Measurement of the Cabibbo-Kobayashi-Maskawa Angle $\gamma$ in $B^+ \rightarrow D^{(*)} K^{+}$ Decays with a Dalitz Analysis of $D \rightarrow K^0_S \pi^- \pi^+$

We report on a measurement of the Cabibbo-Kobayashi-Maskawa CP-violating phase $\gamma$ through a Dalitz analysis of neutral $D$ decays to $K_0^{*0}\pi^-\pi^+$ in the processes $B^- \rightarrow D^{(*)}K^-$, $D^{*} \rightarrow D^{0}\pi^0$, $D^{0}\gamma$. Using a sample of $227 \times 10^6$ $BB$ pairs collected by the BABAR detector, we measure the amplitude ratios $r_B = 0.12 \pm 0.08 \pm 0.03$ and $r_B' = 0.17 \pm 0.10 \pm 0.03$, the relative strong phases $\delta_B = (104 \pm 45 \pm 17 \pm 16)^\circ$ and $\delta_B' = (-64 \pm 41 \pm 14 \pm 15)^\circ$ between the amplitudes $\mathcal{A}(B^- \rightarrow D^{(*)}K^-)$ and $\mathcal{A}(B^- \rightarrow D^{(*)}K^-)$. As a consequence of parity and angular momentum conservation in the $\bar{D}^0\pi^0$ decay, the factor $\kappa$ takes the value $+1$ for $B^- \rightarrow D^0K^-$ and $B^- \rightarrow \bar{D}^{(*)}(\bar{D}^0\pi^0)K^-$, and $-1$ for $B^- \rightarrow \bar{D}^{0}(\bar{D}^0\gamma)K^-$. We first determine $\mathcal{A}_D(m_2^2, m_3^2)$ through a Dalitz analysis of a high-statistics sample of tagged $D^0$ mesons from inclusive $D^{*+} \rightarrow D^0\pi^+$ decays reconstructed in data. We then perform a simultaneous fit to the $|\mathcal{A}^{(*)}(m_2^2, m_3^2)|^2$ and $|\mathcal{A}^{(*)}(m_2^2, m_3^2)|^2$ distributions for the $B^- \rightarrow \bar{D}^{(*)}K^-$ samples to determine the CP parameters $r_B^{(*)}$, $\delta_B^{(*)}$, and $\gamma$. We emphasize that in this analysis the Dalitz amplitude is only a means to extract the CP parameters.

$B^-$ candidates are formed by combining a mass-constrained $D^{(*)}$ candidate with a track identified as a kaon [9]. We accept $K_S^0 \rightarrow \pi^+\pi^-$ candidates that have a two-pion invariant mass within 9 MeV/$c^2$ of the $K_S^0$ mass [4] and a cosine of the angle between the line connecting the $D^0$ and $K_S^0$ decay vertices and the $K_S^0$ momentum (in the plane transverse to the beam) greater than 0.99. $D^0$ candidates are selected by requiring the $K_S^0\pi^+\pi^-$ invariant mass to be within 12 MeV/$c^2$ of the $D^0$ mass [4]. The $\pi^0$ candidates from $D^{(*)} \rightarrow D^0\pi^0$ are formed from pairs of photons with invariant mass in the range $[115, 150]$ MeV/$c^2$, and with photon energy greater than 30 MeV. Photon candidates from $D^{(*)} \rightarrow D^0\gamma$ are selected if their energy and $\mathcal{A}(B^- \rightarrow \bar{D}^{(*)}K^-)$. The curves represent the fit projections for signal plus background (solid lines) and background (dotted lines). The peaking structure of the background is due to remaining $B^- \rightarrow \bar{D}^{(*)}\pi^-$ events.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1}
\caption{(color online). Distributions of $m_{ES}$ for (a) $B^- \rightarrow \bar{D}^0K^-$, (b) $B^- \rightarrow \bar{D}^{0}(\bar{D}^0\pi^0)K^-$, and (c) $B^- \rightarrow \bar{D}^{0}(\bar{D}^0\gamma)K^-$.}
\end{figure}
is greater than 100 MeV. $D^{*0} \rightarrow D^0 \pi^0(D^0 \gamma)$ candidates are required to have a $D^{*0} \rightarrow D^0$ mass difference within 2.5(10) MeV/$c^2$ of its nominal value [4].

The beam-energy substituted $B$ mass $m_{ES}$ [12] (Fig. 1) and the difference $\Delta E$ between the reconstructed energy of the $B^-$ candidate and the beam energy in the $e^+ e^-$ c.m. frame are used to identify signal $B^-$ decays. We require $m_{ES} > 5.2$ GeV/$c^2$ and $|\Delta E| < 30$ MeV. Since the background is dominated by random combinations of tracks arising from $e^+ e^- \rightarrow q\bar{q}$, $q = [u, d, s, c]$ (continuum) events, we require $|\cos \theta_0^*| < 0.8$, where $\theta_0^*$ is the c.m. angle between the thrust axis of the $B^-$ candidate and that of the remaining particles in the event. The reconstruction efficiencies (purities in the signal region $m_{ES} > 5.272$ GeV/$c^2$) are 18% (63%), 5.9% (86%), and 81.1% (52%) for the $B^- \rightarrow \bar{D}^0 K^-$, $B^- \rightarrow \bar{D}^{*0}(\bar{D}^0 \pi^0)K^-$, and $B^- \rightarrow \bar{D}^{*0}(\bar{D}^0 \gamma)K^-$ decay modes, respectively. The cross feed among the different samples is negligible.

The $D^0$ decay amplitude is determined from an unbinned maximum-likelihood Dalitz fit to a high-purity (97%) sample of 81 496 $D^{*+} \rightarrow D^0 \pi^+$ decays reconstructed in 91.5 fb$^{-1}$ of data (Fig. 2). We use the isobar formalism described in Ref. [13] to express $\mathcal{A}_D$ as a sum of two-body decay-matrix elements (subscript $r$) and a nonresonant (subscript NR) contribution,

$$\mathcal{A}_D(m_1^2, m_2^2) = \sum_r a_r e^{i\phi_r} \mathcal{A}_r(m_1^2, m_2^2) + a_{NR} e^{i\phi_{NR}},$$

where each term is parameterized with an amplitude $a_r$ and a phase $\phi_r$. The function $\mathcal{A}_r(m_1^2, m_2^2)$ is the Lorentz-invariant expression for the matrix element of a $D^0$ meson decaying into $K^0_S \pi^- \pi^+$ through an intermediate resonance $r$, parameterized as a function of the position in the Dalitz plane.

Table I summarizes the values of $a_r$ and $\phi_r$ obtained using a model consisting of 16 two-body elements comprising 13 distinct resonances and accounting for efficiency variations across the Dalitz plane and the small background contribution. For $r = \rho(770), \rho(1450)$ we use the functional form suggested in Ref. [14], while the remaining resonances are parameterized by a spin-dependent relativistic Breit-Wigner distribution. For intermediate states with a $K^+$, the regions of interference between DCS and CA decays are particularly sensitive to $\gamma$, and we include the DCS component when a significant contribution is expected. In addition, we find that the inclusion of the scalar $\pi\pi$ resonances $\sigma$ and $\sigma'$ significantly improves the quality of the fit [15]. Since the two $\sigma$ resonances are not well established and are only introduced to improve the description of our data, the uncertainty on their existence is considered in the systematic errors. We estimate the goodness of fit through a two-dimensional $\chi^2$ test and obtain $\chi^2 = 3824$ for 3054–32 degrees of freedom.

We simultaneously fit the $B^- \rightarrow \bar{D}^{*0} K^-$ samples using an unbinned extended maximum-likelihood fit to extract the CP-violating parameters along with the signal and background yields. Three different background compo-

![FIG. 2 (color online).](image-url)
nents are considered: continuum events, $B^- \rightarrow D^{(*)} \pi^-$ and $Y(4S) \rightarrow BB$ (other than $B^- \rightarrow D^{(*)} \pi^-$) decays. In addition to $m_{ES}$, the fit uses $\Delta E$ and a Fisher discriminant [12] to distinguish signal from $B^- \rightarrow D^{(*)} \pi^-$ and continuum background, respectively. The log-likelihood is

$$\ln L = -\sum_c N_c + \sum_j \ln \left[ \sum_c N_c \mathcal{P}_c(\xi_j) \mathcal{P}^{Dalitz}_c(\bar{\eta}_j) \right],$$

where $\xi_j = \{m_{ES}, \Delta E, \mathcal{F}\}$ and $\bar{\eta}_j = (m_e^2, m_\pi^2)$, characterize the event $j$. Here, $\mathcal{P}_c(\xi)$ and $\mathcal{P}^{Dalitz}_c(\bar{\eta})$ are the probability density functions (PDF’s), and $N_c$ the event yield for signal or background component $c$. For signal events, $\mathcal{P}^{Dalitz}_c(\bar{\eta})$ is given by $|A_c^0(\bar{\eta})|^2$ corrected by the efficiency variations. All PDF shape parameters used to describe signal, continuum, and $B^- \rightarrow D^{(*)} \pi^-$ components are determined directly from $B^- \rightarrow D^{(*)} K^-$ and $B^- \rightarrow D^{(*)} \pi^-$ signal, sideband regions, and off-peak data, and are fixed in the final fit for $CP$ parameters and event yields. Only the $m_{ES}$, $\Delta E$, and Dalitz PDF’s for $BB$ background events are determined from a detailed Monte Carlo simulation. The results for the $CP$ fit yields $282 \pm 20, 90 \pm 11$, and $44 \pm 8$ signal $D^{0}K^-$, $D^{(*)} (D^{0} \pi^0) K^-$, and $D^{(*)} (D^{0} \gamma) K^-$ candidates, respectively, consistent with expectations based on measured branching fractions and efficiencies estimated from Monte Carlo simulation. The results for the $CP$-violating parameters $z^{(e)}_\pm = (x^{(e)}_\pm, y^{(e)}_\pm)$, where $x^{(e)}_\pm$ and $y^{(e)}_\pm$ are defined as the real and imaginary parts of the complex amplitude ratios $r^{(e)}_B \equiv e^{i(\delta^{(e)}_B + \gamma)}$, respectively, are summarized in Table II. Here, $r^{(e)}_B$ is the amplitude ratio between the amplitudes $b \rightarrow u$ and $b \rightarrow c$, separately for $B^+$ and $B^-$. The only nonzero statistical correlations involving the $CP$ parameters are for the pairs $z_\pm$, $z_\mp$, $z^*_\pm$, and $z^*_\mp$, which amount to 3%, 6%, $-17\%$, and $-27\%$, respectively. The $z^{(e)}_\pm$ variables are more suitable fit parameters than $r^{(e)}_B$, $\delta^{(e)}_B$, and $\gamma$ because they are better behaved near the origin, especially in low-statistics samples. Figures 3(a) and 3(b) show the one- and two-standard deviation confidence-level contours (statistical only) in the $z^{(e)}_\pm$ planes for $D^0 K^-$ and $D^{(*)} K^-$, and separately for $B^-$ and $B^+$. The separation between the $B^-$ and $B^+$ regions in these planes is an indication of direct $CP$ violation.

The largest single contribution to the systematic uncertainties in the $CP$ parameters comes from the choice of the Dalitz model used to describe the $D^0 \rightarrow K^0_s \pi^+ \pi^-$ decay amplitudes. To evaluate this uncertainty we use the nominal Dalitz model (Table I) to generate large samples of pseudoexperiments. We then compare experiment by experiment the values of $z^{(e)}_\pm$ obtained from fits using the nominal model and a set of alternative models. We find that removing different combinations of $K^*$ and $\rho$ resonances (with low fit fractions), or changing the functional form of the resonance shapes, has little effect on the total $\chi^2$ of the fit, or on the values of $z^{(e)}_\pm$. However, models where one or both of the $\sigma$ and $\rho$ resonances are removed lead to a significant increase in the $\chi^2$ of the fit. We use the average variations of $z^{(e)}_\pm$ corresponding to this second set of alternative models as the systematic uncertainty due to imperfect knowledge of $A_D$.

The experimental systematic uncertainties include the errors on the $m_{ES}$, $\Delta E$, and $\mathcal{F}$ PDF parameters for signal and background, the uncertainties in the knowledge of the Dalitz distribution of background events, the efficiency variations across the Dalitz plane, and the uncertainty in the fraction of events with a real $D^0$ produced in a back-to-
back configuration with a negatively charged kaon. Less significant systematic uncertainties originate from the imprecise knowledge of the fraction of real $D^0$'s, the invariant mass resolution, and the statistical errors in the Dalitz amplitudes and phases from the fit to the tagged $D^0$ sample. The possible effect of $CP$ violation in $B^- \to \bar{D}^{(*)0} \pi^-$ decays and $B\bar{B}$ background was found to be negligible.

A frequentist (Neyman) construction of the confidence regions of $p = (r_B, r_{B'}^\epsilon, \delta_B, \delta_{B'}^\epsilon, \gamma)$ based on the constraints on $Z^{(\epsilon)}_\gamma$ has been adopted [4]. Using a large number of pseudoexperiments corresponding to the nominal $CP$ fit model but with many different values of the $CP$ fit parameters, we construct an analytical (Gaussian) parameterization of the PDF of $Z^{(\epsilon)}_\gamma$ as a function of $p$. For a given $p$, the five-dimensional confidence level $C = 1-\alpha$ is calculated by integrating over all points in the fit parameter space closer (larger PDF) to $p$ than the fitted data values. The one- (two-)standard deviation region of the $CP$ parameters is defined as the set of $p$ values for which $\alpha$ is smaller than 3.7% (45.1%).

Figures 3(c) and 3(d) show the two-dimensional projections in the $r_{B'}^\epsilon-\gamma$ planes, including systematic uncertainties, for $\bar{D}^{(*)0}K^-$ and $\bar{D}^{(*)0}K^-$. The figures show that this Dalitz analysis has a twofold ambiguity, $(\gamma, \delta_{B'}^\epsilon) \to (\gamma + 180^\circ, \delta_{B'}^\epsilon + 180^\circ)$. The significance of direct $CP$ violation, obtained by evaluating $C$ for the most probable $CP$ conserving point, corresponds to 1.6, 2.1, and 2.4 standard deviations, for $\bar{D}^{(*)0}K^-$ and $\bar{D}^{(*)0}K^-$, and their combination, respectively. Similar results are obtained using a Bayesian technique with uniform a priori probability distributions for $r_{B'}^\epsilon$, $\delta_{B'}^\epsilon$, and $\gamma$.

In summary, we have measured the direct $CP$-violating parameters in $B^- \to \bar{D}^{(*)0}K^-$ using a Dalitz analysis of $\bar{D}^{(*)} \to K^0\pi^-\pi^+$ decays, obtaining $r_B = 0.12 \pm 0.08 \pm 0.03 \pm 0.04 [0,0.28]$, $r_{B'} = 0.17 \pm 0.10 \pm 0.03 \pm 0.03 [0,0.35]$, $\delta_B = (104 \pm 45_{-21}^{+17_{-14}^{+24}})^\circ$, $\delta_{B'} = (64 \pm 41_{-12}^{+14_{-15}^{+31}})^\circ$, and $\gamma = (70 \pm 31_{-10}^{+23_{-13}^{+14}})^{\circ,\circ,\circ}$ [129$^\circ$, 137$^\circ$]. The first error is statistical, the second is the experimental systematic uncertainty, and the third reflects the Dalitz model uncertainty. The values inside square brackets indicate the two-standard deviation intervals. The results for $\gamma$ from $B^- \to \bar{D}^{(*)0}K^-$ and $B^- \to \bar{D}^{(*)0}K^-$ alone are $(70 \pm 38)^\circ$ and $(71 \pm 35)^\circ$, respectively (statistical errors only). The constraint on $\gamma$ is consistent with that reported by the Belle Collaboration [8], which has a slightly better statistical precision since our $r_B$ constraint favors smaller values.

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[5] Reference to the charge-conjugate state is implied here and throughout the text unless otherwise stated.
[15] The $\sigma$ and $\sigma'$ masses and widths are determined from the data. We find (in MeV/$c^2$) $M_{\sigma} = 484 \pm 9$, $\Gamma_{\sigma} = 383 \pm 14$, $M_{\sigma'} = 1014 \pm 7$, and $\Gamma_{\sigma'} = 88 \pm 13$. Errors are statistical.