Planck intermediate results. L. Evidence for spatial variation of the polarized thermal dust spectral energy distribution and implications for CMB B-mode analysis


(Affiliations can be found after the references)

Preprint online version: June 24, 2016

ABSTRACT

The characterization of the Galactic foregrounds has been shown to be the main obstacle in the challenging quest to detect primordial B-modes in the polarized microwave sky. We make use of the Planck HFI 2015 data release at high frequencies to place new constraints on the properties of the polarized thermal dust emission at high Galactic latitudes. Here, we specifically study the spatial variability of the dust polarized spectral energy distribution (SED), and its potential impact on the determination of the tensor-to-scalar ratio, r. We use the correlation ratio of the Cℓ/μ angular power spectra between the 217- and 353-GHz channels as a tracer of these potential variations, computed on different high Galactic latitude regions, ranging from 80% to 20% of the sky. The new insight from Planck is a departure of the correlation ratio from unity that cannot be attributed to a spurious decorrelation due to the cosmic microwave background, instrumental noise, or statistical systematics. The effect is marginally detected on each region, but the statistical combination of all the regions gives more than 99% confidence for this variation in polarized dust properties. In addition, we show that the decorrelation increases when there is a decrease in the mean column density of the region of the sky being considered, and we propose a simple power-law empirical model for this dependence, which matches what is seen in the Planck data. We explore the effect that this measured decorrelation has on simulations of the BICEP2-Keck Array/Planck analysis and show that the 2015 constraints from those data still allow a decorrelation between the dust at 150 and 353 GHz of the order of the one we measure. Finally, using simplified models, we show that either spatial variation of the dust SED or of the dust polarization angle could produce decorrelations similar to those we observe in the data.

Key words. Interstellar medium: dust – Submillimeter: ISM – Polarization – Cosmic background radiation

1. Introduction

Since the combined BICEP2-Keck Array and Planck analysis (BICEP2-Keck Array and Planck Collaborations

Corresponding authors:
L. Montier, Ludovic.Montier@irap.omp.eu
J. Aumont, jonathan.aumont@ias.u-psud.fr

Planck (http://www.esa.int/Planck) is a project of the European Space Agency (ESA) with instruments provided by two scientific consortia funded by ESA member states and led by Principal Investigators from France and Italy, telescope reflectors provided through a collaboration between ESA and a scientific consortium led and funded by Denmark, and additional contributions from NASA (USA).
Characterization of these foregrounds is today the main limitation in the quest for the gravitational wave signature in the $B$-mode cosmic microwave background (CMB) power spectrum, i.e., for measuring the tensor-to-scalar ratio, $r$. This endeavour has been led by the analysis of the Galactic dust foregrounds carried out using Planck data (Planck Collaboration Int. XXX 2016, hereafter PIPXXX), and more recently of the synchrotron foregrounds using Planck and WMAP data (Planck Collaboration XXV 2016; Choi & Page 2015; Krachmalnicoff et al. 2015). It appears that Galactic thermal dust currently represents the major contaminant at high latitude in the spectral bands mainly adopted to search for the CMB signal by ground-based and balloon-borne experiments, i.e., between 100 and 220 GHz (Planck Collaboration Int. XXII 2015; Planck Collaboration X 2016).

The efficiency of Galactic dust cleaning for the cosmological $B$-mode analyses is based on two main factors: the accuracy of the Galactic polarized dust template; and the way it is extrapolated from the submillimetre to the millimetre bands. While recent forecasts of cosmological $B$-modes detection for ground-based, balloon-borne, and satellite experiments (Creminell et al. 2015; Errard et al. 2015) appear to allow for very good sensitivity (down to $r \approx 2 \times 10^{-3}$) with only a few observational bands, when assuming a simple modelling of the foreground emission, other studies have shown that even in a global component separation framework, accurate modelling of the polarized dust spectral energy distribution (SED) is needed in order to reach the required very low levels of contamination of the cosmological $B$-modes by Galactic dust residuals (Armitage-Caplan et al. 2012; Remazeilles et al. 2015). Incorrect modelling of the dust SED would lead to biased estimates of the $r$ parameter.

Several investigations to quantify the spatial variability of the polarized dust SED were initiated using the Planck data. A first estimate of the polarized dust spectral index was discussed in Planck Collaboration Int. XXII (2015) for regions at intermediate Galactic latitudes. Over 39% of the sky the averaged value has been shown to be slightly larger than the dust spectral index in intensity, with a value of $\beta_T = 1.59$, compared with $\beta_T = 1.51$. More interestingly, an upper limit of its spatial dispersion was obtained by computing the standard deviation of the mean polarized dust spectral index estimated on 352 patches of 10° diameter, yielding a dispersion of 0.17. However, this estimate was shown to be dominated by the expected Planck noise, and does not allow us to build a reliable model of these spatial variations.

A second early approach, in PIPXXX, investigated the correlation ratio between the 217- and 353-GHz Planck bands, which is a statistical measurement of the dust SED spatial variation, as we will discuss thoroughly in the following sections. This ratio was computed on large fractions of the sky and led to an upper limit of 7% decorrelation between 217 and 353 GHz. An initial estimate of the impact of a possible dust polarization decorrelation between frequencies, due to a spatial variation of its SED, was performed in the BKP15 analysis. A loss of 10% in the 150 GHz $\times$ 353 GHz cross-spectrum in the joint analysis, due to the decorrelation, was estimated to produce a positive bias of 0.018 on the determination of $r$.

In this new study, we present an improved analysis of the correlation ratio between the 353-GHz and 217-GHz Planck bands as a tracer of the spatial variations of the polarized dust SED. We derive new constraints on these variations and look at the impact they would have on the determination of the cosmological $B$-mode signal.

This paper is organized as follows. We first present the data used in this work in Sect. 2. The analysis of the dust polarization correlation ratio is described in Sect. 3. The impact of the polarized dust decorrelation on the determination of the tensor-to-scalar ratio $r$ is illustrated in Sect. 4. We discuss the possible origin of the spatial variations of the polarized dust SED in Sect. 5, before concluding in Sect. 6.

2. Data and region selection

2.1. Planck data

In this paper, we use the publicly available Planck High Frequency Instrument (HFI) data at 217 GHz and 353 GHz (Planck Collaboration I 2016). The signal-to-noise ratio of dust polarization in the Planck-HFI maps at lower frequencies does not allow us to derive significant results in the framework adopted for this study and consequently the other channels are not used. These data consist of a set of maps of the Stokes $Q$ and $U$ parameters at each frequency, projected onto the HEALPix pixelization scheme (Górski et al. 2005). The maps and their properties are described in detail in Planck Collaboration VIII (2016).

We also use subsets of these data in order to exploit the statistical independence of the noise between them. As described in Planck Collaboration VIII (2016), the data were split into either the time or the detector domains. In this paper we use two particular data splits.

- The so-called “detector-set” maps (hereafter DS, also sometimes called “DetSets”). Planck-HFI measures the sky polarization thanks to polarization sensitive bolometers (PSBs), which are each sensitive to one direction of polarization. PSBs are assembled in pairs, the angle between two PSBs in a pair being 90° and the angle between two pairs being 45°, allowing for the reconstruction of Stokes $I$, $Q$, and $U$. Eight such pairs are available at 217 and 353 GHz. These were split into two subsets of four pairs to produce two noise-independent sets of $Q$ and $U$ maps called “detector-sets,” for each frequency band.

- The so-called “half-mission” maps (hereafter HM). Planck-HFI completed five independent full-sky surveys over 30 months. Surveys 1 and 2 constitute the Planck “nominal mission” and this was repeated a second time during Surveys 3 and 4. A fifth survey was performed with a different scanning strategy, but was not included in the released half-mission maps. Thus, Planck-HFI data can be split into two noise-independent sets of $Q$ and $U$ maps, labelled “HM1” and “HM2,” for each frequency band.

2.2. Region selection

In order to focus on the most diffuse areas of the Galactic dust emission, we have performed our analysis on various fractions of the sky using the set of science regions introduced in PIPXXX. These have been constructed to reject regions with CO line brightness $I_{CO} \geq 0.4 \text{ K km s}^{-1}$.
The complement to this mask by itself defines a preliminary region that retains a sky fraction $f_{\text{sky}} = 0.8$. In increments of $f_{\text{sky}} = 0.1$, the retained regions can be identified by the colours yellow (0.3) to blue (0.8), inclusively. The LR63N and LR63S regions are displayed in black and white, respectively, and the LR63 region is the union of the two. Also shown is the (unapodized) point source mask used.

As previously stated in PIPXXX, the dust polarization angular power spectra computed on these regions can be considered as approximately statistically independent because most of the power arises from the brightest 10% of a given region, the same 10% that differentiates one (preliminary) region from another.

More specifically, for the $C_\ell^{BB}$ power spectra computed with the 353-GHz Planck data, the difference (non-overlapping) region $LR_x - LR_{x-1}$, $i > 1$ (about 10% of the sky, by definition), contains more than 75% of the power computed on $LR_x$, for $x \in \{24, 33, 42, 53, 63, 72\}$. LR63N and LR63S are of course independent of one another, but not of LR63.

We do not use the non-overlapping regions (which would be fully statistically independent), preferring the LR regions in order to be able to relate the present work to the previous Planck analyses in general and to PIPXXX in particular.

### 2.3. Simulations

The simulated polarization maps presented in this work have been built using a simplified 2-component model consisting of dust plus CMB, both simulated as stochastic realizations with a Gaussian distribution. CMB maps are defined as realizations based on the Planck Collaboration best-fit ΛCDM model (Planck Collaboration XIII 2016), assuming a tensor-to-scalar ratio $r = 0$ and an optical depth $\tau = 0.06$ (Planck Collaboration et al. 2016a,b). The dust component maps at 353 GHz are defined as Gaussian realizations using a power-law model of the $EE$ and $BB$ angular power spectra (with a spectral index equal to $-0.42$ and amplitudes matching Table 1 of PIPXXX), following the prescriptions of PIPXXX, and normalized for each region of the sky introduced in Sect. 2.2. Dust maps at other frequencies are scaled with a constant modified blackbody spectrum with $B_\nu^{\text{dust}} = 1.59$ and $T_d = 19.6$ K (Planck Collaboration Int. XVII 2014; Planck Collaboration Int. XXII 2015).

The instrumental noise component is then introduced in each pixel using the Planck $Q$ and $U$ covariance maps at 217 and 353 GHz, associated with the DS and HM data setups. We checked using a set of 1000 simulations that in the range of multipoles $\ell = 50$–700 the instrumental noise built from covariances was consistent with the Full Focal Plane Monte Carlo noise simulations, namely FFP8 (Planck Collaboration XII 2016) at 217 and 353 GHz, which are publicly available through the Planck Legacy Archive.² For each of the regions described in Sect. 2.2, we built 1000 independent dust, CMB and noise realizations with these properties.

We note that we did not directly use the FFP8 maps in this analysis because of two issues with the polarized dust component in the Planck Sky Model (PSM, Delabrouille et al. 2013) used to build these simulations. First, the polarized dust maps include Planck instrumental noise because they are computed from the Planck 353-GHz maps smoothed to 30'. This contribution significantly enhances the dust signal at high Galactic latitudes, where the signal-to-noise ratio of the data is low at this resolution. The second issue comes from the fact that the polarized dust component in the PSM already includes spatial variations of the polarized dust spectral index. Thus, it cannot be used to perform a null-test for no spatial variation of the SED.

### 3. The dust polarization correlation ratio

#### 3.1. Definition

We statistically estimate the spatial variation of the dust SED by constructing the correlation ratio for the dust between 217 and 353 GHz, as a function of the multipole $\ell$. This is defined as the ratio of the cross-spectrum between these bands and the geometric mean of the two auto-spectra in the same bands, i.e.,

$$R_{XX}^{XY} = \frac{C_{XX}^{XY}(353 \times 217)}{\sqrt{C_{XX}^{XX}(353 \times 353) C_{XX}^{XY}(217 \times 217)}}, \quad (1)$$

where $X \in \{E, B\}$. If the maps at 217 and 353 GHz contain only dust and the dust SED is constant over the region for which the power spectra are computed, then $R_{XX}^{XY} = 1$. However, if the dust is not the only component to contribute to the sky polarization or if the dust SED varies spatially, then the ratio is expected to deviate from unity. Nevertheless, the spatial variations of the SED, as we will see in the next subsection, do not affect the ratio $R_{XX}^{XY}$ at the largest scales. This is the reason why the study in PIPXXX (where the ratio $R_{XX}^{XY}$ was computed from the fitted amplitude of power laws dominated by the largest scales)

²http://www.cosmos.esa.int/web/planck/pla
found no significant deviation from unity. In what follows we conduct this analysis for $\ell > 50$ to avoid any significant contribution from Planck systematics effects at low $\ell$ (Planck Collaboration VIII 2016).

In order to avoid any bias issues due to the noise auto-correlation, and in order to minimize the systematic effects, the auto- and cross-spectra are computed using independent sets of Planck data, namely the half-mission and detector-set maps (Planck Collaboration VIII 2016). Hence the auto-spectra are computed for frequency $\nu$ as

$$C_\ell^{\nu \nu}(v \times v) \equiv C_\ell^{\nu \nu}(D_1 \times D_2^2),$$

where $D_1$ and $D_2$ are the two independent sets of data, e.g., DS1 and DS2, or HM1 and HM2 (see Sect. 2.1). Similarly, the cross-spectra between two frequencies, $\nu_1$ and $\nu_2$, are given by

$$C_\ell^{\nu_1 \nu_2}(v_1 \times v_2) \equiv \frac{1}{4} \sum_{i,j} C_\ell^{\nu_1 \nu_2}(D_i \times D_j^2),$$

where the indices $i$ and $j$ take the values 1 and 2. The results will be labelled “HM” or “DS” when obtained using the half-mission or detector-set maps, respectively.

The spectra have been computed using Xpol (Tristram et al. 2005), which is a polarization pseudo-$C_\ell$ estimator that corrects for incomplete sky coverage and pixel and beam window functions.

3.2. Planck measurements

The $EE$ and $BB$ dust correlation ratios obtained in four multipoles ranges ($\ell = [50, 160], [160, 320], [320, 500], and [500, 700]$) are shown in Fig. 2 for the Planck HM and DS data versions, and a fraction of the sky $f_{\text{sky}} = 0.7$, i.e., the LR63 region.

Since the definition of the correlation ratio, $R^{XX}_\ell$, uses nonlinear operations (such as ratio and square root), the associated uncertainties are not trivial to estimate and so are determined from the median absolute deviation of 1000 Monte Carlo noise realizations, including Planck instrumental noise, as detailed in Sect. 2.3.

The correlation ratios between the 217- and 353-GHz bands that might be expected are computed using the simulation setup introduced in Sect. 2.3, and defined by a 2-component modelling of dust plus CMB signals, assuming no spatial variations of the dust SED and no noise. If we suppose that the Planck maps at 217 and 353 GHz are a sum of CMB and dust components, then (in thermodynamic units and assuming no instrumental noise) we have

$$M_{353} = M_{\text{dust}} + M_{\text{CMB}},$$

$$M_{217} = \alpha M_{\text{dust}} + M_{\text{CMB}},$$

where $M_{\text{dust}}$ is the dust map at 353 GHz, $M_{\text{CMB}}$ is the CMB map, $\alpha$ is a constant scaling coefficient representing the dust SED, and $M$ can represent the Stokes parameters $Q$ and $U$. Then, combining Eqs. (1) and (4) and assuming that the dust and CMB components are not spatially correlated, the expected correlation ratio becomes

$$R^{XX}_\ell = \frac{\alpha^2 C^{XX}_{\ell,\text{dust}} + C^{XX}_{\ell,\text{CMB}}}{\alpha^2 (C^{XX}_{\ell,\text{dust}})^2 + (C^{XX}_{\ell,\text{CMB}})^2 + (1 + \alpha^2) C^{XX}_{\ell,\text{dust}} C^{XX}_{\ell,\text{CMB}}}^{1/2},$$

where $X \in \{E, B\}$. It can clearly be seen that even if $\alpha$ is a constant, the CMB component will make the correlation ratio $R \neq 1$. The model power spectra corresponding to Eq. (5) are also displayed in Fig. 2.

The Planck $EE$ data match the expected $EE$ correlation ratio and are strongly dominated by the CMB signal. Two approaches have been considered for potentially removing the CMB component from the correlation ratio in order to see the effect of the dust decorrelation in the $EE$ spectra, namely analysis in either pixel space or multipole space. The noise on the CMB template, subtracted from the Planck maps, would produce an auto-correlation of the noise when building the correlation ratios and would strongly impact our analysis in polarization; this argues against using the first (pixel-based) option. Moreover, the second option, which consists of correcting the 217- and 353-GHz Planck cross-spectra by subtracting a model of the CMB power spectrum, is affected by the cosmic variance of the CMB, which is dominant compared to the dust component in the
353 GHz. Since the denominator of levels expressed in terms of son, we cannot give symmetrical error bars and significance
CMB, and noise can be highly non-Gaussian. For this rea-
zero, the distribution of the correlation ratios from dust,
shape of the distribution. As we will see, the
Planck
situity to exceed (PTE), which makes no assumption about the
ration ratios are available for all multipole ranges and re-
results averaged over the lowest multipole bin are shown as a function of the mean column
density (see Table 1).

3.3. Significance of the Planck measurements

The significance of the Planck data correlation ratios can be quantified using the distribution of \(R_{\text{BB}}\) computed with simulations including Planck instrumental noise at 217 and 353 GHz. Since the denominator of \(R_{\text{BB}}\) can get close to zero, the distribution of the correlation ratios from dust, CMB, and noise can be highly non-Gaussian. For this reason, we cannot give symmetrical error bars and significance levels expressed in terms of \(\sigma\). Instead we use the probability to exceed (PTE), which makes no assumption about the shape of the distribution. As we will see, the Planck measurements of the correlation ratios in the DS and HM data sets appear systematically smaller than the most probable correlation ratio in our simulations that include instrument-
noise.

The impact of the CMB and the noise on the correlation ratio can be seen in Fig. 3 via the grey vertical histograms. These are based on the correlation ratios obtained on a set of 1 000 Monte Carlo realizations, including Gaussian Planck noise on top of the simulated CMB and dust components (see Sect. 2.3). The distributions of the simulated correlation ratios are available for all multipole ranges and re-
ions in Appendix A (in Figs. A.1–A.9). While noise barely affects the most probable ratios in the first multipole bin \(\ell = 50–160\) when compared to the theoretical expectation (blue), it can create an important level of decorrelation in
the other bins, particularly in the smallest regions (up to an additional 30 % in the fourth multipole bin for LR16).

In order to quantify the significance of the Planck measurements with respect to the simulated decorrelation from CMB and noise, we compute the PTE, defined as the probability of a simulation having more decorrelation (i.e., smaller correlation ratio) than the data. This is computed as the fraction of the 1 000 realizations having a correla-
tion ratio smaller than the Planck measurements, for each multipole bin and HM/DS case. These PTEs are reported in Table 1. We also compute a combined PTE over all multipole bins, defined as the probability to obtain correlation ratios smaller than Planck measurements simultaneously in all four multipole bins \((50 < \ell < 700)\).

The combined PTEs (for \(50 < \ell < 700\)) are < 1.5 % for the DS case and < 0.1 % for the HM case. When focusing on individual multipole bins, the detection level is not as strong; the PTE values range between 0.6 % and 11.2 % in the first multipole bin \((50 < \ell < 160)\) for both DS and HM cases. In the second multipole bin \((160 < \ell < 320)\), the DS correlation ratio PTEs range from 11.5 to 47.4 %, while for the HM case they range from 1.1 to 39.9 % (with significant PTEs on several regions though). The third and fourth multipole bins \((320 < \ell < 700)\) show no significant evidence for decorrelation.

The significant excess of decorrelation in the Planck data between 217 and 353 GHz, especially in the first multi-
pole bin \((50 < \ell < 160)\) is consequently very unlikely to be attributable to CMB or to instrumental noise (or to system-
atic effects, as discussed in the next section). We therefore conclude that this excess is a statistical measurement of the spatial variation of the polarized dust SED.

3.4. Impact of systematic effects

Since we have restricted our analysis to multipoles \(\ell > 50\), the 217- and 353-GHz cross-spectra are assumed not to be significantly affected by those systematic effects that are most important at low multipoles, such as the ADC nonlinearity correction or the dipole and calibration uncertainties

<table>
<thead>
<tr>
<th>(\ell) range</th>
<th>LR16</th>
<th>LR24</th>
<th>LR33</th>
<th>LR42</th>
<th>LR53</th>
<th>LR63N</th>
<th>LR63</th>
<th>LR63S</th>
<th>LR72</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTE_{DS} [%]</td>
<td>50–700</td>
<td>1.7</td>
<td>1.6</td>
<td>10.8</td>
<td>3.0</td>
<td>1.6</td>
<td>11.2</td>
<td>3.2</td>
<td>6.2</td>
</tr>
<tr>
<td>50–160</td>
<td>2.9</td>
<td>46.5</td>
<td>46.4</td>
<td>42.7</td>
<td>16.3</td>
<td>11.5</td>
<td>14.8</td>
<td>47.4</td>
<td>21.7</td>
</tr>
<tr>
<td>160–320</td>
<td>84.9</td>
<td>72.8</td>
<td>54.4</td>
<td>42.8</td>
<td>19.2</td>
<td>41.9</td>
<td>73.5</td>
<td>39.4</td>
<td></td>
</tr>
<tr>
<td>320–500</td>
<td>48.6</td>
<td>34.6</td>
<td>36.6</td>
<td>36.6</td>
<td>61.1</td>
<td>24.5</td>
<td>18.1</td>
<td>18.6</td>
<td></td>
</tr>
<tr>
<td>500–700</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

Planck Collaboration: Spatial variation of the polarized dust spectral energy distribution

Table 1. Properties of the nine regions used in this analysis, in terms of effective sky fraction and column density based on dust opacity (see Sect. 3.5). The probability to exceed (PTE) values obtained for each multipole bin and sky region are reported for the HM and DS cases. They are defined as the probability to obtain correlation ratios smaller than the Planck measurements, based on 1 000 simulations with dust plus CMB signals and Gaussian noise, and expressed as a percentage.
However, the Planck cross-spectra in the multipole range $50 < \ell < 700$ could be affected by beam systematics. Thanks to its definition, the correlation ratio should be approximately independent of the beam uncertainty, because of the presence of the same beam functions, $B_{353}$ and $B_{217}$, in the numerator and denominator. The only remaining issue could come from the difference between the beam function of the $353 \times 217$ cross-spectra, $B_{353}B_{217}$, and the product of the independent beam functions, $B_{353} \times B_{217}$. We have checked that this ratio exhibits a very low departure from unity, at the $10^{-5}$ level, which cannot reproduce the amplitude of the observed Planck correlation ratio. We also checked that the correction of the bandpass mismatch in the 217- and 353-GHz bands does not affect the correlation ratio. The same analysis has been reproduced using two versions of the bandpass mismatch corrections (see Planck Collaboration VII 2016), yielding results consistent down to 0.1%.

We use the two splits, HM and DS, as an indicator of the level of residuals due to systematic effects in our analysis. This can be assessed by examining Figs. A.1 to A.9. While the HM and DS correlation ratios are very consistent in the first and last multipole bins ($\ell = 50–160$, and $\ell = 500–700$), they are in less agreement for the second multipole bin, ($\ell = 160–320$). This apparent discrepancy is not explained by the current knowledge of any systematic effects in Planck, and so indicates the need for some caution.

### 3.5. Dependence on column density

For CMB polarization studies, it is important to characterize the dependence of the observed decorrelation ratio on column density. The Planck correlation ratios in the first multipole bin ($\ell = [50, 160]$), obtained on the various science regions, are shown in Fig. 3 as a function of the mean column density, $N_{\text{HI}}$, computed for each region as the average over the unmasked pixels of the Planck column density map, assuming a constant opacity $\tau/N_{\text{HI}}$ (Planck Collaboration XI 2014). The HM and DS measurements of the Planck BB correlation ratio can be compared to the theoretical expectation (blue segment) and the dispersion due to noise (grey histograms) computed over 1 000 Monte Carlo realizations including Galactic noise (see Sect. 2.3). We recall (Sect. 2.2) that measurements of the correlation ratio obtained in different regions can be considered as statistically independent to a good approximation (except for LR63 with respect to LR63N or LR63S).

In this first multipole bin ($50 < \ell < 160$), where the primordial $B$-mode signal is expected, the $BB$ correlation ratio of the DS and HM cases can be well described by a power law of $N_{\text{HI}}$:

$$r_{50–160}^{BB} = 1 - K_{50–160}^{BB} \left( \frac{N_{\text{HI}}}{10^{20}} \right)^{\gamma} ,$$

with $K_{50–160}^{BB} = 0.40 \pm 0.32$ and $\gamma = -1.76 \pm 0.69$. Hence the more diffuse the Galactic foregrounds, the stronger the
decorrelation between 217 and 353 GHz (this trend is also observed in the last multipole bin with a lower statistical significance, but is less obvious in other bins). This is an important issue for CMB analyses, which mainly focus on the most diffuse regions of the sky in order to minimize the contamination by Galactic dust emission.

4. Impact on the CMB B-modes

In Sect. 3 we showed that the correlation of the dust polarization B-modes between 217 and 353 GHz can depart significantly from unity for multipoles \( \ell \gtrsim 50 \). Such a decorrelation, as previously noted by, e.g., Tassis & Pavlidou (2015) or BKP15, will impact the search for CMB primordial B-modes that assume a constant dust polarization SED over the region of sky considered. In order to quantify this effect, we use toy-model simulations of the BICEP2/Keck and Planck data, introducing a decorrelation between the 150- and the 353-GHz channels that matches our results in Sect. 3. This analysis is intended to be illustrative only, and not to be an exact reproduction of the work presented in Sect. 3.5 when dealing with small regions, since we expect to have a decorrelation and that our dust amplitude is higher. The value we find for \( A_d \) is similar to that of the BKP15 analysis.

Finally, we repeat the same analysis on simulations corresponding to BICEP2 and Keck Array Collaborations (2015), where we add the 95-GHz data and increase the sensitivity in the 150-GHz channel with respect to BKP15. Unlike in BICEP2 and Keck Array Collaborations (2015), we do not parametrize a synchrotron component. The posterior from these simulations, labelled “BKP+95),” are also displayed in Fig. 4. The maximum likelihood values are \( r = 0.014 \pm 0.027 \) (or \( r < 0.07 \) at 95% CL), and \( A_d = 3.54 \pm 0.77 \mu K^2 \). Even without a synchrotron component in the model of the data, the positions of the peak in the posteriors on \( r \) and \( A_d \) with respect to BKP15 are shifted in the same direction as found in BICEP2 and Keck Array Collaborations (2015).

The region of the sky observed by the BICEP and Keck instruments presented in BKP15 has a mean column density of \( N_{HI} = 1.6 \times 10^{20} \text{cm}^{-2} \), very similar to the one for our region LR24. In the multipole range 50 < \( \ell < 160 \), using the empirical relation derived in Sect. 3.5 (Eq. 6), we expect to have a decorrelation \( R_{BB}^{0.42}(217, 353) = 0.85 \) between 217 and 353 GHz. Introducing the latter decorrelation in our simple simulations shifts the \( r \) posterior towards higher values, making a decorrelation as high as the one we measure on the LR24 mask very unlikely, given the BKP15 data. This shows the limitation of the empirical relation we derived in Sect. 3.5 when dealing with small regions, since the properties of the decorrelation might be very variable over the sky. A specific analysis of these data could quantitatively confirm the amount of decorrelation that is already allowed or excluded by the data.

The results presented in this section stress that a decorrelation between the dust polarization at any two frequencies will result in a positive bias in the \( r \) posterior, in the absence of an appropriate modelling in the likelihood parameterization or in any component separation. The current BICEP2/Keck and Planck limits on \( r \) still leave room for a decorrelation of the dust polarization among frequen-
cies that could be enough to lead to spurious detections for future Stage-III or Stage-IV CMB experiments (see, e.g., Wu et al. 2014; Errard et al. 2016).

5. Discussion

We now quantify how the observed decorrelation of the BB power spectrum between the 217- and 353-GHz bands can be explained by spatial variations of the polarized dust SED using two toy-models presented in Sects. 5.1 and 5.2. This simplified description in characterizing spatial variations of the dust SED is a first step, which ignores correlations between dust properties and the structure of the magnetized interstellar medium. Correlations between matter and the Galactic magnetic field have been shown to be essential to account for statistical properties of dust polarization at high Galactic latitudes (Clark et al. 2015; Planck Collaboration Int. XXXVIII 2016). In Sect. 5.3, we use the framework introduced by Planck Collaboration Int. XLIV (2016) to discuss why such correlations are also likely to be an essential element of any physical account of the variations of the dust SED in polarization.

5.1. Spectral index variations

In a first approach, we assume that the variations of the polarized dust SED can be fully explained by spatial variations of the polarized dust spectral index applied simultaneously to the Stokes Q and U components.

We make a simplifying approximation by assuming that the polarized dust spectral index follows a Gaussian distribution centred on the mean value $\tilde{\beta}_d^d = 1.59$ (Planck Collaboration Int. XXII 2015), with a single dispersion, $\Delta \beta_d^d$, at all scales over the whole sky. Specifically, the dust polarization spectral index is given by

$$\beta_d^d(\tilde{\mu}) = N(\tilde{\beta}_d^d, \Delta \beta_d^d)(\tilde{\mu}),$$

where $N(x_0, \sigma)$ is a Gaussian distribution centred on $x_0$ with a standard deviation $\sigma$, and $\Delta \beta_d^d$ is the dispersion of the spectral index map, defined as the standard deviation after smoothing at a resolution of 1'. The dust temperature is kept constant over the sky and equal to $T_d = 19.6 K$ (Planck Collaboration Int. XXII 2015).

A main caveat of this approach comes from the power introduced by the spatial variations of the spectral index, which alters the power spectrum of dust polarization. This effect remains small between 217 and 353 GHz for low values of $\Delta \beta_d^d$ at $\ell < 150$, but can lead to dust power spectra that are inconsistent with Planck observations when extrapolated to further bands. This effect is inherent to this modelling approach, which must only be considered as illustrative. However, here the simulated maps at 217 and 353 GHz are built from maps at an intermediate frequency ($\sqrt[2]{217 \times 353} \approx 277 \text{ GHz}$) to minimize the addition of power.

The BB correlation ratio model between 217 and 353 GHz is constructed as follows. We start with a set of $Q$ and $U$ dust template maps at 277 GHz, appropriately normalized to Planck data as detailed in Sect. 2.3. The polarization dust maps at 217 and 353 GHz are extrapolated from 277 GHz using a Gaussian realization of the polarized dust spectral index, given a level of the dispersion $\Delta \beta_d^d$. These simulated maps do not include noise at this point. The correlation model is finally obtained by averaging 100 realizations of the correlation ratio computed on a pair of simulated maps.

This model is illustrated in Fig. 5 for three non-zero values of $\Delta \beta_d^d$ and compared to the Planck HM and DS measurements in the LR42 region. In order to match the Planck data (including uncertainties), an indicative value of $\Delta \beta_d^d$ around 0.07 is suggested by this simple analysis.

We compare our estimate of the spectral index variations for dust polarization to those measured for the total dust intensity. We use the Commander (Eriksen et al. 2006, 2008) dust component maps derived from a modified blackbody fit to Planck data (Planck Collaboration X 2016), providing two separate maps of dust temperature...
and dust spectral index. From the ratio $I_{353}/I_{217}$ computed from these two maps, we derive an equivalent all-sky map of the intensity dust spectral index, $β_0^d$, assuming a constant dust temperature of $T_d = 19.6 \, \text{K}$. We use the two half-mission maps of the dust spectral index, computed from $1°$ resolution maps, instead of the full-survey map, in order to reduce the impact of noise and systematic effects when computing the covariance, and we derive an estimate of the standard deviation of the dust spectral index in intensity, $\Delta β^d_0 \approx 0.045$ in LR42, about half the value measured for polarization in the same region. We note that this value is not corrected for the contribution of the data noise and the anisotropies of the cosmic infrared background (Planck Collaboration Int. XXII 2015). Both of these contributions are much smaller than the empirical value of $0.17$ found in Planck Collaboration Int. XXII (2015), which is dominated by noise.

### 5.2. Polarization angle variations

In a second approach, we assume that the decorrelation of the BB power spectrum between 217 and 353 GHz can be explained by spatial variations of the polarization angle, keeping the dust temperature and polarized spectral index constant over the whole sky. Unlike the first model, this modelling approach conserves the total power in the power spectrum. It is motivated by the nature of the polarized signal, which can be considered as the sum of spin-2 quantities over multiple components with varying spectral dependencies along the line of sight. Physical interpretation of polarization angle variations are further discussed in Sect. 5.3.

The spatial variations of the polarization angle are assumed to follow a circular normal distribution (or von Mises distribution) around 0, given by

$$f(θ | κ) = \frac{e^{κ \cos(θ)}}{2πI_0(κ)}, \quad (9)$$

where $I_0(κ)$ is the modified Bessel function of order 0, and $κ$ is analogous to $1/σ^2$ for the normal distribution. While the circular normal distribution allows us to define random angles in the range $[-π, π]$, we re-scale the realizations of angular variations to match the definition for the range of polarization angles lying between $-π/2$ and $π/2$. Again we start from a set of $Q$ and $U$ dust maps at 277 GHz, which are then extrapolated to 217 and 353 GHz following a modified blackbody spectrum using $T_0 = 19.6 \, \text{K}$ and $β_0^d = 1.59$. The polarization pseudo-vector obtained from $Q$ and $U$ is rotated independently at each frequency using two different realizations of the polarization angle variations. The correlation ratio model is then obtained by averaging 100 realizations of the correlation ratio computed from a pair of simulated maps.

This model is illustrated Fig. 6 for three finite values of $κ$ in the LR42 region. The case $κ = 2$, which matches the Planck data quite well, represents a $1 \, σ$ dispersion of the polarization angle of about $2°$ between the 217- and 353-GHz polarization maps, after smoothing at $1°$ resolution.

### 5.3. Origin on the spatial variations of the polarized SED

The spatial variations of the Galactic dust SED are found to be larger for dust polarization than for dust intensity. This difference could result from the lack of correlation between spectral variation and emission features in our modelling in Sect. 5.1. It could also reflect the different nature of the two observables. While variations in the dust SED tend to average out in intensity (because it is a scalar quantity), they may not average out as much in polarization (because it is a pseudo-vector). In other words, dust polarization depends on the magnetic field structure, while dust intensity does not. In this section, we describe further details of this interpretation of the decorrelation.

The analysis of Planck data offers several lines of evidence for the imprint of interstellar magneto-hydrodynamical (MHD) turbulence on the dust polarization sky. Firstly, Planck Collaboration Int. XIX (2015) reported an anti-correlation between the polarization fraction and the local dispersion of the polarization angle. Secondly, large filamentary depolarization patterns observed in the Planck 353-GHz maps are associated, in most cases, with large, local, fluctuations of the po-
Planck Collaboration: Spatial variation of the polarized dust spectral energy distribution

Fig. 7. Illustration of the decomposition of the line of sight polarization pseudo-vector into a random walk process through four turbulent layers. This is shown for two neighbouring pixels at two frequencies, $\nu_1$ and $\nu_2$. The integrated polarization pseudo-vectors are affected by the polarization angle fluctuations, leading to a decorrelation between the two frequencies.

of the turbulent component of the magnetic field along the line of sight.

A quantitative modelling of this perspective on dust polarization is required to assess its ability to reproduce the results of our analysis of the Planck data and its impact on the dust/CMB component-separation task. To our knowledge, none of the studies carried out so far to quantify limits set by polarized dust foregrounds on future CMB experiments in order to search for primordial $B$-modes (except for the work of Tassis & Pavlidou 2015) have considered the frequency decorrelation that arises from the interplay between dust properties and interstellar MHD turbulence.

6. Conclusions

We have used the 217- and 353-GHz Planck 2015 data to investigate the spatial variation of the polarized dust SED at high Galactic latitudes. We computed $R^{BB}_{150,353}$—the $BB$ cross-spectrum between the Planck polarization maps in these bands divided by the geometric mean of the two auto-spectra in the same bands—over large regions of the sky at high latitudes, for multipoles $50 < \ell < 700$. $R^{BB}_{150,353}$ was computed with distinct sets of data to control systematics.

The ratio $R^{BB}_{150,353}$ has been shown to be significantly lower than what is expected purely from the presence of CMB and noise in the data, with a confidence, estimated using data simulations, larger than 99%. We interpret this result as evidence for significant spatial variations of the dust polarization SED.

In the multipole bin $50 < \ell < 160$ that encompasses the recombination bump of the primordial $B$-mode signal, $R^{BB}_{150,353}$ values are consistent for the distinct Planck data splits we used. The measured values exhibit a systematic trend with column density, where $R^{BB}_{150,353}$ decreases for decreasing mean column densities $N_{\text{H}}$, as $1 - R^{BB}_{150,353}(N_{\text{H}}/10^{20} \text{ cm}^{-2})^\gamma$, with $\gamma = -1.76 \pm 0.69$ and $R^{BB}_{150,353} \approx 0.40 \pm 0.32$. This suggests that, statistically speaking, the cleaner a sky area is from Galactic foregrounds, the more challenging it may be to extrapolate dust polarization from submm to CMB bands.

The spatial variations of the dust SED are shown to be stronger in polarization than in intensity. This difference may reflect the interplay, encoded in the data, between the polarization properties of dust grains, including grain alignment, the Galactic magnetic field, and the density structure of interstellar matter.

We have proposed two toy-models to quantify the observed decorrelation, based on spatial Gaussian variations of the polarized dust spectral index, and spatial Gaussian variations of the polarization angle. Both models reproduce the general trend observed in Planck data, with reasonable accuracy given the noise level of the Planck measurements. They represent a first step in characterizing variations of the dust SED, as a prelude to further work on a physically motivated model, which will take into account the expected correlation between the spatial variations of the SED and structures in the dust polarization maps.

Spatial variations of the dust SED can lead to biased estimates of the tensor-to-scalar ratio, $r$, because of inaccurate extrapolation of dust polarization when cleaning or modelling the $B$-mode signal at microwave frequencies. As an illustration, we have shown that in a region such as that observed by the BICEP2-Keck Array, a decorrelation of 15% between the 353- and 150-GHz bands could re-
Planck Collaboration: Spatial variation of the polarized dust spectral energy distribution

sult in a likelihood posterior on the tensor-to-scalar ratio of \( r = 0.046 \pm 0.036 \), comparable to the joint BICEP2 and Keck Array/Planck Collaboration result.

It appears essential now to place tighter constraints on the spectral dependence of polarized dust emission in the sub-mm, in order to properly propagate the information on the Galactic foregrounds into the CMB bands. More specifically, the spatial variations of the polarized dust SED need to be mapped and modelled with improved accuracy in order to be able to confidently reach a level of Galactic dust residual lower than \( r \approx 10^{-2} \).

Acknowledgements. The Planck Collaboration acknowledges the support of: ESA; CNES, and CNRS/INSU-IN2P3-INP (France); ASI, CNR, and INAF (Italy); NASA and DoE (USA); STFC and UKSA (UK); CSIC, MINECO, JA, and RES (Spain); Tekes, AoF, and CSC (Finland); DLR and MPG (Germany); CSA (Canada); DTU Space (Denmark); SER/SSO (Switzerland); RCN (Norway); SFI (Ireland); FCT/MCTES (Portugal); ERC and PRACE (EU). A description of the Planck Collaboration and a list of its members, indicating which technical or scientific activities they have been involved in, can be found at http://www.cosmos.esa.int/web/planck/planck-collaboration. The research leading to these results has received funding from the European Research Council under the European Union’s Seventh Framework Programme (FP7/2007-2013) / ERC grant agreement No. 267934.

References


Appendix A: Distribution of the $R^{BB}_\ell$ from Planck simulations.

We present in this appendix the distribution of the ratio $R^{BB}_\ell$ for our simulations of the Planck 217- and 353-GHz maps, including Gaussian CMB, dust, and Planck noise, which are described in Sect. 3.3. Since the quantity $R^{BB}_\ell$ is a ratio, we expect distributions that are strongly non-Gaussian, even for Gaussian simulations, as soon as the denominator gets close to zero.

In Figs. A.1–A.9, we show the histogram of the $R^{BB}_\ell$ correlation ratio from 1000 realizations of detector-set and half-mission simulations. The histogram “occurrences” are the fractions of simulations that fall in each $R^{BB}_\ell$ bin. These are compared to the values obtained from the Planck data. Since we compute only noise-independent cross-spectra, when $R^{BB}_\ell$ has a negative denominator, the data value is absent. The probability to exceed (PTE) for each data set, computed as the percentage of simulations that have a smaller $R^{BB}_\ell$ than the data, is reported in Table 1.

1 APC, AstroParticule et Cosmologie, Université Paris Diderot, CNRS/IN2P3, CEA/IRfu, Observatoire de Paris, Sorbonne Paris Cité, 10, rue Alice Domon et Léonie Duquet, 75205 Paris Cedex 13, France
2 Aalto University Metsähovi Radio Observatory and Dept of Radio Science and Engineering, P.O. Box 13000, FI-00076 AALTO, Finland
3 African Institute for Mathematical Sciences, 6-8 Melrose Road, Muizenberg, Cape Town, South Africa
4 Agenzia Spaziale Italiana Science Data Center, Via del Politecnico snc, 00133, Roma, Italy
5 Aix Marseille Université, CNRS, LAM (Laboratoire d’Astrophysique de Marseille) UMR 7326, 13388, Marseille, France
6 Astrophysics Group, Cavendish Laboratory, University of Cambridge, J J Thomson Avenue, Cambridge CB3 0HE, U.K.
7 Astrophysics & Cosmology Research Unit, School of Mathematics, Statistics & Computer Science, University of KwaZulu-Natal, Westville Campus, Private Bag X54001, Durban 4000, South Africa
8 CITA, University of Toronto, 60 St. George St., Toronto, ON M5S 3H8, Canada
9 CNRS, IRAP, 9 Av. colonel Roche, BP 44346, F-31028 Toulouse cedex 4, France
Fig. A.4. Same as Fig. A.1, for the LR42 region.

Fig. A.5. Same as Fig. A.1, for the LR53 region.

Fig. A.6. Same as Fig. A.1, for the LR63N region.

Fig. A.7. Same as Fig. A.1, for the LR63S region.

10 California Institute of Technology, Pasadena, California, U.S.A.
11 Computational Cosmology Center, Lawrence Berkeley National Laboratory, Berkeley, California, U.S.A.
12 DTU Space, National Space Institute, Technical University of Denmark, Elektrovej 327, DK-2800 Kgs. Lyngby, Denmark
13 Département de Physique Théorique, Université de Genève, 24, Quai E. Ansermet, 1211 Genève 4, Switzerland
14 Departamento de Astrofísica, Universidad de La Laguna (ULL), E-38206 La Laguna, Tenerife, Spain
15 Departamento de Física, Universidad de Oviedo, Avda. Calvo Sotelo s/n, Oviedo, Spain
16 Department of Astrophysics/IMAPP, Radboud University Nijmegen, P.O. Box 9010, 6500 GL Nijmegen, The Netherlands
17 Department of Physics & Astronomy, University of British Columbia, 6224 Agricultural Road, Vancouver, British Columbia, Canada
18 Department of Physics and Astronomy, Dana and David Dornsife College of Letter, Arts and Sciences, University of Southern California, Los Angeles, CA 90089, U.S.A.
19 Department of Physics and Astronomy, University College London, London WC1E 6BT, U.K.
20 Department of Physics, Gustaf Hällströmin katu 2a, University of Helsinki, Helsinki, Finland
21 Department of Physics, Princeton University, Princeton, New Jersey, U.S.A.
22 Department of Physics, University of California, Santa Barbara, California, U.S.A.
23 Department of Physics, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, Illinois, U.S.A.
Planck Collaboration: Spatial variation of the polarized dust spectral energy distribution

Fig. A.8. Same as Fig. A.1, for the LR63 region.

Fig. A.9. Same as Fig. A.1, for the LR72 region.

24 Dipartimento di Fisica e Astronomia G. Galilei, Università degli Studi di Padova, via Marzolo 8, 35131 Padova, Italy
25 Dipartimento di Fisica e Astronomia, Alma Mater Studiorum, Università degli Studi di Bologna, Viale Berti Pichat 6/2, I-40127, Bologna, Italy
26 Dipartimento di Fisica e Scienze della Terra, Università di Ferrara, Via Saragat 1, 44122 Ferrara, Italy
27 Dipartimento di Fisica, Università La Sapienza, P.le A. Moro 2, Roma, Italy
28 Dipartimento di Fisica, Università degli Studi di Milano, Via Celoria, 16, Milano, Italy
29 Dipartimento di Fisica, Università degli Studi di Trieste, via A. Valerio 2, Trieste, Italy
30 Dipartimento di Matematica, Università di Roma Tor Vergata, Via della Ricerca Scientifica, 1, Roma, Italy
31 Discovery Center, Niels Bohr Institute, Copenhagen University, Blegdamsvej 17, Copenhagen, Denmark
32 European Space Agency, ESAC, Planck Science Office, Camino bajo del Castillo, s/n, Urbanización Villarfranca del Castillo, Villanueva de la Cañada, Madrid, Spain
33 European Space Agency, ESTEC, Keplerlaan 1, 2201 AZ Noordwijk, The Netherlands
34 Gran Sasso Science Institute, INFN, viale F. Crispi 7, 67100 L’Aquila, Italy
35 HGSFP and University of Heidelberg, Theoretical Physics Department, Philosophenweg 16, 69120, Heidelberg, Germany
36 Haverford College Astronomy Department, 370 Lancaster Avenue, Haverford, Pennsylvania, U.S.A.
37 Helsinki Institute of Physics, Gustaf Hallström katu 2, University of Helsinki, Helsinki, Finland
38 INAF - Osservatorio Astronomico di Padova, Vicolo dell’Osservatorio 5, Padova, Italy
39 INAF - Osservatorio Astronomico di Roma, via di Frascati 33, Monte Porzio Catone, Italy
40 INAF - Osservatorio Astronomico di Trieste, Via G.B. Tiepolo 11, Trieste, Italy
41 INAF/IASF Bologna, Via Gobetti 101, Bologna, Italy
42 INAF/IASF Milano, Via E. Bassini 15, Milano, Italy
43 INFN - CNAF, viale Berti Pichat 6/2, 40127 Bologna, Italy
44 INFN, Sezione di Bologna, viale Berti Pichat 6/2, 40127 Bologna, Italy
45 INFN, Sezione di Ferrara, Via Saragat 1, 44122 Ferrara, Italy
46 INFN, Sezione di Roma 1, Università di Roma Sapienza, Piazzale Aldo Moro 2, 00185, Roma, Italy
47 INFN, Sezione di Roma 2, Università di Roma Tor Vergata, Via della Ricerca Scientifica, 1, Roma, Italy
48 INFN/National Institute for Nuclear Physics, Via Valerio 2, I-34127 Trieste, Italy
49 IUCAA, Post Bag 4, Ganeshkhind, Pune University Campus, Pune 411 007, India
50 Imperial College London, Astrophysics group, Blackett Laboratory, Prince Consort Road, London, SW7 2AZ, U.K.
51 Institut d’Astrophysique Spatiale, CNRS, Univ. Paris-Sud, Université Paris-Saclay, Bât. 121, 91405 Orsay cedex, France
52 Institut d’Astrophysique de Paris, CNRS (UMR7095), 98 bis Boulevard Arago, F-75014, Paris, France
53 Institute of Astronomy, University of Cambridge, Madingley Road, Cambridge CB3 0HA, U.K.
54 Institute for Theoretical Astrophysics, University of Oslo, Blindern, Oslo, Norway
55 Instituto de Astrofísica de Canarias, C/Vía Láctea s/n, La Laguna, Tenerife, Spain
56 Instituto de Fisica de Cantabria (CSIC-Universidad de Cantabria), Avda. de los Castros s/n, Santander, Spain
57 Istituto Nazionale di Fisica Nucleare, Sezione di Padova, via Marzolo 8, I-35131 Padova, Italy
58 Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, Pasadena, California, U.S.A.
59 Jodrell Bank Centre for Astrophysics, Alan Turing Building, School of Physics and Astronomy, The University of Manchester, Oxford Road, Manchester, M13 9PL, U.K.
60 Kavli Institute for Cosmological Physics, University of Chicago, Chicago, IL 60637, USA
61 Kavli Institute for Cosmology Cambridge, Madingley Road, Cambridge CB3 0HA, U.K.
62 LAL, Université Paris-Sud, CNRS/IN2P3, Orsay, France
63 LERMA, CNRS, Observatoire de Paris, 61 Avenue de l’Observatoire, Paris, France
64 Laboratoire AIM, IRFU/Service d’Astrophysique - CEA/DSM - CNRS - Université Paris Diderot, Bât. 709, CEA-Saclay, F-91191 Gif-sur-Yvette Cedex, France
65 Laboratoire Traitement et Communication de l’Information, CNRS (UMR 5141) and Télécom ParisTech, 46 rue Barrault F-75634 Paris Cedex 13, France