Measurement of transverse energy–energy correlations in multi-jet events in \( pp \) collisions at \( \sqrt{s} = 7 \) TeV using the ATLAS detector and determination of the strong coupling constant \( \alpha_s(m_Z) \)

ATLAS Collaboration

1. Introduction

The study of jet production at the LHC provides a quantitative test of Quantum Chromodynamics, QCD, at the highest momentum transfers. Theoretical calculations for jet cross-sections in hadronic collisions have been carried out up to next-to-leading order (NLO) accuracy in the strong coupling constant \( \alpha_s \) \cite{1,2,3} and extensively compared with the data \cite{4,5,6,7,8,9,10}. These calculations are valid for configurations with up to four jets in the final state.

Event shape variables have been measured in all major \( e^+e^- \) experiments, as well as in experiments at the electron–proton collider HERA. These studies were recently extended to hadron colliders with measurements of the transverse thrust and the transverse minor \cite{11,12} at the Tevatron \cite{13} and the LHC \cite{14,15}.

Energy–energy correlations (EEC), i.e. measurements of the energy-weighted angular distributions of hadron pairs produced in \( e^+e^- \) annihilation, were proposed in Refs. \cite{16,17} as an alternative event shape variable not based on the determination of the thrust principal axis \cite{18} or the sphericity tensor \cite{19}. The EEC function and its asymmetry, AEEC, were subsequently calculated in \( O(\alpha_s^3) \) \cite{20,21,22}, and their measurements \cite{23,24,25,26} have had significant impact on the precision tests of perturbative QCD and in the determination of the strong coupling constant in \( e^+e^- \) annihilation experiments; a recent review is given in Ref. \cite{27}. The EEC are by construction not affected by soft divergences, and as a consequence of this they are calculable at high orders.

The transverse energy–energy correlation function, TEEC, and its asymmetry, ATEE, were proposed as the analogous variables at hadron collider experiments in Ref. \cite{30}, where predictions to leading order (LO) were also presented. The NLO corrections were calculated recently in Ref. \cite{31} using NLOJET++ \cite{2,2.3}. These calculations allow for a numerical determination of the NLO predictions for the TEEC and ATEE, i.e. the coefficients of the second order polynomials in the strong coupling constant. They are used in this paper for quantitative precision tests of QCD including a determination of the strong coupling constant. The TEEC is defined as:

\[
\frac{1}{\sigma} \frac{d\Sigma}{d(\cos \phi)} = \frac{1}{\sigma} \sum_{ij} \int \frac{d\sigma}{dx_{T_i} dx_{T_j} d(\cos \phi)} x_{T_i} x_{T_j} dx_{T_i} dx_{T_j},
\]

where the sum runs over all pairs of jets in the final state with azimuthal\(^1\) angular difference \( \phi = \Delta \phi_{ij} \) and \( x_{T_i} = E_{T_i}/E_T \) is the transverse energy carried by jet \( i \) in units of the sum of jet transverse energies \( E_T = \sum_i E_{T_i} \). In order to cancel uncertainties that

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\(^1\) ATLAS uses a right-handed coordinate system with its origin at the nominal interaction point (IP) in the centre of the detector and the z-axis along the beam pipe. The x-axis points from the IP to the centre of the LHC ring, and the y-axis points upward. Cylindrical coordinates \((r, \phi)\) are used in the transverse plane, \( \phi \) being the azimuthal angle around the beam pipe. The pseudorapidity is defined in terms of the polar angle \( \theta \) as \( \eta = -\ln(\tan(\theta/2)) \).
are constant over $\cos \phi \in [-1, 1]$, it is useful to define the azimuthal asymmetry of the TEEC (ATEEC) as
\[
\frac{1}{\sigma} \frac{d \Sigma^{\text{asym}}}{d \cos(\phi)} = \frac{1}{\sigma} \frac{d \Sigma}{d \cos(\phi)} |_\phi - \frac{1}{\sigma} \frac{d \Sigma}{d \cos(\phi)} |_{\pi - \phi}.
\]
This Letter presents a measurement of the TEEC and its associated asymmetry using high-energy jets.

2. The ATLAS detector

The ATLAS detector [39] is a multi-purpose particle physics detector with a forward–backward symmetric cylindrical geometry and a solid angle coverage of almost $4\pi$.

The inner tracking system covers the pseudorapidity range $|\eta| < 2.5$, and consists of a silicon pixel detector, a silicon microstrip detector, and, for $|\eta| < 2.0$, a transition radiation tracker. It is surrounded by a thin superconducting solenoid providing a 2 T magnetic field along the beam direction. A high-granularity liquid-argon sampling electromagnetic calorimeter covers the region $|\eta| < 3.2$. An iron/scintillator tile hadronic calorimeter provides coverage in the region $|\eta| < 1.7$. The endcap and forward regions, spanning $1.5 < |\eta| < 4.9$, are instrumented with liquid-argon calorimeters for electromagnetic and hadronic measurements. The muon spectrometer surrounds the calorimeters. It consists of three large air-core superconducting toroid systems and separate triggers and high-precision tracking chambers providing accurate muon tracking for $|\eta| < 2.7$.

The trigger system [40] has three consecutive levels: level 1 (L1), level 2 (L2) and the event filter (EF). The L1 triggers are hardware-based and use coarse detector information to identify regions of interest, whereas the L2 triggers are software-based and perform a fast online data reconstruction. Finally, the EF uses reconstruction algorithms similar to the offline versions with the full detector granularity.

3. Monte Carlo samples

Multi-jet production in $pp$ collisions is represented by the convolution of the production cross-sections for parton–parton scattering with the parton distribution functions. Monte Carlo (MC) generators differ in the approximations used to calculate the underlying short-distance QCD process, in the way parton showers are built to take into account higher-order effects and in the fragmentation scheme responsible for long-distance effects. For this analysis, two different MC approaches are used, depending on whether the underlying hard process is considered to be $2 \to 2$ or multi-legged. The generated events are then processed with the ATLAS full detector simulation [41] based on GEANT4 [42].

The baseline MC samples are generated using PYTHIA 6.423 [43] with the matrix elements for the underlying $2 \to 2$ processes calculated at LO using the MRST2007LO* parton distribution functions (PDF) [44] and matched to transverse-momentum-ordered parton showers. The AUET2B tune [45,46] is used to model the underlying event (UE) and the hadronisation follows the Lund string model [47].

Additional samples are generated with HERWIG++ 2.5.1 [48], using the CTEQ6.6 PDF [49] and the UE7000 tune for the underlying event [50]. HERWIG++ uses angular-ordered parton showers, a cluster hadronisation scheme and its own underlying-event parameterisation given by Jimmy [51].

A different approach to simulate multi-jet final states is followed by ALPGEN [52]. This approach is based on LO matrix-element calculations for $2 \to n$ multi-parton final states, with $n \leq 6$, interfaced with HERWIG+JIMMY [53,51] to provide the parton shower, hadronisation and underlying-event models. ALPGEN is known to provide a good description of the multi-jet final states as measured by ATLAS [54].

4. Event selection and jet calibration

The data used in this analysis were recorded in 2011 at $\sqrt{s} = 7$ TeV and collected using a single-jet trigger. It requires at least one jet, reconstructed with the anti-$k_t$ algorithm [55] with radius parameter $R = 0.4$ as implemented in FastJet [56]. The jet transverse energy, $E_T = E \sin \theta$, is required to be greater than 135 GeV at the trigger level. This trigger is fully efficient at reconstructed transverse energies above 240 GeV. Taking into account the prescale factor of this trigger, the data collected correspond to an effective integrated luminosity of $L_{\text{int}} = 158 \text{ pb}^{-1}$ [57].

Events are required to have at least one primary vertex, with five or more associated tracks with transverse momentum $p_T > 400$ MeV. If there is more than one primary vertex, the vertex maximising $\sum p_T^2$ is chosen. MC simulated events are subject to a reweighting algorithm in order to match the average number of interactions per bunch-crossing observed in the data.

In the analysis, jets are reconstructed with the same algorithm as used in the trigger, the anti-$k_t$ algorithm with radius parameter $R = 0.4$. The input objects to the jet algorithm are topological clusters of energy deposits in the calorimeters [58]. The baseline calibration for these clusters corrects their energy using local hadronic calibration [59,60]. The four-momentum of an uncalibrated jet is defined as the sum of the four-momenta of its constituent clusters, which are considered massless. The resulting jets are massive. However, the effect of this mass is marginal for jets in the kinematic range considered in this paper.

The jet calibration procedure includes energy corrections for multiple $pp$ interactions in the same or neighbouring bunch crossings, termed “pileup” in the following, as well as angular corrections to ensure that the jet originates from the primary vertex. Effects due to energy losses in inactive material, shower leakage, the magnetic field, as well as inefficiencies in energy clustering and jet reconstruction, are taken into account. This is done using an MC-based correction, in bins of $\eta$ and $p_T$, derived from the relation of the reconstructed jet energy to the energy of the corresponding hadron-level jet, not including muons or non-interacting particles. In a final step, an in situ calibration corrects for residual differences in the jet response between the MC simulation and the data using momentum-balance techniques for dijet, $\gamma +$ jet, $Z +$ jet and multi-jet final states. This so-called jet energy scale (JES) [61] is subject to uncertainties including those affecting the energy of well-measured objects, like $Z$ bosons and photons. The total JES uncertainty is given by a set of independent sources, correlated in $p_T$. The uncertainty in the $p_T$ of individual jets due to the JES increases from $(1-4\%)$ for $|\eta| < 1.8$, to 5% for $1.8 < |\eta| < 4.5$.

The selected events must have at least two jets with transverse momentum $p_T > 50$ GeV and pseudorapidity $|\eta| < 2.5$. The two leading jets are further required to fulfil $p_T + p_T > 500$ GeV. In addition, jets are required to satisfy quality criteria that reject beam-induced backgrounds [62], as well as criteria for the fraction of the momentum of tracks within the jet which arise from the primary interaction vertex. The number of selected events in data is $3.8 \times 10^5$, with an average jet multiplicity ($N_{\text{jets}}$) = 2.6. The resulting distribution for $(p_T + p_T)/2$ extends up to 1.3 TeV with an average value of 305 GeV.

5. Results at the detector level

The selected events are used to measure the TEEC and its associated asymmetry ATEEC, as defined in Equations (1) and (2). The
TEEC distribution for a sample of $N$ events is obtained by calculating the cosines of the angles in the transverse plane between all possible pairs of jets in each event. Every pair $(i, j)$ represents an entry in the distribution, which is then weighted with the normalised product of the transverse energies. The weights $w_{ij}$ are defined as

$$w_{ij} = x_{i1} x_{j1} = \frac{E_{i1} E_{j1}}{(\sum_k E_{k1})^2},$$

such that for a given event their sum is always unity, as the self correlations $i = j$ are also taken into account. The resulting distribution is then divided by the number of events, which normalises it to unit area. This weighting procedure reduces the sensitivity to the jet energy scale and resolution.

Fig. 1 shows the TEEC and ATEEC distributions along with comparisons to detector-level PYTHIA, HERWIG and ALPGEN expectations. The TEEC exhibits peaks at $\cos \phi = 1$ (self correlations) and near $\cos \phi = -1$, with a rather flat central region around $\cos \phi = 0$. These features are similar to those observed in $e^+ e^-$ annihilation, as described in Ref. [31]. The central region is expected to be dominated by hard radiation processes while multiple soft radiation is expected to be important in the $\cos \phi \approx \pm 1$ regions.

The description of the TEEC is good in the back-to-back region $\cos \phi \approx -1$ for both PYTHIA 6 and ALPGEN. Differences up to 10% are observed in the central part, while the region of small angles shows differences as large as about 15%. The description by HERWIG++ is poorer. The ATEEC exhibits a steep fall-off, which is reproduced by both PYTHIA 6 and ALPGEN. HERWIG++ shows some discrepancies as large as 30%.

6. Correction to particle level

The data are corrected to the particle level in order to take into account detector efficiencies and resolutions. This allows a direct comparison with theoretical calculations, as well as with measurements of other experiments.

Particle-level jets are reconstructed in MC events using all particles with average lifetime $\tau > 10^{-11}$ s, including muons and neutrinos. The kinematic selection criteria are the same as for the detector-level distribution. The unfolding relies on a bin-by-bin correction given by the ratios of the particle-level to detector-level distributions in the PYTHIA AUET2B sample, which is then applied to the detector-level distributions in data. To check the effect of bin migrations on the unfolding procedure, an iterative Bayesian method [63] as implemented in RooUnfold [64] is also used. The convergence criteria is fulfilled when the linear sum over all bins of the absolute relative differences from one iteration to the next drops below $10^{-2}$. The method converges after five iterations. The differences between the two approaches are negligible, compared to the statistical uncertainties, in the full range of $\cos \phi$. This is expected due to the high azimuthal resolution of the jet axis, which is 10 mrad.

The following experimental sources of uncertainty are considered for this measurement:

- **Jet energy scale**: The uncertainty due to the jet energy scale (JES) [61] is calculated using MC techniques by varying each jet energy and momentum by one standard deviation for each of the 63 independent sources of the JES uncertainty, and propagated to the TEEC. These uncertainties depend on the jet transverse momentum and pseudorapidity. The total uncertainty due to the JES is calculated as the sum in quadrature of all independent uncertainties. In order to investigate the effect of possible correlations between JES sources in the analysis, two alternative scenarios with weaker and stronger correlations have been considered [61]. The impact of the change of correlation configurations, as well as of the number of JES independent sources, on the value of $\alpha_s(m_Z)$ and its experimental error is found to be negligible.

The values of the JES uncertainty are typically asymmetric for both the TEEC and ATEEC distributions, although the values for
this asymmetry are small. Thus, the positive and negative parts of the uncertainty are independently summed in quadrature. The TEEC distribution has a total uncertainty of up to 3.5\% from the JES sources, the largest contributions being due to close-by jets and to the different response to jets initiated by gluons or quarks. This is the dominant experimental systematic uncertainty in the analysis.

- **Jet energy resolution**: The uncertainty in the jet energy resolution [65] is propagated to the TEEC by smearing each jet transverse momentum by a \( p_T \) and \( \eta \)-dependent factor accounting for the resolution uncertainty. The size of this uncertainty is below 1\% for both the TEEC and the ATEEC distributions.

- **Pileup**: The pileup uncertainty is estimated by comparing the ratio of the detector-level TEEC and ATEEC distributions obtained in samples with reduced (\( \mu < 6 \)) and enhanced pileup activity (\( \mu > 6 \)). Here \( \mu \) is the average number of interactions per bunch crossing [57]. These ratios are formed in both data and MC simulation and the difference assigned as the pileup systematic uncertainty, which is as large as 2\% (4\%) for the TEEC (ATEEC). The size of this dedicated estimate is larger than what is predicted by the sum of the two sources of uncertainty due to pileup included in the JES uncertainty. The envelope of the two different estimates is used.

- **Parton shower modelling**: To estimate the uncertainty due to the parton shower modelling, the data unfolded with PYTHIA 6 and HERWIG++ are compared. The parton shower and hadronisation models in the two generators are different, as is the implementation of UE effects. The size of this uncertainty is as large as 3.5\% (2.5\%) for the TEEC (ATEEC).

- **Unfolding**: To estimate the uncertainty associated with the unfolding procedure, a data-driven method is used to test its stability. This method relies on the reweighting of the particle-level projection of the unfolding transfer matrix so that the agreement between the detector-level projection and the data is enhanced. This modified detector-level distribution is then unfolded using the correction factors described above. The difference between the modified particle-level distribution and the nominal one is then taken as the uncertainty. This uncertainty is smaller than 0.5\% for the full cosine range.

Other possible sources of uncertainty are also studied, such as the jet angular resolution and jet quality selection procedure. They are found to be at the per mille level, much smaller than the statistical uncertainty on the corrected data, and are therefore neglected. To reduce the effect of statistical fluctuations, all the independent systematic uncertainties discussed here are smoothed separately.

**Fig. 2** shows the breakdown of the systematic uncertainties for both the TEEC and the ATEEC, together with the total, obtained as the sum in quadrature of every independent source discussed above.

The TEEC and ATEEC distributions, once corrected for detector effects, are shown in **Fig. 3**, together with their total uncertainties, while numerical values are given in Tables 1 and 2.

As already seen in the detector-level distributions, PYTHIA 6 and ALPGEN give a fair description of the data both for the TEEC and ATEEC. The back-to-back region \( \cos \phi \sim -1 \) is well described, while small discrepancies, at the level of 10\%, are observed in the central region of the TEEC and for large \( \cos \phi \) values. The description by HERWIG++ is poorer.

The shape of the ATEEC is very similar to that observed at \( e^+e^- \) colliders, see Refs. [23–35], and well reproduced by PYTHIA 6 and ALPGEN.

### 7. Theoretical predictions and uncertainties

In perturbative QCD (pQCD), according to the factorisation theorem [66], final-state observables can be expressed as a convolution of the partonic cross-sections, \( \sigma_i \), with the parton distribution functions. Thus, in this particular case, the TEEC distribution to leading order in the strong coupling constant, can be expressed as the three-jet, energy-weighted, differential cross-section in \( \cos \phi \), normalised to the integrated two-jet cross-section. This can be schematically expressed as

\[
\frac{1}{\sigma_i} \frac{d\Sigma}{d(\cos \phi)} = \frac{\sum_{a_i b_i} f_{a_i}(x_1) f_{a_2}(x_2) \otimes \hat{\Sigma}_{a_1 b_1} f_{a_1}(x_1) f_{a_2}(x_2) \otimes \hat{a}_{a_1 b_1} b_{a_2 b_2}}{\Sigma_{a_i b_i} f_{a_i}(x_1) f_{a_2}(x_2) \otimes \hat{\Sigma}_{a_1 b_1} f_{a_1}(x_1) f_{a_2}(x_2) \otimes \hat{a}_{a_1 b_1} b_{a_2 b_2}},
\]

where \( \sum_{a_1 b_1} f_{a_1}(x_1) f_{a_2}(x_2) \otimes \hat{a}_{a_1 b_1} b_{a_1 b_2} \) is the transverse energy–energy weighted partonic cross-section, \( x_i (i = 1, 2) \) are the fractional longitudinal momenta of the initial-state partons, \( f_{a_i}(x_i) \) and \( f_{a_2}(x_2) \) are the PDF, and \( \otimes \) denotes a convolution over the appropriate variables. The denominator of Eq. (4) is the integrated dijet cross-section used to normalise the TEEC.

The pQCD NLO calculations of the TEEC and ATEEC distributions are performed using NLOjet++ [2,3] interfaced with the MSTW 2008 [67], CT10 [68], NNPDF 2.3 [69] and HERAPDF 1.5 [70] parton distribution functions at NNLO. Typically, \( \mathcal{O}(10^9) \) events are generated for these calculations. This involves the calculation of the \( 2 \rightarrow 3 \) partonic subprocesses at NLO accuracy and of the \( 2 \rightarrow 4 \) partonic subprocesses at tree level. In order to avoid the double collinear singularities appearing in the latter [38], the angular range is restricted to \( |\cos \phi| < 0.92 \).
Fig. 3. The unfolded distributions for transverse energy–energy correlation (left) and its asymmetry (right) along with comparisons to MC expectations. The statistical uncertainties are shown with error bars, while the total experimental uncertainties are shown in a shaded band.

Table 1
Values of the transverse energy–energy correlation function (TEEC). The statistical and systematic uncertainties, due to Jet Energy Scale and Resolution (JES and JER), shower modelling as well as pileup and unfolding, are shown in the subsequent columns. Uncertainties marked with a dash (–) are smaller than 0.00005.

<table>
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<th>cos φ</th>
<th>TEEC</th>
<th>Stat.</th>
<th>JES</th>
<th>JER</th>
<th>Shower</th>
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The renormalisation and factorisation scales, inherent in any pQCD calculation, are usually taken to reflect the typical transverse momentum of the process under investigation. For the TEEC and ATEEC calculations, they are taken to be

$$\mu_R = \mu_F = \frac{p_{T1} + p_{T2}}{2},$$

where $p_{T1}$ and $p_{T2}$ are the transverse momenta of the two leading jets. This is also the choice in Ref. [71]. The value of the strong coupling constant at a given scale is connected to $\alpha_s(m_Z)$ using the two-loop beta function [2,3].

The NLO theoretical predictions are subsequently corrected for non-perturbative effects such as hadronisation and the underlying event. This correction is calculated using the leading-logarithm parton shower generators PYTHIA 6 and HERWIG++ interfaced with different tunes. The full MC generator particle-level predictions with these effects switched on are compared with the parton-level predictions before hadronisation and without UE effects. From this comparison a bin-by-bin correction factor is calculated as the ratio of the two predictions, which is then used to correct the NLOJET++ output. They are found to deviate from unity by about 1% for both PYTHIA 6 and HERWIG++ for most of the $\{\cos \phi\} < 0.92$ range.

Three main theoretical uncertainties are considered for the analysis: those corresponding to the renormalisation and factorisation scale variations, those corresponding to the PDF, and those on the non-perturbative corrections.

- **Scale uncertainty**: The ambiguity in the choice of the renormalisation and factorisation scales gives rise to a scale uncertainty. To estimate it, the scales $\mu_R$ and $\mu_F$ are varied by a factor of two up and down, with the additional requirement that $0.5 \leq \mu_R/\mu_F \leq 2$. From all those variations, the largest uncertainty is obtained when both $\mu_R$ and $\mu_F$ are varied simultaneously by the same factor from the nominal scale. These two combinations are used to define the envelope of the scale uncertainty for both the TEEC and ATEEC. The size of the scale uncertainty is highly asymmetric and is at most about 8% for the TEEC distribution, and somewhat smaller for the ATEEC.

- **PDF uncertainty**: The CT10 parton distribution functions provide 50 variations for the 25 fitted parameters at the 90% confidence level. Each of the 25 parameters are varied up and down following the CT10 recommendations in Ref. [68], and are combined for each bin of the TEEC and ATEEC distributions following the prescription given in Ref. [72]. The size of the PDF uncertainty, once scaled at 68% confidence level, is about 1.5%. A similar procedure is used for the MSTW2008, NNPDF 2.3 and HERAPDF 1.5 parton distribution functions.

8. **Determination of the strong coupling $\alpha_s(m_Z)$**

The evaluation of $\alpha_s(m_Z)$ is made by minimising a $\chi^2$ function taking into account correlations between the systematic uncertainties using nuisance parameters $\lambda_k$, one for each source of uncertainty. These nuisance parameters are normalised to zero mean and unit variance. The minimum of the $\chi^2$ function is found in a 66-dimensional space, one dimension corresponding to $\alpha_s(m_Z)$ and the rest to the nuisance parameters associated with the experimental errors. The function to be minimised is defined as

$$\chi^2(\alpha_s, \lambda) = \sum_i \frac{(x_i - F_i(\alpha_s, \lambda))^2}{\Delta x_i^2 + \Delta t_i^2} + \sum_k \lambda_k^2,$$

where the NLOJET++ predictions are varied according to

$$F_i(\alpha_s, \lambda) = \psi_i(\alpha_s) \left(1 + \sum_k \lambda_k \sigma_k^{(i)} \right).$$

In these expressions, $x_i$ corresponds to the data points in each distribution (TEEC or ATEEC), and $\Delta x_i$ are their statistical uncertainties. $\Delta t_i$ are the statistical errors on the NLOJET++ predictions, while $\sigma_k^{(i)}$ correspond to the $k$-th source of experimental uncertainty in the bin $i$.

The functions $\psi_i(\alpha_s)$ are analytical expressions parameterising the dependence of each observable (TEEC or ATEC) on the strong coupling constant. They are obtained by fitting the predictions for each bin as a function of $\alpha_s(m_Z)$. This function is chosen to be a parabola, as the theoretical predictions account for terms quadratic in $\alpha_s$. The quality of the fit to the NLO theoretical predictions is found to be excellent for each bin of the TEEC and ATEEC. The uncertainties from these fits are negligible.
The theoretical uncertainties on the predictions are treated by varying the theoretical distributions by each independent source of uncertainty (scale, all independent PDF uncertainties and non-perturbative corrections) and repeating the fit using the modified theoretical input.

The fit to the TEEM data exhibits shifts in a few nuisance parameters, which are always compatible with the ±1σ band. The results for the strong coupling constant obtained using different parameterisations of the PDF are summarised in Table 3, together with the experimental uncertainties and the values of $\chi^2/N_{\text{ dof}}$

The final value for the TEEM fits is chosen to be the one obtained using CT10, since its PDF uncertainty is largest and serves as an envelope covering the variations with different PDF sets as shown in Table 3:

\[
\alpha_s(m_Z) = 0.1173 \pm 0.0010 \text{ (exp.)} ^{+0.0009}_{-0.0001} \text{ (scale)}
\]

\[\pm 0.0017 \text{ (PDF)} \pm 0.0002 \text{ (NPC)}.\] (8)

The fit to the ATEEC data does not show any significant shift in the values of the nuisance parameters. In this case, the fit results in the values for the strong coupling constant which are summarised in Table 4.

The final value for the ATEEC fit is also chosen to be the one obtained using the CT10 parton distribution functions:

\[
\alpha_s(m_Z) = 0.1195 \pm 0.0018 \text{ (exp.)} ^{+0.0060}_{-0.0015} \text{ (scale)} \pm 0.0016 \text{ (PDF);} \] (9)

The agreement between the fitted theoretical NLO predictions, including non-perturbative corrections, and the data is good as shown in Fig. 4 and indicated by the $\chi^2$ values given in Tables 3 and 4. Restricting the angular region in the fits to (−0.72, 0.72), yield values of the strong coupling constant which vary within experimental uncertainties. The values of $\alpha_s(m_Z)$ found in this analysis are in agreement with the world average $\alpha_s(m_Z) = 0.1185 \pm 0.0006$ [73], as well as with other determinations of the strong coupling constant from the data collected at the LHC [71,10,74].

Calculations beyond NLO accuracy, which are already available for processes such as top-quark pair [75] or Higgs boson production [76], are needed for multi-jet production at LHC energies. They are expected to reduce the scale uncertainties, which are the limiting factor in this determination of the strong coupling constant.

### Table 3

<table>
<thead>
<tr>
<th>PDF</th>
<th>$\alpha_s(m_Z)$ value</th>
<th>$\chi^2/N_{\text{dof}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSTW 2008</td>
<td>0.1175 ± 0.0010 (exp.) $^{+0.0009}_{-0.0001}$ (scale) ± 0.0006 (PDF) ± 0.0002 (NPC)</td>
<td>29.0/21</td>
</tr>
<tr>
<td>CT10</td>
<td>0.1173 ± 0.0010 (exp.) $^{+0.0009}_{-0.0001}$ (scale) ± 0.0017 (PDF) ± 0.0002 (NPC)</td>
<td>28.4/21</td>
</tr>
<tr>
<td>NNPDF 2.3</td>
<td>0.1183 ± 0.0010 (exp.) $^{+0.0009}_{-0.0001}$ (scale) ± 0.0009 (PDF) ± 0.0002 (NPC)</td>
<td>29.3/21</td>
</tr>
<tr>
<td>HERAPDF 1.5</td>
<td>0.1167 ± 0.0007 (exp.) $^{+0.00040}_{-0.00008}$ (scale) ± 0.0007 (PDF) ± 0.0001 (NPC)</td>
<td>28.7/21</td>
</tr>
</tbody>
</table>

### Table 4

<table>
<thead>
<tr>
<th>PDF</th>
<th>$\alpha_s(m_Z)$ value</th>
<th>$\chi^2/N_{\text{dof}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSTW 2008</td>
<td>0.1195 ± 0.0017 (exp.) $^{+0.0055}_{-0.0015}$ (scale) ± 0.0006 (PDF)</td>
<td>12.7/10</td>
</tr>
<tr>
<td>CT10</td>
<td>0.1195 ± 0.0018 (exp.) $^{+0.0060}_{-0.0005}$ (scale) ± 0.0016 (PDF)</td>
<td>12.6/10</td>
</tr>
<tr>
<td>NNPDF 2.3</td>
<td>0.1205 ± 0.0018 (exp.) $^{+0.0077}_{-0.0011}$ (scale) ± 0.0009 (PDF)</td>
<td>12.2/10</td>
</tr>
<tr>
<td>HERAPDF 1.5</td>
<td>0.1182 ± 0.0013 (exp.) $^{+0.0041}_{-0.0008}$ (scale) ± 0.0007 (PDF) ± 0.0005 (PDF)</td>
<td>12.1/10</td>
</tr>
</tbody>
</table>

### 9. Summary

First measurements of the TEEM and ATEEC functions are presented using 158 pb$^{-1}$ of $pp$ collision data at 7 TeV recorded by the ATLAS experiment at the LHC. For this purpose, multi-jet final states are selected requiring jets, reconstructed with the anti-$k_t$ algorithm and radius parameter $R = 0.4$, with $p_T > 50$ GeV and $|\eta| < 2.5$ and such that the scalar sum of the transverse momenta of the two leading jets is above 500 GeV. The TEEM and ATEEC data are fairly well described by Pythia 6 and ALPGEN, while the HERWIG++ MC simulation shows some discrepancies which can be as large as 30%. The TEEM and the ATEEC at the particle level are compared to perturbative QCD predictions at NLO accuracy. The renormalisation and factorisation scales are chosen to be $(p_{T1}^2 + p_{T2}^2)/2$, ranging from 250 to 1300 GeV and with an average value of 305 GeV. Through their construction, both the TEEM and ATEEC functions are less affected by experimental effects such as the jet energy scale and resolution or pileup than absolute cross-section measurements. Similarly, the PDF uncertainties in their theoretical predictions, as given by Eq. (4), cancel to a large extent. This renders these observables well suited to determine the strong coupling constant. The data for $|\cos \phi| < 0.92$ are fitted to the QCD predictions obtained with NLOJet++ to determine the value of the strong coupling constant. For the TEEM, which provides the experimentally more accurate determination, the result of the fit using the CT10 PDF yields

\[
\alpha_s(m_Z) = 0.1173 \pm 0.0010 \text{ (exp.)} ^{+0.00063}_{-0.00005} \text{ (scale)} \pm 0.0017 \text{ (PDF)} \pm 0.0002 \text{ (NPC)}.\] (10)

The present determination of $\alpha_s(m_Z)$ is limited by the uncertainties due to the choice of renormalisation and factorisation scales.

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Fig. 4. The unfolded distributions for transverse energy–energy correlation (left) and its asymmetry (right) compared with the results of a fit to pQCD NLO calculations including non-perturbative corrections. The green shaded band indicates the uncertainty on the theoretical predictions, which includes the sum in quadrature of uncertainties associated with scale, αS, PDF and NPC. The statistical uncertainties on the predictions are indicated by green error bars, appreciable only on the tail of the ATEEC. The solid error bars on the data points (in black) indicate the experimental uncertainties taking into account the correlations between them. The fitted values of the strong coupling constant are αS(NL) = 0.1173 (TEEC) and αS(NL) = 0.1195 (ATEEC). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

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