Search for Lepton-Flavor and Lepton-Number Violation in the Decay $\tau^- \to \ell^- h^+ h^-$

A search for lepton-flavor and lepton-number violation in the decay of the tau lepton into one charged lepton and two charged hadrons is performed using 221.4 fb⁻¹ of data collected at an $e^+e^-$ center-of-mass energy of 10.58 GeV with the BABAR detector at the SLAC PEP-II storage ring. In all 14 decay modes considered, the observed data are compatible with background expectations, and upper limits are set in the range $B(\tau \rightarrow \ell hh') < (0.7 - 4.8) \times 10^{-7}$ at 90% confidence level.

**TABLE I.** Efficiency estimates, the number of expected background events ($N_{bgd}$) in the signal region (with total uncertainties), the number of observed events ($N_{obs}$) in the signal region, and the 90% C.L. upper limit on the branching fraction for each decay mode.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Efficiency [%]</th>
<th>$N_{bgd}$</th>
<th>$N_{obs}$</th>
<th>UL at 90% C.L.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+K^-K^-$</td>
<td>3.77 ± 0.16</td>
<td>0.22 ± 0.06</td>
<td>0</td>
<td>$1.4 \times 10^{-7}$</td>
</tr>
<tr>
<td>$e^+K^+\pi^-$</td>
<td>3.08 ± 0.13</td>
<td>0.32 ± 0.08</td>
<td>0</td>
<td>$1.7 \times 10^{-7}$</td>
</tr>
<tr>
<td>$e^-\pi^+K^-$</td>
<td>3.10 ± 0.13</td>
<td>0.14 ± 0.06</td>
<td>1</td>
<td>$3.2 \times 10^{-7}$</td>
</tr>
<tr>
<td>$e^-\pi^+\pi^-$</td>
<td>3.30 ± 0.15</td>
<td>0.81 ± 0.13</td>
<td>0</td>
<td>$1.2 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\mu^-K^-K^-$</td>
<td>2.16 ± 0.12</td>
<td>0.24 ± 0.07</td>
<td>0</td>
<td>$2.5 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\mu^-K^-\pi^-$</td>
<td>2.97 ± 0.16</td>
<td>1.67 ± 0.29</td>
<td>2</td>
<td>$3.2 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\mu^-\pi^+K^-$</td>
<td>2.87 ± 0.16</td>
<td>1.04 ± 0.18</td>
<td>1</td>
<td>$2.6 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\mu^-\pi^+\pi^-$</td>
<td>3.40 ± 0.19</td>
<td>2.99 ± 0.41</td>
<td>3</td>
<td>$2.9 \times 10^{-7}$</td>
</tr>
<tr>
<td>$e^+K^-K^-$</td>
<td>3.85 ± 0.16</td>
<td>0.04 ± 0.04</td>
<td>0</td>
<td>$1.5 \times 10^{-7}$</td>
</tr>
<tr>
<td>$e^+K^-\pi^-$</td>
<td>3.19 ± 0.14</td>
<td>0.16 ± 0.06</td>
<td>0</td>
<td>$1.8 \times 10^{-7}$</td>
</tr>
<tr>
<td>$e^+\pi^+\pi^-$</td>
<td>3.40 ± 0.15</td>
<td>0.41 ± 0.10</td>
<td>1</td>
<td>$2.7 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\mu^-K^-K^-$</td>
<td>2.06 ± 0.11</td>
<td>0.07 ± 0.10</td>
<td>1</td>
<td>$4.8 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\mu^-K^-\pi^-$</td>
<td>2.85 ± 0.16</td>
<td>1.54 ± 0.25</td>
<td>1</td>
<td>$2.2 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\mu^-\pi^+\pi^-$</td>
<td>3.30 ± 0.18</td>
<td>1.46 ± 0.27</td>
<td>0</td>
<td>$0.7 \times 10^{-7}$</td>
</tr>
</tbody>
</table>
One of the charged particles found in the 3-prong hemisphere must be identified as either an electron or muon candidate. Electrons are identified using the ratio of observed EMC energy to track momentum $(E/p)$, the shape of the shower in the EMC, and the ionization loss in the tracking system $(dE/dx)$. Muons are identified by hits in the IFR and small energy deposits in the EMC. Each of the other two charged particles found in the 3-prong hemisphere must be identified as either a pion or a kaon, using information from the DIRC and $dE/dx$.

After event topology and particle identification requirements, there are significant backgrounds from light quark $q\bar{q}$ production and SM $\tau\tau$ events (without LFV), as well as small contributions from Bhabha, $\mu^+\mu^-$, and two-photon production of four charged particles. Additional selection criteria, largely the same for all 14 signal channels, are applied as follows. No photon candidates, identified as EMC energy deposits unassociated to a track, with $E_\gamma > 100$ MeV are allowed. This restriction removes $q\bar{q}$ backgrounds and SM $\tau\tau$ events. The total transverse momentum of the event in the c.m. frame must be greater than 0.2 GeV/c, while the polar angle of the missing momentum in the laboratory frame is required to be in the range $[0.25, 2.4]$ radians. These two requirements are effective at reducing two-photon and Bhabha backgrounds. The mass of the 1-prong hemisphere calculated from the four-momentum of the track in the 1-prong hemisphere and the missing momentum in the event, is required to be in the range $[0.6, 1.9]$ GeV/$c^2$ for $e\mu\mu$ candidates and $[0.8, 1.9]$ GeV/$c^2$ for $\mu\mu\mu$ candidates. The 1-prong mass requirement is particularly effective at removing $q\bar{q}$ backgrounds as well as the remaining two-photon contribution. To reduce Bhabha backgrounds, the momentum of the 1-prong track in the c.m. frame is required to be less than 4.5 GeV/c for the $e\pi\pi$ candidates. In addition, particle identification vetoes are applied to selected production channels. For all decay modes, lepton and pion candidates must not pass the kaon identification as well. For the $e\mu\mu$ decay modes, except for $eKK$, the 1-prong track must not be identified as an electron. This requirement is useful to reduce possible contamination from Bhabhas.

To further reduce backgrounds, candidate signal events are required to have an invariant mass and total energy in the 3-prong hemisphere consistent with the neutrinoless decay of a tau lepton. These quantities are calculated from the observed track momenta assuming the corresponding lepton and hadron masses for each decay mode. The mass difference and energy difference are defined as $\Delta M = m_{rec} - m_\tau$ and $\Delta E = E_{rec} - E_{beam}$, where $m_{rec}$ is the reconstructed 3-prong invariant mass, $m_\tau = 1.777$ GeV/$c^2$ is the tau mass [14], $E_{rec}$ is the reconstructed 3-prong total energy in the c.m. frame, and $E_{beam}$ is the c.m. beam energy. Rectangular signal regions are defined separately for each decay mode in the $(\Delta M, \Delta E)$ plane. For the $\mu\mu\mu$ modes, $\Delta M$ is required to be in the range $[-20, +20]$ MeV/$c^2$, while for the $e\mu\mu$ modes the range is $[-30, +20]$ MeV/$c^2$ to account for radiative losses. For all 14 decay modes, $\Delta E$ must be in the range $[-100, +50]$ MeV.

These signal region boundaries are optimized to provide the smallest expected upper limits on the branching fractions in the background-only hypothesis. These expected upper limits are estimated using only MC simulations, not candidate events in data. To avoid bias, a blind analysis procedure was adopted with the number of data events in the signal region remaining unknown until the selection criteria were finalized and all systematic studies had been performed. Figure 1 shows the observed data for all 14 selection channels, along with the signal region boundaries and the expected signal distributions.

The dominant remaining backgrounds are low multiplicity $q\bar{q}$ events and SM $\tau\tau$ events. These background classes have unique distributions in the $(\Delta M, \Delta E)$ plane: $q\bar{q}$ events populate the plane uniformly, while $\tau\tau$ backgrounds are restricted to negative values of both $\Delta M$ and $\Delta E$. Backgrounds from Bhabha, $\mu\mu$, and two-photon

![Bar graph](image)

**FIG. 1.** Observed data shown as dots in the $(\Delta M, \Delta E)$ plane and the boundaries of the signal region for each decay mode. The dark and light shading indicates contours containing 50% and 90% of the selected MC signal events, respectively.
events are found to be negligible. For each background class, a probability density function (PDF) describing the shape of the background distribution in the ($\Delta M$, $\Delta E$) plane is determined by fitting an analytic function to the Monte Carlo prediction as described in more detail below. These PDFs are then combined with normalization coefficients determined from an unbinned maximum likelihood fit to the observed data in the ($\Delta M$, $\Delta E$) plane in a sideband (SB) region. The resulting function describes the event rate observed in the SB region and is used to predict the expected background rate in the signal region. The SB region is defined as the rectangle, excluding the signal region, bounding $\Delta M$ in the range $[-0.7, +0.4]$ GeV/$c^2$ for $ehh'$ final states and $[-0.4, +0.4]$ GeV/$c^2$ for $\mu hh'$ final states, while $\Delta E$ must be in the range $[-0.7, +0.4]$ GeV. The PDF shape determinations and SB fits are performed separately for each of the 14 decay modes.

For the $q\bar{q}$ backgrounds, a PDF is constructed from the product of two functions $P_M$ and $P_E$, where the coordinates ($\Delta M$, $\Delta E$) have been rotated slightly from ($\Delta M$, $\Delta E$) to better fit the expected distributions. The function $P_M(\Delta M')$ is a Gaussian and the function $P_E(\Delta E') = (1-x/\sqrt{1+x^2})(1 + a_1 x + a_2 x^2 + a_3 x^3)$ where $x = (\Delta E' - a_3)/a_5$ and $a_i$ are fit parameters. The resulting $q\bar{q}$ PDF is described by eight fit parameters, including the rotation angle, which are determined by fits to MC $q\bar{q}$ background samples for each decay mode. For the $\tau\tau$ PDF, the function $P_M(\Delta M')$ is the sum of two Gaussians with different widths above and below the peak, while the functional form of $P_E(\Delta E')$ is the same as the $q\bar{q}$ PDF above. To properly model the wedge-shaped kinematic limit in tau decays, a coordinate transformation of the form $\Delta M' = \cos \beta_1 \Delta M + \sin \beta_1 \Delta E$ and $\Delta E = \cos \beta_2 \Delta E - \sin \beta_2 \Delta M$ is performed. In total there are 12 free parameters describing this PDF, and all are determined by fits to MC $\tau\tau$ samples.

With the shapes of the two background PDFs determined, an unbinned maximum likelihood fit to the data in the SB region is used to find the expected rate of each background type in the signal region. Extensive MC studies show that these PDF functions adequately describe the predicted background shapes near the signal regions. The accuracy of these predictions is verified by comparing to data in regions neighboring the signal region in the ($\Delta M$, $\Delta E$) plane where no signal is expected. Expected backgrounds are shown in Table I, and an example of the background prediction compared to the observed data is shown in Fig. 2.

The efficiency of the selection for signal events is estimated with a MC simulation of neutrinoless tau decays. About 40% of the MC signal events pass the initial 1–3 topology requirement, and 20% to 70% of these preselected events pass the particle identification (PID) criteria, depending upon the signal mode. The final efficiency for signal events to be found in the signal region after all requirements is shown in Table I for each decay mode and ranges from 2.1% to 3.8%. This efficiency includes the 85% branching fraction for 1-prong tau decays [11].

The PID selection efficiencies and misidentification rates are measured directly using tracks in kinematically-selected data control samples. These values are parameterized as a function of particle momentum, charge, polar angle, and azimuthal angle in the laboratory frame. The lepton-identification criteria have been designed to give very low misidentification rates at the expense of some efficiency loss. The electron ID is expected to be 81% efficient in signal $ehh'$ events, with a mis-ID rate of 0.1% for pions and 0.2% for kaons in generic $\tau\tau$ events. The muon ID is 44% efficient for $\mu hh'$ signal events, with a mis-ID rate of 1.0% for pions and 0.4% for kaons. The hadronic identification is designed to classify the hadronic candidates as pions or kaons, but is not intended to distinguish hadrons from leptons. The pion ID is 92% efficient with a mis-ID rate of 12% for kaons, while the kaon ID is 81% efficient with a 1.4% mis-ID rate for pions.

The largest systematic uncertainty for the signal efficiency is the uncertainty in measuring particle ID efficiencies. This uncertainty (all uncertainties quoted are relative) is dominated by the statistical precision of the PID control samples, and ranges from 0.7% for $e^{-}\pi^{+}\pi^{-}$ to 3.8% for $\mu^{+}K^{+}K^{-}$. The modeling of the tracking efficiency contributes an uncertainty of 2.5%, while the restriction on extra photons leads to an additional uncertainty of 2.4%. All other sources of uncertainty are found to be small, including the modeling of radiative effects, track momentum resolution, trigger performance, observables used in the selection criteria, and knowledge of the tau 1-prong branching fractions. No uncertainty is assigned for possible model dependence of the signal decay. The selection efficiency is found to be uniform within 20% across the Dalitz plane, provided the invariant mass for any pair of particles is less than 1.4 GeV/$c^2$.

Since the background levels are extracted directly from the data, systematic uncertainties on the background esti-
mation are directly related to the background normalization, parametrization, and the fit technique used. The finite data available in the SB region used to determine the background rates dominates the background uncertainty. Additional uncertainties of 10% are estimated by varying the fit procedure and changing the functional form of the background PDFs. The uncertainty on the branching fraction of SM tau decays with one or two kaons is also evaluated, and contributes less than 15% for all final states.

The numbers of events observed ($N_{\text{obs}}$) and the background expectations ($N_{\text{bgd}}$) are shown in Table I, with no significant excess observed. Upper limits on the branching fractions are calculated according to $B_{90}^{UL} = N_{90}^{UL}/(2e\mathcal{L}\sigma_{\tau\tau})$, where $N_{90}^{UL}$ is the 90% C.L. upper limit for the number of signal events when $N_{\text{obs}}$ events are observed with $N_{\text{bgd}}$ background events expected. The quantities $\varepsilon$, $\mathcal{L}$, and $\sigma_{\tau\tau}$ are the selection efficiency, luminosity, and $\tau^+\tau^-$ cross section, respectively. The branching fraction upper limits are calculated including all uncertainties using the technique of Cousins and Highland [15] following the implementation of Barlow [16]. The estimates of $\mathcal{L}$ and $\sigma_{\tau\tau}$ are correlated [17], and the uncertainty on the product $\mathcal{L}\sigma_{\tau\tau}$ is 2.3%. The 90% C.L. upper limits on the $\tau \to \ell hh'$ branching fractions, shown in Table I, are in the range $(0.7-4.8) \times 10^{-7}$. These limits represent an order of magnitude improvement over the previous experimental bounds [7].

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‡Deceased.

[6] Throughout this Letter, charge conjugate decay modes are also implied.
[17] The luminosity is measured using the observed $\mu^+\mu^-$ rate, and the $\mu^+\mu^-$ and $\tau^+\tau^-$ cross sections are both estimated with KK2F.