Measurements of Branching Fraction, Polarization, and Charge Asymmetry of $B^{\pm} \to \rho^{\pm} \rho^{0}$ and a Search for $B^{\pm} \to \rho^{\pm} f_0(980)$

We measure the branching fraction \( (B) \), polarization \( (f_L) \), and CP asymmetry \( (A_{CP}) \) of \( B^+ \rightarrow \rho^+ \rho^0 \) decays and search for the decay \( B^+ \rightarrow \rho^+ f_0(980) \) based on a data sample of \( 2.318 \times 10^6 \ Y(4S) \rightarrow BB \) decays collected with the BABAR detector at the SLAC PEP-II asymmetric-energy \( B \) factory. In \( B^+ \rightarrow \rho^+ \rho^0 \) decays we measure \( B = (16.8 \pm 2.2 \pm 2.3) \times 10^{-6} \), \( f_L = 0.905 \pm 0.042 \pm 0.025 \), and \( A_{CP} = -0.12 \pm 0.13 \pm 0.10 \), and find an upper limit on the branching fraction of \( B^+ \rightarrow \rho^+ f_0(980)(\rightarrow \pi^+ \pi^-) \) decays of 1.9 \( \times 10^{-6} \) at 90% confidence level.

The measurement of the CP-violating phase of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix [1] is an important part of the present program in particle physics. Violation of CP symmetry is manifested as a nonzero area of the CKM unitarity triangle [2]. In this Letter we report the measurement of the branching fraction, polarization, and CP asymmetry of the \( B^+ \rightarrow \rho^+ \rho^0 \) decay mode, which is needed for the \( \rho \rho \) isospin analysis used to extract \( \alpha = \arg \left(-V_{ud} V_{ub}^* / V_{td} V_{tb}^* \right) \) [3]. We also set an upper limit on the unknown branching fraction of \( B^+ \rightarrow \rho^+ f_0(980)(\rightarrow \pi^+ \pi^-) \), which is measured to control this background to the \( B^+ \rightarrow \rho^+ \rho^0 \) analysis.

In \( B^0(\bar{B}^0) \rightarrow \rho^+ \rho^- \) decays [4] the interference between the \( BB \) oscillations which depend on \( V_{ud} \) and the dominating tree-level amplitude \( b \rightarrow u \bar{u} d \bar{d} \) causes a time-dependent CP asymmetry that depends on \( \sin(2\alpha) \). The presence of loop (penguin) amplitudes leads to a shift \( \delta \alpha = |\alpha - \alpha_{penguin}| \), between the physical weak phase \( \alpha \) and the effective one \( \alpha_{eff} \), experimentally measured in \( B^0 \rightarrow \rho^+ \rho^- \) decays [5,6]. However, the penguin amplitudes in these decays are known to contribute at a very low level because of the small upper limit of 1.1 \( \times 10^{-6} \) at 90% confidence level (C.L.) [7], obtained from the branching fraction of the penguin dominated mode \( B^0 \rightarrow \rho^0 \rho^0 \). The size of \( \delta \alpha \) can be extracted from the full isospin analysis combining all \( B \rightarrow \rho \rho \) modes [3].

In \( B \rightarrow \rho \rho \) decays, a spin zero particle decays into two spin one particles. The final state is therefore a superposition of two transversely polarized modes (helicity \( \pm 1 \)) and one longitudinal mode (helicity 0), which can be measured through an angular analysis. The longitudinal polarization fraction \( f_L \) is defined as the fraction of decays to the helicity zero state \( f_L = \Gamma_L / \Gamma \), where \( \Gamma \) is the total decay rate and \( \Gamma_L \) is the decay rate to the longitudinally polarized final state. The transverse polarization is a mixed CP state, while the longitudinal state is pure CP even. The previous measurements of \( f_L \) [8,9] showed the decay is consistent with being fully longitudinally polarized.

Our analysis is performed in the helicity frame [10] as a function of the two helicity angles \( \theta_\perp \) and \( \theta_0 \), where the helicity angle of a \( \rho^+ (\rho^0) \) meson is defined as the angle between its daughter \( \pi^+ (\pi^-) \) and the direction opposite to the B meson in the \( \rho^+ (\rho^0) \) rest frame. The polarization \( f_L \) can be extracted from the differential decay rate:

\[
\frac{1}{\Gamma} \frac{d^2 \Gamma}{d \cos \theta_\perp d \cos \theta_0} = \frac{9}{4} \left[ f_L \cos^2 \theta_\perp \cos^2 \theta_0 + \frac{1}{4} (1 - f_L) \sin^2 \theta_\perp \sin^2 \theta_0 \right].
\]

Here we integrate over the angle between the \( \rho \)-meson decay planes.

The measurements presented in this Letter are based on data collected with the BABAR detector [11] at the SLAC PEP-II asymmetric-energy \( e^+ e^- \) collider. The analyzed data sample of \( 2.318 \pm 2.6 \times 10^6 \ BB \) pairs produced at the \( Y(4S) \) resonance corresponds to an integrated luminosity of 210.5 fb\(^{-1}\).

To reconstruct \( B^\pm \rightarrow \rho^\pm \rho^0 \) and \( B^\pm \rightarrow \rho^\pm f_0 \) decays, we select events with at least three charged tracks and one neutral pion candidate. Charged tracks are required to originate from the interaction point and have particle identification information inconsistent with kaon, electron, and proton hypotheses. We form \( \pi^0 \rightarrow \gamma \gamma \) candidates from pairs of calorimeter showers, each with a photonlike lateral spread and a minimum energy of 50 MeV. The invariant mass of \( \pi^0 \) candidates is required to fall in the range \( 0.10 < m_{\gamma \gamma} < 0.16 \) GeV/c\(^2\).

The mass of charged \( \rho^\pm \) candidates must satisfy \( 0.396 < m_{\pi^\pm \rho^\mp} < 1.146 \) GeV/c\(^2\) where the lowside requirement on the \( \pi^+ \pi^- \) mass is chosen to exclude \( K^0_L \rightarrow \pi^+ \pi^- \) decays. Neutral final state meson candidates \( (\rho^0, f_0) \) must satisfy \( 0.520 < m_{\pi^+ \pi^-} < 1.146 \) GeV/c\(^2\). In order to suppress backgrounds with low momentum pions, the helicity angles are required to fall in the ranges \( -0.8 < \cos \theta_\perp < 0.95 \) and \( \cos \theta_0 < 0.95 \). Backgrounds from \( D^0 \rightarrow K^- \pi^+ \pi^0 \) and \( D^0 \rightarrow \pi^- \pi^+ \pi^0 \) decays are reduced by requiring the candidate \( D^0 \) invariant mass to be at least 40 MeV/c\(^2\) away from the \( D^0 \) mass.

About 20% of the selected events have multiple \( B \) candidates and the one that has the reconstructed \( \pi^0 \) mass closest to the \( \pi^0 \) mass is kept. In the case that more than one candidate has the same reconstructed \( \pi^0 \) mass, we select one at random.

Continuum decays represent the largest source of background and are reduced by requiring \( |\cos \theta_T| < 0.8 \), where \( \theta_T \) is the cosine of the angle between the \( B \) thrust axis and that from the rest of the event (ROE). To further discriminate signal from continuum, we also use a neural network built out of five event-shape variables: a Fisher discrimi-
nant combining the 0th and 2nd order monomials [12] for charged particles and neutral clusters of the ROE; the cosine of the angle between the direction of the B and the collision axis (z) in the center-of-mass (c.m.) frame; the cosine of the angle between the B thrust axis and the z axis; the variable $|\cos \theta_j|$ defined above; and the sum of transverse momenta in the ROE relative to the z axis. The output is transformed into a variable $x_{NN}$ which has roughly Gaussian signal and background distributions. We select candidates in a range of $x_{NN}$ that removes 54% of continuum background events while retaining 90% of the signal. After these selections, about 85% of the remaining events are from continuum decays.

Signal event candidates are further identified based on two kinematic variables: the beam-energy-substituted mass $m_{ES} = \sqrt{(s/2 + p_t \cdot p_B)^2/E_i^2 - p_B^2}$, using the total initial $e^+e^-$ 4-momentum $(E_i, \mathbf{p}_i)$, c.m. energy $(\sqrt{s})$ and the B momentum $(\mathbf{p}_B)$, and $\Delta E = E_B - \sqrt{s}/2$, the difference between the reconstructed $B$ energy in the c.m. frame $(E_B)$ and the beam energy. Events are selected if $m_{ES} > 5.26 \text{ GeV}/c^2$ and $|\Delta E| < 150 \text{ MeV}$.

After the selection criteria are applied, the efficiency is 8.4% for longitudinal and 18.6% for transverse polarized $B^\pm \rightarrow \rho^\pm \rho^0$ decays. The selection efficiency is 16.6% for $B^\pm \rightarrow \rho^\pm f_0$ decays. Any possible interference effects between the $B^\pm \rightarrow \rho^\pm \rho^0$ and $B^\pm \rightarrow \rho^\pm f_0$ are neglected.

An unbinned extended maximum likelihood fit is applied to the selected sample of $N_{tot} = 74293$ events in order to measure the $B^\pm \rightarrow \rho^\pm \rho^0$ event yield, polarization, and charge asymmetry as well as the $B^\pm \rightarrow \rho^\pm f_0$ event yield. The likelihood function is

$$L = \frac{1}{N_{tot}!} \exp \left(-\sum_{k=1}^{M} n_k \prod_{i=1}^{N_{tot}} \sum_{j=1}^{M} n_j \mathcal{P}_j(\tilde{x}_i) \right),$$

(2)

where $M$ is the number of hypotheses (signal, mis-reconstructed signal, continuum, and B-background classes), and $n_j$ represents the number of measured events for each hypothesis determined by maximizing the likelihood function. $\mathcal{P}_j(\tilde{x}_i)$ is the product of the probability density functions (PDFs) of hypothesis $j$ evaluated at the $i$th event’s measured variables, $\tilde{x}_i = (m_{ES}, \Delta E, m_{\tau^0}, m_{\tau^+}, m_{\tau^-}, \cos \theta_\tau, \cos \theta_0, x_{NN})$. In addition, the charge asymmetry, obtained from the measured $B^+$ and $B^-$ signal candidate decay yields, $A_{CP} = \frac{N_{CP}^- - N_{CP}^+}{N_{CP}^- + N_{CP}^+}$, is determined in the fit to the data.

Each discriminating variable in the likelihood function is modeled with a PDF extracted either from the data, or from high statistics Monte Carlo (MC) simulated data samples. The correlations between the variables are assumed to be small and the PDFs independent. This is checked with systematic error studies, and corrections are applied where necessary.

The continuum background $\Delta E$, $m_{ES}$, and $x_{NN}$ distributions are modeled with one-dimensional parametrized distributions taken from fits to the data. Correlations are observed between the $m_{\pi^\pm}$ and $\cos \theta_\tau$ distributions for both $\rho$-meson candidates, which are taken into account with two-dimensional PDFs. The signal component is modeled with one-dimensional parametrized distributions for each of six variables; $m_{ES}$ is modeled with a nonparametric PDF [13]. The signal PDF shapes are obtained from fits to signal MC sample after the selection is applied. Events with a true $B^\pm \rightarrow \rho^\pm \rho^0$ decay but with wrong tracks or calorimeter clusters assigned to the final state are referred to as self cross feed (SCF) events. They make up 35% and 14% of the selected longitudinally and transversely polarized signal samples, respectively. The longitudinal and transverse SCF components and B-background PDFs are determined in a similar manner using high statistics MC samples and modeled with nonparametric PDFs [13] for each variable.

To understand the backgrounds from other B decay modes we use MC simulated events. There are two types of B background: “charmed” (decays involving $b \rightarrow c$ transitions), and “charmless” (all other b decays). Altogether 16 B-background categories plus the two SCF components are included in the fit. The SCF yields and polarization are fixed in the final fit at values that match those fitted for the signal in previous iterations of the fit. Four specific charmed background modes are included: $B^\pm \rightarrow D^0\pi^\pm$, $B^\pm \rightarrow D^0\rho^\pm$, $B^\pm \rightarrow D^{*0}\pi^\pm$, and $B^\pm \rightarrow D^{*0}\rho^\pm$. Other charmed backgrounds are combined into two generic classes of events for charged and neutral charmed B decays. For the charmless B backgrounds, separate MC samples of eight modes were used: neutral B decaying to $\rho^0\rho^0$ and charged B decaying to $\rho^\pm f_0(980)$, $\eta\rho^\pm$, $K^{*0}\rho^\pm$, $\rho_i \pi^\pm$, $\rho_i \pi^\pm$, $\rho_i \rho^0$, and $\rho_i \rho^0$ with the decays $\rho_i \rightarrow (\rho \pi)^0$ and $\rho_i \rightarrow (\rho \pi)^\pm$. For B decaying to vector mesons, only the longitudinal component of the decay is considered. Two generic categories, one for 5-body modes and one for all “other charmless” decays, complete the B-background model.

The number of “other charmless” events and the $B^\pm \rightarrow \rho^\pm f_0$ yield were determined from the data fit. The other 14 backgrounds had their yields fixed in the fit. We use the following branching fractions: $\mathcal{B}(B^0 \rightarrow \rho^+ \rho^-) = (26.2 \pm 3.7) \times 10^{-6}$ [14], $\mathcal{B}(B^\pm \rightarrow \eta \rho^\pm) = (12.9 \pm 6.5) \times 10^{-6}$ [15], $\mathcal{B}(\eta' \rightarrow \rho^0\gamma) = 0.295 \pm 0.010$ [16], $\mathcal{B}(B^+ \rightarrow K^{*0}\rho^+) = (10.5 \pm 1.8) \times 10^{-6}$ [14], and $\mathcal{B}(K^{*0} \rightarrow K^+\pi^-) = 2.3$. The decays $B^\pm \rightarrow (\rho \pi)^0$ and $B^\pm \rightarrow (\rho \pi)^\pm$ have few experimental constraints [17,18]. We adopt the following $B^\pm$ branching ratios, in units of $10^{-6}$, and assume a 100% systematic uncertainty: $a_{i0}^0 \pi^0 = 12$, $a_{i0}^0 \pi^0 = 6$, $a_{i1}^0 \rho^0 = a_{i1}^0 \rho^0 = 48$.

Table I shows the results of the fit, where the quoted errors are statistical errors only. Projection plots for $m_{ES}$ and $\Delta E$ are shown in Fig. 1.
We considered systematic effects from biases in the fit model, which are due to the imperfect modeling of some correlations between fitted variables in the likelihood function. Tests of the fit are made by using a large number of MC samples containing the amounts of signal, continuum, and $B$-background events measured or fixed in the data fit, and where correlations between variables are modeled for signal and $B$ backgrounds. The fits to these samples should reproduce the number of MC events generated. A shift of the fitted values with respect to the generated ones indicates a bias of $-49$ events on the $\rho^+\rho^0$ yield, $+0.009$ on $f_L$, and $-4.6$ events on the $\rho^+f_0$ yield in the fitting procedure. We use the same technique to study the effects of correlations between the neural net and helicity variables in the $q\bar{q}$ continuum and observe a fit bias of $+24$ events on the $\rho^+\rho^0$ yield, $-0.001$ on $f_L$, and $-19$ events on the $\rho^+f_0$ yield. No other significant correlations were observed between the other discriminating variables. These two biases are corrected from the fit measurements, and half of each separate fit bias is taken as the systematic error (cf. Table II).

Many of the $B$-background rates are poorly known. The effect of uncertainties in these values is evaluated by varying the number of events in each background category within the range allowed by the error on the branching fraction. Fourteen nonresonant backgrounds that are not in the default fit are tested by adding them singly to the fit fraction. Fourteen nonresonant backgrounds that are not in the range allowed by the error on the branching fraction. The impact of the uncertainty on the measurement of the $f_0$ mass and width [19] has also been evaluated. The values of the systematic errors described above are given in Table II.

Systematic uncertainties in the reconstruction and calibration procedure introduce a systematic error of 3% after a correction of $-2.5\%$ on the $\pi^0$ reconstruction efficiency, 3.9% after a correction of $-1.5\%$ on the track reconstruction efficiency, and a systematic error of 1.1% from the particle identification. The uncertainty on the efficiency ratio between longitudinal and transverse events is found to be negligible. The error on $A_{CP}$ includes a 0.45% uncertainty in the charged track reconstruction asymmetry, a 4% uncertainty from the detector’s intrinsic charged particle identification asymmetry, and a 9% uncertainty which is the largest single shift obtained when assuming a uniform probability for the charge asymmetry of every $B$-background individually.

The systematic error associated with misreconstructed signal is evaluated by taking the difference between the default fit and the one for which these events are not modeled, and therefore mostly absorbed into the “other charmless” background category. We consider the error due to the uncertainty on the signal, $B$ background, and continuum PDF shapes and estimate a systematic error by varying the parameters obtained from MC calculations that govern these shapes within their statistical uncertainty. The values of the systematic errors described above are given in Table II.

### Table I. Summary of the results of the fit with statistical errors (before correction for fit biases).

<table>
<thead>
<tr>
<th>Observables</th>
<th>Fitted value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^\pm \rightarrow \rho^+\rho^0$ yield</td>
<td>390 ± 49 events</td>
</tr>
<tr>
<td>Polarization $f_L$</td>
<td>0.897 ± 0.042</td>
</tr>
<tr>
<td>Charge asymmetry $A_{CP}$</td>
<td>$-0.12 \pm 0.13$</td>
</tr>
<tr>
<td>$B^\pm \rightarrow \rho^+f_0$ yield</td>
<td>51 ± 30 events</td>
</tr>
</tbody>
</table>

### Table II. Summary of the systematic uncertainties on the $B^\pm \rightarrow \rho^+\rho^0$ yield, the polarization $f_L$, and the $B^\pm \rightarrow \rho^+f_0$ yield.

<table>
<thead>
<tr>
<th>Source</th>
<th>$\rho^+\rho^0$ yield</th>
<th>$f_L$</th>
<th>$\rho^+f_0$ yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fit bias uncertainty</td>
<td>27.3</td>
<td>0.005</td>
<td>9.8</td>
</tr>
<tr>
<td>$B$-background rates</td>
<td>11.0</td>
<td>0.007</td>
<td>2.8</td>
</tr>
<tr>
<td>Nonresonant backgrounds</td>
<td>12.0</td>
<td>0.009</td>
<td>3.0</td>
</tr>
<tr>
<td>Amount of SCF</td>
<td>24.0</td>
<td>0.010</td>
<td>0.6</td>
</tr>
<tr>
<td>PDF shapes</td>
<td>+21.1</td>
<td>+0.017</td>
<td>+7.9</td>
</tr>
<tr>
<td></td>
<td>-22.5</td>
<td>-0.022</td>
<td>-13.5</td>
</tr>
<tr>
<td></td>
<td>+0.9</td>
<td>0.000</td>
<td>3.9</td>
</tr>
<tr>
<td>Total</td>
<td>+45</td>
<td>-46</td>
<td>-18</td>
</tr>
</tbody>
</table>

The systematic errors on the $m_{ES}$, $\Delta E$, $m_{\pi^+\pi^-}$, and $m_{\pi^+\pi^-}$ with a cut on the ratio of the signal and background likelihoods that selects about 40% of the signal. For (a) and (b), the observable plotted is excluded from the fit in calculating the likelihood used for the enrichment selection. For (c) and (d), only the $m_{ES}$, $\Delta E$, and $x_{NN}$ variables have been used in calculating the likelihood. Points represent on-resonance data, dashed lines the continuum and $BB$ backgrounds PDFs, and solid lines the likelihood function with yields taken from the fit where all variables have been used.
In summary, we measure the branching fraction, longitudinal polarization, and $CP$ asymmetry of the decay $B^{\pm} \to \rho^{\pm} \rho^0$, using a dataset of about $231.8 \times 10^6 \ BB$ pairs, to be

$$\mathcal{B}(B^{\pm} \to \rho^{\pm} \rho^0) = (16.8 \pm 2.2 \pm 2.3) \times 10^{-6},$$

$$f_L(B^{\pm} \to \rho^{\pm} \rho^0) = 0.905 \pm 0.042^{+0.023}_{-0.027},$$

$$A_{CP}(B^{\pm} \to \rho^{\pm} \rho^0) = -0.12 \pm 0.13 \pm 0.10.$$  

The measurement of the branching fraction has improved by a factor of about 2 with respect to the previous $BABAR$ measurement [8], and supersedes it. The isospin relations between branching ratios are consistent between this measurement and those of $\rho^+ \rho^-$ and $\rho^0 \rho^0$ [14], validating the approach [3] used to constraint $a$. Moreover, our measurements confirm that this mode is largely longitudinally polarized. They also confirm that the charge asymmetry is consistent with zero as expected for decays proceeding through one decay channel only; this suggests the contributions of electroweak penguins are small in the $B \to \rho \rho$ system.

In addition we measure

$$\mathcal{B}(B^{\pm} \to \rho^0(980)(\to \pi^+ \pi^-)) = (0.7 \pm 0.8 \pm 0.5) \times 10^{-6}$$

with a significance of $0.4\sigma$. We set an upper limit on the branching fraction of $1.9 \times 10^{-6}$ at 90% confidence level by finding the yield ($N$) that satisfies $\int_0^{0.3} \mathcal{L}(n)dn / \int_0^{0.3} \mathcal{L}(n)dn = 0.9$ taking into account systematic uncertainties.

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[4] Charge conjugation is implied throughout this document, unless explicitly stated.