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Competing effects of nuclear deformation and density dependence of the $\Lambda N$ interaction in $B_\Lambda$ values of hypernuclei

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Competitive effects of nuclear deformation and density dependence of $\Lambda N$ interaction in $\Lambda$ binding energies $B_\Lambda$ of hypernuclei are studied systematically on the basis of the baryon-baryon interaction model ESC (extended soft core) including many-body effects. By using the $\Lambda N$ $G$-matrix interaction derived from ESC, we perform microscopic calculations of $B_\Lambda$ in $\Lambda$ hypernuclei within the framework of the antisymmetrized molecular dynamics under the averaged-density approximation. The calculated values of $B_\Lambda$ reproduce experimental data within a few hundred keV in the wide mass regions from 9 to 51. It is found that competitive effects of nuclear deformation and density dependence of $\Lambda N$ interaction work decisively for fine-tuning of $B_\Lambda$ values.

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I. INTRODUCTION

Basic quantities in hypernuclei are $\Lambda$ binding energies, $B_\Lambda$, from which a potential depth, $U_\Lambda$, in nuclear matter can be evaluated. The early success in reproducing the $U_\Lambda$ value was achieved by Nijmegen hard-core models [1], where the most important role was played by the $\Lambda N$-$\Sigma N$ coupling term. Medium and heavy $\Lambda$ hypernuclei have been produced by counter experiments such as $(\pi^+, K^\pm)$ reactions. Accurate data of $B_\Lambda$ values in ground and excited states of hypernuclei have been obtained by $\gamma$-ray observations and $(e,e'K^\pm)$ reactions. With the increase of experimental information [2], precise interaction models have been constructed. In the Nijmegen group, the soft-core models have been developed with continuous efforts so as to reproduce reasonably hypernuclear data [3–6]. In the recent versions of the extended-soft-core (ESC) models [5,6], two-meson and meson-pair exchanges are taken into account explicitly, while these effects are implicitly and roughly described by exchanges of “effective bosons” in one-boson exchange (OBE) models. The latest model ESC08C aims to reproduce consistently all features of the $S = −1$ and $−2$ systems. The parameter fitting has been improved continuously, and the final version has to be submitted soon. In Ref. [9], they used successfully the version of 2012 in the early stage of parameter fitting [7], denoted as ESC08C(2012). This version is used also in the present work.

Recently, the dependence of $B_\Lambda$ on structures of core nuclei, in particular, nuclear deformations, has been discussed in $p$-shell [8] and $sd$-$pf$-shell [9–11] hypernuclei theoretically. Generally, values of $B_\Lambda$ are related to nuclear structure in two ways. One is that an increase of deformation reduces the overlap of the densities between a $\Lambda$ particle and the core nucleus, which makes $B_\Lambda$ smaller. Such effects are seen in $sd$-$pf$ shell hypernuclei. In Refs. [9,11], the antisymmetrized molecular dynamics for hypernuclei (HyperAMD) [12,13] was applied to several $sd$-$pf$ shell hypernuclei such as $^{28}_{16}$Ca and $^{40}_{20}$Sc. It was found that $B_\Lambda$ values in deformed states were decreased, reflecting smaller overlaps.

The other effect is due to the density dependence of the $\Lambda N$ effective interaction. In light hypernuclei and/or dilute states like cluster states, the density overlap between a $\Lambda$ and nucleons is significantly decreased, which can affect the $B_\Lambda$ through the density dependence. For example, in Be hypernuclei having a $2\alpha$-cluster structure with surrounding neutrons, it was discussed that the overlap becomes much smaller in the well-pronounced $2\alpha$-cluster states [8]. When the $\Lambda N$ effective interaction derived from the $G$-matrix calculation is designed to depend on the nuclear Fermi momentum $k_F$, the smaller overlap makes the relevant value of $k_F$ small, i.e., less Pauli-blocking, resulting in the increase of $B_\Lambda$. Considering this effect, it is expected that appropriate values of $k_F$ in finite systems are reduced as overlaps become small with mass numbers, which would affect the mass dependence of $B_\Lambda$.

$\Lambda N$ interactions are related intimately to the recent topic of heavy neutron stars (NS). The stiff equation of state (EoS) giving the large NS mass necessitates the strong three-nucleon repulsion in the high-density region, the existence of which has been established by many works [14] in nuclear physics. However, the hyperon mixing in neutron-star matter brings about the remarkable softening of the EoS, canceling this repulsive effect. A possible way to solve such a problem is to assume that strong repulsions exist universally in three-baryon channels. More specifically, it is assumed that the $\Lambda NN$ repulsion works in $\Lambda$ hypernuclei as well as the three-nucleon repulsion. A $\Lambda NN$ three-body effect, which is generally a hyperonic many-body effect (MBE), has to appear as an additional density dependence of the $\Lambda N$ effective interaction. It is important to study MBEs by analyzing the experimental data of $B_\Lambda$ systematically.

The aim of the present work is to reveal how the density dependence of the $\Lambda N$ effective interaction affects the mass dependence of $B_\Lambda$. Because the $p$-, $sd$-, and $pf$-shell hypernuclei have various structures in the ground states, they would affect the values of $B_\Lambda$ through the density dependence of the $\Lambda N$ interaction. To investigate it, we use the HyperAMD combined with the $\Lambda N$ $G$-matrix interaction, which
successfully describes various structures of hypernuclei without assumptions on specific clustering and deformations [12,13].

This paper is organized as follows. In the next section, the $\Lambda N$ $G$-matrix interaction is explained as well as treatment of MBEs. In Sec. III, we explain how to describe hypernuclei, namely, the theoretical framework of HyperAMD. In Sec. IV, we show the calculated values of $B_\Lambda$ including MBEs and discuss effects from core structures on $B_\Lambda$. Section V summarizes this paper.

II. $\Lambda N$ $G$-Matrix Interaction

We start from esc08c(2012), which was used in the analysis of $\Lambda$ hypernuclei based on the HyperAMD most successfully [9]. One should be careful, however, that the main conclusion in this work has to be valid qualitatively also for other realistic interaction models including $\Lambda N$-$\Sigma N$ coupling terms which lead to strong density dependencies of the $\Lambda N$ effective interactions. Hereafter, esc08c(2012) is denoted as ESC simply. As a model including an additional density dependence due to a hyperonic MBE, we adopt the model given in Ref. [15]. Here, the multipomerone exchange repulsion (MPP) is added into ESC together with the phenomenological three-body attraction (TBA), where both of them are represented as density-dependent two-body interactions. Using ESC + MPP + TBA, $G$-matrix calculations are performed with the continuous choice for off-shell single-particle potentials: Contributions of MPP and TBA are renormalized into $\Lambda N G$ matrices. The MPP part is given as

$$V_{\text{MPP}}^{(N)}(r; \rho) = g_p N \rho^{N-2}(m_p/\sqrt{2\pi})^3 \exp\left(-\frac{1}{2}m_p^2r^2\right),$$

(1)

corresponding to triple ($N = 3$) and quartic ($N = 4$) pomerone exchange. The values of the two-body pomerone strength $g_p$ and the pomerone mass $m_p$ are the same as those in ESC. A scale mass $M$ is taken as the proton mass. The TBA part is assumed as

$$V_{\text{TBA}}(r; \rho) = V_0 \exp\left(-r^2/(2\rho_0^2)\right)^2 \rho \exp(-3.5\rho)(1 + P)/2,$$

(2)

with $P$ being a space-exchange operator. In Refs. [15,16], these interactions were assumed to be universal in all baryonic channels. Namely, the parameters $g_p^{(3)}$, $g_p^{(4)}$, and $V_0$ in hyperonic channels were taken to be the same as those in nucleon channels, assuring the stiff EoS of hyperon-mixed neutron-star matter. Three sets with different strengths of MPPs were used in Refs. [15,16]. In the case of the set MPa, for instance, the parameters were taken as $g_p^{(3)} = 2.34$, $g_p^{(4)} = 30.0$, and $V_0 = -32.8$. In the present analysis, however, such a choice leads to a too strong density dependence of the $\Lambda N G$-matrix interaction for reproducing the mass dependence of $B_\Lambda$ values: In the case of esc08c(2012), the mass dependence of $B_\Lambda$ values is reproduced rather well without the additional MBE. Then, the values of $g_p^{(3)}$ and $g_p^{(4)}$ may be much smaller than the above values so that the additional density dependence is not strong. Here, the parameters are determined so that calculated results of $B_\Lambda$ values in the present framework are consistent with the experimental data. They are taken as $g_p^{(3)} = 0.39$, $g_p^{(4)} = 0.0$, and $V_0 = -5.0$: MPP (TBA) is far less repulsive (attractive) than those in the above case. In this case, the calculated value of $B_\Lambda$ is 13.0 MeV in $^{16}_\Lambda O$, which is consistent with the observed value (see Table III). Thus, MBE is represented by MPP + TBA, having only minor effects on the results in this work.

$\Lambda N$ $G$-matrix interactions $V_{\Lambda N}$ for ESC are constructed in nuclear matter with Fermi momentum $k_F$ [17]. They are represented in coordinate space and parametrized in a three-range Gaussian form [17]:

$$V_{\Lambda N}(r; k_F) = \sum_{i=1}^3 \left(a_i + b_i k_F + c_i k_F^2\right) \exp\left(-r^2/\beta_i^2\right).$$

(3)

The parameters $(a_i, b_i, c_i)$ are determined so as to simulate the calculated $G$ matrix for each spin-parity state. The procedures to fit the parameters are given in Ref. [17], and the determined parameters for ESC are given in Ref. [9].

Contributions from MBE (MPP + TBA) to $G$ matrices are represented by modifying the second-range parts of $V_{\Lambda N}(k_F, r)$ for ESC by $\Delta V_{\Lambda N}(k_F, r) = (a + b k_F + c k_F^2) \exp\left(-r^2/\beta_i^2\right)$ (3). It should be noted that the values of parameters $g_p^{(3)}, g_p^{(4)}$ and $V_0$ are connected to the values of $a$, $b$, and $c$ through this procedure. The values of parameters are given in Table I.

In applications of nuclear-matter $G$-matrix interactions $V_{\Lambda N}(r; k_F)$ to finite systems, a basic problem is how to choose $k_F$ values in each system: An established manner is to use so called local-density and averaged-density approximations, etc., based on physical insight. As the better choice to describe a single-particle (s.p.) states, we adopt an averaged-density approximation (ADA) [17], where the averaged value of $k_F$ is defined by

$$k_F = \left(\frac{3\pi^2 \rho}{2}\right)^{1/3}. \quad \langle \rho \rangle = \int d^3 r \rho_N(r) \rho_\Lambda(r).$$

(4)

In the case of local-density approximation (LDA), $k_F$ values are obtained from $(\rho_N(r) + \rho_\Lambda(r))/2$ as a function of $r$. We compare ADA and LDA by calculating $B_\Lambda$ values for $^{89}_\Lambda Y$ and $^{16}_\Lambda O$ with use of the $\Lambda$-nucleus folding model in which $\Lambda N$ $G$-matrix interactions $V_{\Lambda N}(k_F, r)$ are folded into density distributions [17]. For spherical-core systems, the results calculated with the $G$-matrix folding model are similar to those with the HyperAMD used in the following section. In Table II, the result is shown in the case of using ESC without MBE. It is demonstrated here that the $B_\Lambda$ values in $^{89}_\Lambda Y$ are reproduced nicely in both cases of ADA and LDA with no adjustable parameter. On the other hand, in $^{16}_\Lambda O$, the value of $B_\Lambda$ obtained with LDA is found to be smaller than

<table>
<thead>
<tr>
<th>$E$</th>
<th>$\delta E$</th>
<th>$\delta O$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>4.809</td>
<td>2.701</td>
</tr>
<tr>
<td>$b$</td>
<td>-11.09</td>
<td>-7.743</td>
</tr>
<tr>
<td>$c$</td>
<td>5.264</td>
<td>8.004</td>
</tr>
</tbody>
</table>
that obtained with ADA. Thus, the $B_\Lambda$ values with LDA are similar to (smaller than) those with ADA in heavy (light) systems, and eventually the mass dependence of $B_\Lambda$ values can be reproduced better using ADA than LDA. Hence, ADA is employed in the present work as an approximate way to use nuclear-matter $G$-matrix interactions in finite systems.

### III. ANALYSIS BASED ON HYPERAMD

In this study, we apply the HyperAMD to $p$-, $sd$-, and $pf$-shell $\Lambda$ hypernuclei, namely, from $^9\Lambda$Li up to $^{59}\Lambda$Fe, to describe various structures of these hypernuclei such as an $\alpha$ clustering and prolate, oblate, and triaxial deformations in ground states. Combined with the generator coordinate method (GCM), we perform the systematic analysis of $B_\Lambda$. 

#### A. Hamiltonian and wave function

The Hamiltonian used in this study is

$$H = T_N + T_\Lambda + T_g + V_{NN} + V_C + V_{\Lambda N},$$  

(5)

where $T_N$, $T_\Lambda$, and $T_g$ are the kinetic energies of the nucleons, $\Lambda$ particle, and center-of-mass motion, respectively. We use Gogny D1S [20,21] as the effective nucleon-nucleon interaction $V_{NN}$, and the Coulomb interaction $V_C$ is approximated by the sum of seven Gaussians. As for the $\Lambda N$ interaction $V_{\Lambda N}$, we use the $G$-matrix interaction discussed above.

The variational wave function of a single $\Lambda$ hypernucleus is described by the parity-projected wave function, $\Psi^\pm = \hat{\rho}_\pm \{\varphi_1, \ldots, \varphi_\Lambda\} \otimes \varphi_\Lambda$, where

$$\varphi_i \propto e^{-\sum m} v_m (r_m - Z_m)^2 \otimes (ui \chi_t + vi \chi_\perp) \otimes (p \text{ or } n),$$  

(6)

$$\varphi_\Lambda \propto \sum_{m=1}^M c_m e^{-\sum m} v_m (r_m - Z_m)^2 \otimes (a_m \chi_\perp + b_m \chi_\perp).$$  

(7)

Here the s.p. wave packet of a nucleon $\varphi_i$ is described by a single Gaussian, while that of $\Lambda$, $\varphi_\Lambda$, is represented by a superposition of Gaussian wave packets. The variational parameters are $Z_t$, $Z_\perp$, $v_\sigma$, $u_t$, $v_t$, $a_m$, $b_m$, and $c_m$. In the actual calculation, the energy variation is performed under the constraint on the nuclear quadrupole deformation parameters $(\beta, \gamma)$ in the same way as in Ref. [13]. By the frictional cooling method, the variational parameters in $\Psi^\pm$ are determined for each set of $(\beta, \gamma)$, and the resulting wave functions are denoted as $\Psi^\pm(\beta, \gamma)$. 

#### B. Angular momentum projection and generator coordinate method

After the variation, we project out the eigenstate of the total angular momentum $J$ for each set of $(\beta, \gamma)$ (angular momentum projection, AMP):

$$\Psi_{MK}^J(\beta, \gamma) = \frac{2J + 1}{8\pi^2} \int d\Omega D_J^{\ast MK}(\Omega) R(\Omega)\Psi^\pm(\beta, \gamma).$$  

(8)

The integrals over the three Euler angles $\Omega$ are performed numerically. Then the wave functions with differing values of $K$ and $(\beta, \gamma)$ are superposed (GCM):

$$\Psi^J_n = \sum_{p} \sum_{K = -J}^{J} c_{npK} \Psi_{MK}^{J \pm}(\beta_p, \gamma_p).$$  

(9)

The coefficients $c_{npK}$ are determined by solving the Griffin-Hill-Wheeler equation [13].

#### C. $B_\Lambda$ and analysis of wave function

The $B_\Lambda$ values are calculated as the energy difference between the ground states of a hypernucleus ($^{A+1}_Z$) and the core nucleus ($^A_Z$) as $B_\Lambda = E(^A_Z; J^+) - E(^{A+1}_Z; J^+)$, where $E(^A_Z; J^+)$ and $E(^{A+1}_Z; J^+)$ are calculated by the GCM.

We also calculate squared overlap between the $\Psi_{MK}^J(\beta, \gamma)$ and the GCM wave function $\Psi^J_n$,

$$O_{MK}^J(\beta, \gamma) = \left|\left\langle \Psi_{MK}^J(\beta, \gamma)|\Psi^J_n\right\rangle\right|^2,$$  

(10)

which we call the GCM overlap. $O_{MK}^J(\beta, \gamma)$ shows the contribution of $\Psi_{MK}^J(\beta, \gamma)$ to each state $J^\pm$, which is useful to estimate the deformation of each state. In this study, we regard $(\beta, \gamma)$ corresponding to the maximum value of the GCM overlap as the nuclear deformation of each state.

### IV. RESULTS AND DISCUSSIONS

#### A. $B_\Lambda$ in $p$-, $sd$-, and $pf$-shell $\Lambda$ hypernuclei

The calculated values of $B_\Lambda$ for ESC including MBEs are summarized in Table III together with the values of $k_F$ and $(\rho)$ and compared with those calculated by using ESC only (in parentheses) and observed values of $B_\Lambda$ ($B_{\uparrow\Lambda}$). Here, the $k_F$ values are calculated by Eq. (4) on the basis of ADA. In Table III, we also show $(\beta, \gamma)$, which gives the maximum value of the GCM overlap defined by Eq. (10).

Recently, in Ref. [28], it has been discussed that the $B_{\uparrow\Lambda}$ measured by the $(p^+, K^+)$ experiments are systematically shallower by 0.54 MeV on average than the emulsion data for $^7\Lambda$Li, $^9\Lambda$Be, $^{10}\Lambda$B, and $^{12}\Lambda$C. It indicates that the reported binding energy of $^{12}\Lambda$C [24] would be shallower by 0.54 MeV, which is used for the binding energy measurements as the reference in the $(p^+, K^+)$ experiments. Therefore, in Table III, the values of $B_{\uparrow\Lambda}$ measured by the $(p^+, K^+)$ or $(K^-, \pi^+)$ experiments (with dagger) are shifted by 0.54 MeV deeper from the values reported by Refs. [2,18,19,22,26]. Despite this correction, calibration ambiguities in the $(p^+, K^+)$ data still remain. One should be mindful of this problem when the calculated values of $B_\Lambda$ are compared with these data.
TABLE III. $-B_{\Lambda}$ (MeV) calculated with ESC + MBE together with $\langle \rho \rangle$ (fm$^{-3}$) and $k_F$ (fm$^{-1}$) defined by Eq. (4), and nuclear quadrupole deformation ($\beta, \gamma$) for each hypernucleus. Values in parentheses are calculated with E08C(2012) only in units of MeV. Observed values $B^\text{exp}_{\Lambda}$ are taken from Refs. [2,18,19,22–28]. Values of $B^\text{exp}_{\Lambda}$ with a dagger are also explained in the text.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\langle \rho \rangle$</th>
<th>$k_F$</th>
<th>$-B^\text{cal}_{\Lambda}$</th>
<th>$-B^\text{exp}_{\Lambda}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^9$Li</td>
<td>±0.50</td>
<td>2$^o$</td>
<td>0.072 1.02</td>
<td>-8.1(-7.9)</td>
<td>-8.50±0.12 [24]</td>
<td></td>
</tr>
<tr>
<td>$^9$Be</td>
<td>0.87</td>
<td>1$^o$</td>
<td>0.060 0.96</td>
<td>-8.1(-7.9)</td>
<td>-6.71±0.04 [25]</td>
<td></td>
</tr>
<tr>
<td>$^{10}$B</td>
<td>0.45</td>
<td>2$^o$</td>
<td>0.072 1.02</td>
<td>-8.2(-8.0)</td>
<td>-8.29±0.18 [24]</td>
<td></td>
</tr>
<tr>
<td>$^{10}$Be</td>
<td>±0.57</td>
<td>1$^o$</td>
<td>0.077 1.04</td>
<td>-9.0(-8.7)</td>
<td>-9.11±0.22 [23]</td>
<td></td>
</tr>
<tr>
<td>$^{11}$B</td>
<td>0.68</td>
<td>1$^o$</td>
<td>0.075 1.04</td>
<td>-9.2(-8.9)</td>
<td>-8.89±0.12 [25]</td>
<td></td>
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<tr>
<td>$^{12}$B</td>
<td>0.50</td>
<td>2$^o$</td>
<td>0.081 1.05</td>
<td>-10.1(-9.8)</td>
<td>-10.24±0.05 [25]</td>
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</tr>
<tr>
<td>$^{12}$C</td>
<td>0.39</td>
<td>44$^o$</td>
<td>0.083 1.07</td>
<td>-11.3(-11.0)</td>
<td>-11.37±0.06 [25]</td>
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<tr>
<td>$^{13}$C</td>
<td>0.41</td>
<td>34$^o$</td>
<td>0.086 1.08</td>
<td>-11.0(-10.7)</td>
<td>-10.76±0.19 [24]</td>
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<tr>
<td>$^{14}$C</td>
<td>0.45</td>
<td>60$^o$</td>
<td>0.090 1.10</td>
<td>-11.6(-11.3)</td>
<td>-11.69±0.19 [23]</td>
<td></td>
</tr>
<tr>
<td>$^{15}$C</td>
<td>0.52</td>
<td>22$^o$</td>
<td>0.093 1.11</td>
<td>-12.5(-12.4)</td>
<td>-12.17±0.33 [24]</td>
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</tr>
<tr>
<td>$^{16}$N</td>
<td>0.28</td>
<td>60$^o$</td>
<td>0.098 1.13</td>
<td>-12.9(-12.6)</td>
<td>-13.59±0.15 [25]</td>
<td></td>
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<tr>
<td>$^{17}$O</td>
<td>0.02</td>
<td>-0.105 1.16</td>
<td>-13.0(-12.7)</td>
<td>-12.96±0.05 [19]</td>
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<tr>
<td>$^{18}$O</td>
<td>0.30</td>
<td>3$^o$</td>
<td>0.110 1.18</td>
<td>-14.3(-14.0)</td>
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<tr>
<td>$^{20}$Ne</td>
<td>0.46</td>
<td>0$^o$</td>
<td>0.106 1.16</td>
<td>-15.4(-15.1)</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>$^{24}$Mg</td>
<td>0.478</td>
<td>21$^o$</td>
<td>0.116 1.20</td>
<td>-16.1(-15.8)</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>$^{25}$Mg</td>
<td>0.36</td>
<td>36$^o$</td>
<td>0.125 1.23</td>
<td>-16.3(-16.4)</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>$^{28}$Si</td>
<td>0.32</td>
<td>53$^o$</td>
<td>0.125 1.23</td>
<td>-16.6(-16.4)</td>
<td>-17.1±0.02 [2]</td>
<td></td>
</tr>
<tr>
<td>$^{32}$S</td>
<td>0.23</td>
<td>16$^o$</td>
<td>0.130 1.24</td>
<td>-17.6(-17.4)</td>
<td>-18.0±0.5 [22]</td>
<td></td>
</tr>
<tr>
<td>$^{40}$K</td>
<td>0.01</td>
<td>-0.136 1.26</td>
<td>-19.4(-19.2)</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{40}$Ca</td>
<td>0.03</td>
<td>-0.136 1.26</td>
<td>-19.4(-19.2)</td>
<td>-19.2±1.1 [26]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{48}$Ca</td>
<td>0.13</td>
<td>12$^o$</td>
<td>0.136 1.26</td>
<td>-19.6(-19.4)</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>$^{48}$K</td>
<td>0.01</td>
<td>-0.141 1.27</td>
<td>-20.2(-20.1)</td>
<td>-</td>
<td></td>
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</tr>
<tr>
<td>$^{56}$V</td>
<td>0.18</td>
<td>2$^o$</td>
<td>0.151 1.31</td>
<td>-20.4(-20.4)</td>
<td>-20.51±0.13 [18]</td>
<td></td>
</tr>
<tr>
<td>$^{59}$Fe</td>
<td>0.26</td>
<td>23$^o$</td>
<td>0.142 1.28</td>
<td>-21.4(-21.3)</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

Let us discuss the calculated values of $B_{\Lambda}$ shown in Table III. As mentioned in Sec. II, we determine the parameters of MPP and TBA in Eqs. (1) and (2) so as to reproduce $B^\text{exp}_{\Lambda}$ in $^{16}$O in the HyperAMD calculation with ESC + MPP + TBA. It is seen that the $B_{\Lambda}$ with ESC + MPP + TBA reproduces the observed data within about 200 keV except for $^{9}$Be, $^{12}$N, and $^{28}$Si, which is achieved owing to the $k_F$ dependence of the $\Delta N$ $G$-matrix interaction used. As seen in Table III, the $k_F$ values become small with decreasing mass number, which means that the $\Delta N$ $G$-matrix interaction becomes attractive. The main origin of the $k_F$ dependence is from the $\Delta N$-$\Sigma N$ coupling terms included in ESC.

B. Effects of core deformation

For the fine agreement of $B_{\Lambda}$ values to the experimental data, it is very important to describe properly the core structures, in particular, nuclear deformations.Recently, many authors have been studying deformations of hypernuclei in $p$-shell [29–32], $sd$-shell [9,10,29–34], and $pf$-shell [9,10,29] mass regions. In this study, we take into account deformations of hypernuclei by performing GCM calculations in which intrinsic wave functions with various ($\beta, \gamma$) deformations, $\Phi(\beta, \gamma)$, are diagonalized.

To study the importance of core deformations in the systematic calculations of $B_{\Lambda}$ values, we perform the GCM calculation by using the spherical wave functions $\Psi_{MK}(\beta=0.0)$ in Eq. (9) [case (B)], whereas Table III summarizes the GCM results with various deformations [case (A)]. In case (B), the $k_F$ value is determined independently from case (A) with $\Psi_{MK}(\beta=0.0)$ by Eq. (4) for each hypernucleus. By using the $k_F$ values determined in case (B), we also perform the GCM calculations with various ($\beta, \gamma$) deformations [case (C)].

Table IV shows the calculated values of $B_{\Lambda}$ in cases (A)–(C) in the typical $p$-shell hypernuclei $^{11}$B, $^{12}$B, and $^{13}$C. Comparing cases (A) and (B), we find the considerable discrepancy of $B_{\Lambda}$, i.e., the $B_{\Lambda}$ values in case (B) are shallower than those in case (A), which indicates that the $B_{\Lambda}$ values become smaller, if the core nuclei are spherical. This is mainly due to the larger $k_F$ value in case (B) compared with that in case (A), which comes from the increase of $\langle \rho \rangle$ in a spherical state [see Eq. (4)]. For example, in the case of $^{11}$B, the obtained value of $B_{\Lambda}$ is 9.7 MeV with $k_F = 1.16$ fm$^{-1}$ in case (B), whereas $B_{\Lambda}$ is 11.3 MeV with $k_F = 1.07$ fm$^{-1}$ in case (A) [cf. $B^\text{exp}_{\Lambda} = 11.4 \pm 0.02$ MeV [27]]. The same difference between cases (A) and (B) is seen in the other hypernuclei, in particular, in light hypernuclei with $A < 16$, as shown in Figs. 1(a) and 1(b).

In Table IV, it is also found that the values of $B_{\Lambda}$ in case (C) are shallower than those in case (B), which deviate greatly from those in case (A) and the observations. This is because the deformation of the core nuclei decreases the overlap between the $\Lambda$ and core nuclei. Because we use the same $k_F$ in cases (B) and (C), the smaller overlap with deformation in case
(C) makes \( B_\Lambda \) shallower. Therefore, it can be said that the consistent descriptions of the deformation and the values of \( k_F \) determined in deformed states are essential to reproduce the observations. \( B_\Lambda \) values are given by the balance of two competitive effects: (i) the deformation makes the \( \Lambda \) s.p. energy (\( k_F \) value) shallower (smaller), and (ii) the smaller value of \( k_F \) makes the \( \Lambda \) s.p. energy deeper due to the density dependence of the \( \Lambda N \) interaction. In the \( A > 16 \) region, generally, deformations make \( B_\Lambda \) values smaller because effect (ii) is not sufficiently remarkable to cancel effect (i). On the other hand, in the \( A < 16 \) region, deformations make \( B_\Lambda \) values larger due to effect (ii).

Let us confirm whether the core deformation is successfully described under the present AMD framework with the Gogny D1S interaction. It can be done essentially by comparing the \( E2 \) transition probabilities, \( B(E2) \), of the core nuclei with the observations, which are quite sensitive to the nuclear deformation. For example, in \( ^{12}_\Lambda B \), we calculate \( B(E2) \) in \( ^{11}_\Lambda B \) as \( B(E2; 5/2^-_1 \rightarrow 3/2^-_1) = 16 \, e^2 \, fm^4 \) by performing the GCM calculation with various \( (\beta, \gamma) \) deformations following Refs. [35,36], which is consistent with the experimental value \( B(E2; 5/2^-_1 \rightarrow 3/2^-_1) = 14 \pm 3 \, e^2 \, fm^4 \) [37]. On the basis of the structure calculation for \( ^{11}_\Lambda B \), we obtain a very reasonable value of \( B_\Lambda \) in \( ^{12}_\Lambda B \) by the addition of a \( \Lambda \) particle. Then, it is confirmed that our calculations for \( B_\Lambda \) are performed in the model space to describe core deformations properly.

Here, we compare the deformation of hypernuclei with that predicted by Ref. [30], in which \( ^{12}_\Lambda C \) and \( ^{28}_\Lambda Si \) are predicted to be spherical within the framework of relativistic mean field, whereas the core nuclei \( ^{12}C \) and \( ^{27}Si \) are oblate deformations. This means that the addition of a \( \Lambda \) particle makes the core nucleus spherical. In the present work, we also find the reduction of the core deformation by the addition of a \( \Lambda \) particle. However, the degree of deformation change is rather small. Thus these hypernuclei are still deformed as shown in Table III, while \( (\beta, \gamma) = (0.50, 59^\circ) \) in \( ^{12}C \) and \( (\beta, \gamma) = (0.35, 55^\circ) \) in \( ^{27}Si \). This difference between the present work and Ref. [30] mainly comes from the effects of rotational motions, which are included by performing the AMP [see Eq.(8)]. In fact, it is also found that the deformation of \( ^{12}C \) becomes spherical before performing the AMP [32], which is the same trend as predicted by Ref. [30]. In the present calculation, not only rotational motions but also configuration mixing and shape fluctuations are taken into account by performing the AMP and GCM, which can affect the deformation of hypernuclei.

### C. Deviation of \( B_\Lambda \) in several hypernuclei

We comment on the large deviation of \( B_\Lambda \) in \( ^{9}_\Lambda Be, ^{15}_\Lambda N, \) and \( ^{28}_\Lambda Si \). In \( ^{9}_\Lambda Be \), it is considered that the Gogny D1S force [20,21] overestimates the size of each \( \alpha \) particle of the 2\( \alpha \)-cluster structure of the core \(^8Be \) due to the zero-range density-dependent term, as pointed out in Ref. [38], which would cause the overestimation of \( B_\Lambda \) by the decrease of \( k_F \) through Eq. (4). It is found that the \( k_F \) value, which reproduces the \( B_\Lambda^{exp} \) of \(^9Be \) (\( k_F = 1.08 \, fm^{-1} \)), is much larger than that shown in Table III (\( k_F = 0.96 \, fm^{-1} \)). The smallness of the latter value of \( k_F \) is due to the overestimation of the size of \( \alpha \) with Gogny D1S. It is also found that the same phenomenon appears in the \( \Lambda \) hypernuclei with \( A < 9 \) having an \( \alpha \)-cluster structure by using Gogny D1S. Therefore, we exclude them from being the subject of the present analysis. In such cases, it would be necessary to use appropriate effective \( NN \) interactions instead of Gogny D1S. In \( ^{15}_\Lambda N \), the \( B_\Lambda^{exp} \) measured by the emulsion experiment [25] seems to be deviating from those of the neighboring hypernuclei in Fig. 1(b). This might be due to the difficulties of the analysis and smaller numbers of events in the emulsion experiments. Therefore, we hope to perform a new analysis of the emulsion measurements with a large statistic in the future. In \( ^{28}_\Lambda Si \), the value of \( B_\Lambda \) is underestimated in case (A), whereas that in case (B) (17.3 MeV) is much closer to the experimental value. This might be due to an overestimation of the core deformation, which is seen in the comparison of the electric quadrupole moment \( Q \) in the ground state \(^{27}Si \) of \(^{27}Si \), namely, \( Q(5/2^+, AMD) = 10 \, e \, fm^2 \), whereas \( Q(5/2^+, exp) = 6.1 \pm 0.4 \, e \, fm^2 \) [39]. Because the calculated values of \( k_F \) are almost the same in cases (A) and (B) (1.23 \( fm^{-1} \)), the value of \( B_\Lambda \) would be in between the values of these cases, if the deformation of \(^{27}Si \) were smaller than the present result.

### D. \( B_\Lambda \) and strength of many-body force

Finally, we also comment on the relation between \( B_\Lambda \) and the strength of MPP and TBA. In the present study, the parameters \( g_F^{(13)} \) and \( g_F^{(4)} \) in Eq. (1) [\( V_0 \) in Eq.(2)] are taken as far smaller (less attractive) than those in Refs. [15,16]. They are determined so as to improve the fitting of \( B_\Lambda \) values to the experimental data. As seen in Table III, the calculated values of \( B_\Lambda \) with ESC only reproduce rather well the experimental ones. Therefore, there remains only a small room to introduce MBE on the basis of ESC. On the other hand, in the case of MPA [15,16], the parameters of MPP and TBA in hyperonic

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**TABLE IV.** Comparison of \( B_\Lambda \) with cases (A), (B), and (C) in \(^{11}_\Lambda B, ^{13}_\Lambda B, \) and \(^{13}_\Lambda C \). The value of \( k_F \) calculated by Eq. (4) in each case is also shown. \( (\beta, \gamma) \) giving the maximum values of the GCM overlap [Eq. (10)] are also shown in cases (A) and (C).

<table>
<thead>
<tr>
<th>( ^{11}_\Lambda B )</th>
<th>( ^{13}_\Lambda B )</th>
<th>( ^{13}_\Lambda C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case (A)</td>
<td>Case (B)</td>
<td>Case (C)</td>
</tr>
<tr>
<td>( -B_\Lambda )</td>
<td>(-10.1)</td>
<td>(-9.0)</td>
</tr>
<tr>
<td>( k_F )</td>
<td>(1.05)</td>
<td>(1.13)</td>
</tr>
<tr>
<td>( (\beta, \gamma) )</td>
<td>((0.50, 29^\circ))</td>
<td>((0.50, 29^\circ))</td>
</tr>
<tr>
<td>( -B_\Lambda^{exp} )</td>
<td>(-10.24 \pm 0.05 [25])</td>
<td>(-11.37 \pm 0.06 [25])</td>
</tr>
</tbody>
</table>
 channels are taken to be the same as those in nucleon channels assuming the stiff EoS of hyperon mixed neutron-star matter. It is found that values of $B_A$ are overestimated if the parameter set of MPa is used combined with ESC. For example, $B_A$ with MPa are 13.0 MeV for $^{15}\text{C}$ (cf. $B_{\text{exp}}^{15}\text{C} = 11.69 \pm 0.19$ MeV) and 14.2 MeV for $^{16}\text{O}$ (cf. $B_{\text{exp}}^{16}\text{O} = 12.96 \pm 0.05$ MeV). This indicates that the strength of MPP and TBA in MPa is too strong to reproduce the observations, when MPa is used together with ESC. It is known that two-body $\Lambda N$ effective interactions still have ambiguities, and thus potential depth and $k_F$ dependence are different among models. The dependence of MBE on two-body $\Lambda N$ effective interaction models will be discussed in a following paper. Here, for instance, a strong MPP such as MPa is shown to be allowable in the case of the latest version of ESC08C.

V. SUMMARY

On the basis of the baryon-baryon interaction model ESC including MBE, competitive effects of nuclear deformation and density dependence of the $\Lambda N$ interaction are investigated. By using the $G$-matrix interaction derived from ESC, we perform microscopic calculations of $B_A$ within the framework of HyperAMD with the ADA treatment for the hypernuclei with $9 \leq A \leq 59$. It is found that the calculated values of $B_A$ reproduce the experimental data within a few hundred keV, when the additional density dependence by MBE is taken into account. This is achieved by the competition between the nuclear deformation and density dependence of $\Lambda N$ interaction. Generally, the overlap between the $\Lambda$ and nucleons varies depending on the degree of core deformation. In the light hypernuclei with $A \leq 16$, it is found that the $B_A$ becomes larger by the density dependence of the $\Lambda N$ interaction, because the overlap rapidly decreases for increasing deformation, which mainly comes from the $\Lambda N-\Sigma N$ coupling. On the other hand, in $sd$-pf-shell hypernuclei, the change of the overlap is rather small even if the core deformation is enhanced. Therefore, the density dependence does not affect the $B_A$ significantly. Instead, increasing deformation makes $B_A$ smaller by decreasing the overlap. Thus, both the taking into account the core deformations and the treatment of the density dependence of the $\Lambda N$ interaction are essential to understand the systematic behavior of $B_A$.

The Fortran code ESC08C2012.f can be found on the permanent open-access website NN-Online: http://nn-online.org.

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