PDF hosted at the Radboud Repository of the Radboud University Nijmegen

The following full text is a publisher's version.

For additional information about this publication click this link.
http://hdl.handle.net/2066/129117

Please be advised that this information was generated on 2017-09-24 and may be subject to change.
Spin asymmetries $A_1$ and structure functions $g_1$ of the proton and the deuteron from polarized high energy muon scattering


We present the final results of the spin asymmetries $A_1$ and the spin structure functions $g_1$ of the proton and the deuteron from polarized high energy muon scattering.

We use a new method which minimizes the radiative background by selecting events with at least one hadron as well as a muon in the final state. We find that this hadron method gives smaller errors for the deuteron in the kinematic range $0.0008 < x < 0.02$, so it is combined with the usual method to provide the optimal set of results.
I. INTRODUCTION

Polarized deep inelastic lepton-nucleon scattering is an important tool to study the spin structure of the nucleon. Measurements with proton, deuteron, and helium-3 targets have determined the spin structure functions of the nucleon and have verified the Bjorken sum rule [1], which is a fundamental relation of QCD.

In the last five years, the Spin Muon Collaboration (SMC) at CERN has reported experimental results on the spin structure of the proton [2–8] and of the deuteron [3, 5, 8–11], measured in inelastic muon scattering at beam energies of 100 and 190 GeV. Thus far our published results for the virtual photon-proton and virtual photon-deuteron cross section asymmetries \( A_1^p(x, Q^2) \) and \( A_1^d(x, Q^2) \) and for the spin-dependent structure functions \( g_1^p(x, Q^2) \) and \( g_1^d(x, Q^2) \) have been obtained from inclusive scattering events. These results are updated in this paper, principally with a final value for the muon beam polarization.

Since the inclusive scattering events include a large radiative background at low \( x \), we now employ a new and alternative method of determining the asymmetries which requires at least one hadron as well as a muon in the final state. This hadron method removes the background due to elastic and quasielastic scattering accompanied by a high energy bremsstrahlung photon, and improves the statistical accuracy of the measurement at low \( x \). A similar method has been applied successfully by the New Muon Collaboration (NMC) [12] and the E665 [13] analyses of \( F_2 \) structure function ratios.

Our final results for the asymmetries \( A_1^p \) and \( A_1^d \) are based on both the inclusive and the hadron methods and cover the kinematic region of 0.0008 < \( x \) < 0.7 and \( Q^2 > 0.2 \) GeV\(^2\). An optimal set is defined with the inclusive method being used for \( x > 0.02 \) and the hadron method for \( x < 0.02 \). In the low \( x \) region the statistical errors from the hadron method are smaller than those from the inclusive method. The range of reduction varies from 1 to 0.6 with decreasing \( x \). For \( Q^2 > 1 \) GeV\(^2\) the lowest \( x \) reached is 0.003 where the reduction factor is 0.8. Results presented here stem from 15.6 and 19.0 million events accepted after all cuts for the \( A_1^p \) and the \( A_1^d \) determinations, respectively.

The outline of this paper is as follows. Section II gives the formulae for the asymmetry determination and explains the update of the beam polarization, while Sec. III describes in detail the hadron method. In Sec. IV, after showing the updated result for the \( A_1 \) measurement with the inclusive method, we give the results for the hadron method, compare both, and finally define the optimal data set by using the hadron method at low \( x \) and the inclusive one at high \( x \).

Section V presents the structure functions \( g_1 \) and Sec. VI their integrals in the measured \( x \) range as well as their first moments with contributions from the unmeasured region taken from the QCD analysis (see our following paper [14]). In Sec. VII we calculate the nonsinglet combination \( g_1^p - g_1^d \), compare it to the corresponding unpolarized combination \( F_1^p - F_1^d \), and compute its integral in the measured range and its first moment. Section VIII contains a summary. The detailed discussion of the first moments \( \Gamma_1^{p,d} \) and the Bjorken sum rule is presented in our following paper [14]. The Appendix gives a parametrization of the world data on the spin

---

\^a Now at The Royal Library, 102 41 Stockholm, Sweden. \^b Deceased. \^c Now at Ericsson Infotech AB, Karlstad, Sweden. \^d Now at University of Wisconsin, Department of Physics, Boston, MA 02115. \^e Now at SLAC, Stanford CA 94309. \^f Now at University of Mainz, Institute of Nuclear Physics, D-55099, Germany. \^g Permanent address: Miyazaki University, Faculty of Engineering, 889-21 Miyazaki-Shi, Japan. \^h Permanent address: Paul Scherrer Institut, 5232 Villigen, Switzerland. \^i Permanent address: The Institute of Physical and Chemical Research (RIKEN), Wako 351-01, Japan. \^j Permanent address: University of California, Institute of Particle Physics, Santa Cruz, CA 95064. \^k Permanent address: KEK, Tsukuba-Shi, 305 Ibaraki-Ken, Japan. \^l Now at University of Michigan, Ann Arbor MI 48109. \^m Now at SBC Warburg Dillon Read, CH-4002 Basel, Switzerland. \^n Permanent address: Rice University, Bonner Laboratory, Houston, TX 77251-1892. \^o Now at Penn. State University, 303 Osmond Lab, University Park, PA 16802. \^p Permanent address: University of Buenos Aires, Physics Department, 1428 Buenos Aires, Argentina.
TABLE I. Main characteristics of different measurements in the SMC experiment: beam energy, target material, and average target polarization with the relative accuracy of its measurement. The last column refers to publications concerning the experiments.

<table>
<thead>
<tr>
<th>Year</th>
<th>Beam energy (GeV)</th>
<th>Target</th>
<th>\langle P_\mu \rangle</th>
<th>\Delta P_\mu / P_\mu (%)</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>100</td>
<td>C_5D_5OD</td>
<td>0.40</td>
<td>±5</td>
<td>[9,5,8,11]</td>
</tr>
<tr>
<td>1993</td>
<td>190</td>
<td>C_5H_6OH</td>
<td>0.86</td>
<td>±3.0</td>
<td>[2,4,6,7,5,8]</td>
</tr>
<tr>
<td>1994</td>
<td>190</td>
<td>C_5D_5OD</td>
<td>0.86</td>
<td>±5.4</td>
<td>[10,11,5,8]</td>
</tr>
<tr>
<td>1995</td>
<td>190</td>
<td>C_5D_5OD</td>
<td>0.50</td>
<td>±2.1</td>
<td>[11,8]</td>
</tr>
<tr>
<td>1996</td>
<td>190</td>
<td>NH_3</td>
<td>0.89</td>
<td>±2.7</td>
<td>[7,8]</td>
</tr>
</tbody>
</table>

The asymmetries \( A_1^{p,d} \) and the spin-dependent structure functions \( g_1^{p,d} \) are related to the virtual photon-proton (deuteron) asymmetries \( A_1^{p,d} \) and \( A_2^{p,d} \) [19,20] by

\[
A_1^{p,d} = D(A_1^{p,d} + \eta A_2^{p,d}),
\]

\[
g_1^{p,d} = \frac{F_{\mu,d}^{p,d}}{2\chi(1+R)} (A_1^{p,d} + \gamma A_2^{p,d}),
\]

where the factors \( \eta \) and \( \gamma \) depend only on kinematic variables. The depolarization factor \( D \) depends in addition on the ratio of the photoabsorption cross sections for longitudinally and transversely polarized virtual photons. The virtual photon-proton asymmetries are defined as

\[
A_1^p = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}}, \quad A_1^d = \frac{2\sigma_{TL}}{\sigma_{1/2} + \sigma_{3/2}},
\]

\[
A_2^p = \frac{\sigma_0^T - \sigma_2^T}{\sigma_0^T + \sigma_2^T}, \quad A_2^d = \frac{\sigma_0^{TL} + \sigma_1^{TL}}{\sigma_1^{TL}},
\]

where \( \sigma_1^{TL} \) (\( \sigma_3^{TL} \)) is the photoabsorption cross section of a transversely polarized virtual photon by a proton (a deuteron), with total spin projection \( \frac{1}{2} (\frac{3}{2}) \) in the photon direction and \( \sigma_j^{TL} \) is a term arising from the interference between transverse and longitudinal amplitudes. For more details regarding the kinematic factors \( \eta, \gamma \), and \( D \) the reader is referred to Ref. [6]. Corresponding formulas for the deuteron are

\[
A_1^d = \frac{1}{2} (\sigma_0^T - \sigma_2^T)/\sigma_T,
\]

\[
A_2^d = \frac{1}{2} (\sigma_0^{TL} + \sigma_1^{TL})/\sigma_T.
\]

Here \( \sigma_T = \frac{1}{2} (\sigma_0^T + \sigma_1^T + \sigma_2^T) \) is the transverse photoabsorption cross section, \( \sigma_j^T \) is the cross section for absorption of a virtual photon by a deuteron with total spin projection \( J \) in the photon direction, and \( \sigma_j^{TL} \) results from the interference between transverse and longitudinal amplitudes for \( J = 0,1 \).

In the kinematic region of our measurement \( \eta \) and \( \gamma \) are small. The asymmetries \( A_1^d \) and \( A_2^d \) were measured and found to be consistent with zero [4,11,21]. For these reasons we neglect the \( A_2 \) terms in Eq. (3) and estimate the systematic uncertainty in \( A_1 \) due to a possible contribution of \( A_2 \) [7,11].

### III. THE HADRON METHOD

#### A. Description of the procedure

In previous publications the determination of \( A_1 \) from SMC data was done using an inclusive event selection, requiring only a scattered muon. In addition to deep inelastic scattering events, the resulting sample includes scattering events which are elastic on free target nucleons, or elastic or quasielastic on target nuclei and which are accompanied by the radiation of a hard photon. These radiative events do not carry any information on the spin structure of the nucleon and only degrade the statistical accuracy of the measurement. Elastic \( \mu-e \) interactions also do not carry any information on the nucleon spin; they are peaked at \( x = m_e/m_\mu = 0.0005 \) and give for \( x > 0.0008 \) only a small contribution, which is not
considered in the following discussion. The described radiative events dilute the spin effects in the cross section for the inclusive sample, similarly to the nonpolarizable nuclei in the target, accounted for by the dilution factor $f$. The effective dilution factor $f'$,  

$$f' = \frac{\sigma_{p,d}^{p,d}}{\sigma_{\text{tot}}^{p,d}} f = \frac{\rho_{p,d} \sigma_{1\gamma}^{p,d}}{\sum \alpha_{AA} \sigma_{\text{tail}}^{P}}, \tag{6}$$

accounts for both diluting sources. The sum runs over all types of target nuclei. Essentially only protons or deuterons are polarized in the target. For the description of a small correction to the asymmetry due to the polarized background of $^{14}\text{N}$ for the NH$_3$ target and of protons for the deuterated butanol target, see Refs. [2, 7].

The total cross section $\sigma_{\text{tot}}$ and the one-photon-exchange (Born) cross section $\sigma_{1\gamma}$ are related by $\sigma_{\text{tot}} = \lambda \sigma_{1\gamma} + \sigma_{\text{tail}} + \sigma_{\text{inel}}$, where the $\sigma_{\text{tail}}$ terms are the cross sections from the radiative tails (elastic, quasielastic, and inelastic reactions). The factor $\lambda$, which does not depend on the polarization, corrects for higher order contributions: virtual (vacuum and vertex corrections) and soft real photon radiation [6]. For an effective measurement the dilution factor $f'$ should be large.

In the new method of analyzing the data we use only events for which at least one hadron track has been reconstructed; then these hadron-tagged events do not include any contribution from $\sigma_{\text{tail}}$ and $\sigma_{\text{inel}}$ since the recoil proton cannot be observed in our spectrometer due to its small energy. The total cross section for hadron-tagged events thus reduces to  

$$\sigma_{\text{tot}}^{\text{tagged}} = \lambda \sigma_{1\gamma} + \sigma_{\text{inel}}. \tag{7}$$

In the calculation of the effective dilution factor $f'$ for hadron-tagged events, $\sigma_{\text{tot}}^{\text{tagged}}$ replaces $\sigma_{\text{tot}}$ in Eq. (6) and the effective dilution factor increases accordingly, in particular at low $x$, as can be seen in Fig. 1.

The fraction of deep inelastic events which would not be selected as hadron-tagged events with $Q^2 > 1$ GeV$^2$ for our spectrometer was estimated by a Monte Carlo simulation to be in the range of 2–7% for $x < 0.02$ and to increase at higher $x$. This loss of events worsens the statistical accuracy only with a square root dependence while the increase in the dilution factor improves it linearly. The result is that the hadron method gives a net gain in statistical accuracy for $x < 0.02$.

**B. Event selection**

As for the inclusive method, events have to satisfy the following kinematic cuts: energy of the scattered muon $E'_{\mu} > 19$ GeV, $\nu = E_{\mu} - E'_{\mu} > 15$ GeV, $y = \nu/E'_{\mu} < 0.9$, and scattering angle $\theta > 2$ mrad. Events are then labeled inelastic when at least one hadron is found in the final state. As only tracks of charged particles are reconstructed in our spectrometer we can observe neutral hadrons indirectly via their charged decay products, or in the case of a $\pi^0$ meson through converted photons from its decay.

For hadron-tagged events we require, in addition to a scattered muon either one or more tracks pointing to the muon interaction vertex, or a pair of tracks with positive and negative charge from a secondary vertex. The sample, selected in this way, still contains some unwanted radiative events in which the bremsstrahlung photon is converted. These unwanted events occur at large $y$ and at a small angle $\alpha$ between the direction of the produced particle and the direction of the muon momentum loss $\vec{p}_{\mu} - \vec{p}'_{\mu}$, which for radiative elastic and quasielastic events is very close to the direction of the bremsstrahlung photon. An enhancement of events at small $\alpha$ and large $y$ is indeed seen in the data; it disappears if a signature for a charged hadron is required in the calorimeter [22]. Also, such an enhancement is not present in a Monte Carlo simulation which includes only deep inelastic scattering (DIS) events. To remove these radiative events from the sample, but not events with $\pi^0$ mesons, additional conditions were applied: to keep an event we require that tracks, giving a calorimeter response compatible with that for electrons, have $\alpha > 4$ mrad or belong to an event with $y < 0.6$. The same is required for a pair of tracks from a secondary vertex compatible with photon conversion. The events surviving all of these cuts define the sample of hadron-tagged events.

**C. Tests of the procedure**

As a first test of the procedure of asymmetry extraction with hadron tagging the fraction of inclusive events selected as hadron-tagged events is compared with the expected one. The latter is calculated from the ratio of the corresponding effective dilution factors and the probability of detecting at least one hadron in DIS events. This probability was estimated with the Monte Carlo simulation mentioned before.
The comparison is presented in Fig. 2 for events with $Q^2 > 1 \text{ GeV}^2$ for which the fragmentation into hadrons is reliably described in the simulation. In the case of inefficient removal of radiative events, the fraction of inclusive events selected as hadron-tagged events would be larger than expected. Figure 2 shows that this is not the case.

The sensitivity of the measured asymmetry to the selection with tagging was checked by varying the tagging criteria as follows: keeping only tracks giving a good vertex fit, removing all tracks with an energy deposit in the calorimeter consistent with that expected for an electron, applying the cut on $\alpha$ to all tracks, or changing this cut from 4 to 2 mrad. The resulting differences in the asymmetries are compatible with zero for all $x$ bins. For $x < 0.02$, where we will apply hadron tagging (see Sec. IV), the corresponding $\chi^2$ probabilities are in the range of 5–70% for the proton and 30–89% for the deuteron.

Possible biases on $A_1$ introduced by hadron-tagging were also studied with a dedicated Monte Carlo simulation for $Q^2 > 1 \text{ GeV}^2$. The program POLDIS [23] was used to generate events, and the spectrometer acceptance for hadrons was approximated by requiring forward produced hadrons with momentum $p_h > 5 \text{ GeV}$ and $z = E_h / \nu > 0.1$, where $E_h$ is the hadron energy. The asymmetries were calculated for events with such hadrons and compared to those obtained for all events. The differences are shown as a function of $x$ in Fig. 3 for the proton and the deuteron. For the proton, the asymmetries calculated from hadron-tagged events are larger at high $x$. This is to be expected because in this region of $x$ the total energy of the hadronic final state $W$ is not very high and the observed hadron is most likely to be the leading one. Since the detection efficiency for charged hadrons, which are more abundant in $u$-quark than in $d$-quark fragmentation, is higher than for neutral hadrons, the hadron-tagged sample is enriched with scattering on $u$ quarks compared to the inclusive sample. From semi-inclusive measurements [8] it is known that the polarization of the valence $u$ quarks is positive whereas that of the valence $d$ quarks is negative. Therefore, one expects higher values of $A_1$ for the hadron-tagged event sample. If the hadron selection is relaxed ($z > 0.05$ and $p_h > 3 \text{ GeV}$) more nonleading hadrons are accepted and the asymmetry gets closer to the one for inclusive events, as can be seen in Fig. 3. At low $x$ the available energy is large and the tagging no longer favors scattering on $u$ quarks. The asymmetries for hadron-tagged and inclusive events should therefore be the same. Indeed, in this region the estimated differences are negligibly small. For the deuteron the effect of hadron tagging on $A_1$ is very small, as can be seen in Fig. 3. This is expected from isospin invariance. The hadron method is applied to the data at low $x$, also for $Q^2 < 1 \text{ GeV}^2$, where we do not expect a bias since $W$ is large.

**FIG. 2.** (a) Fraction $\phi$ of the inclusive events selected as hadron tagged observed in the data, for the ammonia target, compared with the expectation (see text). (b) Difference $\Delta$ of the two fractions. Errors show the systematic uncertainty of the expected fraction of hadron-tagged events.

**FIG. 3.** The differences of $A^1_i - A^h_i$ calculated from Monte Carlo for all generated DIS events ($A^1_i$) and for events with at least one forward hadron surviving cuts on $z$ and on the hadron momentum ($A^h_i$). The results are shown for two sets of cuts for the proton and for the deuteron.
IV. RESULTS FOR $A_1$ ASYMMETRIES

A. Updated $A_1$ with inclusive event selection

We have updated our previously published results on $A_1$ [7,11] for the proton and the deuteron using the new value of the beam polarization, given in Eq. (2). This leads to a 4% reduction of the $A_1$ values compared to the previous ones. In addition, there were other improvements which are discussed below.

The proton data collected in 1993 have been reprocessed with several improvements introduced since the original analysis. The most important was that information from an additional tracking chamber placed inside the spectrometer magnet just prior to the 1993 run was included in the track reconstruction. Also, the small angle triggers were treated in an improved way in the reconstruction. These changes, among others, resulted in a gain of approximately 10% in the number of events, mainly at low $x$. The new combined proton asymmetries are shown in Fig. 4(a) along with the values from our previous publication [7].

The updated result for $A_1$ of the deuteron has been obtained using a new parametrization for $F_2^d$ obtained in a similar way as the parametrization for $F_2^p$ used in Ref. [7]. These $F_2$ fits are described in the Appendix. The parametrization for $R$ used for $x<0.12$ is based on recently published NMC [24] data, while for $x>0.12$ we use the $R$ parametrization from SLAC [25], as before. The new values of $R$ change the depolarization factor at low $x$, while $F_2^d$ and $R$ enter in the effective dilution factor and also in the polarized radiative corrections. The overall effect of these changes is small. Figure 4(b) presents the updated results compared with the results from our previous publication [11].

![Figure 4](image4.png)

**FIG. 4.** The values of $A_1$ for (a) proton and (b) deuteron, updated as discussed in the text, in comparison with previously published results of Refs. [7] and [11]. Statistical errors are shown as error bars, while the shaded bands below indicate the systematic uncertainty.

![Figure 5](image5.png)

**FIG. 5.** The values of $A_1$ for the two types of event selections, inclusive and hadron tagged. The upper shaded bands indicate the systematic uncertainty of $A_1$ for the hadron-tagged selection, while the lower shaded bands indicate this for the inclusive selection.
B. $A_1$ for hadron tagged events

The SMC data on polarized protons and polarized deuterons were also analyzed using only hadron-tagged events. The results are presented in Fig. 5 as a function of $x$.

Most of the systematic errors were treated in the same way as for the inclusive analysis [7]. They arise from the uncertainties of the target and the beam polarizations, the polarized background, the value of $R$, the neglect of the $A_2$ contribution, and the momentum resolution. In addition, the uncertainties in the effective dilution factor and the radiative corrections include the uncertainty in $a_{\text{mol}}^\text{inel}$, which is taken as 30% of its value. This accounts for events with hard photon radiation, where the available energy for fragmentation into hadrons is reduced, and which may not be tagged. The uncertainty due to acceptance variation with time includes the effect of changes in the acceptance for both the scattered muon and for the hadrons.

C. Comparison of $A_1$ for inclusive and hadron-tagged events

The $A_1$ asymmetries for the two types of event selections, inclusive and hadron tagged, are compared in Fig. 5. The differences are small except for the two lowest $x$ points for the proton data.

As explained before, the results for the event selection with hadron tagging have smaller statistical errors at low $x$, while the inclusive event selection gives more precise results for high $x$. This can be seen in Fig. 6, which gives the ratio of the statistical errors for $A_1$ obtained with the two types of event selections as a function of $x$.

D. Optimal set of $A_1$ from SMC data

Figure 6 demonstrates that for $x<0.02$ the more accurate results for $A_1$ are obtained by using hadron-tagged events, while for $x>0.02$ the inclusive events give the more precise result. We therefore take as the optimal set of $A_1$ values the results from the hadron method for $x<0.02$ and the results from the inclusive method for $x>0.02$. This leads to the $A_1$ values in bins of $x$ presented in Fig. 7 and Tables II and III. The hadron method is used for the lowest 6 $x$ bins for the data shown in Fig. 7. Contributions to the systematic error are detailed in Tables IV and V for each $x$ bin and their quadratic sum is shown as a band in Fig. 7.

The weak $Q^2$ dependence of $A_1^p$ and $A_1^d$ in each bin of $x$ is presented in Figs. 8 and 9 and Tables VI and VII. From perturbative QCD a different $Q^2$ behavior is expected for the structure functions $F_1$ and $g_1$, hence $A_1 = g_1 / F_1$ should be $Q^2$ dependent. This dependence follows from the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equations [26]. It was determined in our QCD analysis, performed in next-to-leading order (NLO), which is presented in the following paper [14]. The results are shown as the solid
TABLE II. Optimal set of asymmetries $A_1^Q(x)$ from SMC data. 
The first error is statistical and the second is systematic. The first 
three bins have $Q^2>2.0$ GeV$^2$, while the remaining ones have $Q^2>
1$ GeV$^2$. Only the $Q^2>1$ GeV$^2$ bins are used in the QCD analysis 
mentioned in Sec. VI.

<table>
<thead>
<tr>
<th>$x$ range</th>
<th>$\langle x \rangle$</th>
<th>$\langle Q^2 \rangle$ (GeV$^2$)</th>
<th>$A_1^Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0008–0.0012</td>
<td>0.001</td>
<td>0.3</td>
<td>$-0.004\pm0.025\pm0.002$</td>
</tr>
<tr>
<td>0.0012–0.002</td>
<td>0.002</td>
<td>0.5</td>
<td>$0.021\pm0.018\pm0.003$</td>
</tr>
<tr>
<td>0.002–0.003</td>
<td>0.002</td>
<td>0.7</td>
<td>$0.014\pm0.017\pm0.003$</td>
</tr>
<tr>
<td>0.003–0.006</td>
<td>0.005</td>
<td>1.3</td>
<td>$0.029\pm0.014\pm0.003$</td>
</tr>
<tr>
<td>0.006–0.010</td>
<td>0.008</td>
<td>2.1</td>
<td>$0.026\pm0.014\pm0.003$</td>
</tr>
<tr>
<td>0.010–0.020</td>
<td>0.014</td>
<td>3.6</td>
<td>$0.036\pm0.013\pm0.003$</td>
</tr>
<tr>
<td>0.020–0.030</td>
<td>0.025</td>
<td>5.7</td>
<td>$0.059\pm0.017\pm0.004$</td>
</tr>
<tr>
<td>0.030–0.040</td>
<td>0.035</td>
<td>7.8</td>
<td>$0.068\pm0.021\pm0.004$</td>
</tr>
<tr>
<td>0.040–0.060</td>
<td>0.049</td>
<td>10.4</td>
<td>$0.101\pm0.018\pm0.006$</td>
</tr>
<tr>
<td>0.060–0.100</td>
<td>0.077</td>
<td>14.9</td>
<td>$0.170\pm0.018\pm0.011$</td>
</tr>
<tr>
<td>0.100–0.150</td>
<td>0.122</td>
<td>21.3</td>
<td>$0.252\pm0.024\pm0.015$</td>
</tr>
<tr>
<td>0.150–0.200</td>
<td>0.173</td>
<td>27.8</td>
<td>$0.296\pm0.033\pm0.018$</td>
</tr>
<tr>
<td>0.200–0.300</td>
<td>0.242</td>
<td>35.6</td>
<td>$0.368\pm0.034\pm0.023$</td>
</tr>
<tr>
<td>0.300–0.400</td>
<td>0.342</td>
<td>45.9</td>
<td>$0.544\pm0.055\pm0.036$</td>
</tr>
<tr>
<td>0.400–0.700</td>
<td>0.480</td>
<td>58.0</td>
<td>$0.625\pm0.075\pm0.048$</td>
</tr>
</tbody>
</table>

lines in Figs. 8 and 9 and give a good description of the data.
Also the assumption of $A_1$ having no $Q^2$ dependence, shown 
as the dashed lines in these figures, describes the data well.

V. CALCULATION OF $g_1$

We evaluate $g_1$ from Eq. (3), using our results for $A_1$ 
from Tables II and III, neglecting the contribution from $A_2$. 
The unpolarized structure function $F_2$ and the ratio $R$ 
are evaluated at the $x$ and $Q^2$ values of our measurement of $A_1$.

VI. FIRST MOMENTS OF $g_1^p$ AND $g_1^d$

We use our data in the kinematic region $Q^2>1$ GeV$^2$ 
therefore $x>0.003$) to calculate the first moments of $g_1^{p,d}(x;Q^2_0)$ at a fixed value of $Q^2=Q^2_0$. The values of

<table>
<thead>
<tr>
<th>$x$ range</th>
<th>$\langle x \rangle$</th>
<th>$\langle Q^2 \rangle$ (GeV$^2$)</th>
<th>$A_1^d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0008–0.0012</td>
<td>0.001</td>
<td>0.3</td>
<td>$0.001\pm0.026\pm0.002$</td>
</tr>
<tr>
<td>0.0012–0.002</td>
<td>0.002</td>
<td>0.5</td>
<td>$-0.016\pm0.020\pm0.003$</td>
</tr>
<tr>
<td>0.002–0.003</td>
<td>0.002</td>
<td>0.7</td>
<td>$-0.005\pm0.020\pm0.002$</td>
</tr>
<tr>
<td>0.003–0.006</td>
<td>0.005</td>
<td>1.3</td>
<td>$-0.018\pm0.016\pm0.002$</td>
</tr>
<tr>
<td>0.006–0.010</td>
<td>0.008</td>
<td>2.1</td>
<td>$-0.020\pm0.016\pm0.003$</td>
</tr>
<tr>
<td>0.010–0.020</td>
<td>0.014</td>
<td>3.5</td>
<td>$-0.027\pm0.015\pm0.003$</td>
</tr>
<tr>
<td>0.020–0.030</td>
<td>0.025</td>
<td>5.5</td>
<td>$-0.009\pm0.020\pm0.003$</td>
</tr>
<tr>
<td>0.030–0.040</td>
<td>0.035</td>
<td>7.5</td>
<td>$-0.013\pm0.024\pm0.003$</td>
</tr>
<tr>
<td>0.040–0.060</td>
<td>0.049</td>
<td>10.0</td>
<td>$0.075\pm0.021\pm0.006$</td>
</tr>
<tr>
<td>0.060–0.100</td>
<td>0.077</td>
<td>14.4</td>
<td>$0.017\pm0.021\pm0.003$</td>
</tr>
<tr>
<td>0.100–0.150</td>
<td>0.121</td>
<td>20.6</td>
<td>$0.069\pm0.028\pm0.006$</td>
</tr>
<tr>
<td>0.150–0.200</td>
<td>0.172</td>
<td>26.8</td>
<td>$0.178\pm0.041\pm0.013$</td>
</tr>
<tr>
<td>0.200–0.300</td>
<td>0.241</td>
<td>34.3</td>
<td>$0.238\pm0.044\pm0.015$</td>
</tr>
<tr>
<td>0.300–0.400</td>
<td>0.342</td>
<td>43.9</td>
<td>$0.190\pm0.073\pm0.014$</td>
</tr>
<tr>
<td>0.400–0.700</td>
<td>0.479</td>
<td>54.8</td>
<td>$0.316\pm0.102\pm0.022$</td>
</tr>
</tbody>
</table>
TABLE V. Contributions to the systematic error for $A_1^d(x)$, otherwise same explanations as for Table IV, except that $\Delta P_{bg}$ now refers to the contribution from protons in the deuterated butanol target.

<table>
<thead>
<tr>
<th>$\langle x \rangle$</th>
<th>$\Delta A_{\text{false}}$</th>
<th>$\Delta P_{\tau}$</th>
<th>$\Delta P_{\mu}$</th>
<th>$\Delta f'$</th>
<th>$\Delta r_c$</th>
<th>$\Delta A_2$</th>
<th>$\Delta R$</th>
<th>$\Delta M R$</th>
<th>$\Delta P_{bg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0010</td>
<td>0.0017</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0009</td>
<td>0.0009</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>0.0016</td>
<td>0.0017</td>
<td>0.0009</td>
<td>0.0004</td>
<td>0.0008</td>
<td>0.0009</td>
<td>0.0010</td>
<td>0.0005</td>
<td>0.0000</td>
<td>0.0002</td>
</tr>
<tr>
<td>0.0025</td>
<td>0.0017</td>
<td>0.0000</td>
<td>0.0002</td>
<td>0.0003</td>
<td>0.0009</td>
<td>0.0013</td>
<td>0.0002</td>
<td>0.0000</td>
<td>0.0002</td>
</tr>
<tr>
<td>0.005</td>
<td>0.0016</td>
<td>0.0002</td>
<td>0.0004</td>
<td>0.0008</td>
<td>0.0010</td>
<td>0.0004</td>
<td>0.0006</td>
<td>0.0000</td>
<td>0.0002</td>
</tr>
<tr>
<td>0.008</td>
<td>0.0018</td>
<td>0.0011</td>
<td>0.0006</td>
<td>0.0010</td>
<td>0.0010</td>
<td>0.0006</td>
<td>0.0006</td>
<td>0.0000</td>
<td>0.0002</td>
</tr>
<tr>
<td>0.014</td>
<td>0.0020</td>
<td>0.0014</td>
<td>0.0006</td>
<td>0.0011</td>
<td>0.0010</td>
<td>0.0007</td>
<td>0.0009</td>
<td>0.0000</td>
<td>0.0002</td>
</tr>
<tr>
<td>0.025</td>
<td>0.0019</td>
<td>0.0007</td>
<td>0.0003</td>
<td>0.0002</td>
<td>0.0010</td>
<td>0.0012</td>
<td>0.0004</td>
<td>0.0001</td>
<td>0.0002</td>
</tr>
<tr>
<td>0.035</td>
<td>0.0019</td>
<td>0.0003</td>
<td>0.0005</td>
<td>0.0007</td>
<td>0.0010</td>
<td>0.0014</td>
<td>0.0006</td>
<td>0.0001</td>
<td>0.0003</td>
</tr>
<tr>
<td>0.049</td>
<td>0.0020</td>
<td>0.0029</td>
<td>0.0020</td>
<td>0.0016</td>
<td>0.0010</td>
<td>0.0016</td>
<td>0.0033</td>
<td>0.0002</td>
<td>0.0004</td>
</tr>
<tr>
<td>0.077</td>
<td>0.0021</td>
<td>0.0007</td>
<td>0.0004</td>
<td>0.0000</td>
<td>0.0012</td>
<td>0.0004</td>
<td>0.0008</td>
<td>0.0005</td>
<td>0.0006</td>
</tr>
<tr>
<td>0.121</td>
<td>0.0022</td>
<td>0.0031</td>
<td>0.0019</td>
<td>0.0016</td>
<td>0.0012</td>
<td>0.0005</td>
<td>0.0027</td>
<td>0.0008</td>
<td>0.0008</td>
</tr>
<tr>
<td>0.172</td>
<td>0.0024</td>
<td>0.0083</td>
<td>0.0045</td>
<td>0.0029</td>
<td>0.0013</td>
<td>0.0006</td>
<td>0.0071</td>
<td>0.0010</td>
<td>0.0011</td>
</tr>
<tr>
<td>0.241</td>
<td>0.0025</td>
<td>0.0084</td>
<td>0.0060</td>
<td>0.0038</td>
<td>0.0014</td>
<td>0.0018</td>
<td>0.0101</td>
<td>0.0012</td>
<td>0.0015</td>
</tr>
<tr>
<td>0.342</td>
<td>0.0026</td>
<td>0.0069</td>
<td>0.0050</td>
<td>0.0041</td>
<td>0.0012</td>
<td>0.0021</td>
<td>0.0089</td>
<td>0.0013</td>
<td>0.0021</td>
</tr>
<tr>
<td>0.479</td>
<td>0.0027</td>
<td>0.0094</td>
<td>0.0074</td>
<td>0.0041</td>
<td>0.0014</td>
<td>0.0024</td>
<td>0.0176</td>
<td>0.0014</td>
<td>0.0027</td>
</tr>
</tbody>
</table>

FIG. 8. $A_1^p$ as a function of $Q^2$ for different bins of $x$ for the SMC data, where the value of $x$ is the average value in each bin. The EMC and E143 results are also shown for comparison. Error bars show statistical uncertainties. The solid line is a result of the QCD analysis described in our next paper [14] and used in Sec. VI, while the dashed line is the fit assuming no $Q^2$ dependence.

FIG. 9. $A_1^d$ as a function of $Q^2$ for different bins of $x$ for the SMC data, where the value of $x$ is the average value in each bin. The E143 results are also shown for comparison. Other explanations as for Fig. 8.
TABLE VI. Optimal set of asymmetries $A_i^0(x,Q^2)$ from SMC data. The errors are statistical only.

<table>
<thead>
<tr>
<th>$\langle x \rangle$</th>
<th>$\langle Q^2 \rangle$ (GeV$^2$)</th>
<th>$A_i^0$</th>
<th>$\langle x \rangle$</th>
<th>$\langle Q^2 \rangle$ (GeV$^2$)</th>
<th>$A_i^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0009</td>
<td>0.25</td>
<td>-0.023±0.037</td>
<td>0.0339</td>
<td>4.23</td>
<td>0.032±0.068</td>
</tr>
<tr>
<td>0.0010</td>
<td>0.30</td>
<td>-0.023±0.043</td>
<td>0.0342</td>
<td>5.80</td>
<td>0.130±0.048</td>
</tr>
<tr>
<td>0.0011</td>
<td>0.34</td>
<td>0.062±0.051</td>
<td>0.0344</td>
<td>7.77</td>
<td>0.034±0.033</td>
</tr>
<tr>
<td>0.0014</td>
<td>0.38</td>
<td>0.056±0.028</td>
<td>0.0359</td>
<td>10.14</td>
<td>0.094±0.039</td>
</tr>
<tr>
<td>0.0016</td>
<td>0.46</td>
<td>0.051±0.033</td>
<td>0.0472</td>
<td>4.29</td>
<td>0.076±0.101</td>
</tr>
<tr>
<td>0.0018</td>
<td>0.55</td>
<td>-0.057±0.034</td>
<td>0.0479</td>
<td>7.83</td>
<td>0.103±0.038</td>
</tr>
<tr>
<td>0.0022</td>
<td>0.59</td>
<td>0.006±0.029</td>
<td>0.0485</td>
<td>10.95</td>
<td>0.091±0.027</td>
</tr>
<tr>
<td>0.0025</td>
<td>0.70</td>
<td>0.032±0.030</td>
<td>0.0527</td>
<td>14.72</td>
<td>0.123±0.040</td>
</tr>
<tr>
<td>0.0028</td>
<td>0.82</td>
<td>-0.002±0.031</td>
<td>0.0737</td>
<td>5.47</td>
<td>0.168±0.094</td>
</tr>
<tr>
<td>0.0035</td>
<td>0.89</td>
<td>0.055±0.023</td>
<td>0.0744</td>
<td>7.88</td>
<td>0.138±0.056</td>
</tr>
<tr>
<td>0.0042</td>
<td>1.14</td>
<td>0.003±0.019</td>
<td>0.0750</td>
<td>11.08</td>
<td>0.181±0.036</td>
</tr>
<tr>
<td>0.0050</td>
<td>1.44</td>
<td>0.059±0.024</td>
<td>0.0762</td>
<td>16.30</td>
<td>0.170±0.028</td>
</tr>
<tr>
<td>0.0056</td>
<td>1.71</td>
<td>0.025±0.038</td>
<td>0.0856</td>
<td>23.10</td>
<td>0.172±0.043</td>
</tr>
<tr>
<td>0.0069</td>
<td>1.44</td>
<td>-0.047±0.040</td>
<td>0.1189</td>
<td>7.40</td>
<td>0.335±0.098</td>
</tr>
<tr>
<td>0.0071</td>
<td>1.76</td>
<td>-0.007±0.029</td>
<td>0.1196</td>
<td>11.14</td>
<td>0.309±0.065</td>
</tr>
<tr>
<td>0.0075</td>
<td>2.04</td>
<td>0.073±0.027</td>
<td>0.1200</td>
<td>16.48</td>
<td>0.225±0.045</td>
</tr>
<tr>
<td>0.0083</td>
<td>2.34</td>
<td>0.060±0.032</td>
<td>0.1205</td>
<td>24.82</td>
<td>0.239±0.041</td>
</tr>
<tr>
<td>0.0090</td>
<td>2.64</td>
<td>0.069±0.041</td>
<td>0.1293</td>
<td>34.31</td>
<td>0.254±0.057</td>
</tr>
<tr>
<td>0.0095</td>
<td>2.94</td>
<td>-0.098±0.059</td>
<td>0.1711</td>
<td>10.18</td>
<td>0.179±0.096</td>
</tr>
<tr>
<td>0.0114</td>
<td>1.75</td>
<td>-0.021±0.109</td>
<td>0.1715</td>
<td>16.51</td>
<td>0.253±0.076</td>
</tr>
<tr>
<td>0.0119</td>
<td>2.07</td>
<td>0.032±0.070</td>
<td>0.1717</td>
<td>24.89</td>
<td>0.194±0.065</td>
</tr>
<tr>
<td>0.0123</td>
<td>2.36</td>
<td>0.003±0.052</td>
<td>0.1718</td>
<td>34.94</td>
<td>0.427±0.069</td>
</tr>
<tr>
<td>0.0125</td>
<td>2.66</td>
<td>0.032±0.043</td>
<td>0.1770</td>
<td>45.47</td>
<td>0.371±0.077</td>
</tr>
<tr>
<td>0.0126</td>
<td>2.96</td>
<td>0.015±0.037</td>
<td>0.2368</td>
<td>10.53</td>
<td>0.317±0.125</td>
</tr>
<tr>
<td>0.0131</td>
<td>3.30</td>
<td>0.009±0.030</td>
<td>0.2392</td>
<td>21.49</td>
<td>0.288±0.059</td>
</tr>
<tr>
<td>0.0145</td>
<td>3.74</td>
<td>0.046±0.030</td>
<td>0.2398</td>
<td>34.94</td>
<td>0.391±0.080</td>
</tr>
<tr>
<td>0.0163</td>
<td>4.43</td>
<td>0.084±0.027</td>
<td>0.2462</td>
<td>52.75</td>
<td>0.438±0.054</td>
</tr>
<tr>
<td>0.0183</td>
<td>5.44</td>
<td>0.022±0.043</td>
<td>0.3383</td>
<td>15.25</td>
<td>0.413±0.150</td>
</tr>
<tr>
<td>0.0231</td>
<td>2.78</td>
<td>0.132±0.104</td>
<td>0.3404</td>
<td>25.00</td>
<td>0.491±0.142</td>
</tr>
<tr>
<td>0.0236</td>
<td>3.31</td>
<td>0.227±0.099</td>
<td>0.3407</td>
<td>34.97</td>
<td>0.691±0.145</td>
</tr>
<tr>
<td>0.0235</td>
<td>3.77</td>
<td>-0.008±0.072</td>
<td>0.3436</td>
<td>61.83</td>
<td>0.553±0.074</td>
</tr>
<tr>
<td>0.0237</td>
<td>4.54</td>
<td>0.093±0.039</td>
<td>0.4688</td>
<td>21.85</td>
<td>0.845±0.170</td>
</tr>
<tr>
<td>0.0241</td>
<td>5.75</td>
<td>0.058±0.028</td>
<td>0.4751</td>
<td>34.98</td>
<td>0.366±0.218</td>
</tr>
<tr>
<td>0.0263</td>
<td>7.41</td>
<td>0.028±0.032</td>
<td>0.4843</td>
<td>72.10</td>
<td>0.614±0.090</td>
</tr>
</tbody>
</table>

$g_1(x,Q^2)$ at the fixed $Q^2$ are determined from $g_1(x,Q^2)$ at the measured $x$ and $Q^2$ as

$$g_1(x,Q^2) = g_1(x,Q^2) + \{ g^{th}_1(x,Q^2) - g^{th}_1(x,Q^2) \},$$

where $g^{th}_1$ is a result of our NLO QCD analysis. This analysis is presented in Ref. [14]. We choose $Q^2_0=10$ GeV$^2$ since it is close to the average $Q^2$ of our data. The resulting values of $g_1(x,Q^2)$ are given in Tables VIII and IX. In the measured range $0.003 < x < 0.7$ the contributions to the first moments of the proton and the deuteron structure functions are calculated neglecting the $x$ dependence of $A_i$ within a given $x$ bin.

The results at $Q^2_0=10$ GeV$^2$ are

$$\int_{0.003}^{0.7} g_1^0(x,Q^2_0) dx = 0.131 \pm 0.005 \pm 0.006 \pm 0.004, \quad (9)$$

$$\int_{0.003}^{0.7} g_1^0(x,Q^2_0) dx = 0.037 \pm 0.006 \pm 0.003 \pm 0.003, \quad (10)$$

where the first uncertainty is statistical, the second is systematic and the third is due to the uncertainty in the $Q^2$ evolution. The errors of $g_1$ are correlated between $x$ bins and this correlation was taken into account when calculating systematic and theoretical uncertainties of the integrals. The contributions from different sources of uncertainty, detailed in
TABLE VII. Optimal set of asymmetries $A_1^d(x,Q^2)$ from SMC data. The errors are statistical only.

<table>
<thead>
<tr>
<th>$\langle x \rangle$</th>
<th>$\langle Q^2 \rangle$</th>
<th>$A_1^d$</th>
<th>$\langle x \rangle$</th>
<th>$\langle Q^2 \rangle$</th>
<th>$A_1^d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0009</td>
<td>0.25</td>
<td>-0.067±0.040</td>
<td>0.0342</td>
<td>3.57</td>
<td>-0.042±0.108</td>
</tr>
<tr>
<td>0.0010</td>
<td>0.30</td>
<td>0.052±0.046</td>
<td>0.0342</td>
<td>4.54</td>
<td>-0.129±0.089</td>
</tr>
<tr>
<td>0.0011</td>
<td>0.34</td>
<td>0.046±0.052</td>
<td>0.0342</td>
<td>5.80</td>
<td>-0.036±0.056</td>
</tr>
<tr>
<td>0.0014</td>
<td>0.38</td>
<td>-0.028±0.032</td>
<td>0.0344</td>
<td>7.78</td>
<td>0.033±0.038</td>
</tr>
<tr>
<td>0.0016</td>
<td>0.46</td>
<td>-0.069±0.037</td>
<td>0.0359</td>
<td>10.13</td>
<td>-0.023±0.045</td>
</tr>
<tr>
<td>0.0018</td>
<td>0.55</td>
<td>0.052±0.037</td>
<td>0.0476</td>
<td>2.63</td>
<td>0.257±0.187</td>
</tr>
<tr>
<td>0.0022</td>
<td>0.59</td>
<td>0.076±0.035</td>
<td>0.0476</td>
<td>3.59</td>
<td>0.322±0.140</td>
</tr>
<tr>
<td>0.0025</td>
<td>0.70</td>
<td>-0.043±0.035</td>
<td>0.0479</td>
<td>4.52</td>
<td>0.034±0.108</td>
</tr>
<tr>
<td>0.0027</td>
<td>0.82</td>
<td>-0.049±0.035</td>
<td>0.0477</td>
<td>5.83</td>
<td>0.047±0.069</td>
</tr>
<tr>
<td>0.0038</td>
<td>0.65</td>
<td>0.020±0.073</td>
<td>0.0480</td>
<td>7.82</td>
<td>0.101±0.044</td>
</tr>
<tr>
<td>0.0035</td>
<td>0.90</td>
<td>0.034±0.029</td>
<td>0.0484</td>
<td>10.95</td>
<td>0.093±0.032</td>
</tr>
<tr>
<td>0.0042</td>
<td>1.14</td>
<td>-0.015±0.023</td>
<td>0.0527</td>
<td>14.72</td>
<td>-0.006±0.047</td>
</tr>
<tr>
<td>0.0050</td>
<td>1.44</td>
<td>-0.024±0.028</td>
<td>0.0744</td>
<td>3.95</td>
<td>-0.019±0.120</td>
</tr>
<tr>
<td>0.0056</td>
<td>1.71</td>
<td>-0.025±0.045</td>
<td>0.0743</td>
<td>5.82</td>
<td>0.034±0.108</td>
</tr>
<tr>
<td>0.0074</td>
<td>1.09</td>
<td>-0.074±0.066</td>
<td>0.0746</td>
<td>7.85</td>
<td>0.026±0.062</td>
</tr>
<tr>
<td>0.0071</td>
<td>1.47</td>
<td>0.026±0.052</td>
<td>0.0753</td>
<td>11.05</td>
<td>0.090±0.041</td>
</tr>
<tr>
<td>0.0071</td>
<td>1.77</td>
<td>-0.043±0.034</td>
<td>0.0760</td>
<td>16.30</td>
<td>-0.025±0.033</td>
</tr>
<tr>
<td>0.0075</td>
<td>2.04</td>
<td>-0.053±0.031</td>
<td>0.0855</td>
<td>23.07</td>
<td>-0.004±0.051</td>
</tr>
<tr>
<td>0.0083</td>
<td>2.34</td>
<td>0.035±0.037</td>
<td>0.1187</td>
<td>5.00</td>
<td>-0.062±0.162</td>
</tr>
<tr>
<td>0.0090</td>
<td>2.64</td>
<td>-0.005±0.047</td>
<td>0.1194</td>
<td>10.23</td>
<td>0.056±0.063</td>
</tr>
<tr>
<td>0.0095</td>
<td>2.94</td>
<td>-0.010±0.069</td>
<td>0.1201</td>
<td>16.43</td>
<td>0.069±0.054</td>
</tr>
<tr>
<td>0.0128</td>
<td>1.59</td>
<td>-0.018±0.064</td>
<td>0.1203</td>
<td>24.82</td>
<td>0.076±0.050</td>
</tr>
<tr>
<td>0.0131</td>
<td>2.06</td>
<td>0.016±0.074</td>
<td>0.1289</td>
<td>34.25</td>
<td>0.093±0.069</td>
</tr>
<tr>
<td>0.0128</td>
<td>2.36</td>
<td>-0.019±0.061</td>
<td>0.1709</td>
<td>9.72</td>
<td>0.231±0.106</td>
</tr>
<tr>
<td>0.0125</td>
<td>2.66</td>
<td>-0.024±0.050</td>
<td>0.1714</td>
<td>16.47</td>
<td>0.062±0.091</td>
</tr>
<tr>
<td>0.0125</td>
<td>2.96</td>
<td>-0.033±0.043</td>
<td>0.1716</td>
<td>24.84</td>
<td>0.249±0.081</td>
</tr>
<tr>
<td>0.0130</td>
<td>3.30</td>
<td>-0.082±0.035</td>
<td>0.1739</td>
<td>39.62</td>
<td>0.171±0.065</td>
</tr>
<tr>
<td>0.0144</td>
<td>3.74</td>
<td>-0.008±0.035</td>
<td>0.2368</td>
<td>10.06</td>
<td>0.264±0.140</td>
</tr>
<tr>
<td>0.0163</td>
<td>4.44</td>
<td>-0.003±0.031</td>
<td>0.2386</td>
<td>16.52</td>
<td>0.205±0.111</td>
</tr>
<tr>
<td>0.0184</td>
<td>5.44</td>
<td>-0.023±0.050</td>
<td>0.2393</td>
<td>24.86</td>
<td>0.093±0.096</td>
</tr>
<tr>
<td>0.0237</td>
<td>2.13</td>
<td>-0.067±0.110</td>
<td>0.2391</td>
<td>34.93</td>
<td>0.265±0.105</td>
</tr>
<tr>
<td>0.0239</td>
<td>2.82</td>
<td>0.071±0.091</td>
<td>0.2454</td>
<td>52.73</td>
<td>0.294±0.072</td>
</tr>
<tr>
<td>0.0242</td>
<td>3.30</td>
<td>-0.063±0.102</td>
<td>0.3388</td>
<td>14.77</td>
<td>0.194±0.178</td>
</tr>
<tr>
<td>0.0239</td>
<td>3.76</td>
<td>-0.004±0.084</td>
<td>0.3404</td>
<td>29.55</td>
<td>0.084±0.132</td>
</tr>
<tr>
<td>0.0237</td>
<td>4.54</td>
<td>-0.079±0.045</td>
<td>0.3431</td>
<td>61.80</td>
<td>0.244±0.102</td>
</tr>
<tr>
<td>0.0241</td>
<td>5.75</td>
<td>0.008±0.032</td>
<td>0.4706</td>
<td>21.18</td>
<td>0.185±0.208</td>
</tr>
<tr>
<td>0.0263</td>
<td>7.41</td>
<td>0.013±0.037</td>
<td>0.4763</td>
<td>34.87</td>
<td>0.558±0.289</td>
</tr>
<tr>
<td>0.0341</td>
<td>2.59</td>
<td>-0.042±0.138</td>
<td>0.4827</td>
<td>71.76</td>
<td>0.317±0.129</td>
</tr>
</tbody>
</table>

Table X, were added in quadrature when computing the total errors. In addition to the uncertainties for $A_1$ given in Tables IV and V, for the calculation of the first moments we consider also contributions from the kinematic resolution and the error due to the approximations in the asymmetry evaluation procedure. The latter was estimated with a Monte Carlo simulation of this procedure. In our previous publications the central values for the integrals in Eqs. (9) and (10) were 0.130 [7] and 0.041 [11], respectively. The difference is mainly due to the updated beam polarization.

The first moments of $g_1$ are

$$\int_0^1 g_1^p(x,Q^2_0)dx = 0.120 \pm 0.005 \pm 0.006 \pm 0.014,$$

(11)

$$\int_0^1 g_1^d(x,Q^2_0)dx = 0.019 \pm 0.006 \pm 0.003 \pm 0.013.$$  

(12)

They are obtained by combining the results from Eqs. (9) and (10) with the contributions from the unmeasured ranges, which were calculated from the parametrizations of parton...
distributions from our NLO QCD analysis [14]. In the calcu-
lization of the total error we have taken into account that the 
value in the measured region affects the contributions from 
the unmeasured regions.

VII. THE NONSINGLET STRUCTURE FUNCTION $g_{1}^{\text{NS}}$

The flavor nonsinglet combination of the spin-dependent 
structure functions $g_{1}^{\text{NS}}=g_{1}^{u}-g_{1}^{d}$ is an interesting quantity 
because a rigorous QCD prediction exists for its first mo-
moment. This sum rule was derived, in the limit of infinite 
momentum transfer, by Bjorken [1] using current algebra 
and isospin symmetry.

A. Comparison of $g_{1}^{u}-g_{1}^{d}$ and $F_{1}^{u}-F_{1}^{d}$

In our experiment $g_{1}^{u}(x,Q^{2})$ and $g_{1}^{d}(x,Q^{2})$ are measured 
in the same bins of $x$ and $Q^{2}$. We evaluate $g_{1}^{\text{NS}}(x,Q^{2})$ from

$$g_{1}^{\text{NS}}(x,Q^{2})=2 \left[ g_{1}^{u}(x,Q^{2}) - g_{1}^{d}(x,Q^{2}) \right] / \left[ 1 - (3/2)\omega_{D} \right],$$

(13)

where $\omega_{D}$ is the probability of the deuteron to be in the $D$
state. As in our previous publications we have used $\omega_{D} = 0.05 \pm 0.01$, which covers most of the published values
[27].

The results are given in Table XI with statistical and sys-
tematic errors. In calculating the systematic error the contribu-
tions from the beam polarization, the dilution factor, and $R$
were treated as correlated between proton and deuteron, 
whereas the other contributions to the systematic error were 
treated as uncorrelated [28].

The results for $g_{1}^{\text{NS}}$ are shown in Fig. 11, together with $g_{1}^{\text{NS}}$ from the E143 experiment calculated from their values 
of $g_{1}^{u}$ and $g_{1}^{d}$ [29]. For both data sets the points are shown at 
the measured $Q^{2}$. In the same figure we show the nonsinglet

<table>
<thead>
<tr>
<th>TABLE VIII</th>
<th>x range</th>
<th>$\langle x \rangle$</th>
<th>$\langle Q^{2} \rangle$ (GeV$^2$)</th>
<th>$g_{1}^{u}$</th>
<th>$g_{1}^{u}(Q_{0}^{2}=10 \text{ GeV}^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0008–0.003</td>
<td>0.002</td>
<td>0.5</td>
<td>0.49 ± 0.42 ± 0.13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.003–0.006</td>
<td>0.005</td>
<td>1.3</td>
<td>0.75 ± 0.36 ± 0.07</td>
<td>1.19 ± 0.36 ± 0.07 ± 0.56</td>
<td></td>
</tr>
<tr>
<td>0.006–0.010</td>
<td>0.008</td>
<td>2.1</td>
<td>0.48 ± 0.26 ± 0.05</td>
<td>0.72 ± 0.26 ± 0.05 ± 0.25</td>
<td></td>
</tr>
<tr>
<td>0.010–0.020</td>
<td>0.014</td>
<td>3.6</td>
<td>0.43 ± 0.15 ± 0.03</td>
<td>0.59 ± 0.15 ± 0.03 ± 0.07</td>
<td></td>
</tr>
<tr>
<td>0.020–0.030</td>
<td>0.025</td>
<td>5.7</td>
<td>0.43 ± 0.13 ± 0.03</td>
<td>0.50 ± 0.13 ± 0.03 ± 0.02</td>
<td></td>
</tr>
<tr>
<td>0.030–0.040</td>
<td>0.035</td>
<td>7.8</td>
<td>0.36 ± 0.11 ± 0.02</td>
<td>0.39 ± 0.11 ± 0.02 ± 0.01</td>
<td></td>
</tr>
<tr>
<td>0.040–0.060</td>
<td>0.049</td>
<td>10.4</td>
<td>0.38 ± 0.07 ± 0.02</td>
<td>0.38 ± 0.07 ± 0.02 ± 0.00</td>
<td></td>
</tr>
<tr>
<td>0.060–0.100</td>
<td>0.077</td>
<td>14.9</td>
<td>0.41 ± 0.04 ± 0.02</td>
<td>0.39 ± 0.04 ± 0.02 ± 0.00</td>
<td></td>
</tr>
<tr>
<td>0.100–0.150</td>
<td>0.122</td>
<td>21.3</td>
<td>0.35 ± 0.03 ± 0.02</td>
<td>0.33 ± 0.03 ± 0.02 ± 0.00</td>
<td></td>
</tr>
<tr>
<td>0.150–0.200</td>
<td>0.173</td>
<td>27.8</td>
<td>0.28 ± 0.03 ± 0.01</td>
<td>0.27 ± 0.03 ± 0.01 ± 0.00</td>
<td></td>
</tr>
<tr>
<td>0.200–0.300</td>
<td>0.242</td>
<td>35.6</td>
<td>0.21 ± 0.02 ± 0.01</td>
<td>0.22 ± 0.02 ± 0.01 ± 0.01</td>
<td></td>
</tr>
<tr>
<td>0.300–0.400</td>
<td>0.342</td>
<td>45.9</td>
<td>0.17 ± 0.02 ± 0.01</td>
<td>0.18 ± 0.02 ± 0.01 ± 0.00</td>
<td></td>
</tr>
<tr>
<td>0.400–0.700</td>
<td>0.480</td>
<td>58.0</td>
<td>0.07 ± 0.01 ± 0.00</td>
<td>0.09 ± 0.01 ± 0.00 ± 0.00</td>
<td></td>
</tr>
</tbody>
</table>
spin-independent structure function $F_{1i}^{d} = F_{1i}^{u} - F_{1i}^{d}$ calculated from the measurements of the ratio $F_{2i}^{d}/F_{2i}^{u}$ [30], a fit to the data for $F_{2i}^{d}$, described in the Appendix, and the values of the function $R$ [24, 25]. The $Q^2$ range of the $F_{1i}^{NS}$ points corresponds approximately to the range covered by the SMC data. The nonsinglet $g_{1}^{NS}$ ($F_{1}^{NS}$) is proportional to the difference of the polarized (unpolarized) $u$-valence quark and $d$-valence quark distributions. There may also be a flavor symmetry violating contribution from the nucleon sea, as has been observed in the unpolarized case [31–34]. A possibility that the mechanism of flavor symmetry violations in polarized data may be related to that of the observed violations in the unpolarized case has been discussed in Ref. [35]. It is interesting that the shapes of the nonsinglet part of the polarized and unpolarized structure functions are very similar. The consequences of this similarity for parton distributions in LO and NLO have been discussed in Ref. [36]. It should be noted that the polarized nonsinglet distribution is not bounded by the unpolarized nonsinglet but by $F_{1}^{p} + F_{1}^{n}$. We observe that $g_{1}^{NS}$ is larger than $F_{1}^{NS}$.

### B. $Q^2$ evolution of $g_{1}^{NS}$

The flavor nonsinglet combination $g_{1}^{NS}$ decouples from the singlet and the gluon sectors, and therefore evolves in a different way than $g_{1}^{p}$ and $g_{1}^{n}$ separately. To calculate its $Q^2$ evolution only the parametrization of $g_{1}^{NS}(x)$ is needed. The evolution to a common $Q_{0}^{2}$ was done by three different methods. The first used the $Q^2$ dependence of the more accurately measured $F_{1i}^{NS}$. The $Q^2$ evolution of $g_{1}^{NS}$ and $F_{1i}^{NS}$ is expected to be the same since the $x$ distributions are similar and the unpolarized and polarized nonsinglet splitting functions are identical. The second method evolved the data using the nonsinglet part from the NLO QCD fit [14] already used in Sec. VI to evolve $g_{1}^{d}$ to the common $Q_{0}^{2}$. The third method used a simpler QCD fit, restricted to the nonsinglet sector [14].

Figure 12 shows $g_{1}^{NS}(x, Q^2)$ in each $x$ bin at its average value of $Q^2$ and evolved to $Q_{0}^{2} = 10$ GeV$^2$ using the nonsinglet fit (method 3) mentioned above. The changes of $g_{1}^{NS}$ due to the $Q^2$ evolution are small (compared to the statistical errors). The values of $g_{1}^{NS}(x, Q_{0}^{2})$ obtained with the third method are given in Table XI. The evolution calculated with methods 1 and 2 gave values very close to those obtained with method 3. The systematic errors due to $Q^2$ evolution given in Table XI cover the results from the three methods.

### C. First moment of $g_{1}^{NS}$

The first moment of $g_{1}^{NS}$ is calculated in three parts: from our data in the measured region 0.003 $< x < 0.7$ and those

---

**Table IX.** The spin-dependent structure function $g_{1}^{d}$ at the measured $Q^2$ and for $Q^2 > 1$ GeV$^2$, where the QCD evolution is applicable, $g_{1}^{d}$ evolved to $Q_{0}^{2} = 10$ GeV$^2$. Other explanations as for Table VIII.

<table>
<thead>
<tr>
<th>$x$ range</th>
<th>$(x)$</th>
<th>$\langle Q^2 \rangle$ (GeV$^2$)</th>
<th>$g_{1}^{d}$</th>
<th>$g_{1}^{d}(Q_{0}^{2} = 10$ GeV$^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0008–0.003</td>
<td>0.002</td>
<td>0.5</td>
<td>$-0.30 \pm 0.48 \pm 0.12$</td>
<td>$-0.30 \pm 0.42 \pm 0.06 \pm 0.49$</td>
</tr>
<tr>
<td>0.003–0.006</td>
<td>0.005</td>
<td>1.3</td>
<td>$-0.47 \pm 0.42 \pm 0.06$</td>
<td>$-0.30 \pm 0.42 \pm 0.06 \pm 0.49$</td>
</tr>
<tr>
<td>0.006–0.010</td>
<td>0.008</td>
<td>2.1</td>
<td>$-0.37 \pm 0.30 \pm 0.04$</td>
<td>$-0.22 \pm 0.30 \pm 0.04 \pm 0.22$</td>
</tr>
<tr>
<td>0.010–0.020</td>
<td>0.014</td>
<td>3.5</td>
<td>$-0.30 \pm 0.17 \pm 0.03$</td>
<td>$-0.22 \pm 0.17 \pm 0.03 \pm 0.06$</td>
</tr>
<tr>
<td>0.020–0.030</td>
<td>0.025</td>
<td>5.5</td>
<td>$-0.06 \pm 0.14 \pm 0.02$</td>
<td>$-0.02 \pm 0.14 \pm 0.02 \pm 0.02$</td>
</tr>
<tr>
<td>0.030–0.040</td>
<td>0.035</td>
<td>7.5</td>
<td>$-0.07 \pm 0.12 \pm 0.01$</td>
<td>$-0.05 \pm 0.12 \pm 0.01 \pm 0.01$</td>
</tr>
<tr>
<td>0.040–0.060</td>
<td>0.049</td>
<td>10.0</td>
<td>$0.27 \pm 0.08 \pm 0.02$</td>
<td>$0.27 \pm 0.08 \pm 0.02 \pm 0.00$</td>
</tr>
<tr>
<td>0.060–0.100</td>
<td>0.077</td>
<td>14.4</td>
<td>$0.04 \pm 0.05 \pm 0.01$</td>
<td>$0.03 \pm 0.05 \pm 0.01 \pm 0.00$</td>
</tr>
<tr>
<td>0.100–0.150</td>
<td>0.121</td>
<td>20.6</td>
<td>$0.09 \pm 0.04 \pm 0.01$</td>
<td>$0.08 \pm 0.04 \pm 0.01 \pm 0.00$</td>
</tr>
<tr>
<td>0.150–0.200</td>
<td>0.172</td>
<td>26.8</td>
<td>$0.15 \pm 0.03 \pm 0.01$</td>
<td>$0.14 \pm 0.03 \pm 0.01 \pm 0.00$</td>
</tr>
<tr>
<td>0.200–0.300</td>
<td>0.241</td>
<td>34.3</td>
<td>$0.12 \pm 0.02 \pm 0.01$</td>
<td>$0.12 \pm 0.02 \pm 0.01 \pm 0.00$</td>
</tr>
<tr>
<td>0.300–0.400</td>
<td>0.342</td>
<td>43.9</td>
<td>$0.05 \pm 0.02 \pm 0.00$</td>
<td>$0.05 \pm 0.02 \pm 0.00 \pm 0.00$</td>
</tr>
<tr>
<td>0.400–0.700</td>
<td>0.479</td>
<td>54.8</td>
<td>$0.03 \pm 0.01 \pm 0.00$</td>
<td>$0.04 \pm 0.01 \pm 0.00 \pm 0.00$</td>
</tr>
</tbody>
</table>

---

**Table X.** The sources of uncertainties for the integrals of $g_{1}^{d}$ and $g_{1}^{d}$ in the measured region 0.003 $< x < 0.7$.

<table>
<thead>
<tr>
<th>Source of the error</th>
<th>$\Delta g_{1}^{d}$</th>
<th>$\Delta g_{1}^{d}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target polarization</td>
<td>0.0037</td>
<td>0.0012</td>
</tr>
<tr>
<td>Beam polarization</td>
<td>0.0029</td>
<td>0.0008</td>
</tr>
<tr>
<td>Dilution factor</td>
<td>0.0027</td>
<td>0.0006</td>
</tr>
<tr>
<td>Uncertainty in $F_{2}$</td>
<td>0.0023</td>
<td>0.0010</td>
</tr>
<tr>
<td>Acceptance variation</td>
<td>0.0015</td>
<td>0.0014</td>
</tr>
<tr>
<td>Radiative corrections</td>
<td>0.0007</td>
<td>0.0008</td>
</tr>
<tr>
<td>Asymmetry evaluation</td>
<td>0.0006</td>
<td>0.0006</td>
</tr>
<tr>
<td>Neglect of $A_{2}$</td>
<td>0.0005</td>
<td>0.0006</td>
</tr>
<tr>
<td>Polarized background</td>
<td>0.0005</td>
<td>0.0004</td>
</tr>
<tr>
<td>Kinematic resolution</td>
<td>0.0003</td>
<td>0.0003</td>
</tr>
<tr>
<td>Momentum measurement</td>
<td>0.0003</td>
<td>0.0001</td>
</tr>
<tr>
<td>Uncertainty on $R$</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Total systematic error</td>
<td>0.0062</td>
<td>0.0026</td>
</tr>
<tr>
<td>Evolution</td>
<td>0.0036</td>
<td>0.0027</td>
</tr>
<tr>
<td>Statistics</td>
<td>0.0052</td>
<td>0.0057</td>
</tr>
</tbody>
</table>
TABLE XI. The nonsinglet structure function $g_1^{NS}$ and their uncertainties (shown only with 2 significant digits after the decimal points) calculated from the measured $g_1^p$ and $g_1^n$ at the measured $Q^2$ and evolved to $Q_0^2 = 10$ GeV$^2$. The first error is statistical and the second is systematic. In the last column the third error indicates the uncertainty in the QCD evolution.

<table>
<thead>
<tr>
<th>$x$ range</th>
<th>$\langle x \rangle$</th>
<th>$\langle Q^2 \rangle$ (GeV$^2$)</th>
<th>$g_1^{NS}$</th>
<th>$g_1^{NS} (Q_0^2 = 10$ GeV$^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.003–0.006</td>
<td>0.005</td>
<td>1.3</td>
<td>2.53 ± 1.17 ± 0.21</td>
<td>3.04 ± 1.17 ± 0.21 ± 0.01</td>
</tr>
<tr>
<td>0.006–0.100</td>
<td>0.008</td>
<td>2.1</td>
<td>1.96 ± 0.83 ± 0.16</td>
<td>2.06 ± 0.83 ± 0.16 ± 0.04</td>
</tr>
<tr>
<td>0.010–0.020</td>
<td>0.014</td>
<td>3.6</td>
<td>1.52 ± 0.47 ± 0.12</td>
<td>1.66 ± 0.47 ± 0.12 ± 0.02</td>
</tr>
<tr>
<td>0.020–0.030</td>
<td>0.025</td>
<td>5.6</td>
<td>1.00 ± 0.40 ± 0.07</td>
<td>1.05 ± 0.40 ± 0.07 ± 0.01</td>
</tr>
<tr>
<td>0.030–0.040</td>
<td>0.035</td>
<td>7.6</td>
<td>0.87 ± 0.35 ± 0.06</td>
<td>0.88 ± 0.35 ± 0.06 ± 0.00</td>
</tr>
<tr>
<td>0.040–0.060</td>
<td>0.049</td>
<td>10.2</td>
<td>0.18 ± 0.21 ± 0.05</td>
<td>0.18 ± 0.21 ± 0.05 ± 0.00</td>
</tr>
<tr>
<td>0.060–0.100</td>
<td>0.077</td>
<td>14.6</td>
<td>0.73 ± 0.13 ± 0.04</td>
<td>0.72 ± 0.13 ± 0.04 ± 0.00</td>
</tr>
<tr>
<td>0.100–0.150</td>
<td>0.122</td>
<td>21.0</td>
<td>0.51 ± 0.10 ± 0.03</td>
<td>0.50 ± 0.10 ± 0.03 ± 0.00</td>
</tr>
<tr>
<td>0.150–0.200</td>
<td>0.173</td>
<td>27.3</td>
<td>0.23 ± 0.10 ± 0.03</td>
<td>0.23 ± 0.10 ± 0.03 ± 0.00</td>
</tr>
<tr>
<td>0.200–0.300</td>
<td>0.242</td>
<td>34.9</td>
<td>0.17 ± 0.06 ± 0.02</td>
<td>0.18 ± 0.06 ± 0.02 ± 0.00</td>
</tr>
<tr>
<td>0.300–0.400</td>
<td>0.342</td>
<td>44.9</td>
<td>0.23 ± 0.05 ± 0.02</td>
<td>0.24 ± 0.05 ± 0.02 ± 0.01</td>
</tr>
<tr>
<td>0.400–0.700</td>
<td>0.480</td>
<td>56.4</td>
<td>0.09 ± 0.03 ± 0.01</td>
<td>0.10 ± 0.03 ± 0.01 ± 0.00</td>
</tr>
</tbody>
</table>

The first moment of $g_1^{NS}$ thus amounts to

$$\int_{0.003}^{0.7} g_1^{NS} dx = 0.184 ± 0.016 ± 0.014 ± 0.001,$$  \hspace{1cm} (14)

where the first error is statistical, the second is systematic, and the third is an evolution error based on errors given in Table XI. The total error on the integral in the measured range is 12% of its value. The contributions from the unmeasured regions are calculated from the parametrization of $g_1^{NS}$ and $F_1^{NS}$ is the same within the bin. The contributions are summed giving the integral in the measured range at $Q_0^2 = 10$ GeV$^2$

$$\int_{0}^{1} g_1^{NS} dx = 0.198 ± 0.023 \quad (Q_0^2 = 10 \text{ GeV}^2). \quad (15)$$

The value of the nonsinglet first moment given in Eq. (15) is in good agreement with the theoretical prediction of 0.186 ± 0.003 at $Q_0^2 = 10$ GeV$^2$. A more general discussion of the test of the Bjorken sum rule including different evaluations in the framework of perturbative QCD is presented in Ref. [14].

VIII. SUMMARY

This paper concludes the SMC analysis of the virtual photon-proton and virtual photon-deuteron spin asymmetries $A_1^g(x, Q^2)$ and $A_1^d(x, Q^2)$ measured in the deep inelastic scattering of polarized muons on polarized protons and polarized deuterons at incident muon energies of 100 and 190 GeV.

![FIG. 11. The nonsinglet functions $x g_1^{NS}$ and $x F_1^{NS}$. Both functions are presented at the measured $Q^2$ of the experiments. The errors are statistical only.](image1)

![FIG. 12. The nonsinglet function $g_1^{NS}$ as a function of $x$ given at the measured $Q^2$ and evolved to $Q_0^2 = 10$ GeV$^2$ with the method described in the text as the third method. Statistical errors are shown as error bars while the shaded band below indicates systematic uncertainty.](image2)
The final analysis included a reanalysis of the inclusive data and incorporated an asymmetry determination based on the hadron method, where the presence of at least one hadron in the final state of the muon-nucleon interaction was required. Such a selection removes a part of the background at low $x$ and hence improves the statistical accuracy there. The hadron method was thus used for $x<0.02$ while the inclusive method was used for $x>0.02$ in the determination of the final set of results for the asymmetries and the spin-independent structure functions $g_1^p(x,Q^2)$ and $g_1^d(x,Q^2)$.

These final results, which cover the kinematic range 0.0008<$x$<0.7 and $0.2<Q^2<100$ GeV$^2$, have been presented. They are consistent with the previously published SMC results [2,6,7,9–11] and supersede them. The final results have been tabulated in bins of $x$ and $Q^2$, and the individual contributions to the systematic error for $A_1$ have been given in bins of $x$. The analysis of events collected with a special trigger, which requires a signal from the hadron calorimeter in addition to the detection of a scattered muon, and allows measurements down to $x=0.0001$, mainly for $Q^2<1$ GeV$^2$, is in progress.

The spin-dependent flavor nonsinglet structure function $g_1^{\text{NS}}$ at the measured $Q^2$ was compared to the spin-independent nonsinglet structure function $F_1^{\text{NS}}$. Integrals of $g_1^{p,d}(x,Q_0^2=10$ GeV$^2)$ and $g_1^{\text{NS}}(x,Q_0^2=10$ GeV$^2)$ over the measured range were calculated using SMC data with $Q^2>1$ GeV$^2$. The first moments of $g_1^p$, $g_1^d$, and $g_1^{\text{NS}}$, including contributions from the unmeasured range obtained from the QCD analysis [14], have been given.

**ACKNOWLEDGMENTS**

We wish to thank our host laboratory CERN for providing major and efficient support for our experiment and an exciting and pleasant environment in which to do it. In particular, we thank J. V. Allaby, P. Darriulat, F. Dydkak, L. Foa, G. Goggi, H. J. Hilke, and H. Wenninger for substantial support and constant advice. We also wish to thank L. Gatignon and the SPS Division for providing us with an excellent beam, the LHC-ECR group for efficient cryogenics support, and J. M. Demolis for all his technical support. We also thank all those people in our home institutions who have contributed to the construction and maintenance of our equipment, especially A. Daël, J. C. Languillat, and C. Curé from DAPNIA/ Saclay for providing us with the high performance target superconducting magnet, Y. Lefèvre and J. Homma from NIKHEF for their contributions to the construction of the dilution refrigerator, and E. Kok for his contributions to the electronics and the data taking. It is a pleasure to thank G. Altarelli, R. D. Ball, F. E. Close, J. Ellis, D. de Florian, S. Forte, T. Gehmann, B. L. Ioffe, R. L. Jaffe, M. Karliner, J. Kuti, E. Leader, A. H. Mueller, G. Ridolfi, and W. Vogelsang for numerous valuable discussions and encouragement over many years. This work was supported by Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie, partially supported by TUBITAK and the Center for Turkish-Balkan Physics Research and Application (Boğaziçi University), supported by the U.S. Department of Energy, the U.S. National Science Foundation, Monbusho Grant-in-Aid for Science Research (International Scientific Research Program and Specially Promoted Research), the National Science Foundation (NWO) of the Netherlands, the Commissariat à l’Energie Atomique, Comision Interministerial de Ciencia y Tecnología and Xunta de Galicia, the Israel Science Foundation, and Polish State Committee for Scientific Research (KBN) Grant No. 2/P03B/081/14.

**APPENDIX**

A phenomenological fit for the unpolarized structure functions $F_2^p(x,Q^2)$ and $F_2^d(x,Q^2)$ was performed. Results for proton structure functions from BCDMS [37, E665 [38], NMC [24], SLAC [39], H1 [40], and ZEUS [41] were used to perform a fit for $F_2^p$. For the fit of $F_2^d$ the results for deuteron structure functions from BCDMS [37, E665 [38], NMC [24], and SLAC [39] and precise measurements of the ratio $F_2^d/F_2^p$ by the NMC [30] were used.

The $F_2$ parametrization, originally proposed by the BCDMS Collaboration and also used by NMC, is as follows:

$$F_2^\text{fit}(x,Q^2)=A(x) \left[ \frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)} \right]^{B(x)} \left[ 1 + \frac{C(x)}{Q^2} \right], \quad (A1)$$

where

$$A(x)=x^6(1-x)^2(a_3+a_5(1-x)+a_5(1-x)^2)$$

$$+a_4(1-x)^3+a_5(1-x)^4,$$

$$B(x)=b_1+b_2x+b_3/(x+b_4),$$

$$C(x)=c_1x+c_2x^2+c_3x^3+c_4x^4.$$

With $Q_0^2=20$ GeV$^2$ and $\Lambda=0.25$ GeV, this 15 parameter function was fitted to $F_2^p$ and $F_2^d$ data separately.

**TABLE XII.** The values of the parameters of Eq. (A1) for $F_2^p$ and for the upper and lower limits of $F_2^d$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$F_2^p$</th>
<th>Upper limit</th>
<th>Lower limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>-0.24997</td>
<td>-0.24810</td>
<td>-0.25196</td>
</tr>
<tr>
<td>$a_2$</td>
<td>2.3963</td>
<td>2.3632</td>
<td>2.4297</td>
</tr>
<tr>
<td>$a_3$</td>
<td>0.22896</td>
<td>0.23643</td>
<td>0.21913</td>
</tr>
<tr>
<td>$a_4$</td>
<td>0.08498</td>
<td>-0.03241</td>
<td>0.21630</td>
</tr>
<tr>
<td>$a_5$</td>
<td>3.8080</td>
<td>4.2268</td>
<td>3.4645</td>
</tr>
<tr>
<td>$a_6$</td>
<td>-7.4143</td>
<td>-7.8120</td>
<td>-6.9887</td>
</tr>
<tr>
<td>$a_7$</td>
<td>3.4342</td>
<td>3.5822</td>
<td>3.2771</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.11411</td>
<td>0.09734</td>
<td>0.13074</td>
</tr>
<tr>
<td>$b_2$</td>
<td>-2.2356</td>
<td>-2.2254</td>
<td>-2.2465</td>
</tr>
<tr>
<td>$b_3$</td>
<td>0.03115</td>
<td>0.03239</td>
<td>0.02995</td>
</tr>
<tr>
<td>$b_4$</td>
<td>0.02135</td>
<td>0.02233</td>
<td>0.02039</td>
</tr>
<tr>
<td>$c_1$</td>
<td>-1.4517</td>
<td>-1.4361</td>
<td>-1.4715</td>
</tr>
<tr>
<td>$c_2$</td>
<td>8.4745</td>
<td>8.1084</td>
<td>8.9108</td>
</tr>
<tr>
<td>$c_3$</td>
<td>-34.379</td>
<td>-33.306</td>
<td>-35.714</td>
</tr>
<tr>
<td>$c_4$</td>
<td>45.888</td>
<td>44.717</td>
<td>47.338</td>
</tr>
</tbody>
</table>
In the fit, the data points were weighted according to their statistical and uncorrelated systematic errors. Additional parameters were included in the fit to describe correlated shifts within the systematic uncertainties and to describe relative normalization shifts between data sets within the normalization uncertainties quoted by the experiments. All parameters and the complete covariance matrices were determined in the fits for $F_2^d$ and $F_2^d$. We used the parameters and the covariance matrices restricted to the 15 parameters of Eq. (A1) to determine the one standard deviation upper and lower limits of $F_2$. Both upper and lower limit values for $F_2^d$ and $F_2^d$ were parametrized with the same function.

The fitted parameters for the central values and for the upper and lower limits corresponding to the total uncertainties of $F_2^d$ are given in Tables XII and XIII. The fitted parametrizations are only valid in the kinematic range of the data sets, which cover correlated regions in the range of $3.5 \times 10^{-5} < x < 0.85$ and $0.2 < Q^2 < 5000 \text{ GeV}^2$ for $F_2^d$, and of $0.0009 < x < 0.85$ and $0.2 < Q^2 < 220 \text{ GeV}^2$ for $F_2^d$. The uncertainty in $F_2^d$ at low $x$ and $Q^2$ is underestimated due to the fact that the uncertainty of the fitted $F_2^d$ is not taken into account, where the ratio data $F_2^d/F_2^d$ are used. This has a negligible effect on the parameter set which describes the central values of the fitted $F_2^d$, but the total error given by the upper and lower limits is too small for $Q^2/\text{ GeV}^2$. For the calculation of the uncertainty of $g_1^d$ due to $F_2^d$ the effect is found to be negligible. Details of the fitting procedure can be found in Ref. [42].

The above parametrizations of $F_2^d$ must be used with the proper values of $R$ to reproduce the measured cross sections. We used a parametrization of the values of $R$ measured by the NMC [24] for $x<0.12$, and for $x>0.12$ we used the SLAC parametrization given in Ref. [25].

\begin{table}[h]
\centering
\begin{tabular}{llll}
\hline
Parameter & $F_2^d$ & Upper limit & Lower limit \\
\hline
$a_1$ & $-0.28151$ & $-0.28047$ & $-0.28178$ \\
$a_2$ & $1.0115$ & $0.82170$ & $1.1694$ \\
$a_3$ & $0.08415$ & $0.06904$ & $0.09973$ \\
$a_4$ & $-0.72973$ & $-0.60191$ & $-0.85884$ \\
$a_5$ & $2.8647$ & $2.2618$ & $3.4541$ \\
$a_6$ & $-2.5328$ & $-1.6507$ & $-3.3995$ \\
$a_7$ & $0.47477$ & $0.08909$ & $0.86034$ \\
$b_1$ & $0.20040$ & $0.18711$ & $0.20865$ \\
$b_2$ & $-2.5154$ & $-2.4711$ & $-2.5475$ \\
b_3 & $0.02599$ & $0.02802$ & $0.02429$ \\
b_4 & $0.01858$ & $0.01973$ & $0.01760$ \\
c_1 & $-1.3569$ & $-1.3762$ & $-1.3513$ \\
c_2 & $7.8938$ & $7.6113$ & $8.3602$ \\
c_3 & $-29.117$ & $-27.267$ & $-37.710$ \\
c_4 & $37.657$ & $35.100$ & $41.106$ \\
\hline
\end{tabular}
\caption{The values of the parameters of Eq. (A1) for $F_2^d$ and for the upper and lower limits of $F_2^d$.}
\end{table}

[23] A. Bravar, K. Kurek, and R. Windmolders Program POLDIS.