Measurement of Time-Dependent CP Asymmetries in $B^0 \to D^{(*)\pm} \pi^\mp$ Decays and Constraints on $\sin(2\beta + \gamma)$

We present a measurement of CP-violating asymmetries in fully reconstructed $B_0 \rightarrow D^{(*)\pm} \pi^\mp$ decays in approximately $88 \times 10^6 \ Y(4S) \rightarrow B\overline{B}$ decays collected with the BABAR detector at the PEP-II asymmetric-energy $B$ factory at SLAC. From a time-dependent maximum-likelihood fit we obtain the following for the CP-violating parameters: $a = -0.022 \pm 0.038 \ (\text{stat}) \pm 0.020 \ (\text{syst})$.
In the standard model, CP violation in the weak interactions between quarks manifests itself as a nonzero area of the Cabibbo-Kobayashi-Maskawa (CKM) unitarity triangle [1]. While it is sufficient to measure one of its angles $\alpha$, $\beta$, or $\gamma$ to be different from $0^\circ$ or $180^\circ$ to demonstrate the existence of CP violation, the unitarity triangle needs to be overconstrained with different measurements to test the CKM mechanism. Measurements of $\beta$ free from theoretical uncertainties exist [2,3], but there are no such measurements of $\alpha$ and $\gamma$. This Letter reports the measurement of CP-violating asymmetries in $B^0 \to D^{(*)}\pi^\pm$ decays [4] in $Y(4S) \to B\bar{B}$ decays and its interpretation in terms of constraints on $|\sin(2\beta+\gamma)|$ [5,6].

The time evolution of $B^0 \to D^{(*)}\pi^\pm$ decays is sensitive to $\gamma$ because of the interference between the CKM-favored decay $B^0 \to D^{(*)}\pi^-$, whose amplitude is proportional to the CKM matrix elements $V_{cb}V_{ud}$, and the doubly CKM-suppressed decay $B^0 \to D^{(*)}\pi^+$, whose amplitude is proportional to $V_{cd}V_{ud}$. The relative weak phase between the two amplitudes is $\gamma$, which, when combined with $B^0\bar{B}$ mixing, yields a weak phase difference of $2\beta + \gamma$ between the interfering amplitudes.

The decay rate distribution for $B^0 \to D^{(*)}\pi^\pm$ decays is

$$f^\pm(\eta, \Delta t) = \frac{e^{-|\Delta t|/\tau}}{4\tau} \times \left[ 1 \mp S_{\pm} \sin(\Delta m_d|\Delta t|) \mp \eta C \cos(\Delta m_d|\Delta t|) \right].$$

(1)

where $\tau$ is the $B^0$ lifetime, neglecting the decay width difference, $\Delta m_d$ is the $B^0\bar{B}$ mixing frequency, and $\Delta t = t_{\text{rec}} - t_{\text{tag}}$ is the time of the $B^0 \to D^{(*)}\pi^\pm$ decay ($B_{\text{rec}}$) relative to the decay of the other $B$ ($B_{\text{tag}}$). In this equation the upper (lower) sign refers to the flavor of $B_{\text{tag}}$ as $B^0$ ($\bar{B}^0$), while $\eta = +1$ ($-1$) and $\zeta = (+1)$ for the final state $D^{(*)}\pi^+$ ($D^{(*)}\pi^-$). In the standard model, the $S$ and $C$ parameters can be expressed as

$$S_\pm = \frac{2\text{Im}(\lambda_\pm)}{1 + |\lambda_\pm|^2}, \quad C = \frac{1 - r^2}{1 + r^2},$$

(2)

where $\lambda_\pm = r^{\pm1}e^{-i(2\beta+\gamma+\delta)}$. Here $\delta$ is the relative strong phase and $r$ is the magnitude of the ratio of the suppressed and the favored amplitudes. The same equations apply for $B^0 \to D^{(*)}\pi^\pm$ decays, with $r$ and $\delta$ replaced by the parameters $r^\prime$ and $\delta^\prime$, respectively [7].

The analysis strategy is similar to that of the time-dependent mixing measurement performed at BABAR [8]. To identify the flavor of $B_{\text{tag}}$, each event is assigned by a neural network to one of four hierarchical, mutually exclusive tagging categories: one lepton and two kaon categories based on the charges of identified leptons and kaons, and a fourth category for remaining events. The effective tagging efficiency is $(28.1 \pm 0.7)\%$ [2]. The time difference $\Delta t$ is calculated from the separation along the beam collision axis, $\Delta z$, between the $B_{\text{rec}}$ and $B_{\text{tag}}$ decay vertices. We determine the $B_{\text{rec}}$ vertex from its charged tracks. The $B_{\text{tag}}$ decay vertex is obtained by fitting tracks that do not belong to $B_{\text{rec}}$, imposing constraints from the $B_{\text{rec}}$ momentum and the beam-spot location. The $\Delta t$ resolution is approximately $1.1$ ps.

The expected CP asymmetry in these decays is small ($r^{(*)} = |V_{ub}/V_{cd}|/|V_{ud}/V_{cb}| = 0.02$), and therefore this measurement is sensitive to the interference between the $b \to u$ and $b \to c$ amplitudes in the decay of $B_{\text{tag}}$. To account for this effect we use a parametrization different from Eq. (2), which is described in Ref. [9] and summarized here. For each tagging category $(i)$ the interference is parametrized in terms of the effective parameters $r_i$ and $\delta_i$. Neglecting terms of order $r_i^{(*)}$ and $r_i^2$, for each tagging category the $\Delta T$ distribution becomes

$$f_i^{\pm(*)}(\eta, \Delta t) = \frac{e^{-|\Delta t|/\tau}}{4\tau} \times \left[ 1 \mp (a^{(*)} \mp \eta b_i \mp \eta c_i^{(*)}) \sin(\Delta m_d|\Delta t|) \pm \eta \cos(\Delta m_d|\Delta t|) \right].$$

(3)

where, in the standard model,

$$a^{(*)} = 2r^{(*)} \sin(2\beta + \gamma) \cos\delta^{(*)},$$

$$b_i = 2r_i \sin(2\beta + \gamma) \cos\delta^i,$$

$$c_i^{(*)} = 2\cos(2\beta + \gamma)(r^{(*)} \sin\delta^{(*)} - r_i \sin\delta^i).$$

Semileptonic B decays do not have a doubly CKM-suppressed amplitude contribution, and hence $r_{\text{lep}} = 0$. Given that we have two B decay modes and four tagging categories, we use two $a$ parameters (one for each final state), three $b$ parameters (one for each nonlepton tagging category), and eight $c$ parameters (one for each combination of tagging category and final state). Results are quoted only for the four parameters $a^{(*)}$ and $c_{\text{lep}}^{(*)}$, which are independent of the unknowns $r_i^{(*)}$ and $\delta_i$. The other parameters are allowed to float in the fit, but, as they depend on $r_i^{(*)}$ and $\delta_i$, they do not contribute to the interpretation of the result in terms of $\sin(2\beta + \gamma)$.

This measurement is based on $88 \times 10^6$ $Y(4S) \to B\bar{B}$ decays, corresponding to an integrated luminosity of $88 \times 10^6$ $Y(4S) \to B\bar{B}$ decays.
with corresponding purities of $(84.9 \pm 0.5\%)$ and $(94.4 \pm 0.4\%)$ in a $\pm 3\sigma$ region around the nominal $B$ mass. Backgrounds from $B^0$ decays that peak in the $m_{ES}$ signal region were estimated with Monte Carlo simulation to constitute $(0.21 \pm 0.06\%)$ and $(0.13 \pm 0.05\%)$ of the $B^0\rightarrow D^\pm\pi^\mp$ and $B^0\rightarrow D^{\ast\pm}\pi^\mp$ yields, respectively. For backgrounds from $B^+$ decays, the corresponding figures are $(0.93 \pm 0.23\%)$ and $(0.93 \pm 0.10\%)$.

An unbinned maximum-likelihood fit is performed on the selected $B$ candidates using the $\Delta T$ distribution in Eq. (3), convolved with a resolution function composed of three Gaussian distributions. Incorrect tagging dilutes the parameters $a^{(*)}$, $c_i^{(*)}$, and the coefficient of $\cos(\Delta m_d \Delta t)$ by a factor $D_i = 1 - 2w_i$ [2,9], where $w_i$ is the mistag fraction. The resolution function and the parameters associated with flavor tagging are determined from the data and are consistent with previous BABAR analyses [2]. The combinatorial background is parametrized as the sum of a component with zero lifetime and one with an effective lifetime fixed to the value obtained from simulation. The fraction of each component and the $\Delta T$ resolution parameters are left free in the fit to the data. The background coming from $B^\pm$ mesons is modeled with an exponential decay with the $B^\pm$ lifetime, and its size is fixed to the value predicted by simulation. The background from $B^0$ mesons is neglected in the nominal fit, but is considered in evaluating the systematic uncertainties.

The results from the fit to the data are

$$a = -0.022 \pm 0.038\text{ (stat)} \pm 0.020\text{ (syst)},$$

$$a^* = -0.068 \pm 0.038\text{ (stat)} \pm 0.020\text{ (syst)},$$

$$c_{lep} = +0.025 \pm 0.068\text{ (stat)} \pm 0.033\text{ (syst)},$$

$$c_{lep}^* = +0.031 \pm 0.070\text{ (stat)} \pm 0.033\text{ (syst)}.$$  \hspace{1cm} (5)

All other fitted $b$ and $c$ parameters are consistent with zero. Figure 2 shows the fitted $\Delta t$ distributions for events from the lepton tagging category, which has the lowest level of background and mistag probability.

FIG. 2. Distributions of $\Delta t$ for the $B^0\rightarrow D^\pm\pi^\mp$ (a)–(d) and $B^0\rightarrow D^{\ast\pm}\pi^\mp$ (e)–(h) candidates tagged with leptons, split by $B$ tagging flavor and reconstructed final state. The lines are fit projections and hatched regions represent background.
The systematic uncertainties on the parameters in Eq. (5) has been calculated in a manner similar to that used in Ref. [8]. A small bias in the $\Delta t$ measurement could result in a bias on the $c$ parameters in Eq. (3). For instance, a realistic $\Delta t$ bias of 0.024 ps results in a shift in $c_{\text{lep}}^{*}$ of 0.002. We are immune from this effect because we fit for tagging category dependent biases in the resolution function directly on data. Nonetheless, the impact of a possible mismeasurement of $\Delta t$ has been estimated by varying the assumptions on the resolution function, the position of the beam spot, the absolute $z$ scale, the internal alignment of the vertex detector, and quality criteria on the reconstructed vertex. The corresponding error on $a^{(*)}$ is $\sigma_a = 0.015$, while that on $c^{(*)}$ is $\sigma_c = 0.026$. The systematic uncertainties on the fit technique ($\sigma_a = 0.013$, $\sigma_c = 0.020$) include the upper limit on the fit bias estimated from samples of fully simulated events, the uncertainty on the $B^0$ lifetime and $\Delta m_D$ [13], and the impact of neglecting higher order terms in $r^{(*)}$ or $r_1^{(*)}$ in Eq. (3). As a cross check, we performed the same fits on samples of 18233 $B^+ \rightarrow D^{(*)+}\pi^-$ and 17400 $B^0 \rightarrow J/\psi K^{*0}$ candidates, where we find no significant CP asymmetries, as expected. The systematic uncertainties in tagging ($\sigma_a = 0.004$, $\sigma_c = 0.003$) are estimated allowing for different tagging efficiencies between $B^0$ and $\bar{B}^0$ and for different $\Delta t$ resolutions for correctly and incorrectly tagged events. We also account for uncertainties on the background ($\sigma_a = 0.001$, $\sigma_c = 0.003$) by varying the effective lifetimes, dilutions, $m_{\text{ES}}$ shape parameters and signal fractions, and background CP asymmetry up to 5 times the expected CP asymmetry for signal.

The results can be interpreted in terms of $\sin(2\beta + \gamma)$ [Eq. (4)] if the decay amplitude ratios $r^{(*)}$, expected to be $|V_{ub}V_{cd}/V_{ud}V_{cb}| \approx 0.02$, are known. Such small amplitude ratios cannot be determined from $B^0 \rightarrow D^{(*)-}\pi^-$ events directly, because the current data sample is too small. We estimate $r^{(*)}$ using the SU(3) symmetry relation $r^{(*)} = \tan \theta_C \sqrt{B(B^0 \rightarrow D_{s}^{(*)+}\pi^-)/B(B^0 \rightarrow D_{s}^{(*)-}\pi^-)}(f_{D^{(*)}}/f_{D^{*0}})$ [5]. From the measurements of the Cabibbo angle $\tan \theta_C = 0.2250 \pm 0.0027$ [13], the branching fractions $B(B^0 \rightarrow D \pi^-) = (0.30 \pm 0.04\%)$ [13], $B(B^0 \rightarrow D^{+}\pi^-) = (0.276 \pm 0.021\%)$ [13], $B(B^0 \rightarrow D^+_s \pi^-) = (2.7_{-0.6}^{+0.7} \pm 0.8) \times 10^{-5}$ [14], $B(B^0 \rightarrow D^{*+}_s \pi^-) = (1.9_{-0.3}^{+0.6} \pm 0.5) \times 10^{-5}$ [14], and from calculations of the decay constant ratios $f_{D^+_s}/f_{D^0} = 1.11 \pm 0.01$ and $f_{D^{*+}_s}/f_{D^{*0}} = 1.10 \pm 0.02$ [15] we obtain

$$r = 0.019 \pm 0.004, \quad r^{(*)} = 0.017^{+0.005}_{-0.007}.$$  

To obtain $\sin(2\beta + \gamma)$, we minimize the $\chi^2$

$$\chi^2(2\beta + \gamma, \delta^{(*)}, r^{(*)}) = \sum_i (\tilde{x}_i - x_i)/\sigma_i^2 + \Delta(r^{(*)}),$$  

where $x_i = a, a^{*}, c_{\text{lep}}^{*}$ are functions of the physics parameters [Eq. (4)], and $\tilde{x}_i$ are the corresponding measured values. $\Delta(r^{(*)})$ is a continuous function that is set equal to 0 within 30% of the estimated $r^{(*)}$ [Eq. (6)], and is an offset quadratic outside this range, with the errors in Eq. (6). The additional 30% error attributed on $r^{(*)}$ is due to the unknown theoretical uncertainty on the validity of the SU(3) symmetry assumption and to neglecting $W$-exchange contributions to $A(B^0 \rightarrow D^{(*)+}\pi^-)$). This error estimate is consistent with the spread in $r^{(*)}$ obtained using a variety of theoretical models [16]. The $\sigma_j$ are the quadratic sums of the statistical and systematic uncertainties in Eq. (5). Correlations between the $\tilde{x}_i$, at most 28%, have negligible influence on the results of this analysis. The simultaneous analysis of two $B$ decay modes allows one to extract $|\sin(2\beta + \gamma)|$.

Figure 3 shows the minimum $\chi^2$ for each value of $|\sin(2\beta + \gamma)|$. The absolute minimum occurs for $|\sin(2\beta + \gamma)| = 0.98$, where $\chi_{\text{min}}^2/d.o.f. = 0.44/1$. The values of $r^{(*)}$ that minimize the $\chi^2$ are consistent with the input values within their statistical errors. Because of the large uncertainties on the fit parameters and their limited physical range, the $\chi^2$ curve is nonparabolic. Thus to obtain a probabilistic interpretation to the results, we consider, for each of many values of $\sin(2\beta + \gamma)$, a large number of simulated experiments with the same characteristics as the data. We compute the consistency of the data with a given value of $\sin(2\beta + \gamma)$ by counting the fraction of simulated experiments for which $\chi^2(\sin(2\beta + \gamma)) - \chi_{\text{min}}^2$ is smaller than it is in the data. This fraction, the frequentist confidence level, is shown in the lower portion of Fig. 3, from which we read that $|\sin(2\beta + \gamma)| > 0.69$ at 68% C.L. We exclude the hypothesis of no CP violation $|\sin(2\beta + \gamma)| = 0$ at 83% confidence level. In order to study the impact of the assumed theoretical error on $r^{(*)}$, we doubled it to 60% at

![FIG. 3.](image-url)
and we found that the lower limit on $|\sin(2\beta + \gamma)|$ at 68% C.L. drops from 0.69 to 0.60.

In conclusion, we studied the time-dependent $CP$-violating asymmetries in fully reconstructed $B^0 \rightarrow D^{(*)\pm} \pi^\mp$ decays, and measured the $CP$-violating parameters listed in Eq. (5). With some theoretical assumptions, we interpret the result in terms of $|\sin(2\beta + \gamma)|$ and we find that $|\sin(2\beta + \gamma)| > 0.69$ at 68% C.L. and that $\sin(2\beta + \gamma) = 0$ is excluded at 83% C.L.

We are grateful for the excellent luminosity and machine conditions provided by our PEP-II colleagues, and for the substantial dedicated effort from the computing organizations that support BABAR. The collaborating institutions wish to thank SLAC for its support and kind hospitality. This work is supported by DOE and NSF (USA), NSERC (Canada), IHEP (China), CEA and CNRS-IN2P3 (France), BMBF and DFG (Germany), INFN (Italy), FOM (The Netherlands), NFR (Norway), MIST (Russia), and PPARC (United Kingdom). Individuals have received support from the A. P. Sloan Foundation, Research Corporation, and Alexander von Humboldt Foundation.

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[4] Charge conjugation is implied in this Letter, unless otherwise stated. The superscript (*) indicates that a symbol must be considered both with and without the suffix.


[7] According to Ref. [6], the strong phase for $B^0 \rightarrow D^{(*)\pm} \pi^\mp$ is $\delta^* + \pi$.


