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Review

Theoretical review of series resistance determination methods for solar cells

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ABSTRACT

Using a systematic approach, a collection of expressions for the series resistance of a solar cell are derived from the diode model. Many published series resistance determination methods are among them, or are slight variations on them. Some expressions have not yet been described in the literature. Representation of the methods in a two-dimensional array allows for easy comparison and reveals that many of the previously published methods are more alike than might appear at first sight. From a discussion of the various methods, on the basis of the two-dimensional array arrangement, an overview of the required approximations and assumptions for each method is assembled. Taking the effect of these approximations and assumptions into account, it is expected that the method of Wolf & Rauschenbach will provide the most accurate value for the series resistance of a solar cell.

Contents

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1. Introduction

The importance of ohmic losses in a solar cell was already mentioned in the famous 1954 paper by Chapin, Fuller and Pearson from Bell Labs which marked the start of the modern era of photovoltaics [1]. Since the ohmic loss between the collector and the point in the solar cell where an electron–hole pair is generated depends on the location of that point [2–5], and electron–hole pairs are generated throughout an illuminated solar cell, the concept of R_s as a lumped effective series resistance of the solar cell is – by definition – a simplification. Nevertheless, as long as current crowding phenomena are small the series resistance of a solar cell can be well modeled by R_s [6], making R_s a useful concept and an important parameter in the analyses of a solar cell's performance. Unfortunately the value of R_s cannot be measured directly and is, in fact, rather a challenge to determine accurately. Studies concerning this started not long after the start of the modern era of photovoltaics and because of the emergence of new solar cell materials and designs the topic has been readdressed ever since. Recent developments in high-efficiency concentrator cells provide the latest challenge to determine R_s with an accuracy in the m Ω range for cells with surface areas in the mm^2 range. This is required in order to determine the optimal configuration of the grid contact as this has a major impact on the power output of a concentrator photovoltaic (CPV) system.

The many methods to determine R_s that have been described in the literature over the years have resulted in a large number of expressions for R_s in parameters that can be directly determined from a solar cell's *IV*-characteristic. The present study provides a systematic approach towards the derivation of these expressions for R_s for a large collection of methods [7–20], resulting in a framework in which these methods are arranged and compared with each other. This reveals that they are more alike than might appear at first sight. They all follow a derivation involving either an equation based on the diode model of a solar cell (labeled f here), its derivative (labeled f'), its integral (labeled F) or a combination of two of them. The systematic derivation of these equations and their arrangement in a two-dimensional array provides a convenient overview of all possible approaches to determine the series resistance of a solar cell from one or two of the above-mentioned equations. The two-dimensional array arrangement also reveals that there are several approaches to determine R_s which have not yet been described in the literature.

In a subsequent analysis of the methods we determine from a theoretical perspective which method is expected to give the best approximation for R_s. That is, which method uses the least unfavorable approximations and assumptions in the derivation towards its expression for R_s.

2. Theory

A general equivalent circuit of a single junction solar cell with a lumped effective series resistance R_s is displayed in Fig. 1. The associated expression for the current I generated by the cell as a function of voltage V is [21]:

$$I = I_L - \sum_{\alpha=a,b,c,\ldots} (I_{D,\alpha}) - I_{sh}$$
$$= I_L - \sum_{\alpha=a,b,c,\ldots} \left(I_{0,\alpha} \left[\exp\left(\frac{V+IR_s}{n_\alpha V_t} \right) - 1 \right] \right) - \frac{V+IR_s}{R_{sh}}, \quad (1)$$

with I, V being the light induced current I_L (which is proportional to the irradiance E), the current $I_{D,\alpha}$ of diode α,[1] the series

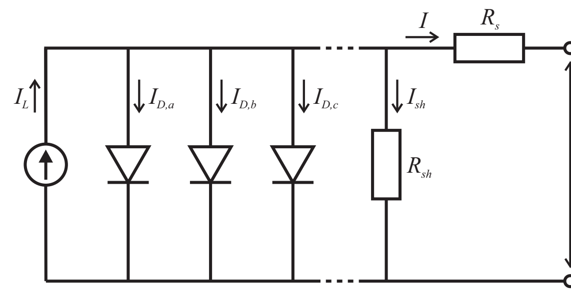

Fig. 1. General equivalent circuit of a solar cell with a lumped effective series resistance.

resistance R_s and the current I_{sh} flowing through shunt resistance R_{sh} all as defined in Fig. 1. $I_{0,\alpha}$ and n_α are the saturation current and ideality factor of diode α. Lastly, there is the thermal voltage V_t, defined as kT/q, with k being the Boltzmann constant, T being the absolute temperature of the solar cell and q being the elemental charge. A list of symbols is provided in Appendix A. With the sign convention used in Eq. (1) the direction in which the light generated current flows is defined as positive and the illuminated *IV*-curve lies in the first quadrant.

Unfortunately Eq. (1) has no general analytic solution. For this reason, the set of diodes is usually represented by a single diode with an associated n and I_0 value. This simplification causes these values to be functions of I and E. The diode ideality factor increases with I and decreases with E as illustrated in Fig. 2 and approaches 1 at high E and/or high V since recombination in the quasi-neutral region dominates under these conditions [22,23]. However, n and I_0 are generally approximated as constants. Another generally used approximation is that $R_{sh} \to \infty$, which is valid for a high quality solar cell. Using these simplifications Eq. (1) can be written as

$$I = I_L - I_0 \left[\exp\left(\frac{V+IR_s}{nV_t} \right) - 1 \right], \quad (2)$$

the so-called single-diode equation. And although this equation still has no general analytic solution, it can be rearranged into the explicit function

$$V = nV_t \ln\left(\frac{I_L + I_0 - I}{I_0} \right) - IR_s. \quad (3)$$

This equation could also be rewritten into an expression for R_s. However, the parameters I_L, I_0 and n are notoriously hard to determine. Therefore, the idea is to find an expression for R_s in terms of cell parameters which are easier to determine such as the short circuit current I_{sc}, the current and voltage at the maximum power point I_{mp} and V_{mp}, the open-circuit voltage V_{oc} and the area A under the *IV*-curve in the first quadrant. Since $I_0 \ll I_L$ in practice, a frequently applied way to avoid having to determine I_0 is to make sure it only appears in a sum together with I_L so that the approximation

$$I_c \equiv I_L + I_0 \approx I_L \quad (4)$$

can be applied. At short circuit conditions, Eq. (3) can be rewritten as

$$I_{sc} = I_L + I_0 - I_0 \exp\left(\frac{I_{sc}R_s}{nV_t} \right), \quad (5)$$

from which it follows that the short circuit current I_{sc} is a good approximation for the photo current I_L and/or I_c, as long as $I_{sc}R_s$ is

[1] Usually the number of constituent diodes is taken to be 2 or 3. Each diode represents a section of the solar cell where a specific recombination mechanism dominates. One diode represents the recombination in the quasi-neutral regions, another the recombination in the depletion region and at the cell surface. A third

(*footnote continued*)
diode is sometimes included to represent Auger recombination in the solar cell, which can be important for in particular silicon cells and III–V cells under very high concentration ratios.

small. In practice this condition is considered to be met as long as the exponential knee of the *IV*-curve remains in the first quadrant. The I_0 that appears in the denominator of the logarithmic term in Eq. (3) can be eliminated using the following equation for open-circuit conditions:

$$V_{oc} = nV_t \ln\left(\frac{I_c}{I_0}\right), \tag{6}$$

and the fact that $\ln((I_c - I)/I_0) = \ln((I_c - I)/I_c) + \ln(I_c/I_0)$, from which it follows that

$$V = nV_t\lambda + V_{oc} - IR_s, \tag{7}$$

with

$$\lambda \equiv \ln\left(\frac{I_c - I}{I_c}\right). \tag{8}$$

Note that here it is assumed that the values of n and I_0 at open-circuit conditions are equal to those at the current of interest. Eq. (7) can be rearranged into the first expression for R_s, expression (12), presented as approach *f*-, in Table 1.

A second expression for R_s can be obtained by first taking the derivative with respect to I of Eq. (3) or (7). The dV/dI term on the left hand side of the resulting equation can also be written as

$$\frac{dV}{dI} = \frac{\frac{dP}{dI} - V}{I}, \tag{9}$$

where $P = VI$ is the power generated by the solar cell under illumination. Since $dP/dI = 0$ at maximum power point (MPP) conditions, $dV/dI|_{MPP} = -V_{mp}/I_{mp}$. Using this substitution, and the assumption that dn/dI and dI_0/dI are zero at the point of interest, the derivative of Eq. (3) or (7) can be rearranged into expression (13) for R_s presented in Table 1 as approach f'-, with

$$I_a \equiv I_c - I_{mp}. \tag{10}$$

A third expression for R_s can be obtained by first integrating Eq. (7) with respect to I over the interval $[0, I_{sc}]$. The integral equals A, the area under the *IV*-curve in the first quadrant. Under the assumption that n and I_0 are constant over the entire *IV*-curve, the resulting equation can be rearranged into expression (14) for R_s, with

$$I_b \equiv 2I_c - I_{sc}. \tag{11}$$

Expression (14) is also displayed in Table 1, as approach *F*-.

Note that expressions (12)–(14) all still contain the unknown parameter n. However, any combination of f, f' and F can be used to obtain an expression for R_s from which n (as well as V_t) is eliminated. This results in the remaining expressions presented in Table 1, labeled by the column and row they are in.

Sets of V and I in Eqs. (12)–(20) can in principle be arbitrarily chosen. However, since n and I_0 are functions of I but are taken as constants, the determined R_s values will depend on I. Therefore, R_s should preferably be determined at the MPP since this is the point at which the cell should be operating. This means V and I in expressions (12)–(20) should be replaced with V_{mp} and I_{mp}. And as mentioned earlier, I_c can be approximated by I_{sc} as long as $I_{sc}R_s$ is small. For the combination of an expression with itself, i.e. approaches ff, $f'f'$ and FF, subscripts 1 and 2 are used to distinguish the variables stemming from each expression. A second *IV*-curve measured under different conditions (usually a different irradiance, under the assumption that n and I_0, and R_s for that matter, are independent of E) is used to supply the required second set of variables.

The above described systematic approach to derive expressions for R_s not only results in a number of expressions that can be found in the literature, but also reveals expressions (15), (17)–(20) which have, to our knowledge, not appeared in the literature before. The others have, and in some cases the author(s) have applied elegant procedures or set some extra requirements to improve the methods as they appear in Table 1. Several examples will be discussed below. An overview of all the assumptions and approximations required for each method is presented in Table 2 in the Discussion and conclusions section.

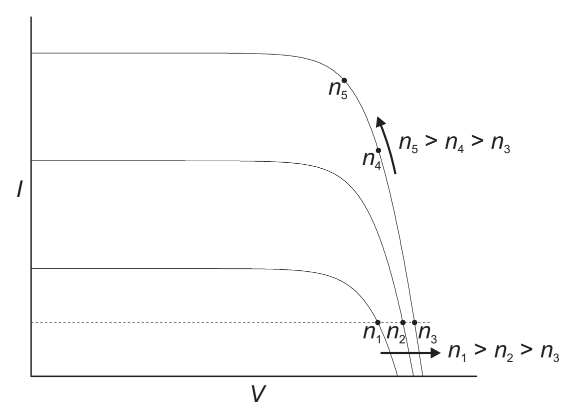

Fig. 2. Variation of diode ideality factor n with current and irradiance when multiple diodes are represented by a single diode (adapted from Hamdy and Call [22]).

3. Evaluation of approaches

For any approach, the accuracy with which the value of R_s of can be determined of course depends on the accuracy with which the *IV*-curve(s) can be obtained experimentally. Only for methods

Table 1
Two-dimensional array arrangement of expressions for R_s, resulting from the single-diode equation (f), its derivative (f'), its integral (F) and combinations thereof.

2nd Index	Category					
	f		***f′***		***F***	
–	$\frac{nV_t\lambda + V_{oc} - V}{I}$	(12)	$\frac{V_{mp}}{I_{mp}} - \frac{nV_t}{I_a}$	(13)	$\frac{2}{I_b}\left(\frac{V_{oc}I_c - A}{I_{sc}} - nV_t\right)$	(14)
f	$\frac{(V_{oc1} - V_1)\lambda_2 - (V_{oc2} - V_2)\lambda_1}{I_1\lambda_2 - I_2\lambda_1}$	(15)	$\frac{I_a\lambda\frac{V_{mp}}{I_{mp}} + V_{oc} - V}{I + I_a\lambda}$	(16)	$\frac{2\lambda}{I_b\lambda + 2I}\left(\frac{V_{oc} - V}{\lambda} + \frac{V_{oc}I_c - A}{I_{sc}}\right)$	(17)
f′	see $f'f$		$\frac{I_{a1}\frac{V_{mp1}}{I_{mp1}} - I_{a2}\frac{V_{mp2}}{I_{mp2}}}{I_{a1} - I_{a2}}$	(18)	$\frac{2}{I_{sc} - 2I_{mp}}\left(\frac{V_{mp}I_a}{I_{mp}} - \frac{V_{oc}I_c - A}{I_{sc}}\right)$	(19)
F	see Ff		see Ff'		$\frac{2}{I_{b1} - I_{b2}}\left(\frac{V_{oc1}I_{c1} - A_1}{I_{sc1}} - \frac{V_{oc2}I_{c2} - A_2}{I_{sc2}}\right)$	(20)

Table 2
Overview of the various assumptions and approximations applied for each R_s determination method. A '×' indicates the use of the particular assumption/approximation and 'n/a' stands for not applicable.

R_s Determination approach	R_s Determination method	Minimum number of required *IV*-curves	Assumption/approximation												
			Single-diode model	$n=1$	$n \neq n(I)$[a] and $I_0 \neq I_0(I)$	$n \neq n(I)$ and $I_0 \neq I_0(I)$, around MPP[b]	$n \neq n(I)$ and $I_0 \neq I_0(I)$, at MPP[c]	$n_{mp} = n_{oc}$ and $I_{0,mp} = I_{0,oc}$	$n \neq n(E)$ and $I_0 \neq I_0(E)$	$I_L + I_0 = I_L$	$R_{sh} \rightarrow \infty$	$R_{s2} = R_{s1}$	$R_{s2} \ll \frac{V_2 - V_1}{I_{L1} - I_1}$	$I_L = I_{sc}$	$I_{L1} - I_{L2} = I_{sc1} - I_{sc2}$[d]
***f*-**	General (fit around MPP)	1	×			×	×	×	n/a	×	×	n/a	n/a	×	×
ff	General	2	×		×[e]	×[e]	×[e]	×	×	×	×	×		×	×
ff	Swanson	2			×[e,f]	×[e,f]	×[e,f]		×[f]			×			×
ff	Aberle et al.	2			×[e,f]	×[e,f]	×[e,f]		×[f]				×	×	×
ff	Wolf & Rauschenbach	2			×[e,f]	×[e,f]	×[e,f]	×[f]	×[f]						×
ff	Mialhe & Charette	2	×			×[g]	×	×	n/a	×	×			×	×
***f′*−**	Warashina & Ushirokawa	1	×			×	×		n/a	×	×	n/a	n/a	×	×
f′f	Picciano	1	×				×	×	n/a	×	×	n/a	n/a	×	×
f′f	Jia et al.	1	×				×	×[h]	n/a	×	×	n/a	n/a	×	×
f′f′	general	2	×		×[e]	×[e]	×		×	×	×	×		×	×
***F*-**	Araujo & Sánchez	1	×	×	×	×	×	×	n/a	×	×	n/a	n/a	×	×
Ff	General	1	×		×	×	×	×	n/a	×	×	n/a	n/a	×	×
Ff′	General	1	×		×	×	×	×	n/a	×	×	n/a	n/a	×	×
FF	General	2	×		×	×	×	×	×	×	×	×		×	×

[a] Note that if the more vigorous approximation $n=1$ is used, the present approximation is automatically applied as well.
[b] Note that if the more vigorous approximations $n \neq n(I)$ and $I_0 \neq I_0(I)$ are used, the present approximations are automatically applied as well.
[c] Note that if the more vigorous approximations $n \neq n(I)$ and $I_0 \neq I_0(I)$, around MPP are used, the present approximations are automatically applied as well.
[d] Note that if the more vigorous approximation $I_L = I_{sc}$ is used, the present approximation is automatically applied as well.
[e] The present approximation applies since $I_{mp\ 2}$ will differ significantly from $I_{mp\ 1}$.
[f] Any I and/or E dependence of n and I_0 will not be caused by the single-diode model approximation.
[g] Since $I_{mp\ 2}$ will differ (only) slightly from $I_{mp\ 1}$ it is only the present approximation that applies, and not the more vigorous approximation that $n \neq n(I)$ and $I_0 \neq I_0(I)$.
[h] Only $I_{0,mp} = I_{0,oc}$ is assumed here, not $n_{mp} = n_{oc}$.

where the determined value of R_s is particularly sensitive to the accuracy of the *IV*-curve(s) will this fact be explicitly stressed.

3.1. F category

A general weak point of the approaches in the *F* category (*F*-, *Ff*, Ff' and *FF*) is that R_s cannot be determined specifically around the MPP, but instead an average of the entire *IV*-curve is given.

3.1.1. F- approach

One approach in the *F* category that can be found in the literature is *F*-, which is known as the (*IV*) Area method and was first described by Araujo and Sánchez [15]. Note that this is an example where the unknown n still appears in the expression. The authors deal with this by the additional simplification of setting $n=1$ (for the entire *IV*-curve[2])with the motivation that this is valid at the illumination levels required for the resistance effects of a concentrator cell to show, at which the emitter diffusion current is dominant along almost the entire *IV*-characteristic. This method requires the cell temperature to be known, since V_t appears in expression (14).

3.2. f' category

3.2.1. f'-approach

The f'-approach in category f' is another example where the unknown n still appears in the expression. However, Warashina and Ushirokawa presented a way to eliminate it in their 1980 paper [12]. To do this expression (13) has to be written in the general form which is also valid for points other than the MPP:

$$R_s = -\frac{dV}{dI} - \frac{nV_t}{I_c - I}. \tag{21}$$

Applying it to two data points of the (single) measured *IV*-curve and eliminating nV_t result in

$$R_s = \frac{(I_c - I_1)\frac{dV}{dI}\Big|_1 - (I_c - I_2)\frac{dV}{dI}\Big|_2}{I_1 - I_2}, \tag{22}$$

which gives R_s in its simplest form. Of course, the same equation would be obtained for the $f'f'$ approach if one imposed the conditions that $I_{L2} = I_{L1}$ and $I_2 \neq I_1$. An easy way to increase the accuracy by which R_s is determined is to make use of (many) more available data points from the single *IV*-measurement. It then becomes a least squares fit of $(dV/dI)(I)$, with I_c taken as a known parameter. This is done by plotting dV/dI against $-(I_c - I)^{-1}$ since the y-intercept of a linear fit equals $-R_s$, which follows from Eq. (21) and is the method presented by Warashina and Ushirokawa [12].[3] The resulting expression for R_s is

$$R_s = \frac{\sum\xi_i \sum\xi_i V_i' - \sum\xi_i^2 \sum V_i'}{N\sum\xi_i^2 - \left(\sum\xi_i\right)^2}, \tag{23}$$

with $\xi_i \equiv -(I_c - I_i)^{-1}$, $V_i' \equiv dV/dI|_i$ and N being the number of data points used. Note that the method assumes n and I_0 to be constant over the fitted range.

An example of a plot of dV/dI against $-(I_c - I)^{-1}$ (with I_c approached by I_{sc}) is displayed in Fig. 3 for the 11 data points surrounding the MPP of the *IV*-curve obtained at a concentration ratio C of 500. The data is obtained from an experimental study of the R_s determination methods applied to a CPV cell [24]. Note that while the linear relation can indeed be observed, the large extrapolation required to reach the dV/dI-axis introduces a considerable uncertainty for the R_s value. It also turns out that the value of R_s obtained with this method is very sensitive to the value of I at which it is obtained, especially around the MPP. This is illustrated in Fig. 4, where R_s is plotted as a function of I/I_{sc}. Consequently, a small uncertainty in the determination of I_{mp} translates into a large uncertainty of R_s. In this particular example, the R_s value derived from the cell's *IV*-characteristic obtained at a concentration ratio of 200 is even negative at the MPP.

Another disadvantage of this method is that it is not entirely straightforward to determine dV/dI in practice (which for this reason was replaced by $-V_{mp}/I_{mp}$ in expression (13)). To determine the data points of Figs. 3 and 4 dV/dI was, therefore, approximated by taking the derivative of the best quadratic polynomial fit for each data point and its two nearest neighbors.[4]

3.2.2. $f'f$ approach

In this approach, n (as well as V_t) is eliminated by combining f'- and f-. An example of the $f'f$ approach can be found in the literature as the Maximum Power Point method and was first described by Picciano [10].[5]

The method Jia et al. presented in 1988 as an improvement on the method of Picciano [18] is an example where an extra requirement was set to the method as it appears in Table 1. For this the authors take the diode ideality factor's dependence on the output current of the solar cell into account by setting n at open circuit conditions (defined as n_{oc} here) equal to 1, while n at the MPP (defined as n_{mp} here) is not set to a specific value.[6,7] This is based on the fact that n approaches 1 at high voltages, as mentioned in Section 2. In this way expression (12) at MPP conditions changes to

$$R_s = \frac{\lambda_{mp} n_{mp} V_t + n_{mp} V_{oc} - V_{mp}}{I_{mp}}, \tag{24}$$

where λ_{mp} is defined as λ at MPP conditions, and expression (13) changes to

$$R_s = \frac{V_{mp}}{I_{mp}} - \frac{n_{mp} V_t}{I_a}. \tag{25}$$

Rearranging Eq. (25) into an expression for n_{mp} and substituting that into Eq. (24) result in

$$R_s = \frac{V_{mp}(I_d - I_{mp})}{I_{mp}(I_d + I_{mp})}, \tag{26}$$

[2] It is also possible to slightly alter the method of Araujo & Sánchez so that n only has to be set equal to 1 for high voltages, for example for the range $V_{mp} \le V \le V_{oc}$. It only requires reducing the upper integration limit from I_{sc} to I_{mp} when integrating Eq. (7), so that one only calculates the area A' under the *IV*-curve below the MPP. Rearranging the resulting expression for A' results in

$$R_s = \frac{2}{2I_c - I_{mp}}\left(\frac{V_{oc}I_c - A' - V_{mp}(I_c - I_{mp})}{I_{mp}} - nV_t\right).$$

[3] Almost a decade later the method was published again by Sites and Mauk [19].

[4] This generally applied approximation can be omitted by deriving the exact expression

$$\frac{\Delta V}{\Delta I} = \frac{\ln\frac{I_c - I_2}{I_c - I_1}}{\Delta I} nV_t - R_s,$$

and applying the technique of Warashina & Ushirokawa to it, i.e. plotting $\Delta V/\Delta I$ against $\ln(I_c - I_2)/(I_c - I_1)/\Delta I$. This method would fall under the *f*- approach.

[5] Although expression (16) might appear to be different from Picciano's expression for R_s, the only differences are that he has already substituted V and I by V_{mp} and I_{mp} and has applied the approximation $I_c \approx I_L$.

[6] They also set $n=2$ at I_{sc}. This, however, is irrelevant since it disappears from the equations once $I_L + I_0$ is approximated by I_{sc} in their derivation.

[7] The same conditions can be applied to other approaches, for example the *ff* approach. This results in

$$R_s = -\frac{(V_t\lambda_1 + V_{oc1})V_2 - (V_t\lambda_2 + V_{oc2})V_1}{(V_t\lambda_1 + V_{oc1})I_2 - (V_t\lambda_2 + V_{oc2})I_1}.$$

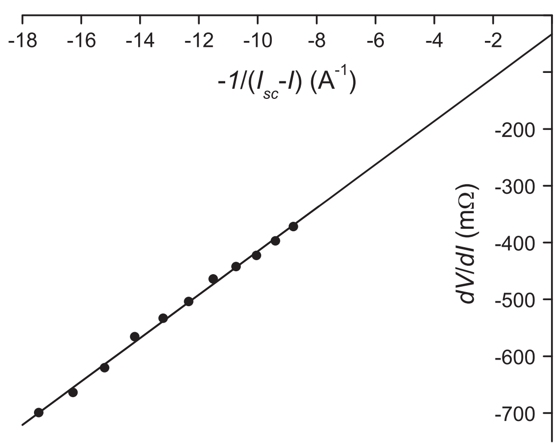

Fig. 3. Plot of dV/dI as a function of $-(I_{sc}-I)^{-1}$ for 11 data points surrounding the MPP of the *IV*-curve measured at a concentration ratio of 500, and the linear fit.

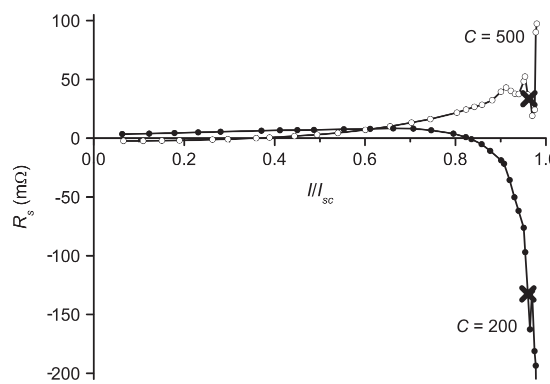

Fig. 4. Plot of R_s, determined using the method of Warashina & Ushirokawa, as a function of I/I_{sc}, for concentration ratios 200 and 500. The diagonal crosses indicate the MPPs.

after some rewriting, with

$$I_d \equiv \left(\lambda_{mp} + \frac{V_{oc}}{V_t}\right) I_a. \tag{27}$$

However, since the adaptation of Picciano's method by Jia et al. is based on the notion that n is not constant throughout the *IV*-curve, it should be noted that in order to obtain Eq. (25) the assumption that $dn/dI|_{MPP} \approx 0$ has been applied, although Jia et al. do not state this explicitly. Without this assumption Eq. (26) becomes

$$R_s = \frac{V_{mp}(I_d - I_{mp})}{I_{mp}(I_d + I_{mp})} + \frac{I_d^2}{I_a(I_d + I_{mp})} \left.\frac{dn}{dI}\right|_{MPP}. \tag{28}$$

Note that $I_d^2/I_a(I_d+I_{mp})$ can in fact be very large compared to R_s, so that this method strongly relies on the above assumption being correct. Also note that this method also assumes that $I_{0,mp} = I_{0,oc}$ and that I_0 is independent of I around the MPP. The method also requires the cell temperature to be known, since $V_t = kT/q$.

3.3. *f* category

3.3.1. *ff* approach

This approach eliminates n (as well as V_t) by combining f with itself. Three popular methods have been reported in the literature [7,8,20] that according to the present study and are adaptations of the *ff* approach. They have in common that they all apply the additional constraint

$$I_{L1} - I_1 = I_{L2} - I_2. \tag{29}$$

Once the first *IV*-curve is measured and I_1 is selected, this constraint dictates either the irradiance at which the second *IV*-curve should be measured or the value of I_2. The easiest way to demonstrate the effect of this constraint on expression (15) is to first rewrite it as

$$R_s = -\frac{V_2\mu_1 - V_1\mu_2}{I_2\mu_1 - I_1\mu_2}, \tag{30}$$

with

$$\mu_i \equiv \ln\left(\frac{I_{c,i} - I_i}{I_0}\right). \tag{31}$$

This equation can be obtained by using Eq. (3) instead of Eq. (7) in a similar derivation as the one that led to expression (15). Applying the constraint leads to $\mu_2 = \mu_1$ and $I_1 - I_2 = I_{L1} - I_{L2}$, simplifying Eq. (30) to

$$R_s = \frac{V_2 - V_1}{I_{L1} - I_{L2}}. \tag{32}$$

The above constraint might seem to simply limit the general applicability of expression (15). However, even though the original papers do not fully explain or even mention this, it does not go unrewarded [7,8,20]. Since it is at exactly this condition that the *ff* approach is valid for solar cell models involving multiple diodes and a finite shunt resistance, i.e. with an $I(V)$ characteristic of the form given by Eq. (1). Because if n_a is eliminated from this equation in the typical fashion for the *ff* approach, then after some rearrangement (see Appendix B) this results in

$$\frac{I_{L1} + I_{0,a} - I_1 - \sum_{\alpha=b,c,\ldots} I_{0,\alpha}\left[\exp\left(\frac{V_1 + I_1 R_{s1}}{n_\alpha V_t}\right) - 1\right] - \frac{V_1 + I_1 R_{s1}}{R_{sh}}}{I_{0,a}} = \left(\frac{I_{L2} + I_{0,a} - I_2 - \sum_{\alpha=b,c,\ldots} I_{0,\alpha}\left[\exp\left(\frac{V_2 + I_2 R_{s2}}{n_\alpha V_t}\right) - 1\right] - \frac{V_2 + I_2 R_{s2}}{R_{sh}}}{I_{0,a}}\right)^{(V_1 + I_1 R_{s1})/(V_2 + I_2 R_{s2})}. \tag{33}$$

Note that for completeness both curves have been given their own value for the series resistance, R_{s1} and R_{s2}, symbolizing a possible irradiance dependence of the series resistance. Now if $I_{L1} - I_1 = I_{L2} - I_2$, then Eq. (33) is valid when $V_1 + I_1 R_{s1} = V_2 + I_2 R_{s2}$. Combining these last two equations results in

$$R_{s1} = \frac{V_2 - V_1 + I_2 R_{s2}}{I_{L1} - I_{L2} + I_2}, \tag{34}$$

which simplifies to Eq. (32) when R_{s1} and R_{s2} are equal.[8]

The difference between the three adaptations lies in the way in which I_{L1}, I_{L2}, I_1 and I_2 are chosen, and will be discussed in detail below. Note that all these methods are described in their simplest form here, i.e. requiring the minimum number of (two) *IV*-curve measurements. In practice an increase of the number of *IV*-curves will generally improve the accuracy with which R_s can be determined. Since the required *IV*-curves are measured at a different irradiance, all three methods rely on the assumption that n_α and $I_{0,\alpha}$ are independent of the irradiance.

3.3.2. Method of Swanson

One adaptation of the *ff* approach found in the literature is the method suggested by Swanson in a private communication with Wolf and Rauschenbach [8]. A few years later it was described by

[8] It can be shown that the same is true for the $f'f'$ case, as long as the condition $dV/dI|_1 R_{s1} = dV/dI|_2 R_{s2}$ is satisfied. However, since R_{s1} and R_{s2} are unknown it is impossible to find such a match. It is possible when one assumes $R_{s2} = R_{s1}$ because then one only has to find the spot on curve 2 where its slope equals the slope of a chosen spot on curve 1. It turns out, however, that this is the exact spot at which $I_{L1} - I_1 = I_{L2} - I_2$ is valid. This means this method is identical to the Method of Swanson, discussed below.

Handy [9] in a graphical approach for more than two curves. It is also the International Standard procedure to determine the series resistance of a silicon solar cell, as defined by the International Electrotechnical Commission in IEC standard 60891 [17]. This standard basically prescribes measuring three *IV*-curves, determining R_s using the method of Swanson for each of the three possible combinations and then taking the average.

Besides the earlier mentioned condition that $I_{L1}-I_1=I_{L2}-I_2$, the method of Swanson assumes R_{s1} and R_{s2} to be equal, so that Eq. (34) simplifies to Eq. (32). This means that if R_s varies with irradiance [6] then the irradiances at which the two curves are measured should not differ by too much, to avoid large deviations. It is best to measure the first curve at an irradiance slightly above the irradiance of interest, and the second one slightly below. Since I_{L1} and I_{L2} cannot be determined, in practice the approximation $I_{L1}-I_{L2}\approx I_{sc1}-I_{sc2}$ is also applied.

3.3.3. Method of Aberle et al.

Another adaptation of the *ff* approach found in the literature is the method presented by Aberle et al. [20], which requires the first curve to be measured at the irradiance of interest while the second *IV*-curve is to be measured under dark conditions. This means $R_{s2}<R_{s1}$, since under dark conditions the current not only flows in the direction opposite to that under illuminated conditions, but also has a different flow pattern with a lower series resistance [20].[9] Since $I_{L2}=0$, $I_2=-(I_{L1}-I_1)$ according to the generally applied constraint given in Eq. (29). This means Eq. (34) becomes

$$R_{s1}=\frac{V_2-V_1}{I_1}-\frac{(I_{L1}-I_1)R_{s2}}{I_1}. \tag{35}$$

Because I_1 should be chosen as I_{mp}, Aberle et al. state that, since $I_{L1}-I_1$ and R_{s2} are both small, the second term on the right hand side of the equation can be neglected. This means the series resistance is approximated as

$$R_s=\frac{V_2-V_1}{I_1}, \tag{36}$$

and in practice the approximation that $I_{L1}\approx I_{sc1}$ will have to be applied in order to determine I_2.

In his dissertation of 2003 [25] Dicker does not ignore the second term on the right hand side of Eq. (35) and presents this as a correction term to the work of Aberle et al. He expresses the unknown R_{s2} as

$$R_{s2}=\frac{V_R-V_{oc1}}{I_{L1}}, \tag{37}$$

with V_R being the voltage of the dark curve for which the current equals $-I_{L1}$. This follows from the substitution of this current, together with $I_{L2}=0, I_1=0$ and $V_1=V_{oc1}$ in Eq. (33), which results in an equation that is only valid when Eq. (37) is satisfied. This is actually the expression for the series resistance under dark conditions as already presented by Rohatgi et al. [26], but derived here for more general conditions. Substituting this expression for R_{s2} in Eq. (35) yields

$$R_{s1}=\frac{V_2-V_1}{I_1}-\frac{I_{L1}-I_1}{I_1}\frac{V_R-V_{oc1}}{I_{L1}}. \tag{38}$$

One should note, however, that if R_{s2} varies with current level I_2, then the expression for R_{s2} in Eq. (37) is not valid for the current of interest $I_2=-(I_{L1}-I_{mp1})$, but for the much lower current $I_2=-I_{L1}$ as illustrated in Fig. 5.

[9] A particular version of the method by Aberle et al., with the additional condition that $I_1=0$ and the assumption that $R_{s2}=R_{s1}$, was published earlier by Rajkanan and Shewchun [11]. However, this assumption is not very realistic considering $R_{s2}<R_{s1}$ as just stated.

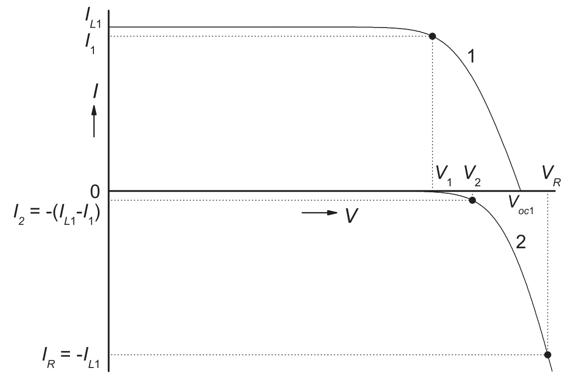

Fig. 5. Representation of the data to be retrieved from an illuminated (1) and a dark *IV*-curve (2) for the method of Aberle et al. and the correction term of Dicker. Indicated are the MPP of curve 1 and the associated point in curve 2 (where $I_2=-(I_{L1}-I_1)$). Also indicated is the point in curve 2 where $I_2=-I_{L1}$.

3.3.4. Method of Wolf & Rauschenbach

A third adaptation of the *ff* approach was first described by Wolf and Rauschenbach [7] and is also known as the Suns-V_{oc} method. The strength of this method is that the unknown R_{s2} is eliminated from Eq. (34) by simply applying the condition $I_2=0$. The first *IV*-curve should then be measured at the irradiance of interest, and the constraint presented in Eq. (29) dictates that the second curve is to be measured at the irradiance for which $I_{L2}=I_{L1}-I_1$. This transforms Eq. (34) into

$$R_s=\frac{V_{oc2}-V_1}{I_1}. \tag{39}$$

The fact that the second curve is measured at an irradiance far below the irradiance of interest might seem to introduce a deviation if R_s is irradiance dependent. However, the second curve is only required for its V_{oc} and this parameter is independent of R_{s2}.

Since I_{L1} and I_{L2} cannot be obtained in practice the approximation $I_{L1}-I_{L2}\approx I_{sc1}-I_{sc2}$ is necessary. The accuracy with which C_2, the concentration ratio of the second *IV*-curve, can be set so that I_{sc2} will have the proper value of $I_{sc1}-I_{mp1}$ will influence the point in the first curve at which R_s is determined. However, from Fig. 6, taken from the experimental study of the R_s determination methods applied to a CPV cell [24], it is clear that C_2, and therefore I_{sc2}, can deviate quite substantially from its intended value before R_s digresses more than 1%.

3.3.5. Method of Mialhe & Charette

There is one last method belonging to the *ff* approach that should be mentioned. Unlike the last three methods discussed, it does not use the general solar cell model from Eq. (1) but the single-diode model from Eq. (2). As mentioned in Section 2 this generally has the disadvantage that n becomes a function of I and E, but is approximated as a constant, introducing errors in the determination of R_s. However, for the *ff* (as well as $f'f'$ and *FF*) approach this error can be minimized by varying the series resistance instead of varying E. This can be achieved by simply connecting a precisely known external resistance R_a in series with the solar cell. This use of an additional series resistance was briefly suggested in 1983 as an exercise for students by Mialhe and Charette [16] but the idea has not been worked out in much detail or picked-up by the solar cell research community.[10]

[10] In 1980, Chaffin and Osbourn already proposed an R_s determination method using two known additional external resistances R_{a1} and R_{a2} [13]. It requires the measurement of two *IV*-curves at the same irradiance, with the cell connected in series with R_{a1} for the first curve, and with R_{a2} for the second curve. Instead of looking at the MPPs, the authors look at the short circuit points, i.e. $V_1=V_2=0$. Eliminating nV_t in the typical fashion for the *ff* approach, but without introducing

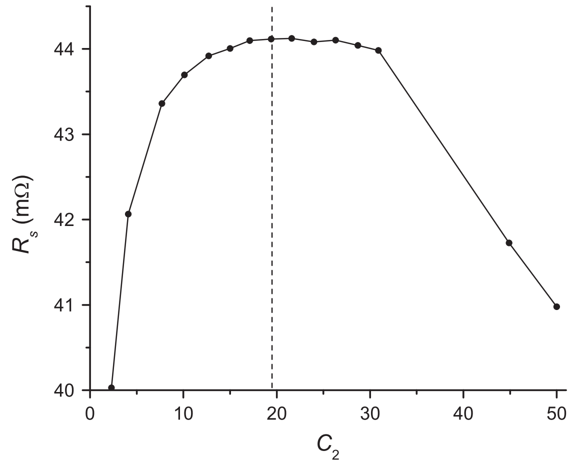

Fig. 6. Plot of R_s for $C_1 = 500$, determined using the method of Wolf & Rauschenbach, for a range of concentration ratio settings for the second curve. The dashed line indicates $C_2 = 19.5$, the value at which $I_{sc2} = I_{sc1} - I_{mp1}$.

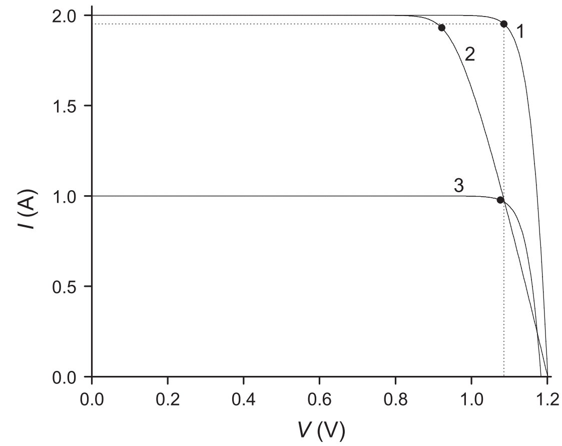

Fig. 7. Three *IV*-curves with their MPP indicated. Each curve was generated using the same parameters, except the 100 mΩ series resistance of curve 2 is ten times as high as that of the other two, and the 1 A photo current of curve 3 is half that of the other two curves.

Utilizing the external series resistance concept means that not only I_L (i.e. E) is kept constant but the I_{mp} values of both curves also differ only slightly, which is illustrated by curves 1 and 2 in Fig. 7. This means that n around the MPP of curve 2 should be virtually the same as n around the MPP of curve 1 (see Fig. 2). In the approach where the second *IV*-curve is measured at a different irradiance (and without an additional series resistance) the I_{mp} values (and therefore, the associated values of n) of the two curves will differ substantially, as illustrated by curves 1 and 3 in Fig. 7.

The very fact that the MPP currents are so similar when using the method of Mialhe & Charette, does make this method very sensitive to deviations in their values, which follows from the denominator in the replacement for Eq. (15) (with points 1 and 2 already taken as the MPPs of the two curves):

$$R_s = \frac{(V_{oc1} - V_{mp1})\lambda_{mp2} - (V_{oc2} - V_{mp2})\lambda_{mp1} + I_{mp2}\lambda_{mp1} R_a}{I_{mp1}\lambda_{mp2} - I_{mp2}\lambda_{mp1}}. \tag{40}$$

This means the value of R_a should be chosen such that there is a good trade-off between I_{mp1} and I_{mp2}: the values should differ enough, but not too much.

4. Discussion and conclusions

The present study provides a systematic approach to derive a large collection of methods to determine the series resistance of a solar cell. Representation of the methods in the two-dimensional array presented in Table 1 allows for easy comparison and reveals that many previously published methods are more alike than might appear at first sight.

(*footnote continued*)
V_{oc} in the equation, gives $Rs = -(\mu_{sc1} I_{sc2} R_{a2} - \mu_{sc2} I_{sc1} R_{a1})/(\mu_{sc1} I_{sc2} - \mu_{sc2} I_{sc1})$, with $\mu_{sci} \equiv \ln((I_c - I_{sci})/I_0)$. By using an irradiance that is high enough to let the exponential knees of both curves lie far into in the second quadrant, $I_{sci} \ll I_c$, so that $\mu_{sc2} \approx \mu_{sc1}$ and $Rs \approx -(I_{sc2} R_{a2} - I_{sc1} R_{a1})/(I_{sc2} - I_{sc1})$. Note that the irradiance required for the approximation to be reasonable is very high and that R_s is not determined at the MPP. The next year Cape, Oliver and Chaffin produced a follow-up paper presenting another method according to similar principles [14]. Basically it comes down to taking the short circuit point, but now according to the $f-$ approach. Under these conditions expression (12) becomes $R_s = (nV_t \lambda_{sc} + V_{oc})/I_{sc} \approx V_{oc}/I_{sc}$ for very high irradiances, because then $I_{sc} \ll I_c$, so $\lambda_{sc} \equiv \ln(I_c - I_{sc})/I_c \approx 0$. Although they present the method in a way that uses an additional series resistance R_a in series with the solar cell, so that $R_s \approx V_{oc}/I_{sc} - R_a$, this is not strictly necessary. It only serves to make the approximation better for a given (very high) irradiance.

Each method requires at least some assumptions and/or approximations, some more influential than others. An overview of all the assumptions and approximations for the discussed methods is presented in Table 2. The approximation that is expected to have the largest influence on the value of R_s to be determined is the one where the general solar cell model is represented by the single-diode model. Since the methods of Swanson, Aberle et al. and Wolf & Rauschenbach do not require this approximation, it is expected that these methods will be the most accurate. Because these three methods all require two *IV*-curves measured at a different irradiance, they do rely on the assumption that n_α and $I_{0,\alpha}$ are independent of E. The method of Swanson also assumes that R_s is independent of E, while the method of Aberle et al. relies on the assumption that $R_{s2} \ll (V_2 - V_1)/(I_{L1} - I_1)$. Since the method of Wolf & Rauschenbach requires neither of these last two assumptions, it is expected to be the most secure of these three methods. From a theoretical point of view it would, therefore, be preferable if the International Electrotechnical Commission adopted this method as its series resistance determination standard, instead of the method of Swanson [17]. One practical advantage that the method of Aberle et al. has over that of Wolf & Rauschenbach is that its second *IV*-curve does not require an (accurate) change in irradiance settings, but is simply measured under dark conditions.

Of course, the methods which only require a single *IV*-curve have even more experimental convenience. However, as mentioned previously, these basically all use the single-diode model approximation. Not only does this ignore the influence of any shunt resistance, it also causes n and I_0 to depend on I and E. The degree to which this dependence is ignored varies per method. All methods from the F category assume n and I_0 to be constant over the entire *IV*-curve. Methods from the f category assume the values of n and I_0 at MPP conditions to be equal to their values at open circuit conditions. And methods from the f' category only assume n and I_0 to be constant around the MPP. One example of this last category is the method of Warashina & Ushirokawa. Unfortunately, the present study indicates that the R_s value that this method determines is extremely sensitive to small deviations in I_{mp}, making this an inferior method to obtain an accurate value of R_s. The method of Araujo & Sánchez is an example of a single curve method in the F category and it even sets n equal to 1 for the entire *IV*-curve. The motivation behind it is based on the assumption that the irradiance required to show the resistance effects of a concentrator solar cell will be high enough for recombination in the quasi-neutral region to dominate. It should be noted, however, that at high irradiances Auger recombination can reduce n to less than 1 [23], so that the method of Araujo & Sánchez can only perform well for a certain irradiance range. The method of Jia et al. is an exception to the rule that methods from the f category assume $n_{mp} = n_{oc}$, because it sets n_{oc} equal to 1 while leaving n_{mp} free to take on another value. Using this approach it only ignores the I dependence of n to a small degree. It does, however, strongly rely on the implicit assumption that n is truly independent of I at the MPP. The method also still assumes that $I_{0,mp} = I_{0,oc}$ and that I_0 is independent of I around the MPP.

The two-dimensional array arrangement also brings to light several methods to determine R_s based on the above stated principle which have not yet been described in the literature: the (general) method of the ff, $f'f'$, FF, Ff and Ff' approaches. However, they all ignore the dependence of n and I_0 on I and E to a high degree and do not have any particular advantages considering the other assumptions and approximations. In addition, the first three methods have no experimental convenience since they require at least two IV-curves measured at a different irradiance and so cannot be considered as an important asset to the collection of series resistance determination methods.

The method of Mialhe & Charette also requires the measurement of two IV-curves and is based on the single-diode model, but has the advantage that its E is constant and the I_{mp} values of both curves only differ slightly, so that the n and I_0 values at those points are not expected to differ much. Still, the method does assume that the values of n and I_0 at MPP conditions are equal to their values at open circuit conditions. And, depending on the accuracy with which the I_{mp} values of the two curves can be determined, a too small difference between I_{mp1} and I_{mp2} will lead to deviations in the determined value of R_s.

From a theoretical perspective we thus conclude that the method of Wolf & Rauschenbach is expected to determine R_s most accurately, followed by the methods of Swanson and Aberle et al. The method of Jia et al. is expected to be the best performing method that only requires a single curve, as long as $dn/dI|_{MPP}$ is really negligible. The verification of these conclusions in an experimental study comparing the here discussed R_s determination methods for CPV cells are published in a contemporary paper [24].

Acknowledgments

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Appendix A

List of symbols

A	area under the IV-curve in the first quadrant
A'	area under the IV-curve below the maximum power point in the first quadrant
C	concentration ratio
E	irradiance
I	current
$I_{0(,\alpha)}$	saturation current of diode (α)
I_a	$\equiv I_c - I_{mp}$
I_b	$\equiv 2I_c - I_{sc}$
I_c	$\equiv I_L + I_0$
I_d	$\equiv (\lambda_{mp} + \frac{V_{oc}}{V_t}) I_a$
$I_{D(,\alpha)}$	current flowing through diode (α)
I_L	light induced current
I_{mp}	current at maximum power point conditions
I_{sc}	short circuit current
I_{sh}	current flowing through the shunt resistance
k	Boltzmann constant
$n_{(\alpha)}$	ideality factor of diode (α)
n_{mp}	n at maximum power point conditions
n_{oc}	n at open circuit conditions
N	number of data points used
P	power generated by the solar cell ($\equiv VI$)
q	elemental charge
R_a	additional resistance, externally connected in series to the solar cell
R_s	series resistance
R_{sh}	shunt resistance
T	absolute temperature of the solar cell
V	voltage
V'_i	$\equiv \frac{dV}{dI}\big\|_i$, with i indicating the ith data point of the IV-curve
V_{mp}	voltage at maximum power point conditions
V_{oc}	open circuit voltage
V_R	voltage of dark IV-curve for which I equals $-I_L$ of an accompanying IV-curve measured under illumination
V_t	thermal voltage ($\equiv \frac{kT}{q}$)
λ	$\equiv \ln(\frac{I_c - I}{I_c})$
λ_{mp}	$\equiv \ln(\frac{I_c - I_{mp}}{I_c})$
μ	$\equiv \ln(\frac{I_c - I}{I_0})$
μ_{sc}	$\equiv \ln(\frac{I_c - I_{mp}}{I_0})$
ξ_i	$\equiv -(I_c - I_i)^{-1}$, with i indicating the ith data point of the IV-curve

For methods using two IV-curves the associated curve is indicated using a numerical subscript.

Appendix B

According to Eq. (1)

$$I = I_L - \left(I_{0,a}\left[\exp\left(\frac{V+IR_s}{n_a V_t}\right) - 1\right]\right) - \sum_{\alpha = b,c,\ldots} \left(I_{0,\alpha}\left[\exp\left(\frac{V+IR_s}{n_\alpha V_t}\right) - 1\right]\right) - \frac{V+IR_s}{R_{sh}}. \tag{B1}$$

Rearranging this into an expression for $n_a V_t$ gives

$$n_a V_t = \frac{V+IR_s}{\ln\left(I_L + I_{0,a} - I - \sum_{\alpha = b,c,\ldots} \left(I_{0,\alpha}\left[\exp\left(\frac{V+IR_s}{n_\alpha V_t}\right) - 1\right]\right) - \frac{V+IR_s}{R_{sh}}\right)}. \tag{B2}$$

Since n_a (as well as the other n_α, $I_{0,\alpha}$ and V_t) is assumed to be independent of irradiance it follows that for two curves measured at a different irradiance

$$\frac{V_1+I_1R_{s1}}{\ln\left(I_{L1} + I_{0,a} - I_1 - \sum_{\alpha = b,c,\ldots} \left(I_{0,\alpha}\left[\exp\left(\frac{V_1+I_1R_{s1}}{n_\alpha V_t}\right) - 1\right]\right) - \frac{V_1+I_1R_{s1}}{R_{sh}}\right)} = \frac{V_2+I_2R_{s2}}{\ln\left(I_{L2} + I_{0,a} - I_2 - \sum_{\alpha = b,c,\ldots} \left(I_{0,\alpha}\left[\exp\left(\frac{V_2+I_2R_{s2}}{n_\alpha V_t}\right) - 1\right]\right) - \frac{V_2+I_2R_{s2}}{R_{sh}}\right)}, \tag{B3}$$

where for completeness both curves have been given their own value for the series resistance, R_{s1} and R_{s2}, symbolizing a possible irradiance dependence of the series resistance. Rearranging gives

$$\ln\left(I_{L1} + I_{0,a} - I_1 - \sum_{\alpha = b,c,\ldots} \left(I_{0,\alpha}\left[\exp\left(\frac{V_1+I_1R_{s1}}{n_\alpha V_t}\right) - 1\right]\right) - \frac{V_1+I_1R_{s1}}{R_{sh}}\right) = \ln\left(I_{L2} + I_{0,a} - I_2 - \sum_{\alpha = b,c,\ldots} \left(I_{0,\alpha}\left[\exp\left(\frac{V_2+I_2R_{s2}}{n_\alpha V_t}\right) - 1\right]\right) - \frac{V_2+I_2R_{s2}}{R_{sh}}\right)\frac{V_1+I_1R_{s1}}{V_2+I_2R_{s2}}. \tag{B4}$$

Taking the exponential on both sides of the above equation results in Eq. (33).

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