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TOPOSES ARE COHOMOLOGICALLY EQUIVALENT TO SPACES

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This purpose of this paper is to prove that for every Grothendieck topos \mathcal{E} there exist a space X and a covering $\varphi: X \rightarrow \mathcal{E}$ which induces an isomorphism in cohomology

$$H^n(\mathcal{E}, A) \xrightarrow{\sim} H^n(X, \varphi^*A) \quad (n \geq 0)$$

for any abelian group A in \mathcal{E} . Moreover for $n = 1$ this is also true for nonabelian A . This implies, by a result of Artin and Mazur, that φ induces an isomorphism of etale homotopy groups.

1. Construction of the cover. Let \mathcal{E} be a Grothendieck topos, and let G be an object of \mathcal{E} . $\text{En}(G)$ is the space (in this paper ‘space’ means space in the sense of [JT], chapter IV, unless explicitly said otherwise) of infinite-to-one partial enumerations of G ; in other words, $\text{En}(G)$ is characterized by the property that for any map $f: \mathcal{F} \rightarrow \mathcal{E}$ of toposes, the points of the induced space $f^*(\text{En}(G))$ in \mathcal{F} correspond to diagrams $\mathbf{N} \leftarrow\leftarrow U \rightarrow\rightarrow f^*G$ in \mathcal{F} with the property that for any $n \in \mathbf{N}$, $U - \{0, \dots, n\} \rightarrow f^*G$ is still epi. We write $\mathcal{E}[\text{En}(G)]$ for the category of sheaves in \mathcal{E} on the space $\text{En}(G)$, and $\varphi: \mathcal{E}[\text{En}(G)] \rightarrow \mathcal{E}$ for the corresponding geometric morphism. The properties of the space $\text{En}(G)$ and the map φ were extensively discussed in [JM]. For the present purpose, we recall the following basic facts. First of all, for a suitable object G of \mathcal{E} , $\mathcal{E}[\text{En}(G)]$ is equivalent to the topos $\text{Sh}(X_{\mathcal{E}})$ of sheaves on a space $X_{\mathcal{E}}$ in Sets , so that φ corresponds to a cover

$$(1) \quad \varphi: \text{Sh}(X_{\mathcal{E}}) \rightarrow \mathcal{E}.$$

Manuscript received 6 November 1988.

¹Supported by a Huygens Fellowship of the NWO.

American Journal of Mathematics 112 (1990), 87–95.

This geometric morphism is connected and locally connected; in particular, $\varphi^*: \mathcal{E} \rightarrow \text{Sh}(X_{\mathcal{E}})$ has a left adjoint $\varphi_!$ such that for any $E \in \mathcal{E}$ and $S \in \text{Sh}(X_{\mathcal{E}})$,

$$(2) \quad \varphi_!(\varphi^*(E) \times S) \cong E \times \varphi_!(S).$$

For any G in \mathcal{E} , there exists a surjective geometric morphism $p: \mathcal{B} \rightarrow \mathcal{E}$ where \mathcal{B} is the category of sheaves on a complete Boolean algebra (Barr's theorem, [B]), such that p^*G is countable (cf. [JT]). \mathcal{B} is a model of set theory, and the induced space $\text{En}(p^*G) \cong p^{\#}(\text{En}(G))$ in \mathcal{B} has enough points, i.e. is an ordinary topological space, which can be described as follows: the points of $\text{En}(p^*(G))$ are functions $\alpha: U \rightarrow p^*G$ with $U \subset \mathbf{N}$ and $\alpha^{-1}(g)$ infinite for all $g \in p^*(G)$; the basic open sets are the sets of the form $V_u = \{\alpha \mid \forall i \in \text{domain}(u): i \in U \text{ and } \alpha(i) = u(i)\}$, where u ranges over all functions $u: K \rightarrow p^*G$ defined on a finite set $K \subset \mathbf{N}$. It is not difficult to prove that each basic open set V_u (in particular, the space itself, V_{ϕ}) is contractible ([JM]).

2. Relative Čech cohomology. In this section, let Y be a space in a topos \mathcal{E} . One can define the relative Čech cohomology groups of Y with coefficients in an abelian group object in $\mathcal{E}[Y]$, i.e. a sheaf (or in fact, just a presheaf) of abelian groups on Y in \mathcal{E} ,

$$\check{H}_{\mathcal{E}}^n(Y, A).$$

These cohomology groups are group objects in \mathcal{E} . Their construction is completely parallel to the usual construction of the Čech cohomology groups of a topological space; indeed, the latter construction immediately translates to the context of a space in a topos \mathcal{E} , by viewing \mathcal{E} as a universe for (constructive) set theory (cf. [BJ]).

More explicitly, let $S \in \mathcal{E}$ and let $\mathcal{U}: S \rightarrow \mathcal{O}(Y)$ be an open cover of Y indexed by S . Let A be a (pre)sheaf of abelian groups on Y in \mathcal{E} ; so A is given by a map $A \rightarrow \mathcal{O}(Y)$ in \mathcal{E} equipped with the structure of a (pre)sheaf. Let

$$\mathcal{U}_p: S_p = S \times \cdots \times S \xrightarrow{-p+1-} \mathcal{U}^{p+1} \xrightarrow{\wedge} \mathcal{O}(Y)^{p+1} \xrightarrow{\wedge} \mathcal{O}(Y)$$

be the map in \mathcal{E} obtained from \mathcal{U} by intersection in Y , and let

$$(3) \quad C^p(\mathcal{U}, A) = \prod_{S_p} (A \times_{\mathcal{O}(Y)} S_p \rightarrow S_p)$$

where $\Pi_{S_p}: \mathcal{C}/S_p \rightarrow \mathcal{C}$ is the right adjoint of the functor $S_p^*: \mathcal{C} \rightarrow \mathcal{C}/S_p$ (cf. [J], p. 36). The $C^p(\mathcal{U}, A)$, $p \geq 0$, give a cochain complex $C^0(\mathcal{U}, A) \rightarrow C^1(\mathcal{U}, A) \rightarrow \dots$ in the usual way, with the differential defined via alternating sums. The cohomology groups of this complex are denoted by $H_{\mathcal{C}}^n(\mathcal{U}, A)$. One may now take the colimit of these groups over the *internal* diagram in \mathcal{C} of all open covers of $\mathcal{O}(Y)$ (so this involves internal covers of Y in \mathcal{C}/E for arbitrary E !), and obtain the relative Čech cohomology groups

$$(4) \quad \check{H}_{\mathcal{C}}^n(Y, A) = \lim_{\rightarrow \mathcal{U}} H_{\mathcal{C}}^n(\mathcal{U}, A) \quad (p \geq 0).$$

Straightforward modifications of the standard argument show that these cohomology groups have the usual properties. For instance, if we write $\varphi: \mathcal{C}[Y] \rightarrow \mathcal{C}$ for the canonical geometric morphism and $e_E: \mathcal{C}/E \rightarrow \mathcal{C}$ for the geometric morphism given by $e_E^* = E^* = (X \mapsto X \times E \xrightarrow{\pi_2} E)$, then for any open cover \mathcal{U} of $e_E^*(Y)$ in \mathcal{C}/E ,

$$(5) \quad H_{\mathcal{C}/E}^0(\mathcal{U}, e_E^*(A)) \cong e_E^* \varphi_* A,$$

where E is any object of \mathcal{C} ; hence

$$(6) \quad H_{\mathcal{C}}^0(Y, A) \cong \varphi_* A.$$

And for an injective object I of the category $\underline{\text{Ab}} \mathcal{C}[Y]$ of abelian sheaves on Y in \mathcal{C} ,

$$(7) \quad H_{\mathcal{C}/E}^n(\mathcal{U}, e_E^* I) = 0 \quad (n > 0)$$

for any E in \mathcal{C} and any open cover \mathcal{U} of $e_E^*(Y)$ in \mathcal{C}/E , so

$$(8) \quad \check{H}_{\mathcal{C}}^n(Y, I) = 0 \quad (n > 0)$$

3. A relative Cartan-Leray spectral sequence. As before, let Y be a space in a topos \mathcal{C} , and let $\varphi: \mathcal{C}[Y] \rightarrow \mathcal{C}$ be the corresponding

geometric morphism. $\mathcal{E}[Y]$ is a subtopos of the topos $\mathcal{E}^{0(Y)^{op}}$ of presheaves on $\mathcal{O}(Y)$ in \mathcal{E} , and we write $i: \mathcal{E}[Y] \hookrightarrow \mathcal{E}^{0(Y)^{op}}$ for the inclusion. The following is a relative version of SGA4, exp V, p. 24.

LEMMA 1. *For any abelian group A in $\mathcal{E}[Y]$, there exists a spectral sequence*

$$E_2^{p,q} = \check{H}_{\mathcal{E}}^p(Y, R^q i_*(A)) \Rightarrow R^{p+q} \varphi_*(A).$$

Proof. Let $0 \rightarrow A \rightarrow I$ be an injective resolution of A in $\underline{\mathbf{Ab}} \mathcal{E}[Y]$. For an open cover \mathcal{U} of Y in \mathcal{E} , one has a double complex of abelian groups $C^{p,q}(\mathcal{U}) = C^p(\mathcal{U}, I^q)$ (cf. (3)). By (5) and (7) above, the cohomology of the total complex is $H^n H^0(C^{**}(\mathcal{U})) = R^n \varphi_*(A)$, so we obtain a spectral sequence

$$(9) \quad E_2^{p,q}(\mathcal{U}) = H^p H^q(C^{**}(\mathcal{U})) = H_{\mathcal{E}}^p(\mathcal{U}, R^q i_* A) \Rightarrow R^{p+q} \varphi_*(A)$$

in the standard way ([G]). The same applies to open covers of $e_E^\#(Y)$ in \mathcal{E}/E for any object E of \mathcal{E} , so by taking the internal colimit in \mathcal{E} over all open covers of Y , we obtain a spectral sequence as stated in the lemma.

Now let $\mathbf{B} \subset \mathcal{O}(Y)$ be a basis for Y in \mathcal{E} which is closed under binary meets. Call \mathbf{B} A -acyclic if for every morphism $B: E \rightarrow \mathbf{B}$ in \mathcal{E} ,

$$(10) \quad \check{H}_{\mathcal{E}/E}^p(B, A|B) = 0. \quad (q > 0)$$

In (10), B stands for the open subspace of $e_E^\#(Y)$ determined by the given morphism $B: E \rightarrow \mathbf{B} \subset \mathcal{O}(Y)$, and $A|B \in \underline{\mathbf{Ab}}((\mathcal{E}/E)[B])$ is the sheaf induced by A .

$$(11) \quad \begin{array}{ccccc} (\mathcal{E}/E)[B] & \hookrightarrow & (\mathcal{E}/E)[e_E^\# Y] & \longrightarrow & \mathcal{E}[Y] \\ & & \downarrow & & \downarrow \varphi \\ & & (\mathcal{E}/E) & \xrightarrow{e_E} & \mathcal{E} \end{array}$$

LEMMA 2. *If \mathbf{B} is an A -acyclic basis for Y as above, then $\check{H}_{\mathcal{E}}^p(Y, A) \cong R^p \varphi_* A$, for all $p \geq 0$.*

Proof. We show by induction on n that $E_2^{p,q} = 0$ for all p and all q with $0 < q < n$, in the spectral sequence of Lemma 1. Suppose this holds for n . Then (cf. [CE], p. 328) $\check{H}_{\mathcal{E}}^i(Y, A) = R^i\varphi_*A$ for $i < n$, and there is an exact sequence $0 \rightarrow \check{H}_{\mathcal{E}}^n(Y, A) \rightarrow R^n\varphi_*A \rightarrow E_2^{0,n} \rightarrow \check{H}_{\mathcal{E}}^{n+1}(Y, A) \rightarrow R^{n+1}\varphi_*A$. But $E_2^{0,n} = \check{H}_{\mathcal{E}}^0(Y, R^n i_*A) \twoheadrightarrow \varphi_* i^* R^n i_*(A) = \varphi_* R^n i^* i_*(A) = \varphi_*(0) = 0$ ($n > 0$), so $\check{H}_{\mathcal{E}}^n(Y, A) \cong R^n\varphi_*A$. Applying this argument not to Y , but to any open subspace B (for any morphism $B: E \rightarrow \mathbf{B}$, cf the diagram (11)), our assumption on \mathbf{B} gives that $R^n i_*(A) \upharpoonright_{\mathbf{B}} = 0$, where $(-)\upharpoonright_{\mathbf{B}}$ denotes the restriction functor $\mathcal{E}^{0(X)^{op}} \rightarrow \mathcal{E}^{\mathbf{B}^{op}}$. Thus if in the spectral sequence (9) above, \mathcal{U} is a cover consisting of basic opens from \mathbf{B} , then $E_2^{p,n}(\mathcal{U}) = H_E^p(\mathcal{U}, 0) = 0$ for all p . Since such covers consisting of basic opens are cofinal in the internal system of all covers, it follows by passing to the colimit that $E_2^{p,n} = 0$ (all p) in the spectral sequence of Lemma 1. So the inductive statement in the beginning of the proof holds for $n + 1$, and Lemma 2 is proved.

Remark. Let Y be a space in \mathcal{E} , as above. Recall (see [JT]) that an open $U \subset Y$ is called surjective if it holds in \mathcal{E} that every cover of U is inhabited. If A is a sheaf on Y and $\{U_\alpha : \alpha \in \mathcal{A}\}$ is a family of opens, then $\Pi\{A(U_\alpha) \mid \alpha \in \mathcal{A}\} \cong \Pi\{A(U_\alpha) \mid \alpha \in \mathcal{A}, U_\alpha \text{ surjective}\}$ (where Π is the internal product $\mathcal{E}/\mathcal{A} \rightarrow \mathcal{E}$, as in Section 2). This is analogous to the fact that for $\mathcal{E} = \mathbf{Sets}$, $A(U) = \{*\}$ if the empty set covers U . Therefore in Lemma 2 it is enough to assume that \mathbf{B} is closed under surjective binary meets (i.e. $B \wedge B' \in \mathbf{B}$ whenever B and $B' \in \mathbf{B}$ and $B \wedge B'$ is surjective), since by this isomorphism, surjective intersections are the only ones that need to be considered in the complexes $C^{p,q}(\mathcal{U})$.

4. The main theorem. Let \mathcal{E} be a Grothendieck topos, and let $X_{\mathcal{E}}$ be the space constructed in Section 1. In the following theorem, $H^q(X_{\mathcal{E}}, -)$ denotes the sheaf cohomology of $X_{\mathcal{E}}$.

THEOREM. *The geometric morphism $\varphi : \mathbf{Sh}(X_{\mathcal{E}}) \rightarrow \mathcal{E}$ has the property that for any abelian group A in \mathcal{E} , $R^q\varphi_*(\varphi^*A) = 0$ for $q > 0$ (for $q = 0$, $R^q\varphi_*(\varphi^*A) \cong A$); consequently, φ induces an isomorphism*

$$H^q(\mathcal{E}, A) \xrightarrow{\sim} H^q(X_{\mathcal{E}}, \varphi^*A)$$

for each $q \geq 0$.

Proof. The second statement follows from the first by the Leray spectral sequence (SGA4, exp V, p. 35). The first statement is a special case (by construction of $X_{\mathcal{E}}$) of the general fact that for any object G in \mathcal{E} , the corresponding geometric morphism $\varphi: \mathcal{E}[\text{En}(G)] \rightarrow \mathcal{E}$ induces isomorphisms $H^q(\mathcal{E}, A) \xrightarrow{\sim} H^q(\mathcal{E}[E(G)], \varphi^*A)$, for any abelian group A in \mathcal{E} and any $q \geq 0$. Let \mathbf{B} be the basis consisting of opens of the form V_u (u a finite partial function from \mathbf{N} to G , cf. [JM]). $\text{En}(G) = V_\phi \in \mathbf{B}$, and $V_u \wedge V_w$ is surjective iff u and w are compatible finite functions, and in that case $V_u \wedge V_w = V_{u \cup w}$, so \mathbf{B} is closed under surjective finite meets (cf. the remark in Section 3).

We will show that for any injective object I of $\underline{\text{Ab}}(\mathcal{E})$ and any $q > 0$

$$(12) \quad R^q \varphi_*(\varphi^*I) = 0.$$

This is enough, because φ is connected, i.e. $\varphi_*\varphi^* \cong \text{id}$, and (12) says that φ^* maps injectives to φ_* -acyclic objects, so there is a spectral sequence $([G])$ for the composition $\varphi^* \circ \varphi_*$, $E_2^{p,q} = (R^p \varphi_*)(R^q \varphi^*)A \Rightarrow R^{p+q}(\varphi_*\varphi^*)A$; φ^* is exact and $\varphi_*\varphi^* \cong \text{id}$, so $E_2^{p,q} = 0$ for $q > 0$ and $E_2^{p,0} = R^p(\text{id})(A) = 0$ for $p > 0$. Thus $R^p \varphi_*(\varphi^*A) = 0$ for $p > 0$.

To prove (12), let I be an injective in $\underline{\text{Ab}} \mathcal{E}$, and let \mathcal{U} be an open cover of $\text{En}(G)$ by basic opens, say $\mathcal{U}: S \rightarrow \mathbf{B} \subset \mathcal{O}(Y)$ as in Section 2. Let us consider the nerve $N(\mathcal{U})$ of \mathcal{U} . This is the simplicial complex in \mathcal{E} defined as follows: $S = (S_p, p \geq 0)$ is a simplicial complex in \mathcal{E} , with as face $d_i: S_p \rightarrow S_{p-1}$ the projection $S^{p+1} \rightarrow S^p$ which deletes the i -th coordinate. The morphism $\mathcal{U}_p: S_p \rightarrow \mathbf{B} \subset \mathcal{O}(Y)$ can be viewed as an S_p -indexed sum of subobjects of the terminal object 1 of $\mathcal{E}[Y]$, and we write $\sum_{S_p} \mathcal{U}_p$ for their internal sum. Then

$$N_p(\mathcal{U}) = \varphi_!(\sum_{S_p} \mathcal{U}_p),$$

and the faces and degeneracies of S give $N(\mathcal{U})$ the structure of a simplicial complex over \mathcal{E} . Moreover,

$$(13) \quad C^p(\mathcal{U}, \varphi^*I) \cong I^{N_p(\mathcal{U})}.$$

(cf. (2)), where the differentials on the left correspond to the differentials obtained on the right by alternating sums from the cofaces of the co-

simplicial object $I^{N(\mathcal{U})}$. We claim that $C^p(\mathcal{U}, \varphi^*I)$ is an acyclic complex. Since I is injective, it suffices to prove that $\text{Free}(N\mathcal{U})$ is an acyclic chain complex in $\underline{\text{Ab}}(\mathcal{E})$, where $\text{Free}(-)$ denotes the free abelian group functor. To this end, let $p: \mathcal{B} \rightarrow \mathcal{E}$ be a Boolean extension as at the end of Section 1, and consider the pullback square

$$\begin{array}{ccc} \mathcal{B}[\text{En}(p^*G)] & \xrightarrow{p} & \mathcal{E}[\text{En}(G)] \\ \psi \downarrow & & \downarrow \varphi \\ \mathcal{B} & \xrightarrow{p} & \mathcal{E}. \end{array}$$

Since φ is locally connected so is ψ , and the Beck-Chevalley condition holds, i.e.

$$p^*\varphi_! \cong \psi_! p^*.$$

Consequently, if we write \mathcal{U}' for the cover of $\text{En}(p^*G)$ induced by \mathcal{U} via pullback along p , we have $p^*(\text{Free}(N\mathcal{U})) \cong \text{Free}(N\mathcal{U}')$. But \mathcal{B} is a model for set theory (with the axiom of choice), so we are now in a position to apply results from classical topology: the cover \mathcal{U}' of $\text{En}(p^*G)$ is a cover by basic opens, and $\text{En}(p^*G)$ as well as each of its basic open subspaces are contractible, so the nerve $N(\mathcal{U}')$ of this cover is a contractible simplicial set, and $\text{Free}(N\mathcal{U}')$ is an acyclic chain complex. Since $p^*(\text{Free}(N\mathcal{U})) = \text{Free}(N\mathcal{U}')$ and p^* is faithful, it follows that $\text{Free}(N\mathcal{U})$ is acyclic, as was to be shown.

Now apply this argument not just to $\text{En}(G)$, but to any basic open $B \subset e_E^\#(\text{En}(G))$ and any $E \in \mathcal{E}$ (cf (11), where $Y = \text{En}(G)$ now). Then we conclude that \mathbf{B} is an I -acyclic basis. (12) now follows by Lemma 2, since the whole space $\text{En}(G)$ is a member of \mathbf{B} . This completes the proof of the theorem.

5. Torsors. Let G be a group in a topos \mathcal{E} . A G -torsor in \mathcal{E} (or principal G -bundle over \mathcal{E}) is an object T of \mathcal{E} equipped with an action $\mu: G \times T \rightarrow T$ of G such that $T \rightarrow 1$ is epi and $(\mu, \pi_2): G \times T \rightarrow T \times T$ is an isomorphism. Recall ([Gi]) that $H^1(\mathcal{E}, G)$ is the pointed set of isomorphism classes of G -torsors (this is a group if G is abelian). For a space X and a sheaf of groups G on X , $H^1(X, G)$ stands for $H^1(\text{Sh}(X), G)$.

THEOREM. *Let \mathcal{E} be a topos, and let $\varphi: \text{Sh}(X_{\mathcal{E}}) \rightarrow \mathcal{E}$ be the cover of Section 1. For any group G in \mathcal{E} , φ induces an isomorphism*

$$H^1(\mathcal{E}, G) \xrightarrow{\sim} H^1(X_{\mathcal{E}}, \varphi^*G)$$

Proof. The functor $\varphi^*: \mathcal{E} \rightarrow \text{Sh}(X_{\mathcal{E}})$ is fully faithful, so it restricts to a fully faithful functor from the category of G -torsors in \mathcal{E} to that of φ^*G -torsors in $\text{Sh}(X_{\mathcal{E}})$. It thus suffices to show that this restriction of φ^* is essentially surjective. By [JM], there is a class $P \subset (X_{\mathcal{E}})^I$ of paths, such that \mathcal{E} is equivalent to the full subcategory of $\text{Sh}(X_{\mathcal{E}})$ consisting of those sheaves on $X_{\mathcal{E}}$ which are constant along the paths in P . Let T be a φ^*G -torsor in $\text{Sh}(X_{\mathcal{E}})$. Then T is locally isomorphic to $\varphi^*(G)$, and $\varphi^*(G)$ is constant along all the paths in P . So T is locally constant along the paths in P , and hence constant along those paths (since the interval I is simply connected).

6. Etale homotopy. Let \mathcal{E} be a locally connected topos, and let p be a point of \mathcal{E} . Artin and Mazur ([AM]) define the etale homotopy groups $\pi_n(\mathcal{E}, p)$ ($n \geq 0$), and prove a Whitehead theorem for toposes: a geometric morphism $(\mathcal{F}, q) \rightarrow (\mathcal{E}, p)$ of pointed locally connected toposes induces isomorphisms of etale homotopy groups iff it induces isomorphisms of cohomology groups with coefficients in a locally constant abelian group A in \mathcal{E} , as well as an isomorphism of the fundamental progroups $\pi_1(\mathcal{F}, q) \rightarrow \pi_1(\mathcal{E}, p)$. Our previous results give:

COROLLARY. *For any locally connected pointed topos (\mathcal{E}, p) there exists a pointed space $(X_{\mathcal{E}}, q)$ and a cover $\varphi: (\text{Sh}(X_{\mathcal{E}}), q) \rightarrow (\mathcal{E}, p)$ which induces isomorphisms in etale homotopy,*

$$\pi_n(X_{\mathcal{E}}, q) \xrightarrow{\sim} \pi_n(\mathcal{E}, p) \quad (n \geq 0)$$

Proof. First of all, we need to modify the construction of the space $X_{\mathcal{E}}$ slightly, in order to lift the point p : if we replace the set \mathbf{N} of natural numbers by an arbitrary infinite set S in the construction of Section 1 (and the space of infinite-to-one enumerations $\mathbf{N} \lll U \rrr G$ by that of infinite-to-one partial maps $\Delta(S) \lll U \rrr G$, where ΔS denotes the constant object of \mathcal{E} corresponding to the set S), we obtain a cover (again called) $\varphi: X_{\mathcal{E}} \rightarrow \mathcal{E}$ with exactly the same properties as before. A straightforward classifying-topos argument shows that if we

choose the cardinality of S sufficiently large (at least that of p^*G) then the given point p can be lifted to a point q of this (modified) space $X_{\mathcal{E}}$. $X_{\mathcal{E}}$ is locally connected since \mathcal{E} is, and φ is a locally connected map. Now the result of Section 5 shows that φ induces an isomorphism in π_1 (since $H^1(\mathcal{E}, G) \cong \text{Hom}(\pi_1(\mathcal{E}, p), G)$, cf [AM], Section 10). The corollary follows by the Whitehead theorem just quoted and the theorem of Section 4.

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