<th>Period</th>
<th>Channel</th>
<th>$f \ell dt$ $(fb^{-1})$</th>
<th>Method</th>
<th>$m_t$ (GeV)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run I</td>
<td>$\ell\ell'$</td>
<td>0.1</td>
<td>Combination of matrix weighting and neutrino weighting</td>
<td>168.4 ± 12.3 (stat) ± 3.6 (syst)</td>
<td>9, 10</td>
</tr>
<tr>
<td>Run I</td>
<td>$\ell + jets$</td>
<td>0.1</td>
<td>Matrix element</td>
<td>180.1 ± 3.6 (stat) ± 3.9 (syst)</td>
<td>11</td>
</tr>
<tr>
<td>Run II</td>
<td>$\ell\ell'$</td>
<td>9.7</td>
<td>Neutrino weighting</td>
<td>173.32 ± 1.36 (stat) ± 0.85 (syst)</td>
<td>12</td>
</tr>
<tr>
<td>Run II</td>
<td>$\ell\ell'$</td>
<td>9.7</td>
<td>Matrix element</td>
<td>173.93 ± 1.61 (stat) ± 0.88 (syst)</td>
<td>13</td>
</tr>
<tr>
<td>Run II</td>
<td>$\ell + jets$</td>
<td>9.7</td>
<td>Matrix element</td>
<td>174.98 ± 0.41 (stat) ± 0.63 (syst)</td>
<td>14, 15</td>
</tr>
</tbody>
</table>

plates that depend upon the value of $m_t$. The templates are compared to the data to extract $m_t$. The ME measurement uses per-event probability densities, based on the reconstructed kinematic information, obtained by integrating over the differential cross sections for the processes contributing to the observed events, using leading order matrix elements for the $t\bar{t}$ production process and accounting for detector resolution. The unmeasured neutrino momentum components are integrated out in this computation. The probability densities from all data events are combined to form a likelihood as a function of $m_t$, which is then maximized to determine $m_t$.

A. Statistical uncertainties and correlation

The statistical uncertainties of the individual NW and ME measurements are given in Table I. Both measurements are carried out using the same full D0 Run II data set, and similar selection criteria. Approximately 90% of the selected events are common to both analyses, and the measurements are therefore statistically correlated. We use an ensemble testing method to estimate these correlations. In the first step, we generate 1000 ensembles of simulated background and signal events with mass $m_t = 172.5$ GeV that pass the criteria of either the NW or the ME selection (see Refs. 12 and 13 for the detailed descriptions of the selections). Each ensemble is generated with the same number of events as observed in data, using the expected signal and background fractions, separately for the $ee$, $\mu\mu$, and $e\mu$ channels. The ME and NW ensembles are then obtained using the individual and slightly more restrictive selection criteria from each analysis, and $m_t$ is extracted following each of the analysis methods. From the two-dimensional distribution of the measured masses shown in Fig. 1, we obtain a statistical correlation of $\rho = 0.64 \pm 0.02$ between the two sets of measurements.

B. Systematic uncertainties in $\ell\ell'$ channel

The different contributions to the systematic uncertainty considered in the NW and ME measurements are reported in Table I. The sources of uncertainty are listed in the following and briefly described when the naming is not self-explanatory. More detailed descriptions are given in Refs. 12 and 13, and in Sec. IV for the signal modeling uncertainties.

In situ light-jet calibration: The statistical uncertainty of the JES calibration, determined in the $\ell + jets$ measurement using light-quark jets, and propagated to the $\ell\ell'$ measurements.

Response to $b$, $q$, and $g$ jets: The part of the JES uncertainty that originates from differences in detector response among $b$, light-quark, and gluon jets.

Model for $b$ jets: The part of the JES uncertainty that originates from uncertainties specific to the modeling of $b$ jets. This includes the dependence on semileptonic branching fractions and modeling of $b$ quark fragmentation.

Light-jet response: The part of the JES uncertainty that affects all jets and includes the dependence of the calibration upon jet energy and pseudorapidity, and the effect of the out-of-cone calorimeter showering correction.

Jet energy resolution
Jet identification efficiency

Multiple interaction model: The systematic uncertainty that arises from modeling the distribution of the number of interactions per Tevatron bunch crossing.

b tag modeling: The uncertainty related to the modeling of the b tagging efficiency for b, c, and light-flavor jets in MC simulation relative to data.

Electron energy resolution

Muon momentum resolution

Lepton momentum scale: The uncertainty arising from the calibration of electron energy and muon momentum scales.

Trigger efficiency: The uncertainties in the estimation of lepton-based trigger efficiencies.

Higher-order corrections: The modeling of higher-order corrections in the simulation of t¯t samples, obtained from the difference between the next-to-leading-order MC@NLO [32] and the leading-order ALPGEN [33] event generators.

Initial and final state radiation: The uncertainty due to the modeling of initial and final state gluon radiation.

Hadronization and underlying events: The uncertainty associated with the modeling of hadronization and the underlying event, estimated from the difference between different hadronization models.

Color reconnection: The uncertainty due to the model of color reconnection.

PDF: The uncertainty from the choice of parton density functions.

Transverse momentum of t¯t system: The uncertainty in the modeling of the distribution of the pT of the t¯t system.

Yield of vector boson + heavy flavor: The uncertainty in the production cross section for Z+b¯b and Z+c¯c relative to Z+jets events.

Background from simulation: The systematic uncertainty on the MC background, which includes the uncertainty from detector effects and the theoretical cross section. It does not include the uncertainties on the ratios of Z+b¯b and Z+c¯c to Z+jets cross sections, which belong to the previous category.

Background based on data: The uncertainties from the modeling of the multijet and W+jets backgrounds estimated using data.

Template statistics: In the NW measurement, this uncertainty arises from the statistical fluctuations of individual bins in signal and background templates. In the ME measurement, there is no such uncertainty as there is no template used to fit the data.

Calibration method: The calibration for both ME and NW measurements is determined using an ensemble testing method. We generate pseudo-experiments with the same number of events as observed in data, using MC events for signal and both MC and data-based samples for backgrounds. Ensembles at different top quark mass hypotheses are generated to determine a linear relation between the uncorrected measurement and the actual MC mass, i.e., to determine slope and offset parameters. The uncertainty in the calibration method arises from the uncertainty in the slope and offset parameters due to the limited size of the MC and data-based samples.

All systematic uncertainties are considered as fully correlated between ME and NW except for the calibration method uncertainty, as the calibrations were performed using almost independent event samples.

The differences between the ME and NW uncertainties reported in Table II are consistent with the expected statistical fluctuations in the various estimates. The fluctuations are ≈ 0.05–0.10 GeV, depending on the source, and their overall contributions are well below the total uncertainties. They therefore have a negligible impact on the overall uncertainties in the individual measurements and their combination.

C. ℓℓ′ combination

To obtain the ME and NW combination through the BLUE method we use the correlations and uncertainties discussed in Sec. III A and Sec. III B.

The result of the BLUE combination is \( m_t = 173.50 \pm 1.31 \text{ (stat)} \pm 0.84 \text{ (syst)} \) GeV. The breakdown of uncertainties is given in Table II. The weights for the NW and ME measurements are 71% and 29%, respectively. The NW and ME measurements agree with a \( \chi^2 \) of 0.2 for one degree of freedom, corresponding to a probability of 65%. As a test of stability, we change the statistical correlation between the two methods from 0.50 to 0.70 to conservatively cover the range of systematic and statistical uncertainty in its determination. The resulting \( m_t \) changes by less than 0.04 GeV.

This combination of the Run II ℓℓ′ measurements is used as an input to the overall combination discussed in the next sections.
TABLE II: Measurements in the $\ell\ell'$ channel with contributions to the uncertainties, and their combination. The total systematic uncertainty and the total uncertainty are obtained by adding the relevant contributions in quadrature. All values are given in GeV. The symbol “n/a” stands for “not applicable”.

<table>
<thead>
<tr>
<th>Source</th>
<th>Run II ME</th>
<th>Run II NW</th>
<th>$\ell\ell'$ combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top quark mass</td>
<td>173.93</td>
<td>173.32</td>
<td>173.50</td>
</tr>
<tr>
<td>In situ light-jet calibration</td>
<td>0.46</td>
<td>0.47</td>
<td>0.47</td>
</tr>
<tr>
<td>Response to $b$, $q$, and $g$ jets</td>
<td>0.30</td>
<td>0.27</td>
<td>0.28</td>
</tr>
<tr>
<td>Model for $b$ jets</td>
<td>0.21</td>
<td>0.10</td>
<td>0.13</td>
</tr>
<tr>
<td>Light-jet response</td>
<td>0.20</td>
<td>0.36</td>
<td>0.31</td>
</tr>
<tr>
<td>Jet energy resolution</td>
<td>0.15</td>
<td>0.12</td>
<td>0.13</td>
</tr>
<tr>
<td>Jet identification efficiency</td>
<td>0.08</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td>Multiple interaction model</td>
<td>0.10</td>
<td>0.06</td>
<td>0.07</td>
</tr>
<tr>
<td>$b$ tag modeling</td>
<td>0.28</td>
<td>0.19</td>
<td>0.22</td>
</tr>
<tr>
<td>Electron energy resolution</td>
<td>0.16</td>
<td>0.01</td>
<td>0.05</td>
</tr>
<tr>
<td>Muon momentum resolution</td>
<td>0.10</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>Lepton momentum scale</td>
<td>0.10</td>
<td>0.01</td>
<td>0.04</td>
</tr>
<tr>
<td>Trigger efficiency</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>Higher-order corrections</td>
<td>0.16</td>
<td>0.33</td>
<td>0.28</td>
</tr>
<tr>
<td>Initial and final state radiation</td>
<td>0.16</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>Hadronization and underlying event</td>
<td>0.31</td>
<td>0.11</td>
<td>0.17</td>
</tr>
<tr>
<td>Color reconnection</td>
<td>0.15</td>
<td>0.22</td>
<td>0.20</td>
</tr>
<tr>
<td>PDF</td>
<td>0.20</td>
<td>0.08</td>
<td>0.11</td>
</tr>
<tr>
<td>Transverse momentum of $t\bar{t}$ system</td>
<td>0.03</td>
<td>0.07</td>
<td>0.06</td>
</tr>
<tr>
<td>Yield of vector boson + heavy flavor</td>
<td>0.06</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>Background from simulation</td>
<td>0.06</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>Background based on data</td>
<td>0.07</td>
<td>0.00</td>
<td>0.02</td>
</tr>
<tr>
<td>Template statistics</td>
<td>n/a</td>
<td>0.18</td>
<td>0.13</td>
</tr>
<tr>
<td>Calibration method</td>
<td>0.03</td>
<td>0.07</td>
<td>0.05</td>
</tr>
</tbody>
</table>

**IV. UNCERTAINTY CATEGORIES IN THE OVERALL COMBINATION**

For the overall combination, the systematic uncertainties are grouped into sources of same or similar origin to form uncertainty categories. We employ categories similar to those used in the Tevatron top quark mass combination \[^{19}\] and use the same naming scheme.

**In situ light-jet calibration**: The part of the JES uncertainty that originates from the in situ calibration procedure using light-quark jets. This uncertainty has a statistical origin. For the Run II $\ell\ell'$ measurement, the uncertainty from transferring the $\ell +$ jets calibration to the dilepton event topology is included in the light-jet response category described below.

**Response to $b$, $q$, and $g$ jets**: As described in Sec. \[^{III\, B}\]

**Model for $b$ jets**: As described in Sec. \[^{III\, B}\]

**Light-jet response**: The part of the JES uncertainty that includes calibrations of the absolute energy-dependent response and the relative $\eta$-dependent response, and, for Run II, the out-of-cone calorimeter showering correction. This uncertainty applies to jets of any flavor.

**Out-of-cone correction**: The part of the JES uncertainty that originates from modeling of uncertainties associated with light-quark fragmentation and out-of-cone calorimeter showering corrections in Run I measurements. For Run II measurements, it is included in the light-jet response category.

**Offset**: This includes the uncertainty arising from uranium noise in the D0 calorimeter and from the corrections to the JES due to multiple interactions. While such uncertainties were sizable in Run I, the shorter integration time in the calorimeter electronics and the in situ JES calibration make them negligible in Run II.

**Jet modeling**: The systematic uncertainties arising from uncertainties in jet resolution and identification.

**Multiple interactions model**: As described in Sec. \[^{III\, B}\]

**$b$ tag modeling**: As described in Sec. \[^{III\, B}\]

**Lepton modeling**: The uncertainties in the modeling of the scale and resolution of lepton $p_T$, which were taken to be negligible in Run I.

**Signal modeling**: The systematic uncertainties arising from $tt$ event modeling, which are correlated across all measurements. This includes the sources described below. In Run I, the breakdown into the first four items could not be performed, because the MC generators used at that time did not have the same flexibility as the more modern generators. Instead, the overall signal modeling uncertainty was estimated by changing the main parameters of a MC generator or comparing results from two different generators.

i. The uncertainty associated with the modeling of initial and final state radiation, obtained by changing the renormalization scale in the scale-setting procedure relative to its default, as suggested in Ref. \[^{34}\]. Studies of $Z \rightarrow \ell\ell$ data indicate that a range of variation between factors of $\frac{1}{2}$ and 2 of this scale covers the mismodeling \[^{13}\].

ii. The uncertainty from higher-order corrections evaluated from a comparison of $tt$ samples generated using MC@NLO \[^{32}\] and ALPGEN \[^{33}\], both interfaced to HERWIG \[^{34}\], \[^{35}\] for the simulation of parton showers and hadronization.
iii. The systematic uncertainty arising from a change in the phenomenological description of color reconnection (CR) among final state partons [37]. It is obtained from the difference between event samples generated using PYTHIA [38] with the Perugia 2011 tune and using PYTHIA with the Perugia 2011NOCR tune [39].

iv. The systematic uncertainty associated with the choice for modeling parton-shower, hadronization, and underlying event. It includes the changes observed when substituting PYTHIA for HERWIG [35, 36] when modeling $t\bar{t}$ signal.

v. The uncertainty associated with the choice of PDF used to generate the $t\bar{t}$ MC events. It is estimated in Run II by changing the 20 eigenvalues of the CTEQ6.1M PDF [40] within their uncertainties. In Run I, it was obtained by comparing CTEQ3M [41] with MRSA [42] for $\ell\ell$, and CTEQ4M [43] with CTEQ5L [44] for $t+\text{jets}$ events.

**Background from theory:** This systematic uncertainty on background originating from theory takes into account the uncertainty in modeling the background sources. It is correlated among all measurements in the same channel, and includes uncertainties on background composition, normalization, and distributions.

**Background based on data:** This includes uncertainties associated with the modeling of multijet background in the $t+\text{jets}$ channel, and multijet and $W+\text{jets}$ backgrounds in the $\ell\ell'$ channel, which are estimated using data. This also includes the effects of trigger uncertainties determined from the data.

**Calibration method:** The uncertainty arising from any source specific to a particular fitting method, includes effects such as the finite number of MC events available to calibrate each method.

Table III summarizes the input measurements and their corresponding statistical and systematic uncertainties.

V. CORRELATIONS

The following correlations are used to combine the measurements:

i. The uncertainties listed as ‘statistical uncertainty’, ‘calibration method’, and ‘background based on data’ are taken to be uncorrelated among the measurements.

ii. The uncertainties in the ‘in situ light-jet calibration’ category are taken to be correlated among the Run II measurements since the $\ell\ell'$ measurement uses the JES calibration determined in the $\ell+\text{jets}$ channel.

iii. The uncertainties in ‘response to $b$, $q$, and $g$ jets’, ‘jet modeling’, ‘$b$ tag modeling’, ‘multiple interaction model’, and ‘lepton modeling’ are taken to be 100% correlated among Run II measurements.

iv. The uncertainties in ‘out-of-cone correction’ and ‘offset’ categories are taken to be 100% correlated among Run I measurements.

v. The uncertainties in ‘model for $b$ jets’ and ‘signal modeling’ categories are taken to be 100% correlated among all measurements.

vi. The uncertainties in ‘light-jet response’ are taken to be 100% correlated among the Run I and the Run II measurements, but uncorrelated between Run I and Run II.

vii. The uncertainties in ‘background from theory’ are taken to be 100% correlated among all measurements in the same channel.

A summary of the correlations among the different systematic categories is shown in Table IV using the inputs from Table III and the correlations specified in Table IV we obtain an overall matrix of correlation coefficients in Table V.

VI. RESULTS

We combine the D0 input measurements of Table III using the BLUE method. The BLUE combination has a $\chi^2$ of 2.5 for 3 degrees of freedom, corresponding to a probability of 47%. The pulls and weights for each of the inputs obtained from the BLUE method are listed in Table VI. Here, the pull associated to each input value $m_i$ with uncertainty $\sigma_i$ is calculated as $\frac{(m_i-m_0)}{\sqrt{\sigma_i^2+\sigma_m^2}}$, where $\sigma_m^2$ is the uncertainty in the combination, and indicates the degree of agreement of the input with the combined value. The weight $w_i$ given to the input measurement $m_i$ is $w_i = \sum_{i=1}^{4} (\text{Cov}^{-1})_{ij}/N$, where Cov is the covariance matrix of the input measurements, and $N$ is a normalization term ensuring $\sum_{i=1}^{4} w_i = 1$. The covariance matrix expressed in terms of the correlation coefficients between the measurements $c_{ij}$ (with the convention $c_{ii} = 0$) is: $\text{Cov}_{ij} = \sigma_i \sigma_j \delta_{ij} + c_{ij}$, where $\delta_{ij}$ is the Kronecker $\delta$. At first order in the correlation coefficients, its inverse is given by $(\text{Cov}^{-1})_{ij} = \frac{1}{\sigma_i \sigma_j} (\delta_{ij} - c_{ij})$, so that the weight $w_i$ can be written as $w_i = \frac{1}{\sigma_i}(1 - \sum_{i} \delta_{ij} c_{ij})/N'$, $N'$ being a normalization term. This expression shows that the weight for the Run I $\ell\ell'$ measurement is negative mainly because the correlation with the Run II $\ell+\text{jets}$ measurement (0.07) is larger than the ratio of their uncertainties (0.76/12.7).
The resulting combined value for the top quark mass is

\[ m_t = 174.95 \pm 0.40 \text{ (stat)} \pm 0.64 \text{ (syst)} \text{ GeV}. \]

Adding the statistical and systematic uncertainties in quadrature yields a total uncertainty of 0.75 GeV, corresponding to a relative precision of 0.43% on the top quark mass. The breakdown of the uncertainties is shown in Table VII. The dominant sources of uncertainty are the statistical uncertainty, the JES calibration, which has statistical origin, and the modeling of the signal. The total statistical and systematic uncertainties are reduced relative to the published D0 and CDF combination [19] due primarily to the latest and most accurate D0 + jets analysis [23]. As a test of stability, we vary the correlation of the dominant source of uncertainties, 'signal modeling', from 100% to 0%, first between Run I and Run II measurements, and in a second check between all measurements. The combined value of \( m_t \) does not change by more than 50 MeV, while the uncertainty changes by no more than 20 MeV. This is due to the fact that the Run II + jets measurement dominates the combination with a weight of 96%. Thus, the combination is not sensitive to the detailed description of the correlation of systematic uncertainties. Due to a much smaller total uncertainty resulting in the large weight for the + jets measurement, the improvement in the combined uncertainty relative to the individual + jets uncertainty is smaller than 10 MeV.

The input measurements and the resulting D0 average mass of the top quark are summarized in Fig. 2 along with the top quark pole mass extracted by D0 from the measurement of the \( t \bar{t} \) cross section [22].

VII. SUMMARY

We have presented the combination of the measurements of the top quark mass in all D0 data. Taking into account the statistical and systematic uncertainties and their correlations, we find a combined average of \( m_t = 174.95 \pm 0.75 \text{ GeV} \). This measurement with, a relative precision of 0.43%, constitutes the legacy Run I
TABLE VII: Combination of D0 measurements of $m_t$ and contributions to its overall uncertainty. The uncertainty categories are defined in the text. The total systematic uncertainty and the total uncertainty are obtained by adding the relevant contributions in quadrature.

<table>
<thead>
<tr>
<th>D0 combined values (GeV)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>top quark mass</td>
<td>174.95</td>
</tr>
<tr>
<td>In situ light-jet calibration</td>
<td>0.41</td>
</tr>
<tr>
<td>Response to $b$, $q$, and $g$ jets</td>
<td>0.16</td>
</tr>
<tr>
<td>Model for $b$ jets</td>
<td>0.09</td>
</tr>
<tr>
<td>Light-jet response</td>
<td>0.21</td>
</tr>
<tr>
<td>Out-of-cone correction</td>
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<td>Jet modeling</td>
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<td>$b$ tag modeling</td>
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<td>Signal modeling</td>
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<td>Background based on data</td>
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<td>Calibration method</td>
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<td>Systematic uncertainty</td>
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<td>Statistical uncertainty</td>
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<td>Total uncertainty</td>
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</table>

and Run II measurement of the top quark mass in the D0 experiment.

VIII. ACKNOWLEDGMENTS

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[14] V. M. Abazov et al. (D0 Collaboration), Precision measurement of the top-quark mass in lepton+jets final...
<table>
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<th>D0</th>
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<th>m_t ± stat ± syst</th>
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<td>Run I Dileptons</td>
<td>0.1 fb^{-1}</td>
<td>168.4 ± 12.3 ± 3.6 GeV</td>
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<td>D0 combined</td>
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<td>9.7 fb^{-1}</td>
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</table>

**FIG. 2:** A summary of the top quark mass measurements used in the D0 combination [9–15], along with the D0 final result, and the top quark pole mass extracted from the D0 cross section measurement [23]. The latter is not used in the combination. The inner red uncertainty bars represent the statistical uncertainties, while the blue bars represent the total uncertainties. For comparison, we also show the preliminary 2014 world average of m_t [20] which includes D0 Run II ℓℓ′ and ℓ + jets measurements that are now superseded. For the top quark pole mass extracted from the D0 cross section measurement, a 1.1 GeV theory uncertainty is included in the systematic uncertainty, and the statistical uncertainty is determined such as its relative contribution to the experimental uncertainty is the same as for the cross-section measurement.
[18] V. Khachatryan et al. (CMS Collaboration), Measurement of the top quark mass using proton-proton data at $\sqrt{s} = 7$ and 8 TeV, Phys. Rev. D 93, 072004 (2016).
[23] V. M. Abazov et al. (D0 Collaboration), Measurement of the inclusive $t\bar{t}$ production cross section in $pp$ collisions at $\sqrt{s} = 1.96$ TeV and determination of the top quark pole mass, Phys. Rev. D 94, 092004 (2016).
[31] The D0 coordinate system is right-handed, with the $z$-axis pointing in the direction of the Tevatron proton beam and the $y$-axis pointing upwards. The pseudorapidity is defined as $\eta = -\ln(\tan(\theta/2))$, where $\theta$ is the polar angle relative to $z$-axis.