Directional Limits on Persistent Gravitational Waves from Advanced LIGO’s First Observing Run

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We employ gravitational-wave radiometry to map the stochastic gravitational wave background expected from a variety of contributing mechanisms and test the assumption of isotropy using data from the Advanced Laser Interferometer Gravitational Wave Observatory’s (aLIGO) first observing run. We also search for persistent gravitational waves from point sources with only minimal assumptions over the 20–1726 Hz frequency band. Finding no evidence of gravitational waves from either point sources or a stochastic background, we set limits at 90% confidence. For broadband point sources, we report upper limits on the gravitational wave energy flux per unit frequency in the range \( F_{\alpha,0}(\nu) < (0.1–56) \times 10^{-8} \text{erg cm}^{-2} \text{s}^{-1} \text{Hz}^{-1} \nu \text{Hz}^{-\alpha} \) depending on the sky location \( \Theta \) and the spectral power index \( \alpha \). For extended sources, we report upper limits on the fractional gravitational wave energy density required to close the Universe of \( \Omega(\nu, \Theta) < (0.39–7.6) \times 10^{-8} \text{s}^{-1} \nu \text{Hz}^{-\alpha} \) depending on \( \Theta \) and \( \alpha \). Directed searches for narrowband gravitational waves from astrophysically interesting objects (Scorpius X-1, Supernova 1987 A, and the Galactic Center) yield median frequency-dependent limits on strain amplitude of \( h_0 < (6.7, 5.5, \text{ and } 7.0) \times 10^{-25} \), respectively, at the most sensitive detector frequencies between 130–175 Hz. This represents a mean improvement of a factor of 2 across the band compared to previous searches of this kind for these sky locations, considering the different quantities of strain constrained in each case.

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Introduction.—A stochastic gravitational-wave background (SGWB) is expected from a variety of mechanisms [1–5]. Given the recent observations of binary black hole mergers GW150914 and GW151226 [6,7], we expect the SGWB to be nearly isotropic [8] and dominated [9] by compact binary coalescences [10–12]. The Laser Interferometer Gravitational Wave Observatory (LIGO) and Virgo Collaborations have pursued the search for an isotropic stochastic background from LIGO’s first observational run [13]. Here, we adopt an eyes-wide-open philosophy and relax the assumption of isotropy in order to allow for the greater range of possible signals. We search for an anisotropic background, which could indicate a richer, more interesting cosmology than current models. We present the results of a generalized search for a stochastic signal with an arbitrary angular distribution mapped over all directions in the sky.

Our search has three components. First, we utilize a broadband radiometer analysis [14,15], optimized for detecting a small number of resolvable point sources. This method is not applicable to extended sources. Second, we employ a spherical harmonic decomposition [16,17], which can be employed for point sources but is better suited to extended sources. Last, we carry out a narrowband radiometer search directed at the sky position of three astrophysically interesting objects: Scorpius X-1 (Sco X-1) [18,19], Supernova 1987 A (SN 1987A) [20,21], and the Galactic Center [22].

These three search methods are capable of detecting a wide range of possible signals with only minimal assumptions about the signal morphology. We find no evidence of persistent gravitational waves, and set limits on broadband emission of gravitational waves as a function of sky position. We also set narrowband limits as a function of frequency for the three selected sky positions.

Data.—We analyze data from Advanced LIGO’s 4 km detectors in Hanford, Washington (H1) and Livingston, Louisiana (L1) during the first observing run (O1), from 15:00 UTC, Sep 18, 2015–16:00 UTC, Jan 12, 2016. During O1, the detectors reached an instantaneous strain sensitivity of \( 7 \times 10^{-24} \text{Hz}^{-1/2} \) in the most sensitive region from 100–300 Hz, and collected 49.67 days of coincident H1L1 data. The O1 observing run saw the first direct detection of gravitational waves and the first direct observation of merging black holes [6,7].

For our analysis, the time-series data are downsampled to 4096 Hz from 16 kH, and divided into 192 s, 50% overlapping, Hann-windowed segments, which are high-pass filtered with a 16th order Butterworth digital filter with knee frequency of 11 Hz (following [13,23]). We apply data quality cuts in the time domain in order to remove segments associated with instrumental artifacts and
hardware injections used for signal validation [24,25]. We also exclude segments containing known gravitational-wave signals. Finally, we apply a standard nonstationarity cut (see, e.g., [26]), to eliminate segments that do not behave as Gaussian noise. These cuts remove 35% of the data. With all vetoes applied, the total live time for 192 s segments is 29.85 days.

The data segments are Fourier transformed and coarse-grained to produce power spectra with a resolution of 1/32 Hz. This is a finer frequency resolution than the 1/4 Hz used in previous LIGO-Virgo stochastic searches [15,17] in order to remove many finely spaced instrumental lines occurring at low frequencies. Frequency bins associated with known instrumental artifacts including suspension violin modes [27], calibration lines, electronic lines, correlated combs, and signal injections of persistent, monochromatic, gravitational waves are not included in the analysis. These frequency domain cuts remove 21% of the observing band. For a detailed description of data quality studies performed for this analysis, see the Supplemental Material [28] of [13].

The broadband searches include frequencies from 20–500 Hz which more than cover the regions of 99% sensitivity for each of the spectral bands (see Table 1 of [13]). The narrowband analysis covers the full 20–1726 Hz band.

Method.—The main goal of a stochastic search is to estimate the fractional contribution of the energy density in gravitational waves \( \Omega_{GW} \) to the total energy density needed to close the Universe \( \rho_c \). This is defined by

\[
\Omega_{GW}(f) = \frac{f}{\rho_c} \frac{d\rho_{GW}}{df},
\]

where \( f \) is frequency and \( d\rho_{GW} \) represents the energy density between \( f \) and \( f + df \) [29]. For a stationary and unpolarized signal, \( \rho_{GW} \) can be factored into an angular power \( P(\Theta) \) and a spectral shape \( H(f) \) [30], such that

\[
\Omega_{GW}(f) = \frac{2\pi^2}{3H_0^2} f^3 H(f) \int d\theta P(\Theta),
\]

with Hubble constant \( H_0 = 68 \, \text{km s}^{-1} \, \text{Mpc}^{-1} \) from [31].

The angular power \( P(\Theta) \) represents the gravitational-wave power at each point in the sky. To express this in terms of the fractional energy density, we define the energy density spectrum as a function of sky position

\[
\Omega(f, \Theta) = \frac{2\pi^2}{3H_0^2} f^3 H(f) P(\Theta).
\]

We define a similar quantity for the energy flux, where

\[
\mathcal{F}(f, \Theta) = \frac{c^3 \pi}{4G} f^2 H(f) P(\Theta)
\]

has units of erg s\(^{-1}\) Hz\(^{-1}\) sr\(^{-1}\) [15,16], \( c \) is the speed of light and \( G \) is Newton’s gravitational constant.

Point sources versus extended sources.—We employ two different methods to estimate \( P(\Theta) \) based on the cross-correlation of data streams from a pair of detectors [17,29]. The radiometer method [14,15] assumes that the cross-correlation signal is dominated by a small number of resolvable point sources. The point source power is given by \( P_{\Theta_0} \) and the angular power spectrum is then

\[
P(\Theta) \equiv P_{\Theta_0} \delta^2(\Theta, \Theta_0).
\]

Although the radiometer method provides the optimal method for detecting resolvable point sources, it is not well suited for describing diffuse or extended sources, which may have an arbitrary angular distribution. Hence, we also implement a complementary spherical harmonic decomposition (SHD) algorithm, in which the sky map is decomposed into components \( Y_{lm}(\Theta) \) with coefficients \( P_{lm} [16] \)

\[
P(\Theta) \equiv \sum_{lm} P_{lm} Y_{lm}(\Theta).
\]

Here, the sum over \( l \) runs from 0 to \( l_{\text{max}} \) and \(-l \leq m \leq l\). We discuss the choice of \( l_{\text{max}} \) below. While the SHD algorithm has comparably worse sensitivity to point sources than the radiometer algorithm, it accounts for the detector response, producing more accurate sky maps.

Spectral models.—In both the radiometer algorithm and the spherical harmonic decomposition algorithm, we must choose a spectral shape \( H(f) \). We model the spectral dependence of \( \Omega_{GW}(f) \) as a power law

\[
H(f) = \left( \frac{f}{f_{\text{ref}}} \right)^{a-3},
\]

where \( f_{\text{ref}} \) is an arbitrary reference frequency and \( a \) is the spectral index (see, also, [13]). The spectral model will also affect the angular power spectrum, so \( P(\Theta) \) is implicitly a function of \( a \).

We can rewrite the energy density map \( \Omega(f, \Theta) \) to emphasize the spectral properties, such that

\[
\Omega(f, \Theta) = \Omega_x(\Theta) \left( \frac{f}{f_{\text{ref}}} \right)^a,
\]

where

\[
\Omega_x(\Theta) = \frac{2\pi^2}{3H_0^2} f_{\text{ref}}^3 P(\Theta)
\]

has units of fractional energy density per steradian \( \Omega_{GW} \) sr\(^{-1}\). The spherical harmonic analysis presents sky maps of \( \Omega_x(\Theta) \). Note that, when \( P(\Theta) = P_{\Theta_0} \) (the monopole moment), we recover a measurement for the energy density of the isotropic gravitational-wave background.
Here, \( \nu \) is an index, which can refer to either individual points on the sky (the pixel basis) or different \( l \)m indices (the spherical harmonic basis). The variable \( C(f, t) \) is the cross-power spectral density measured between the two LIGO detectors at some segment time \( t \). The sum runs over all segment times and all frequency bins. The variable \( \gamma_\nu(f, t) \) is a generalization of the overlap reduction function, which is a function of the separation and relative orientation between the detectors, and characterizes the frequency response of the detector pair [33]; see [16] for an exact definition.

We can think of \( X_\nu \) as a sky map representation of the raw cross-correlation measurement before deconvolving the detector response. The associated uncertainty is encoded in the Fisher matrix

\[
\Gamma_{\mu \nu} = \sum_{f_1} \gamma_\nu^*(f_1, t) \frac{H(f_1)}{P_1(f_1)P_2(f_1)} \gamma_\nu(f_1, t),
\]

where the asterisk denotes complex conjugation.

Once \( X_\nu \) and \( \Gamma_{\mu \nu} \) are calculated, we have the ingredients to calculate both the radiometer map and the SHD map. However, the inversion of \( \Gamma_{\mu \nu} \) is required to calculate the maximum likelihood estimators of GW power \( \hat{P}_\nu = \Gamma_{\nu \nu}^{-1}X_\nu \) [16]. For the radiometer, the correlations between neighboring pixels can be ignored. The radiometer map is given by

\[
\hat{P}_\Theta = (\Gamma_{\Theta \Theta})^{-1}X_\Theta,
\]

where the standard deviation \( \sigma_{\Theta}^{\text{rad}} = (\Gamma_{\Theta \Theta})^{-1/2} \),

\[
\text{All-sky (broadband) Results}
\]

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \Omega_{\text{GW}} )</th>
<th>( H(f) )</th>
<th>( f_s ) (Hz)</th>
<th>( \theta ) (deg)</th>
<th>( l_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>constant ( \propto f^{-3} )</td>
<td>52.50</td>
<td>55</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>2/3</td>
<td>( \propto f^{2/3} )</td>
<td>65.75</td>
<td>44</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>( \propto f^3 )</td>
<td>256.50</td>
<td>11</td>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>

Similarly, the gravitational-wave energy flux can be expressed as

\[
\mathcal{F}(f, \Theta) = \mathcal{F}_a(\Theta) \left( \frac{f}{f_{\text{ref}}} \right)^{-\alpha-1},
\]

where

\[
\mathcal{F}_a(\Theta) = \frac{c^3 \pi}{4G} f_{\text{ref}}^2 \mathcal{P}(\Theta).
\]

In the radiometer case, we calculate the flux in each direction

\[
F_{\alpha, \Theta_0} = \frac{c^3 \pi}{4G} f_{\text{ref}}^2 \mathcal{P}(\Theta_0),
\]

which is obtained by integrating Eq. (11) over the sphere for the point-source signal model described in Eq. (5). This quantity has units of erg cm\(^{-2}\) s\(^{-1}\) Hz\(^{-1}\). Following [13,32], we choose \( f_{\text{ref}} = 25 \) Hz, corresponding to the most sensitive frequency in the spectral band for a stochastic search with the Advanced LIGO network at design sensitivity.

We consider three spectral indices: \( \alpha = 0 \), corresponding to a flat energy density spectrum (expected from models of a cosmological background), \( \alpha = 2/3 \), corresponding to the expected shape from a population of compact binary coalescences, and \( \alpha = 3 \), corresponding to a flat strain power spectral density spectrum [17,32]. The different spectral models are summarized in Table I.

Cross-correlation.—A stochastic background would induce low-level correlation between the two LIGO detectors. Although the signal is expected to be buried in the detector noise, the cross-correlation signal-to-noise ratio (SNR) grows with the square root of integration time [29]. The cross-correlation between two detectors, with (one-sided strain) power spectral density \( P_1(f, t) \) for detector 1, is encoded in what is known as “the dirty map” [16]

\[
X_\nu = \sum_{f_1} \gamma_\nu^*(f_1, t) \frac{H(f_1)}{P_1(f_1)P_2(f_1)} C(f_1, t).
\]

The characteristic frequency \( f_\alpha \), angular resolution \( \Theta \) [Eq. (17)], and corresponding harmonic order \( l_{\text{max}} \) [Eq. (18)] for each \( \alpha \) are also shown. The right hand section of the table shows the maximum SNR, associated significance (\( p \) value) and best upper limit values from the broadband radiometer (BBR) and the SHD. The BBR sets upper limits on energy flux [erg cm\(^{-2}\) s\(^{-1}\) Hz\(^{-1}\) (f/25 Hz)\(^{\alpha-1}\)] while the SHD sets upper limits on the normalized energy density \( [\text{srg}^{-1}(f/25 \text{ Hz})^{\alpha}] \) of the SGWB.
sensitivity (see [16]). The estimator for the SHD and corresponding standard deviation are given by

\[ \hat{P}_{lm} = \sum_{l'm'} (\Gamma^{-1})_{lm,l'm'} X_{l'm'}, \]

\[ \sigma_{\text{SHD}}^2 = [(\Gamma^{-1})_{lm,lm}]^{1/2}, \tag{16} \]

where \( \Gamma \) is the regularized Fisher matrix. We remove 1/3 of the lowest eigenvalues following [16,17].

Angular scale.—In order to carry out the calculation in Eq. (16), we must determine a suitable angular scale, which will depend on the angular resolution of the detector network and vary with spectral index \( \alpha \). The diffraction-limited spot size on the sky \( \theta \) (in radians) is given by

\[ \theta = \frac{c}{2df} \approx \frac{50 \text{ Hz}}{f_\alpha}, \tag{17} \]

where \( d = 3000 \text{ km} \) is the separation of the LIGO detectors. The frequency \( f_\alpha \) corresponds to the most sensitive frequency in the detector band for a power law with spectral index \( \alpha \) given the detector noise power spectra [15]. In order to determine \( f_\alpha \), we find the frequency at which a power law with index \( \alpha \) is tangent to the single-detector “power-law integrated curve” [34]. The angular resolution scale is set by the maximum spherical harmonic order \( l_{\text{max}} \), which we can express as a function of \( \alpha \) since

\[ l_{\text{max}} = \frac{\pi}{\theta} \approx \frac{\pi f_\alpha}{50 \text{ Hz}}. \tag{18} \]

The values of \( f_\alpha, \theta, \) and \( l_{\text{max}} \) for three different values of \( \alpha \) are shown in Table I. As the spectral index increases, so does \( f_\alpha \), decreasing the angular resolution limit, thus, increasing \( l_{\text{max}} \).

Angular power spectra.—For the SHD map, we calculate the angular power spectra \( C_l \), which describe the angular scale of structure in the clean map, using an unbiased estimator [16,17]

\[ \hat{C}_l \equiv \frac{1}{2l+1} \sum_m |\hat{P}_{lm}|^2 - (\Gamma^{-1})_{lm,lm}. \tag{19} \]

Narrowband radiometer.—The radiometer algorithm can be applied to the detection of persistent gravitational waves from narrowband point sources associated with a given sky position [15,17]. We “point” the radiometer in the direction of three interesting sky locations: Sco X-1, the Galactic Center, and the remnant of supernova SN 1987A.

Scorpius X-1 is a low-mass x-ray binary believed to host a neutron star that is potentially spun up through accretion, in which gravitational-wave emission may provide a balancing spin-down torque [18,19,35,36]. The frequency of the gravitational-wave signal is expected to spread due to the orbital motion of the neutron star. At frequencies below ~930 Hz this Doppler line broadening effect is less than 1/4 Hz, the frequency bin width selected in past analyses [15,17]. At higher frequencies, the signal is certain to span multiple bins. Therefore, we combine multiple 1/32 Hz frequency bins to form optimally sized combined bins at each frequency, accounting for the expected signal broadening due to the combination of the motion of the Earth around the Sun, the binary orbital motion, and any other intrinsic modulation. For more detail on the method of combining bins, see the Supplemental Material to this Letter [37].

The possibility of a young neutron star in SN 1987A [20,21] and the likelihood of many unknown, isolated neutron stars in the Galactic Center region [22] indicate potentially interesting candidates for persistent gravitational-wave emission. We combine bins to include the signal spread due to Earth’s modulation. For SN 1987A, we choose a combined bin size of 0.09 Hz. We would be sensitive to spin modulations up to \( |\dot{\nu}| < 9 \times 10^{-9} \text{ Hz s}^{-1} \) within our O1 observation time spanning 116 days. The Galactic Center is at a lower declination with respect to the orbital plane of the Earth. Therefore, the Earth modulation term is more significant, so, for the Galactic Center, we choose combined bins of 0.53 Hz across the band. In this case, we are sensitive to a frequency modulation in the range \( |\dot{\nu}| < 5.3 \times 10^{-8} \text{ Hz s}^{-1} \).

Significance.—To assess the significance of the SNR in the combined bins of the narrowband radiometer spectra, we simulate many realizations of the strain power consistent with Gaussian noise in each individual frequency bin. Combining these in the same way as the actual analysis leaves us with a distribution of maximum SNR values across the whole frequency band for many simulations of noise.

For a map of the whole sky, the distribution of maximum SNR is complicated by the many dependent trials due to covariances between different pixels (or patches) on the sky. We calculate this distribution numerically by simulating many realizations of the dirty map \( X_s \) with expected covariances described by the Fisher matrix \( \Gamma_{\mu\nu} \) [cf. Eqs. (13) and (14), respectively]. This distribution is then used to calculate the significance (or \( p \) value) of a given SNR recovered from the sky maps [17]. We take a \( p \) value of 0.01 or less to indicate a significant result. The absence of any significant events indicates the data are consistent with no signal being detected, in which case we quote Bayesian upper limits at 90% confidence [15,17].

Results.—The search yields four data products: radiometer sky maps, angular power spectra, and radiometer spectra.

Radiometer sky maps optimized for broadband point sources, are shown in Fig. 1. The top row shows the SNR. Each column corresponds to a different spectral index, \( \alpha = 0.2/3, \) and 3, from left to right, respectively. The maximum SNRs are, respectively, 3.32, 3.31, and 3.43.
corresponding to false-alarm probabilities typical of what would be expected from Gaussian noise; see Table I. We find no evidence of a signal and, so, set limits on gravitational-wave energy flux, which are provided in the bottom row of Fig. 1 and summarized in Table I.

SHD sky maps, suitable for characterizing an anisotropic stochastic background, are shown in Fig. 2. The top row shows the SNR and each column corresponds to a different spectral index ($\alpha = 0, 2/3$, and 3, respectively). The maximum SNRs are 2.96, 3.06, and 3.86 corresponding to false-alarm probabilities typical of those expected from Gaussian noise; see Table I. Failing evidence of a signal, we set limits on energy density per unit solid angle, which are provided in the bottom row of Fig. 2 and summarized in Table I. Interactive visualizations of the SNR and upper limit maps are also available online [38].

Angular power spectra are derived from the SHD sky maps. We present upper limits at 90% confidence on the angular power spectrum indices $C_l$ from the spherical harmonic analysis in Fig. 3.

Radiometer spectra, suitable for the detection of a narrowband point source associated with a given sky position, are given in Fig. 4, the main results of which are summarized in Table II. For the three sky locations (Sco X-1, SN 1987A, and the Galactic Center), we
calculate the SNR in appropriately sized combined bins across the LIGO band. For Sco X-1, the loudest observed SNR is 4.58, which is consistent with Gaussian noise. For SN 1987A and the Galactic Center, we observe maximum SNRs of 4.07 and 3.92, respectively, corresponding to false alarm probabilities consistent with noise; see Table II.

Since we observe no statistically significant signal, we set 90% confidence limits on the peak strain amplitude $h_0$ for each optimally sized frequency bin. Upper limits were set using a Bayesian methodology with the constraint that $h_0 > 0$ and validated with software injection studies. The upper limit procedure is described in more detail in the Supplemental Material [37], while the subsequent software injection validation is detailed in [39].

The results of the narrowband radiometer search for the three sky locations are shown in Fig. 4. To avoid setting limits associated with downward noise fluctuations, we take the median upper limit from the most sensitive 1 Hz band as our best strain upper limit. We obtain 90% confidence upper limits on the gravitational-wave strain of $h_0 < 6.7 \times 10^{-25}$ at 134 Hz, $h_0 < 7.0 \times 10^{-25}$ at 172 Hz, and $h_0 < 5.5 \times 10^{-25}$ at 172 Hz from Sco X-1, SN 1987A, and the Galactic Center, respectively, in the most sensitive part of the LIGO band.

**TABLE II.** Results for the narrowband radiometer showing the maximum SNR, corresponding $p$ value and 1 Hz frequency band as well as the 90% gravitational-wave strain upper limits, and corresponding frequency band, for three sky locations of interest. The best upper limits are taken as the median of the most sensitive 1 Hz band.

<table>
<thead>
<tr>
<th>Direction</th>
<th>Max SNR</th>
<th>$p$ value (%)</th>
<th>Frequency band (Hz)</th>
<th>Best UL Frequency (×10^{-25}) band (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sco X-1</td>
<td>4.58</td>
<td>10</td>
<td>616–617</td>
<td>6.7</td>
</tr>
<tr>
<td>SN1987A</td>
<td>4.07</td>
<td>63</td>
<td>195–196617</td>
<td>5.5</td>
</tr>
<tr>
<td>Galactic Center</td>
<td>3.92</td>
<td>87</td>
<td>1347–1348617</td>
<td>7.0</td>
</tr>
</tbody>
</table>

Conclusions.---We find no evidence to support the detection of either pointlike or extended sources and set upper limits on the energy flux and energy density of the anisotropic gravitational-wave sky. We assume three different power law models for the gravitational-wave background spectrum. Our mean upper limits present an improvement over Initial LIGO results of a factor of 8 in flux for the $\alpha = 3$ broadband radiometer and factors of 60 and 4 for the spherical harmonic decomposition method for $\alpha = 0$ and 3, respectively [17,40]. We present the first upper limits for an anisotropic stochastic background dominated by compact binary inspirals (with an $\Omega_{GW} \propto f^{2/3}$ spectrum) of $\Omega_{2/3}(\Theta) < (2–6) \times 10^{-8}$ sr^{-1} depending on sky position. We can directly compare the monopole moment of the spherical harmonic decomposition to the isotropic search point estimate $\Omega_{2/3} = (3.5 \pm 4.4) \times 10^{-8}$ from [13]. We obtain $\Omega_{2/3} = (2\pi^2/3H_0^2)f_{\text{ref}}^3 \sqrt{4\pi \mathcal{P}_{00}} = (4.4 \pm 6.4) \times 10^{-8}$. The two results are statistically consistent. Our spherical harmonic estimate of $\Omega_{2/3}$ has a larger uncertainty than the dedicated isotropic search because of the larger number of (covariant) parameters estimated when $f_{\text{max}} > 0$. We also set upper limits on the gravitational-wave strain from point sources located in the directions of Scorpius X-1, the Galactic Center, and Supernova 1987A. The narrowband results improve on previous limits of the same kind by more than a factor of 10 in strain at frequencies below 50 Hz and above 300 Hz, with a mean improvement of a factor of 2 across the band [17].

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[40] The spherical harmonic analysis uses different values of the maximum order lmax than those used in [17]. The lmax value was decreased by a factor of 2 for α = 0 and increased by 30% for α = 3. The harmonic order resembles extra degrees of freedom, meaning that larger values of lmax tend to reduce the sensitivity in ΩGW and increased by α. The harmonic order resembles extra degrees of freedom, meaning that larger values of lmax tend to reduce the sensitivity in ΩGW and increased by α.

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