Precision Measurement of the Top Quark Mass in Lepton + Jets Final States

We measure the mass of the top quark in lepton + jets final states using the full sample of $p\bar{p}$ collision data collected by the D0 experiment in Run II of the Fermilab Tevatron Collider at $\sqrt{s} = 1.96$ TeV, corresponding to 9.7 fb$^{-1}$ of integrated luminosity. We use a matrix element technique that calculates the probabilities for each event to result from $t\bar{t}$ production or background. The overall jet energy scale is constrained in situ by the mass of the W boson. We measure $m_t = 174.98 \pm 0.76$ GeV. This constitutes the most precise single measurement of the top-quark mass.

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Since its discovery [1,2], the determination of the properties of the top quark has been one of the main goals of the Fermilab Tevatron Collider, recently joined by the CERN Large Hadron Collider. The measurement of the top quark mass $m_t$, a fundamental parameter of the standard model (SM), has received particular attention. Indeed, $m_t$, the mass of the W boson $M_W$, and the mass of the Higgs boson are related through radiative corrections that provide an internal consistency check of the SM [3]. Furthermore, $m_t$ dominantly affects the stability of the SM Higgs potential, which has related cosmological implications [4–6]. Currently, with $m_t = 173.34 \pm 0.76$ GeV, a world-average combined precision of about 0.5% has been achieved [7–9].

In this Letter, we present a measurement of $m_t$ using a matrix element (ME) technique, which determines the probability of observing each event under both the $t\bar{t}$ signal and background hypotheses described by the respective MEs [10]. The overall jet energy scale (JES) is calibrated in situ by constraining the reconstructed invariant mass of the hadronically decaying W boson to $M_W = 80.4$ GeV [11]. The measurement is performed using the full set of $p\bar{p}$ collision data at $\sqrt{s} = 1.96$ TeV recorded by the D0 detector in Run II of the Fermilab Tevatron Collider, corresponding to an integrated luminosity of 9.7 fb$^{-1}$. This is an update of a previous D0 measurement that used 3.6 fb$^{-1}$ of integrated luminosity and measured $m_t = 174.94 \pm 1.14$ (stat + JES) \pm 0.96 (syst) GeV [12]. In the present measurement, we not only use a larger data sample to improve the statistical precision, but also refine the estimation of systematic uncertainties through an updated detector calibration, in particular improvements to the $b$-quark JES corrections [13], and using recent improvements in modeling the $t\bar{t}$ signal. The analysis was performed blinded in $m_t$.

The D0 detector central-tracking system consists of a silicon microstrip tracker and a central fiber tracker, both located within a 1.9 T superconducting solenoidal magnet [14,15], with designs optimized for tracking and vertexing at pseudorapidities $|\eta| < 3$ and $|\eta| < 2.5$, respectively [16]. A liquid-argon calorimeter with uranium absorber plates has a central section covering pseudorapidities up to $|\eta| \approx 1.1$, and two end calorimeters that extend coverage to $|\eta| \approx 4.2$, with all three housed in separate cryostats [17]. An outer muon system, at $|\eta| < 2$, consists of a layer of tracking detectors and scintillation trigger counters in front of 1.8 T iron toroids, followed by two similar layers after the toroids [18].

The top quark decays into a $b$ quark and a $W$ boson with $\approx 100\%$ probability assuming unitarity of the CKM matrix, resulting in a $W^+W^-b\bar{b}$ final state. This analysis is performed using lepton + jets ($\ell + \text{jets}$) final states, where one of the $W$ bosons decays leptonically, and the other hadronically. Here, $\ell$ denotes either an electron ($e$) or a muon ($\mu$), including those from leptonic tau decays. This analysis requires the presence of one isolated electron [19] or muon [20] with transverse momentum $p_T > 20$ GeV and $|\eta| < 1.1$ or $|\eta| < 2$, respectively. In addition, exactly four jets with $p_T > 20$ GeV within $|\eta| < 2.5$, and $p_T > 40$ GeV for the jet of highest $p_T$, are required. Jets are reconstructed using an iterative cone algorithm [21] with a cone parameter of $R = 0.5$. Jet energies are corrected to the...
particle level using calibrations derived from exclusive $\gamma + \text{jet}$, $Z + \text{jet}$, and dijet events [13]. These calibrations account for differences in detector response to jets originating from a gluon, a $b$ quark, and $u$, $d$, $s$, or $c$ quarks. Furthermore, each event must have an imbalance in transverse momentum of $P_T > 20$ GeV expected from the undetected neutrino. Additional selection requirements to suppress background contributions from multijet (MJ) production are discussed in more detail in Ref. [22]. To further reduce background, at least one jet per event is required to be tagged as originating from a $b$ quark ($b$ tagged) through the use of a multivariate algorithm [23]. The tagging efficiency is on average $\approx 65\%$ for $b$-quark jets in this analysis, while the mistag rate for gluons and for light ($u$, $d$, $s$) quark jets is $\approx 5\%$. In total, 1468 and 1124 events are selected in the $e + \text{jet}$ and $\mu + \text{jet}$ channels, respectively, which is consistent with expectation from SM predictions.

The extraction of $m_t$ is based on the kinematic information in the event and performed with a likelihood technique using per-event probability densities (PDs) defined by the MEs of the processes contributing to the observed events. Assuming only two noninterfering contributing processes, $t\bar{t}$ and $W + \text{jets}$ production, the per-event PD is

$$P_{evt} = A(\tilde{x})[fP_{sig}(\tilde{x}; m_t, k_{JES}) + (1 - f)P_{bkg}(\tilde{x}; k_{JES})],$$

(1)

where the observed signal fraction $f$, $m_t$, and the overall multiplicative factor adjusting the energies of jets after the JES calibration $k_{JES}$, are parameters to be determined from data. Here, $\tilde{x}$ represents the measured jet and lepton four-momenta, and $A(\tilde{x})$ accounts for acceptance and efficiencies. The function $P_{sig}$ describes the PD for $t\bar{t}$ production. Similarly, $P_{bkg}$ describes the PD for $W + \text{jets}$ production, which contributes $14\%$ of the data in the $e + \text{jets}$ and $20\%$ in the $\mu + \text{jets}$ channels according to the normalization procedure in Ref. [22]. $W + \text{jets}$ and MJ backgrounds have similar PD in the studied kinematic region, and thus, MJ production is accounted for in $P_{evt}$ via $P_{bkg}$. MJ events contribute $12\%$ to the $e + \text{jets}$ and $5\%$ to the $\mu + \text{jets}$ channels. The combined contribution from all other backgrounds amounts to about $5\%$ in both channels.

In general, the set $\tilde{x}$ of measured quantities will not be identical to the set of corresponding partonic variables $\tilde{y}$ because of finite detector resolution and parton hadronization. Their relationship is described by the transfer function $W(\tilde{x}, \tilde{y}, k_{JES})$, where we assume that the jet and lepton angles are known perfectly. The densities $P_{sig}$ and $P_{bkg}$ are calculated through a convolution of the differential partonic cross section, $d\sigma(\tilde{y})$, with $W(\tilde{x}, \tilde{y}, k_{JES})$ and the PDs for the initial-state partons, $f(q_i)$, where the $q_i$ are the momenta of the colliding partons, by integrating over all possible parton states leading to $\tilde{x}$

$$P_{sig} = \frac{1}{\sigma_{obs}(m_t, k_{JES})} \int d\sigma(\tilde{y}, m_t) d\tilde{q}_1 d\tilde{q}_2 f(q_1) f(q_2) \times W(\tilde{x}, \tilde{y}; k_{JES}).$$

(2)

The sum in the integrand extends over all possible flavor combinations of the initial state partons. The longitudinal momentum parton density functions (PDFs), $f(q_i)$, are taken from the CTEQ6L1 set [24], while the dependencies $f(q_{i,x})$, $f(q_{i,y})$ on transverse momenta are taken from PDFs obtained from the PYTHIA simulation [25]. The factor $\sigma_{obs}(m_t, k_{JES})$, defined as the expected total $t\bar{t}$ cross section, guarantees that $A(\tilde{x})P_{sig}$ is normalized to unity. The differential cross section, $d\sigma(\tilde{y}, m_t)$, is calculated using the leading order (LO) ME for the process $q\bar{q} \rightarrow t\bar{t}$. The integration in Eq. (2) is performed over the masses of the $t$ and $\bar{t}$ quarks which are assumed to be equal, the masses of the $W^\pm$ bosons, the energy $E$ (curvature $1/p_T$) of the electron (muon), and $E_q$ ($E_{q+} + E_{q-}$) for the quarks from the $W \rightarrow q\bar{q'}$ decay. The $M_W = 80.4$ GeV constraint for the in situ JES calibration is imposed by integrating over $W$ boson masses from a Breit-Wigner prior. There are 24 possible jet-parton assignments that are summed with weights based on their consistency with the $b$-tagging information.

The density $P_{sig}$ is calculated by numerical Monte Carlo (MC) integration and is identical to that in Ref. [12], except as described. The transfer function $W(\tilde{x}, \tilde{y}, k_{JES})$ and $\sigma_{obs}(m_t, k_{JES})$ are rederived using improved detector calibrations. Instead of pseudorandom numbers, we utilize the implementation of Bradley and Fox [26] of the Sobol low discrepancy sequence [27] for MC integration, which provides a reduction of about 1 order of magnitude in calculation time. Furthermore, we approximate the exact results of Eq. (2) for a grid of points in $(m_t, k_{JES})$ space by calculating the ME only once for each $m_t$ and multiplying the results with the transfer function $W(\tilde{x}, \tilde{y}, k_{JES})$ to obtain $P_{sig}$ for any $k_{JES}$. This results in another order of magnitude reduction in computation time. Both improvements are verified to provide a performance of the ME technique consistent with that in Ref. [12]. They proved essential to reduce the statistical uncertainty in evaluating most of the systematic uncertainties discussed below.

The differential partonic cross section for $P_{bkg}$ is calculated using the LO $W + 4\text{jets}$ MEs implemented in VECBOS [28]. The initial-state partons are all assumed to have zero transverse momentum $p_T$. As in the case of $P_{sig}$, we apply identical procedures to calculate $P_{bkg}$ to those in Ref. [12], but using the updated transfer function $W(\tilde{x}, \tilde{y}, k_{JES})$ and background normalization factor.

We calculate $P_{sig}$ and $P_{bkg}$ on a grid in $(m_t, k_{JES})$ with spacings of (1 GeV, 0.01). A likelihood function, $L(x_1, x_2, \ldots, x_N; m_t, k_{JES}, f)$, is constructed at each grid point from the product of the individual $P_{evt}$ values for the measured quantities $x_1, x_2, \ldots, x_N$ of the selected events, and $f$ is determined by maximizing $L$ at that grid point. The
likelihood function \( \mathcal{L}(\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_N; m_t, k_{\text{JES}}) \) is then projected onto the \( m_t \) and \( k_{\text{JES}} \) axes by integrating over \( k_{\text{JES}} \) and \( m_t \), respectively. Best unbiased estimates of \( m_t \) and \( k_{\text{JES}} \) and their statistical uncertainties are extracted from the mean and standard deviation (SD) of \( \mathcal{L}(\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_N; m_t) \) and \( \mathcal{L}(\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_N; k_{\text{JES}}) \).

Simulations are used to calibrate the ME technique. Signal \( t\bar{t} \) events, as well as the dominant background contribution from \( W + \) jets production, are generated with ALPGEN [29] using the CTEQ6L1 set of PDFs, interfaced to PYTHIA for parton showering using the MLM matching scheme [30]. Therefore, it is the value of \( m_t \) as defined in the MC generator that is measured, and this value is expected to correspond within \( \approx \)1 GeV to \( m_t \) as defined in the pole mass scheme [31]. The simulation of parton showers with PYTHIA uses modified tune D0 tune A with showers with PYTHIA uses modified tune D0 tune A with

\[ \sigma_{m_t} [\text{GeV}] = 0.49 \pm 0.04 \text{ (stat)} \pm 0.17 \text{ (JES)} \text{ GeV}, \]

\[ \sigma_{k_{\text{JES}}} = 0.025 \text{ (JES)} \text{ GeV}, \]

These values are in agreement with the expectation for the signal yield assuming \( \sigma_{m_t} = 7.24 \text{ pb} \) [33].

With \( f \) determined as above, we proceed to form PEs for the calibration of the method response in \( m_t \) and \( k_{\text{JES}} \). Five sets of PEs are formed, for \( f = 0.5, 0.6, 0.7, 0.8, \) and \( 0.9 \) at \( m_t^{\text{gen}} = 172.5 \text{ GeV} \), \( k_{\text{JES}}^{\text{gen}} = 1 \) to linearly calibrate the response of the ME technique to \( f \). We find \( f = 63\% \) in the \( e + \) jets and \( f = 70\% \) in the \( \mu + \) jets channels, with an absolute uncertainty of 1\% due to the finite size of the data sample and the calibration in \( f \). These values are in agreement with the expectation for the signal yield assuming \( \sigma_{m_t} = 7.24 \text{ pb} \) [33].

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Ref. [22]. The \( t\bar{t} \) signal is normalized to total cross sections of \( \sigma_{\text{tot}} = 7.8 \text{ pb} \) in the \( e + \text{jets} \) and \( \sigma_{\text{tot}} = 7.6 \text{ pb} \) in the \( \mu + \text{jets} \) channel, corresponding to the measured signal fraction.

Systematic uncertainties are evaluated using PEs constructed from simulated signal and background events, for three categories: modeling of signal and background events, uncertainties in the simulation of the detector response, and uncertainties associated with procedures used and assumptions made in the analysis. Contributions from these sources are listed in Table I.

The first four sources of systematic uncertainty in Table I are evaluated for \( m_t \) using different signal models. All other systematic uncertainties are evaluated by rederiving the calibration with simulations reflecting an alternative model, and applying the alternative calibration to data. The statistical components of systematic uncertainties are \( \approx 0.05 \text{ GeV} \) for the former and \( \approx 0.01 \text{ GeV} \) for the latter sources of systematic uncertainty. The statistical components are never larger than the net difference between the default and alternative models for any of the sources of systematic uncertainty. One-sided sources of systematic uncertainties are taken as symmetric in both directions in the total quadrature sum.

We refine the evaluation procedure for several sources of systematic uncertainty compared to Ref. [12] as described below. Details on other, typically smaller, sources of systematic uncertainty can be found in Ref. [12]. The uncertainty from higher order corrections is evaluated by comparing events simulated with MC@NLO [35] to ALPGEN interfaced to HERWIG [36]. The uncertainty due to the modeling of initial and final state radiation is constrained from Drell-Yan events [37]. As indicated by these studies, we change the amount of radiation via the renormalization scale parameter for the matching scale in ALPGEN interfaced to PYTHIA [38] up and down by a factor of 1.5. In addition, we reweight \( t\bar{t} \) simulations in \( p_T \) of the \( t\bar{t} \) system (\( p_T^0 \)) to match data, and combine the two effects in quadrature. The uncertainty originating from the choice of a model for hadronization and underlying event (UE) is evaluated by comparing events simulated with ALPGEN interfaced to either PYTHIA or HERWIG. The JES calibration is derived using PYTHIA with a modified tune D0 tune A [13], and is expected to be valid for this configuration only. Applying it to events that use HERWIG for evolving parton showers can lead to a sizable effect on \( m_t \). However, this effect would not be present if the JES calibration were based on HERWIG. To avoid such double counting of uncertainty sources, we evaluate the uncertainty from hadronization and UE by considering as \( \pm \) the momenta of particle level jets matched in \((\eta, \phi)\) space to reconstructed jets. In this evaluation, we reweight our default \( t\bar{t} \) simulations in \( p_T^0 \) to match ALPGEN interfaced to HERWIG. A potential effect of color reconnection (CR) on \( m_t \) is evaluated by comparing ALPGEN events interfaced to PYTHIA with the Perugia 2011NOCR and Perugia 2011 tunes [39], where the latter includes an explicit CR model. The residual jet energy scale uncertainty from a potential dependence of the JES on \((p_T, \eta)\) is estimated by changing the jet momenta as a function of \((p_T, \eta)\) by the upper limits of JES uncertainty, the lower limits of JES uncertainty, and a linear fit within the limits of JES uncertainty. The maximum excursion in \( m_t \) is quoted as systematic uncertainty. Dedicated calibrations to account for the flavor-dependent response to jets originating from a gluon, a \( b \) quark, and \( u, d, c, \) or \( s \) quarks are now an integral part of the JES correction [13], and the uncertainty on \( m_t \) from these calibrations is evaluated by changing them within their respective uncertainties. This systematic uncertainty accounts for the difference in detector response to \( b \)- and light-quark jets. To evaluate the uncertainty from modeling of \( b \) tagging, differential corrections in \((p_T, \eta)\) to ensure MC—data \( b \)-tagging efficiency agreement are changed within their uncertainties. The uncertainty due to the modeling of multijet events is evaluated by assuming a 100% uncertainty on its contribution to the data sample, i.e., by leaving it out when deriving the alternative calibration. We construct PEs with \( \pm 5\% \) variations on the measured signal fraction, which approximately corresponds to the systematic uncertainty on the measured \( t\bar{t} \) production cross section using D0 data [40], ignoring the uncertainty from integrated luminosity, and construct the PEs according to this 5% change.

### TABLE I. Summary of uncertainties on the measured top quark mass. The signs indicate the direction of the change in \( m_t \) when replacing the default by the alternative model.

<table>
<thead>
<tr>
<th>Source of uncertainty</th>
<th>Effect on ( m_t ) (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Signal and background modeling</strong></td>
<td></td>
</tr>
<tr>
<td>Higher order corrections</td>
<td>+0.15</td>
</tr>
<tr>
<td>Initial and final state radiation</td>
<td>±0.09</td>
</tr>
<tr>
<td>Hadronization and UE</td>
<td>+0.26</td>
</tr>
<tr>
<td>Color reconnection</td>
<td>±0.10</td>
</tr>
<tr>
<td>Multiple ( p\bar{p} ) interactions</td>
<td>−0.06</td>
</tr>
<tr>
<td>Heavy flavor scale factor</td>
<td>±0.06</td>
</tr>
<tr>
<td>( b )-jet modeling</td>
<td>+0.09</td>
</tr>
<tr>
<td>PDF uncertainty</td>
<td>±0.11</td>
</tr>
<tr>
<td><strong>Detector modeling</strong></td>
<td></td>
</tr>
<tr>
<td>Residual jet energy scale</td>
<td>±0.21</td>
</tr>
<tr>
<td>Flavor-dependent response to jets</td>
<td>±0.16</td>
</tr>
<tr>
<td>( b ) tagging</td>
<td>±0.10</td>
</tr>
<tr>
<td>Trigger</td>
<td>±0.01</td>
</tr>
<tr>
<td>Lepton momentum scale</td>
<td>±0.01</td>
</tr>
<tr>
<td>Jet energy resolution</td>
<td>±0.07</td>
</tr>
<tr>
<td>Jet identification efficiency</td>
<td>−0.01</td>
</tr>
<tr>
<td><strong>Method</strong></td>
<td></td>
</tr>
<tr>
<td>Modeling of multijet events</td>
<td>+0.04</td>
</tr>
<tr>
<td>Signal fraction</td>
<td>±0.08</td>
</tr>
<tr>
<td>MC calibration</td>
<td>±0.07</td>
</tr>
<tr>
<td><strong>Total systematic uncertainty</strong></td>
<td>±0.49</td>
</tr>
<tr>
<td><strong>Total statistical uncertainty</strong></td>
<td>±0.58</td>
</tr>
<tr>
<td><strong>Total uncertainty</strong></td>
<td>±0.76</td>
</tr>
</tbody>
</table>
In summary, we have performed a measurement of the mass of the top quark using the matrix element technique in $t\bar{t}$ candidate events in lepton+jets final states using 9.7 fb$^{-1}$ of Run II integrated luminosity collected by the D0 detector at the Fermilab Tevatron $p\bar{p}$ Collider. The result,

$$m_t = 174.98 \pm 0.58\text{(stat+JES)} \pm 0.49\text{(syst)} \text{ GeV}, \quad \text{or} \
$$

$$m_t = 174.98 \pm 0.76 \text{ GeV},$$

is consistent with the values given by the current Tevatron and world combinations of the top quark mass [8,9] and achieves by itself a similar precision. With an uncertainty of 0.43%, it constitutes the most precise single measurement of the top quark mass, with a total systematic uncertainty notably smaller than any other single measurement.

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[16] The D0 coordinate system is right-handed, with the z axis pointing in the direction of the Tevatron proton beam and the y axis pointing upwards. The angles $\phi$ and $\theta$ are the azimuthal and polar angles relative to the x and z axes, respectively. We also use, as an angular variable, the pseudorapidity defined by $\eta = -\ln[\tan(\theta/2)]$.
The pull is defined as $\sum_i (m_i - \langle m_i \rangle)/\Delta m_i$, where $i = 1, 2, \ldots, 1000$ runs over all PEs. We find pull widths of 1.16 and 1.19 for $m_t$ and $k_{\text{JES}}$.