Initial Experiments with TPTP-style Automated Theorem Provers on ACL2 Problems

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This paper reports our initial experiments with using external ATP on some corpora built with the ACL2 system. This is intended to provide the first estimate about the usefulness of such external reasoning and AI systems for solving ACL2 problems.

1 Motivation

We are motivated by the recent development of bridges between ITP systems and their libraries such as Mizar/MML [3], Isabelle/HOL [28], and HOL Light [5]/Flyspeck [4] on one side, and ATP/SMT systems such as E [21], Vampire [13] and Z3 [18] on the other side. The work on such systems in the last decade has shown that many top-level lemmas in the ITP libraries can be re-proved by ATPs after a suitable translation to common ATP formats like TPTP, when given the right previous lemmas as axioms [23, 16, 210]. Automated selection of the right previous lemmas from such large libraries, and also automated construction and selection of the right theorem-proving strategies for proving new conjectures over such large libraries is an interesting AI problem, where complementary AI methods such as machine learning from previous proofs and other heuristic selection methods have turned out to be relatively successful [22, 13, 21, 27, 26, 11]. Since 2008 the CASC LTB (Large-Theory Batch) competition division [22] has been measuring the performance of various ATP/AI methods on batches of problems that usually refer to a large common set of axioms. These problems have been generated from the large libraries of Mizar, HOL Light and Isabelle, and also from the libraries of common-sense reasoning systems such as SUMO [19] and Cyc [20]. This has in the long run motivated the ATP/AI developers to invent and improve methods and strategies that are useful on such classes of problems. Such methods then get integrated into strong advising services (often “cloud-based”) for the ITP users, such as HOL(y)Hammer [7], Sledgehammer [14], and MizAR [25, 8].

The plan of work that we intend to follow with ACL2 is analogous to the procedure that was relatively successful with Mizar, Isabelle and HOL Light: (i) define a translation of the ACL2 formulas to the common TPTP formats (initially FOF), (ii) export all ACL2 theorems and definitions into the TPTP format, (iii) find and export the ACL2-proof dependencies between the ACL2 theorems, (iv) apply machine learning to such proof dependencies to obtain strong recommendations (premise-selection) systems that suggest the lemmas useful for proving a new conjecture, and (v) test the performance of the ATP systems and their strategies on the problems translated from ACL2, either by re-proving the theorems from their exact ACL2 proof dependencies, or by using premise selection to recommend the right lemmas. We present the initial work in these directions and the initial evaluation here.

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2 Translation to TPTP

Our translation to FOF TPTP is based on the description of the ACL2 logic by Kaufman and Moore [12]. We add the following axioms defining the ACL2 primitives to each TPTP problem:

\[
\text{fof(spcax1,axiom, t \neq \text{nil}).} \\
\text{fof(spcax2,axiom, \neg [X,Y]: ((X = Y) \iff \text{acleq}(X,Y) = t)).} \\
\text{fof(spcax3,axiom, \neg [X,Y]: ((X \neq Y) \iff \text{acleq}(X,Y) = \text{nil}})). \\
\text{fof(spcax4,axiom, \neg [B,C]: (\text{if}(\text{nil},B,C) = C)).} \\
\text{fof(spcax5,axiom, \neg [A,B,C]: ((A \neq \text{nil}) \Rightarrow \text{if}(A,B,C) = B)).} \\
\text{fof(spcax6,axiom, \neg [A]: (\text{not}(A) = \text{if}(A, \text{nil}, t)))).} \\
\text{fof(spcax7,axiom, \neg [P,Q]: (\text{implies}(P,Q) = \text{if}(P,\text{if}(Q,t,\text{nil}),t))).} \\
\text{fof(spcax8,axiom, \neg [P,Q]: (\text{iff}(P,Q) = \text{and}(\text{implies}(P,Q),\text{implies}(Q,P)))).} \\
\text{fof(\text{and},axiom, \neg [A,B]: (\text{and}(A,B) = \text{if}(A,B,\text{nil}))).} \\
\text{fof(\text{or},axiom, \neg [A,B]: \text{or}(A,B) = \text{if}(A,A,B)).} \\
\text{fof(\text{consp1}, axiom, \neg [A,B]: \text{acleq}(\text{consp}(\text{cons}(A,B)),t) \neq \text{nil}).} \\
\text{fof(\text{consp2}, axiom, \neg [X]: (\text{or}(\text{acleq}(\text{consp}(X),t),\text{acleq}(\text{consp}(X),\text{nil})) \neq \text{nil}).} \\
\text{fof(\text{consp3}, axiom, \neg [X]: (\text{implies}(\text{consp}(X),\text{acleq}(\text{cons}(\text{car}(X),\text{cdr}(X)),X)) \neq \text{nil}).} \\
\]

and we turn every formula into a TPTP statement by requiring that it is not equal to nil.

The translation from ACL2 to TPTP FOF proceeds in two stages. Initially, for each ACL2 file, after it is loaded and checked by ACL2, all formulas and proof dependencies are written in the common ACL2 (Lisp-based) format. This is done by a small Lisp (ACL2) program. The generated Lisp format is next translated to TPTP by a short Emacs Lisp program.

2.1 Extraction of formulas and proof dependencies

One of the strengths of ACL2 is that the user has the ability to use macros. Many function definitions and theorems are abbreviated through this use of macros. For getting formulas from ACL2, this poses a problem, since the macros can contain anything, while they are not part of any logic. To work around this, the books are loaded in ACL2 using the LD command. After this, we enter raw lisp mode, save a copy of the world, and extract information from it.

We export the PROOF-SUPPORTERS-ALIST to be used as input for the machine learning methods that learn from previous proofs dependencies. In addition, we export the THEOREM and the LEMMAS for all symbols in that list, in the order in which they are added to ACL2’s logical world (oldest to newest). As a consequence, macros do not occur in our export anymore. On the downside, the theorems and definitions that are local to an encapsulate do not occur either. In addition, no theorem will state that functions terminate. To make our export to TPTP a bit easier, lambda terms are written out using a function similar to REMOVE-LAMBDFAS.

An alternative to getting formulas from the ACL2 world, is by using the SET-OVERRIDE-HINTS. Every time the code is called with an empty history, we output the clause to a file. One drawback is that this way of exporting may be overridden, but we found that this rarely happens in practice. We plan to use this data as well for learning which hints should be used for proving a theorem. Such learning could likely be used to directly advise ACL2 (or its users), but perhaps also to advise the TPTP-style ATPs running on the translated data.

As far as we are aware, these are the two main options for extracting theorems and lemmas from ACL2 after they are interpreted. For this paper the data from PROOF-SUPPORTERS-ALIST was used.
2.2 Lisp to Prolog

The macro-expanded and lambda-expanded formulas in the saved world are processed by an external Emacs Lisp program that mainly transforms the Lisp syntax into Prolog (TPTP) syntax. For example the following ACL2 formulas:

\[
\text{(EQUAL (CDR (CONS X Y)) Y)}
\]

\[
\text{(EQUAL (EO-ORDINALP ACL2::X))}
\]

\[
\text{(IF (CONSP ACL2::X))}
\]

\[
\text{(IF (EO-ORDINALP (CAR ACL2::X))}
\]

\[
\text{(IF (EQUAL (CAR ACL2::X) '0))}
\]

\[
\text{'NIL}
\]

\[
\text{(IF (EO-ORDINALP (CDR ACL2::X))}
\]

\[
\text{(IF (CONSP (CDR ACL2::X))}
\]

\[
\text{(IF (EO-ORD-< (CAR ACL2::X))}
\]

\[
\text{(CAR (CDR ACL2::X)))}
\]

\[
\text{'NIL 'T) 'T) 'NIL)) 'NIL)
\]

\[
\text{(IF (INTEGERP ACL2::X))}
\]

\[
\text{(IF (< ACL2::X '0) 'NIL 'T) 'NIL)))
\]

get converted into these TPTP formulas:

\[
\text{fof(cdr_cons,axiom,(acleq(cdr(cons(X,Y)),Y) != nil)).}
\]

\[
\text{fof(e0_ordinalp,axiom,}
\]

\[
\text{(acleq(e0_ordinalp(X)),}
\]

\[
\text{if(consp(X),}
\]

\[
\text{if(e0_ordinalp(car(X)),}
\]

\[
\text{if(acleq(car(X),0),nil,}
\]

\[
\text{if(e0_ordinalp(cdr(X)),}
\]

\[
\text{if(consp(cdr(X)),}
\]

\[
\text{if('e0_ord_<'(car(X),car(cdr(X))),nil,t),t),nil)),nil),}
\]

\[
\text{if(integerp(X),if('<'(X,0),nil,t),nil))}
\]

\[
\text{!= nil)).}
\]

We have briefly experimented with more ambitious encodings that translate ACL2 macros such as AND, OR, IMPLIES, IFF into the corresponding TPTP connectives and which translate the ACL2 equality directly as the TPTP equality, however this leads to various initial problems caused by the fact that everything is an untyped function in ACL2, while in TPTP one cannot apply a function to a predicate. Still, such more advanced encodings might result in better performance in the future. Other issues of the translation to TPTP that we are gradually addressing in less or more complete ways include the treatment of quoting, detection of what is a constant and what is a variable in ACL2, handling of various built-ins such as numbers and strings, etc. A major source of incompleteness mentioned below is our current (non-)treatment of ACL2 encapsulation and functional instances. A particularly useful recently introduced feature of TPTP is the possibility to quote non-Prolog atoms in apostrophes. This allows us to re-use practically all ACL2 names in TPTP without any more complicated syntactic transformations. This is not possible for variables, whose TPTP names get autogenerated when their ACL2 syntax is not usable. We use the tptp4X preprocessing tool to collect and universally quantify all variables in the translated formulas, and also to change the symbol names when one symbol appears with different arities (this
may happen due to quoting in ACL2). All the extracted TPTP formulas and their ACL2 dependencies are available at our web page.1

3 Reproving Experiments

In order to evaluate the completeness of our translation and the initial performance of TPTP provers, the ACL2 exporter was run on all ACL2 books (directory books/), which took about 800 minutes. These books contain 3776 Lisp files, of which 1280 can be successfully processed and translated to TPTP by the current version of our exporter and translator, producing at least one TPTP formula. This results in 25,310 unique TPTP FOF. Some of these formulas originate from ACL2 definitions, hence we use them only as TPTP axioms and do not consider them for reproving.

For each proved problem we generate a TPTP file that includes the problem as a conjecture and includes the statements of all the theorems and definitions used by ACL2 in the proof of the theorem. This gives rise to 23,559 problems. In an initial experiment on the subset of these problems we evaluated a number of ATPs and their versions choosing a set of three complementary ones to run on the whole set. The provers and versions chosen are: Vampire 2.6, E-Prover 1.8 run with alternate strategy scheduling (BliStr) 24, and CVC 4. In order to evaluate the incompleteness of the encoding we also included the finite counter-model finder Paradox 4. All experiments are run with 10s time limit on a 48-core server with AMD Opteron 6174 2.2 GHz CPUs, 320 GB RAM, and 0.5 MB L2 cache per CPU and each problem is assigned one CPU.

The results of the first reproving experiment are presented in Fig. 1. The second and third columns correspond to Theorem and CounterSatisfiable problem status returned by the ATP and the last column shows the state of the art contribution of each prover. We attribute the high number of disproved theorems to the incompleteness of our translation. We have been able to identify three main reasons for this incompleteness. First, we do not handle encapsulation properly. This means that proofs that use functional instances cannot be directly replayed by ATPs. Second, we do not normalize the arities of the functions and predicates. Functions that are applied with different arities in ACL2 are translated to different constants in the TPTP encoding. This means that such proofs cannot be currently replayed. Third, ACL2 uses arithmetic of the underlying system directly. We currently encode all numeric constants as different TPTP constants.

All three issues can be handled efficiently. For the first two, we intend to use a translation to a higher-order representation that is immediately lambda-lifted. For the third issue, it is possible to translate either to SMT solvers or the TFA language and provers such SPASS+T.

![Figure 1: Reproving with 10s (23,559 problems)](http://mws.cs.ru.nl/~urban/acl2/00allfof3) and ![Figure 2: Reproving with 10s (23,559 problems)](http://mws.cs.ru.nl/~urban/acl2/00alldeps2)

<table>
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<tr>
<th>Prover</th>
<th>Proved (%)</th>
<th>Disproved (%)</th>
<th>Unique</th>
<th>SotAC</th>
</tr>
</thead>
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<td>Vampire</td>
<td>4,502 (19.1)</td>
<td>55</td>
<td>0.36</td>
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</tr>
<tr>
<td>CVC</td>
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<td>108</td>
<td>0.37</td>
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<tr>
<td>E-Prover</td>
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<td>3,826 (16.2)</td>
<td>67</td>
<td>0.19</td>
</tr>
<tr>
<td>Paradox</td>
<td>13,866 (58.9)</td>
<td>13,916 (59.1)</td>
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<td>13,916 (59.1)</td>
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<td></td>
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<td>Disproved (%)</td>
<td>Size</td>
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<td>------------</td>
<td>---------------</td>
<td>------</td>
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<td>42.98</td>
<td>1019</td>
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<td>std</td>
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<td>61.12</td>
<td>625</td>
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<td>48.14</td>
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<td>models</td>
<td>6.82</td>
<td>67.80</td>
<td>205</td>
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</tr>
</tbody>
</table>

Figure 2: Reproveing results for different book categories (divided by ACL2 top-level directory)

The reproveing results differ quite significantly, depending on the book. We present the results for particular books, categorized by ACL2 top-level directories in Fig. 2. Only book categories with more than 200 TPTP problems are displayed. Contrary to our expectations, the arithmetic books have an average proved rate; while datastructures, defsort, sorting, or models have quite high disprove rate.

4 Machine Learning Experiments

To learn which theorems may be relevant to proving other theorems, we characterize the formulas by the symbols and terms that occur in them and use such features for measuring the similarity of the formulas. Similarly to the formulas translated from Mizar, HOL Light and Isabelle, we can normalize the variables in the terms in various ways, resulting in various notions of formula similarity, see [10] for details. In our ACL2 initial experiments we have focused on the feature extraction algorithm that performs a unification of variables in each subterm. With this algorithm, we have extracted 107,944 distinct features from the ACL2 books, and average number of features per formula amounts to 17.81. These feature numbers (as well as the numbers of training examples - proofs) are comparable to the numbers that we get from other large libraries, thus the various efficient sparse machine learning methods developed recently, e.g., for real-time learning over the whole Mizar library [9] can be used here without any modification.

For each book category, we separately extract the features, dependencies, and an order of theorems.
<table>
<thead>
<tr>
<th>Book</th>
<th>100-Cover (%)</th>
<th>100-Precision</th>
</tr>
</thead>
<tbody>
<tr>
<td>arithmetic-2</td>
<td>94</td>
<td>6.84</td>
</tr>
<tr>
<td>arithmetic-3</td>
<td>94</td>
<td>6.37</td>
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<td>arithmetic-5</td>
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<td>data-structures</td>
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<td>7.39</td>
</tr>
<tr>
<td>all</td>
<td>89</td>
<td>7.9</td>
</tr>
</tbody>
</table>

Figure 3: Machine learning results for different book categories. For clarity, only books with more than 100 TPTP problems are displayed.

We use k-NN (k-nearest neighbor) with the dependencies of each neighbor weighted by the neighbor’s distance, together with the TF-IDF feature weighting to produce predictions for every problem. The way the experiments are done simulates the way a user or ACL2 is developing the library: at every point, only the previous theorems and their dependencies and features are known. Based on the features of the conjecture, the k-NN produces an ordering of the available premises, and a prefix of this order (the 100 premises that are most likely to help solve the goal) are included in the TPTP problems.

We perform two evaluations of the predictions: first a theoretical machine learning evaluation, second a practical evaluation using TPTP ATPs. For the theoretical evaluation we measured two factors. *Cover*, expresses how much of the whole training set is covered in the first 100 predictions. *Precision*, expresses how many of the first 100 predictions are from the training set. In the exported set of ACL2 books the average size of the training example is 12.6 (average number of dependencies); the 100-cover is 89% and the 100-precision is 7.9. The numbers here again vary quite significantly depending on the book category, and we present a short overview in Fig.3.

Finally we perform an ATP evaluation. The evaluation is performed with the same setup as the reproving described in Section3. For each problem the 100 k-NN predictions based only on the previous
knowledge has been written to a TPTP file and the four ATP provers are evaluated on the file. A summary of the results can be found in Fig. 4. The success rate being about 75–80% of the dependency reprov ing rate is in par with the machine learning premise selection results in other systems. With a larger number of advised premises, the number of problems for which finite models exist has decreased significantly. A number of proofs found by premise selection concerns the problems which were not provable with the original ACL2 dependencies: the union of the problems solvable is 6,333 (26.9%). This likely also means that some of the problems that are currently counter-satisfiable (due to the issues such as encapsulation) have an alternative first-order proof that goes via different lemmas.

5 Conclusion

We have presented the initial experiments with external TPTP-style ATPs on the large set of problems extracted from the ACL2 books. While this work is in an early stage, the experiments show that the ATPs can solve a nontrivial part of the ACL2 lemmas automatically. We hope that the numbers will still substantially improve, as we address the remaining reasons for incompleteness.

We also performed an evaluation of machine learning techniques for premise selection for ACL2 translated problems. The mechanisms that have been used primarily with ITPs so far, behave quite well with the ACL2 problems, even without any adjustment. A further tuning of the algorithms (e.g. adjusting the number of premises, ATP strategies, machine learning methods, feature extraction, etc.) will likely increase the proved theorems rates.

Apart from that, there are a number of possible future steps, such as implementing good proof reconstruction techniques for importing the ATP proofs, trying specialized inductive provers (e.g., the QuodLibet system, etc.), or even employing ACL2 via the TPTP framework as a strong inductive ATP that helps other ITPs with problems where induction is needed. A by-product of this work are the large number of TPTP problems, on which the general ATP and TPTP/CASC community can try to improve their methods.

References


2 All the problems and data generated in our evaluation are available at http://cl-informatik.uibk.ac.at/users/cek/acl2dataset1/ The re-proving problems (using acl2-dependencies) are in the file reprove.tgz, the k-NN prediction problems we evaluated are in the file knn.tgz, and the data packed for each book directory separately are in the file books.tgz.
Initial Experiments with TPTP-style Automated Theorem Provers on ACL2 Problems


