Fractional quantum Hall effect at filling factors up to ν=3
LETTER TO THE EDITOR

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Abstract. Additional gaps in the energy spectrum of a two-dimensional electron gas in a strong magnetic field due to electron–electron interaction seem to be responsible for the fractional quantum Hall effect (FQHE). We have analysed the structures attributed to the FQHE in the components $\rho_{xx}$, $\rho_{xy}$ and $\sigma_{xx}$ of the resistivity and conductivity tensor as a function of the magnetic field at Landau filling factors up to $\nu = 3$. For the first time the values for the energy gaps at $\nu = 4$ and $\nu = 3$ are determined and resistivity values for the corresponding Hall plateaux are reported.

Hall effect measurements on GaAs/Al$_x$Ga$_{1-x}$As heterostructures show at low temperatures plateaux in the Hall resistivity $\rho_{xy}$ as a function of the magnetic field $B$ which are attributed to localised states in energy gaps of the electronic spectrum. Simultaneously, the resistivity $\rho_{xx}$ (measured on samples with Hall geometry) and the conductivity $\sigma_{xx}$ (measured on samples with Corbino geometry) become zero at zero temperature.

Most of the gaps in the energy spectrum of a heterostructure (two-dimensional electron gas) in a strong magnetic field can be explained within the independent electron picture which predicts energy levels at $E = (n + \frac{1}{2}) \hbar \omega_c \pm \frac{1}{2} g \mu_B B$ with $n = 0, 1, 2, \ldots$. The spin splitting $g \mu_B B$ is much smaller than the cyclotron energy $\hbar \omega_c$, since the Landé $g$ factor for electrons in GaAs is relatively small (Englert et al 1982). Therefore the energy gaps between levels with different Landau quantum number $n$ are mainly determined by the cyclotron energy $\hbar \omega_c$, and the analysis of the thermally activated resistivity according to $\rho_{xx}(T) \sim \exp(-\Delta E/2kT)$ leads to an activation energy $\Delta E$ which agrees quite well with the cyclotron energy. (The experimentally observed enhancement of $\Delta E$ with values up to $\Delta E = 1.2 \hbar \omega_c$ may be explained by many-body effects.) Consequently, the analysis of the activated conductivity is generally accepted as a method for an approximate determination of the value of energy gaps.

Energy gaps at the Fermi energy are not only visible at integer filling factors $\nu$ of the
Landau levels \( \nu = n_s \hbar/eB \) \( (n_s \) = two-dimensional carrier density) but also at certain fractions of the filling factor\( (\text{Gossard 1984}). \) Especially for \( \nu = \frac{1}{3} \) and \( \nu = \frac{2}{3} \), well pronounced Hall plateaux were observed. The theory of Laughlin (1983), which includes electron–electron interaction, can explain the appearance of gaps at \( \nu = 1/(2m + 1) \) with \( m = 1, 2, 3, \ldots \) as a result of a condensation of the two-dimensional electron gas in a new type of macroscopic ground state with fractionally charged excited states. Electron–hole symmetry arguments are used to explain the existence of energy gaps, not only at \( \nu = \frac{1}{3}, \frac{2}{3} \) etc., but also at \( \nu = \frac{1}{5}, \frac{2}{5} \) etc.

From the theoretical point of view, the energy gaps at \( \nu = \frac{1}{3} \) and \( \nu = \frac{2}{3} \) should be identical (no interaction between different Landau levels) if measured at the same magnetic field. The scaling of the energy gaps with the magnetic field should be proportional to the inverse magnetic length \( l = (\hbar/eB)^{1/2} \) but recent experimental results disagree with this prediction\( (\text{Kawaji et al 1984}). \) Since the magnetic field values for the plateaux at \( \nu = \frac{1}{3} \) and \( \nu = \frac{2}{3} \) are different by a factor of two, one may argue that the value of the energy gap is not simply proportional to \( \sqrt{B} \).

Measurements of the \text{FQHE} at filling factors \( \nu = \frac{1}{3} \) and \( \nu = \frac{2}{3} \) (which are equivalent to the \( \nu = \frac{1}{3} \) and \( \nu = \frac{2}{3} \) state, but are located nearly at the same magnetic field) should give more information about the origin of the \text{FQHE}.

Our high-quality devices (carrier density \( n_s = 2.3 \times 10^{11} \text{ cm}^{-2} \), mobility at \( T = 1.5 \text{ K} \) higher than \( 10^6 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1} \)) are useful for an analysis of the \text{FQHE} at filling factors \( \nu > 0.48 \) (magnetic fields up to \( B = 20 \text{ T} \)) and the measurements of \( \rho_{xx} \) and \( \rho_{xy} \) in figure 1 show clearly both the integral quantum Hall effect (spin splitting up to \( \nu = 10 \) resolved)
and the fractional quantum Hall effect. The minima in $\rho_{xx}$ at filling factors $\nu = 1.63$, $\nu = 1.32$, $\nu = 0.66$, and $\nu = 0.59$ (filling factors calibrated relative to the centre of the plateaux with $\nu = 1$, $\nu = 2$, $\nu = 4$ and $\nu = 6$) agree quite well with odd denominator fractions, but the minimum at $\nu = 0.76$ deviates substantially from the expected value $\nu = \frac{1}{3}$, and the broad minimum at $\nu = 0.5$ cannot be explained. The structures around $\nu = \frac{2}{3}$ and $\nu = \frac{4}{3}$ are much more pronounced in measurements at $T = 50$ mK as shown in figure 2. The upper curves are $\rho_{xx}$ and $\rho_{xy}$ data obtained on a device with Hall geometry, whereas the lower curve is directly proportional to $\sigma_{xx}$ (Corbino device from the same wafer as the Hall device, but with slightly lower carrier density). The Hall plateaux at fractional filling factors have the value $\rho_{xy} = \frac{3h}{4e^2}$ ($1 \pm 0.9 \times 10^{-3}$) within a magnetic field range of $\Delta B/B = 5.4\%$ and $\rho_{xy} = 3h/5e^2$ ($1 \pm 10^{-3}$) within a magnetic field range of $\Delta B/B = 1.9\%$.

Both the conductivity $\sigma_{xx}$ and the resistivity $\rho_{xx}$ at $\nu = \frac{4}{3}$ show a temperature dependence proportional to $\exp(-W/kT)$ with an activation energy of $W/k = 0.25$ K and $W/k = 0.24$ K respectively. These energies are more than a factor 6 smaller than the corresponding gap at $\nu = \frac{3}{5}$. The measured temperature dependence at $\nu = \frac{4}{3}$ is much weaker than that found for $\nu = \frac{2}{3}$ and cannot be described by only one activation energy. Similar results are obtained for samples from another wafer with nearly the same carrier density but about 20\% higher mobility. Figure 3 shows experimental points for $\sigma_{xx}(T)$ and the activation energies represented by straight lines are $W/k = 0.086$ K ($\nu = \frac{3}{5})$ and

**Figure 2.** Measurement of the resistivity and conductivity components $\rho_{xx}$, $\rho_{xy}$ and $\sigma_{xx}$ as a function of the magnetic field at $T = 50$ mK.
Figure 3. Temperature dependence of the conductivity minima at two different filling factors: \( \nu = \frac{3}{8} \); \( \nu = \frac{3}{4} \).

\( \frac{W}{k} = 0.303 \text{ K} \) (\( \nu = \frac{3}{4} \)). The deviation of the data from a simple activated behaviour at \( T < 70 \text{ mK} \) may originate from uncertainties in our temperature calibration.

We would like to point out that \( \sigma_{xx} \) measurements on Corbino devices always show more structures and smaller plateaux at integer filling factors than the corresponding

Figure 4. Conductivity data \( \sigma_{xx}(B) \) (Corbino geometry) at \( T = 0.55 \text{ K} \) for a device with a mobility \( \mu = 1.22 \times 10^6 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1} \).
ρ_{xx} data. One may identify in figure 2 σ_{xx} minima even at \( ν = \frac{6}{7} \), \( ν = \frac{5}{7} \), \( ν = \frac{4}{7} \), and \( ν = \frac{3}{7} \), but these values are not very reliable even though the structures are visible for devices from different wafers.

The conductivity measurement shown in figure 4 shows structures which cannot be explained on the basis of the existing theories. On the other hand we have never observed the expected structure at a filling factor of \( ν = \frac{3}{8} \) at the correct magnetic field position, and the question arises of whether or not a gap at a filling factor \( ν = \frac{3}{8} \) is responsible for the observations. If this is true, the existing theory for the FQHE has to be modified.

References

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