Measurements of Branching Fractions and CP-Violating Asymmetries in $B^0 \to \rho^\pm K^\mp$ Decays

We present measurements of branching fractions and $CP$-violating asymmetries in $B^0 \to \rho^- \pi^+$ and $B^0 \to \rho^- K^+$ decays. The results are obtained from a data sample of $88.9 \times 10^6 \ Y(4S) \to B\bar{B}$ decays collected with the $BABAR$ detector at the SLAC PEP-II asymmetric-energy $B$ Factory. From a time-dependent maximum likelihood fit we measure the branching fractions $\mathcal{B}(B^0 \to \rho^- \pi^+) = [22.6 \pm 1.8 \text{(stat)} \pm 2.2 \text{(syst)}] \times 10^{-6}$ and $\mathcal{B}(B^0 \to \rho^- K^+) = (7.3^{+1.3}_{-1.2} \pm 1.3) \times 10^{-6}$, and the $CP$-violating charge asymmetries $A_{\rho\pi}^{CP} = -0.18 \pm 0.08 \pm 0.03$ and $A_{\rho K}^{CP} = 0.28 \pm 0.17 \pm 0.08$, the direct $CP$ violation parameter $C_{\rho\pi} = 0.36 \pm 0.18 \pm 0.04$ and the mixing-induced $CP$ violation parameter $S_{\rho\pi} = 0.19 \pm 0.24 \pm 0.03$, and the dilution parameters $\Delta C_{\rho\pi} = 0.28^{+0.18}_{-0.19} \pm 0.04$ and $\Delta S_{\rho\pi} = 0.15 \pm 0.25 \pm 0.03$. 

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One of the central issues in particle physics is whether the single complex phase of the Cabibbo-Kobayashi-Maskawa quark-mixing matrix [1] is sufficient to explain the observed pattern of CP violation. We present here a simultaneous measurement of branching fractions and CP-violating asymmetries in the decays \( B^0 \rightarrow \rho^+ \pi^- \) and \( B^0 \rightarrow \rho^- \pi^+ \) (and their charge conjugates). The BABAR and Belle experiments have performed searches for CP-violating asymmetries in \( B \) decays to \( \pi^+ \pi^- \) [2,3], where the mixing-induced CP asymmetry is related to the angle \( \alpha_{CP} = \arg(-V_{td}^* V_{tb}/V_{td} V_{tb}^{*}) \) of the unitarity triangle as it is for \( \rho^+ \pi^- \). However, unlike \( \pi^+ \pi^- \), \( \rho^+ \pi^- \) is not a CP eigenstate, and four-flavor-charge configurations \[ B^0(\overline{B}^0) \rightarrow \rho^+ \pi^- \] must be considered [4]. Although this leads to a more complicated analysis, it benefits from a branching fraction that is nearly 5 times larger [5,6]. The extraction of \( \alpha \) from \( \rho^+ \pi^- \) is complicated by the interference of decay amplitudes with different strong and weak phases. One strategy for overcoming this problem is to perform an isospin analysis including all \( \rho \pi \) final states [7]. Another approach exploits the dynamical information of the full Dalitz plot to extract \( \alpha \) and the strong amplitudes simultaneously [8].

Following a quasi-two-body approach [9], we restrict the analysis to the two regions of the \( \pi^+ \pi^0 h^\pm \) Dalitz plot \( (h = \pi \text{ or } K) \) that are dominated by either \( \rho^+ h^- \) or \( \rho^- h^+ \). With \( \Delta \tau = t_{ph} - t_{tag} \) defined as the proper time interval between the decay of the reconstructed \( B^0(\overline{B}^0) \) and that of the other meson \( B^0(\overline{B}^0) \), the time-dependent decay rates are given by

\[
\rho^+ h^\pm(\Delta t) = \left(1 \pm A_{CP}^{h\pm} \right) e^{-|\Delta t|/\tau} \times \left[ 1 + Q_{tag}(S_{ph} \pm \Delta S_{ph}) \sin(\Delta m_2 \Delta t) - Q_{tag}(C_{ph} \pm \Delta C_{ph}) \cos(\Delta m_2 \Delta t) \right],
\]

where \( Q_{tag} = 1 \) (−1) when the tagging meson \( B^0(\overline{B}^0) \) is a \( B^0(\overline{B}^0) \), \( \tau \) is the mean \( B^0(\overline{B}^0) \) lifetime, and \( \Delta m_2 \) is the \( B^0(\overline{B}^0) \) oscillation frequency. The time- and flavor-integrated charge asymmetries \( A_{CP}^{h\pm} \) and \( A_{CP}^{h\pm} \) measure direct CP violation. For the \( \rho \pi \) mode, the quantities \( S_{ph} \) and \( C_{ph} \) parametrize mixing-induced CP violation related to the angle \( \alpha \), and flavor-dependent direct CP violation, respectively. The parameters \( \Delta C_{ph} \) and \( \Delta S_{ph} \) are sensitive to CP violation. \( \Delta C_{ph} \) describes the asymmetry between the rates \( \Gamma(B^0 \rightarrow \rho^+ \pi^-) + \Gamma(\overline{B}^0 \rightarrow \rho^- \pi^+) \) and \( \Gamma(B^0 \rightarrow \rho^- \pi^+) + \Gamma(\overline{B}^0 \rightarrow \rho^+ \pi^-) \), while \( \Delta S_{ph} \) is related to the strong phase difference between the amplitudes contributing to \( B^0 \rightarrow \rho \pi \) decays. More precisely, one finds the relations \( S_{ph} \pm \Delta S_{ph} = \sqrt{1 - (C_{ph} \pm \Delta C)^2 \sin(2\alpha_{CP} \pm \delta) \rho \pi} \), where \( 2\alpha_{CP} = \arg[q/p(A_{CP}^h/A_{CP}^\pm)] \), \( \delta = \arg[A_{CP}(A_{CP}^h/A_{CP}^\pm)] \), \( q/p \) is the \( B^0(\overline{B}^0) \) mixing phase, and \( A_{CP}(A_{CP}^h/A_{CP}^\pm) \) are the transition amplitudes of the processes \( B^0(\overline{B}^0) \rightarrow \rho^+ \pi^- \) and \( B^0(\overline{B}^0) \rightarrow \rho^- \pi^+ \), respectively. The angles \( \alpha_{CP} \) are equal to \( \alpha \) in the absence of contributions from penguin amplitudes. For the self-tagging \( \rho K \) mode, the values of the four time-dependent parameters are \( C_{ph} = 0, \Delta C_{ph} = -1, S_{ph} = 0, \) and \( \Delta S_{ph} = 0 \).

The data used in this analysis were accumulated with the BABAR detector [10], at the SLAC PEP-II asymmetric-energy \( e^+e^- \) storage ring. The sample consists of \( (88.9 \pm 1.0) \times 10^6 \) \( BB \) pairs collected at the \( Y(4S) \) resonance ("on-resonance"), and an integrated luminosity of 9.6 fb collected about 40 MeV below the \( Y(4S) \) ("off-resonance"). In Ref. [10] we describe the silicon vertex tracker and drift chamber used for track and vertex reconstruction, the Cherenkov detector (DIRC), the electromagnetic calorimeter (EMC), and their use in particle identification (PID).

We reconstruct \( B^0_{ph} \) candidates from combinations of two tracks and a \( \pi^0 \) candidate. We require that the PID of both tracks be inconsistent with the electron hypothesis, and the PID of the track used to form the \( \rho \) be inconsistent with the kaon hypothesis. The \( \pi^0 \) candidate mass must satisfy \( 0.11 < m(\gamma\gamma) < 0.16 \text{ GeV}/c^2 \), where each photon is required to have an energy greater than 50 MeV in the laboratory frame and to exhibit an EMC cluster profile consistent with an electromagnetic shower. The mass of the \( \rho \) candidate must satisfy \( 0.4 < m(\pi^0 \pi^0) < 1.3 \text{ GeV}/c^2 \). To avoid most of the interference region, the \( B \) candidate is rejected if both the \( \pi^+ \pi^0 \) and \( \pi^- \pi^0 \) pairs satisfy this requirement. Taking advantage of the helicity structure of \( B \rightarrow \rho h \) decays (\( h \) is denoted bachelor track hereafter), we require \( |\cos\theta_\pi| > 0.05 \), where \( \theta_\pi \) is the angle between the \( \pi^0 \) momentum and the negative \( B \) momentum in the \( \rho \) rest frame. The bachelor track from the \( \rho h \) decay must have a \( e^+e^- \) center-of-mass (c.m.) momentum above 2.4 GeV/c.

For 86% of the \( B^0 \rightarrow \rho h \) decays that pass the event selection, the pion from the \( \rho \) has momentum below this value, and thus the charge of the \( \rho \) is determined unambiguously. For the remaining events, the charge of the \( \rho \) is taken to be that of the selected \( \pi^+ \pi^- \) combination. With this procedure, 5% of the selected simulated signal events are assigned an incorrect charge.

To reject background from two-body \( B \) decays, the invariant masses of the \( \pi^+ h^- \) and \( \pi^- h^+ \) combinations must each be less than 5.14 GeV/c^2. Two kinematic variables are used to discriminate between signal-\( B \) decays and combinatorial background. One variable is the mass of the \( B \) candidate and \( \sqrt{s}/2 \), where \( \sqrt{s} \) is the total c.m. energy. The other variable is the beam-energy-substituted mass \( m_{ES} = \sqrt{(s/2 + p_B \cdot p_B)/E_i^2 - p_B^2} \), where the \( B \) momentum \( p_B \) and the four-momentum of the initial state (\( E_i, p_i \)) are defined in the laboratory frame. The \( \Delta E \) distribution for \( \rho \pi \) (\( \rho K \)) signal peaks around 0 (−45) MeV since the
pion mass is always assigned to the bachelor track. We require $5.23 < m_{ES} < 5.29$ GeV/$c^2$ and $-0.12 < \Delta E < 0.15$ GeV, where the asymmetric $\Delta E$ window suppresses higher-multiplicity B background, which leads to mostly negative $\Delta E$ values. Discrimination between $\rho \pi$ and $\rho K$ events is provided by the Cherenkov angle $\theta_C$ and, to a lesser extent, by $\Delta E$.

The dominant continuum background from $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$) events is suppressed through the use of a neural network (NN) combining four discriminating variables: the reconstructed $\rho$ mass, $\cos \theta_\rho$, and the two event-shape variables that are used in the Fisher discriminant of Ref. [2]. The NN is trained in the signal region with off-resonance data and simulated signal events. The final sample of signal candidates is selected with a cut on the NN output that retains $\sim 65\%$ (5\%) of the signal (continuum).

Approximately 23\% (20\%) of simulated $\rho \pi$ ($\rho K$) events have more than one $\rho$/$h$ candidate passing the selection criteria. In these cases, we choose the candidate with the reconstructed $\pi^0$ mass closest to the nominal $\pi^0$ mass. A total of 20 497 events pass all selection criteria. The signal efficiency determined from Monte Carlo (MC) simulation is 20.7\% (18.5\%) for $\rho \pi$ ($\rho K$) events; 31\% (30\%) of the selected events are misreconstructed, mostly due to combinatorial-$\pi^0$ background.

We use MC-simulated events to study the cross feed from other $B$ decays. The charmless modes are grouped into 11 classes with similar kinematic and topological properties. Two additional classes account for the neutral and charged $B \rightarrow c$ decays. For each of the background classes, a component is introduced into the likelihood, with a fixed number of events. Backgrounds from two-, three-, and four-body decays to $\rho \pi$ are dominated by $B^+ \rightarrow \pi^+ \pi^0$, $B^0 \rightarrow \rho^0 \pi^+$, and longitudinally polarized $B^0 \rightarrow \rho^+ \rho^-$ decays. The dominant two-body background for the $\rho K$ sample is $B^+ \rightarrow K^+ \pi^0$, while for three- and four-body sources it is $B \rightarrow K^+ \pi$ and higher kaon resonances, estimated from inclusive $B \rightarrow K \pi \pi$ measurements.

The time difference $\Delta t$ is obtained from the measured distance between the $z$ positions (along the beam direction) of the $B_{\rho h}^0$ and $B_{\text{tag}}^0$ decay vertices, and the boost $\beta \gamma = 0.56$ of the $e^+ e^-$ system [2,11]. To determine the flavor of the $B^0_{\text{tag}}$, we use the tagging algorithm of Ref. [11]. This produces four mutually exclusive tagging categories. We also retain untagged events in a fifth category to improve the efficiency of the signal selection and the sensitivity to charge asymmetries. Correlations between the $B$ flavor tag and the charge of the reconstructed $\rho h$ candidate are observed in various $B$-background channels and evaluated with MC simulation.

We use an unbinned extended maximum likelihood fit to extract the $\rho \pi$ and $\rho K$ event yields, the $CP$ parameters, and the other parameters defined in Eq. (1). The likelihood for the $N_k$ candidates $i$ tagged in category $k$ is

$$L_k = e^{-N_k} \prod_{i=1}^{N_k} \prod_{h} \left[ N_{ih}^0 \epsilon_k \mathcal{P}_{ih} + N_{i^0}^{h} \mathcal{P}_{i^0 h} + \sum_{j=1}^{N_k} \mathcal{L}_{ijh} \right],$$

where $N_k^0$ is the sum of the signal and continuum yields (to be determined by the fit) and the fixed $B$-background yields, $N_{ih}^0$ is the number of signal events of type $\rho h$ in the entire sample, $\epsilon_k$ is the fraction of signal events tagged in category $k$, and $N_{i^0}^{h}$ is the number of continuum background events with bachelor tag of type $h$ that are tagged in category $k$. The total likelihood $L$ is the product of likelihoods for each tagging category.

The probability density functions (PDFs) $\mathcal{P}_{i^0}^{h}$, $\mathcal{P}_{i^0}^{h}$ and the likelihood terms $\mathcal{L}_{ijh}$ are the product of the PDFs of five discriminating variables. The signal PDF is thus given by $\mathcal{P}_{i^0}^{h} = \mathcal{P}_{i^0}^{h}(m_{ES}) \times \mathcal{P}_{i^0}^{h}(\Delta E) \times \mathcal{P}_{i^0}^{h}(N_{q}) \times \mathcal{P}_{i^0}^{h}(P_{k})$, where $\mathcal{P}_{i^0}^{h}(\Delta t)$ contains the measured physics quantities defined in Eq. (1), dilated by the effects of mistagging and the $\Delta t$ resolution. The PDF of the continuum contribution with bachelor tag $h$ is denoted $\mathcal{P}_{i^0}^{h}$. The likelihood term $\mathcal{L}_{ijh} = N_{i^0}^{h} \mathcal{P}_{i^0 h}$ corresponds to the $B$-background contribution $j$ of the $N_k$ classes with an expected event yield $N_{i^0}^{h}$. The signal PDFs are decomposed into three parts with distinct distributions: signal events that are correctly reconstructed, misreconstructed signal events with right-sign $\rho$ charge, and misreconstructed signal events with wrong-sign $\rho$ charge. Their individual fractions are estimated by MC simulation. The $m_{ES}$, $\Delta E$, and NN output PDFs for signal and $B$ background are taken from the simulation except for the means of the signal Gaussian PDFs for $m_{ES}$ and $\Delta E$, which are free to vary in the fit. The continuum PDFs are described by six free parameters. The $\theta_C$ PDF is modeled as in Ref. [2]. The $\Delta t$-resolution function for signal and $B$-background events is a sum of three Gaussian distributions, with parameters determined from a fit to fully reconstructed $B^0$ decays [11]. The continuum $\Delta t$ distribution is parameterized as the sum of three Gaussian distributions with common mean, two relative fractions, and three distinct widths that scale the $\Delta t$ event-by-event error, yielding six free parameters. For continuum, two charge asymmetries and the ten parameters $N_{i^0}^{h}$ are free. A total of 34 parameters, including signal yields and the parameters from Eq. (1), are varied in the fit.

The contributions to the systematic error on the signal parameters are summarized in Table I. The uncertainties associated with $\Delta m_{d}$ and $\tau$ are estimated by varying these parameters within the uncertainty on the world average [12]. The uncertainties due to the signal model (PDF shapes, fraction of misreconstructed events) are obtained from a control sample of fully reconstructed $B^0 \rightarrow D^+ \rho^+$ decays. We perform fits on large MC samples with the measured proportions of $\rho \pi/\rho K$ signal, and
continuum and $B$ backgrounds. Biases observed in these tests are due to imperfections in the PDF model; e.g., unaccounted correlations between discriminating variables. The biases are added in quadrature and assigned as a systematic uncertainty of the fit procedure. The systematic errors due to interference between the doubly-Cabibbo-suppressed (DCS) $B \to 4\text{c}d\bar{u}$ amplitude with the Cabibbo-favored $B \to 4\bar{c}u\bar{d}$ amplitude for tagside $B$ decays have been estimated from simulation by varying freely all relevant strong phases [13].

The main source of systematic uncertainty is the $B$-background model. The expected event yields from the background modes are varied according to the uncertainties in the measured or estimated branching fractions. Systematic errors due to possible nonresonant $B^0 \to \pi^+ \pi^- \pi^0$ decays are derived from experimental limits [5]. Repeating the fit without using the $\rho$-candidate mass and helicity angle gives results that are compatible with those reported here. Since $B$-background modes may exhibit $CP$ violation, the corresponding parameters are varied within their physical ranges.

The maximum likelihood fit results in the event yields $N^{\rho\pi} = 428^{+34}_{-33}$ and $N^{\rho K} = 120^{+21}_{-20}$, where the errors are statistical. Correcting the yields with a small fit bias determined using the MC simulation (3% for $\rho\pi$ and 0% for $\rho K$), we find for the branching fractions

$$B(B^0 \to \rho^+ \pi^-) = (22.6 \pm 1.8 \pm 2.2) \times 10^{-6},$$

$$B(B^0 \to \rho^- K^+) = (7.3^{+1.1}_{-1.2} \pm 1.3) \times 10^{-6},$$

where the first errors are statistical and the second systematic. The systematic errors include an uncertainty of 7.7% for efficiency corrections, dominated by the uncertainty in the $\pi^0$ reconstruction efficiency. Figure 1 shows distributions of $m_{ES}$ and $\Delta E$, enhanced in signal content by cuts on the signal-to-continuum likelihood ratios of the other discriminative variables. Distributions of discriminating variables show satisfactory agreement with the fit. For the $CP$-violating parameters, we obtain

$$A^{\rho\pi}_{CP} = -0.18 \pm 0.08 \pm 0.03,$$

$$C_{\rho\pi} = 0.36 \pm 0.18 \pm 0.04,$$

$$A^{\rho K}_{CP} = 0.28 \pm 0.17 \pm 0.08,$$

$$S_{\rho\pi} = 0.19 \pm 0.24 \pm 0.03.$$  

For the other parameters in the description of the $B^0(\bar{B}^0) \to \rho\pi$ decay-time dependence, we find

$$\Delta C_{\rho\pi} = 0.28^{+0.18}_{-0.19} \pm 0.04,$$

$$\Delta S_{\rho\pi} = 0.15 \pm 0.25 \pm 0.03.$$  

We find the linear correlation coefficients $c_{C,\Delta C} = 0.18$ and $c_{S,\Delta S} = 0.23$, while all other correlations are smaller. As a validation of our treatment of the time dependence we allow $\tau$ and $\Delta m_d$ to vary in the fit. We find $\tau = (1.64 \pm 0.13) \text{ps}$ and $\Delta m_d = (0.52 \pm 0.12) \text{ps}^{-1}$; the remaining free parameters are consistent with the nominal fit. The raw time-dependent asymmetry $A_{B^0/B^0} = (N_{B^0} - N_{B^0})/(N_{B^0} + N_{B^0})$ in the tagging categories dominated by kaons and leptons is represented in Fig. 2.

In summary, we have presented measurements of branching fractions and $CP$-violating asymmetries in $B^0 \to \rho^- \pi^+$ and $\rho^- K^+$ decays. We do not find evidence
for either mixing-induced $CP$ violation in the time-
dependent asymmetry of $B_0 \to \rho^\pm \pi^\mp$ decays, or direct $CP$ violation in $B^{0} \to \rho^\pm K^\mp$. Our measurement of direct $CP$ violation in $B^{0} \to \rho^\pm \pi^\mp$ is consistent with zero within 2.0 standard deviations, when both statistical and systematic errors are taken into account.

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*Also with Università di Perugia, Perugia, Italy.
† Also with Università della Basilicata, Potenza, Italy.
‡ Also with IFIC, Instituto de Física Corpuscular, CSIC-Universidad de Valencia, Valencia, Spain.
§Deceased.