Measurement of the $\bar{B}^0$ lifetime and the $B^0\bar{B}^0$ oscillation frequency using partially reconstructed $B^0 \to D^{**} (\ell^- \bar{\nu}_\ell)$ decays

MEASUREMENT OF THE $B^0$ LIFETIME AND THE $B^0\bar{B}^0$ ...

PHYSICAL REVIEW D 73, 012004 (2006)

21University of Colorado, Boulder, Colorado 80309, USA
22Colorado State University, Fort Collins, Colorado 80523, USA
23Institut für Physik, Universität Dortmund, D-44221 Dortmund, Germany
24Institut für Kern- und Teilchenphysik, Technische Universität Dresden, D-01062 Dresden, Germany
25Ecole Polytechnique, LLR, F-91128 Palaiseau, France
26University of Edinburgh, Edinburgh EH9 3JZ, United Kingdom
27Dipartimento di Fisica and INFN, Università di Ferrara, I-44100 Ferrara, Italy
28Laboratori Nazionali di Frascati dell’INFN, I-00044 Frascati, Italy
29Dipartimento di Fisica and INFN, Università di Genova, I-16146 Genova, Italy
30Harvard University, Cambridge, Massachusetts 02138, USA
31Physikalisches Institut, Universität Heidelberg, Philosophenweg 12, D-69120 Heidelberg, Germany
32Imperial College London, London, SW7 2AZ, United Kingdom
33University of Iowa, Iowa City, Iowa 52242, USA
34Iowa State University, Ames, Iowa 50011-3160, USA
35Laboratoire de l’Accélérateur Linéaire, F-91898 Orsay, France
36Lawrence Livermore National Laboratory, Livermore, California 94550, USA
37University of Liverpool, Liverpool L69 72E, United Kingdom
38Queen Mary, University of London, E1 4NS, United Kingdom
39Royal Holloway and Bedford New College, University of London, Egham, Surrey TW20 0EX, United Kingdom
40University of Louisville, Louisville, Kentucky 40292, USA
41University of Manchester, Manchester M13 9PL, United Kingdom
42University of Maryland, College Park, Maryland 20742, USA
43University of Massachusetts, Amherst, Massachusetts 01003, USA
44Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
45McGill University, Montréal, Quebec, Canada H3A 2T8
46Dipartimento di Fisica and INFN, Università di Firenze, I-50019 Sesto Fiorentino, Italy
47University of Mississippi, University, Mississippi 38677, USA
48Laboratoire de Physique des Particules de Paris, Université Paris VI et VII, F-75252 Paris, France
49University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA
50Dipartimento di Fisica and INFN, Universita di Perugia, I-06100 Perugia, Italy
51Instituto de Fisica e Química, IFFA, University of Porto, 4150-070 Porto, Portugal
52University of Oregon, Eugene, Oregon 97403, USA
53Dipartimento di Fisica e Astronomia, Università di Padova, I-35131 Padova, Italy
54Laboratoire de Physique Nucléaire et de Hautes Energies, Universités Paris VI et VII, F-75252 Paris, France
55University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA
56Dipartimento di Fisica and INFN, Università di Padova, I-35131 Padova, Italy
57Dipartimento di Fisica and INFN, Università di Perugia, I-06100 Perugia, Italy
58Dipartimento di Fisica and INFN, Università di Roma ‘Sapienza’, I-00185 Roma, Italy
59Dipartimento di Fisica e Astronomia, Università di Padova, I-35131 Padova, Italy
60University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA
61University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA
62University of Pittsburgh, Pittsburgh, Pennsylvania 15260, USA
63University of Southern California, Los Angeles, California 90089, USA
64University of Texas at Arlington, Arlington, Texas 76019, USA
65Department of Physics, University of Warwick, Coventry CV4 7AL, United Kingdom
66University of Wisconsin, Madison, Wisconsin 53706, USA
67Yale University, New Haven, Connecticut 06511, USA
(Received 12 July 2005; published 23 January 2006)
We present a simultaneous measurement of the $B^0$ lifetime $\tau_{B^0}$ and $B^0\bar{B}^0$ oscillation frequency $\Delta m_d$. We use a sample of about 50,000 partially reconstructed $B^0 \rightarrow D^{\ast+} \ell^- \bar{\nu}_\ell$ decays identified with the BABAR detector at the PEP-II $e^+e^-$ storage ring at SLAC. The flavor of the other $B$ meson in the event is determined from the charge of another high-momentum lepton. The results are $\tau_{B^0} = (1.504 \pm 0.013(stat)^{+0.007}_{-0.006}(syst))$ ps, $\Delta m_d = (0.511 \pm 0.007(stat)^{+0.007}_{-0.006}(syst))$ ps$^{-1}$.

**I. INTRODUCTION**

The time evolution of $B^0$ mesons is governed by the overall decay rate $\Gamma(B^0) = 1/\tau_{B^0}$ and by the mass difference $\Delta m_d$ of the two mass eigenstates. A precise determination of $\Gamma(B^0)$ reduces the systematic error on the parameter $|V_{cb}|$ of the Cabibbo-Kobayashi-Maskawa quark mixing matrix [1]. The parameter $|V_{td}V_{tb}|$ enters the box diagram that is responsible for $B^0\bar{B}^0$ oscillations and can be determined from a measurement of $\Delta m_d$, although with sizable theoretical uncertainties.

We present a measurement of $\tau_{B^0}$ and $\Delta m_d$ using $B^0 \rightarrow D^{\ast+} \ell^- \bar{\nu}_\ell$ decays [2] selected from a sample of about 88 × $10^6$ $B\bar{B}$ events recorded by the BABAR detector at the PEP-II asymmetric-energy $e^+e^-$ storage ring, operated at or near the $Y(4S)$ resonance. $B\bar{B}$ pairs from the $Y(4S)$ decay move along the beam axis with a nominal Lorentz boost $\langle \beta \gamma \rangle = 0.56$, so that the vertices from the two $B$ decay points are separated on average by about 260 $\mu$m. The $B^0\bar{B}^0$ system is produced in a coherent $P$-wave state, so that flavor oscillation is measurable only relative to the decay of the first $B$ meson. Mixed (unmixed) events are selected by the observation of two equal (opposite) flavor $B$ meson decays. The probabilities of observing mixed ($S^-$) or unmixed ($S^+$) events as a function of the proper time difference $\Delta t$ between decays are

$$S^{\pm}(\Delta t) = \frac{e^{-|\Delta t|/\tau_{B^0}}}{4\tau_{B^0}} (1 \pm 2w \cos(\Delta m_d \Delta t)), \quad (1)$$

where the dilution factor $D$ is related to the fraction $w$ of events with wrong flavor assignment by the relation $D = 1 - 2w$ and $\Delta t$ is computed from the distance between the two vertices projected along the beam direction.

**II. THE BABAR DETECTOR AND DATA SET**

We have analyzed a data sample of 81 fb$^{-1}$ collected by BABAR on the $Y(4S)$ resonance, a sample of 9.6 fb$^{-1}$ collected 40 MeV below the resonance to study the continuum background, and a sample of simulated $B\bar{B}$ events corresponding to about 3 times the size of the data sample.

---

*Also with Università di Perugia, Dipartimento di Fisica, Perugia, Italy.
†Also with Università della Basilicata, Potenza, Italy.
‡Deceased.

The simulated events are processed through the same analysis chain as the real data. BABAR is a multipurpose detector, described in detail in Ref. [3]. The momentum of charged particles is measured by the tracking system, which consists of a silicon vertex tracker (SVT) and a drift chamber (DCH) in a 1.5-T magnetic field. The positions of points along the trajectories of charged tracks measured with the SVT are used for vertex reconstruction and for measuring the momentum of charged particles, including those particles with low transverse momentum that do not reach the DCH due to bending in the magnetic field. The energy loss in the SVT is used to discriminate low-momentum pions from electrons. Higher-energy electrons are identified from the ratio of the energy of their associated shower in the electromagnetic calorimeter (EMC) to their momentum, the transverse profile of the shower, the energy loss in the DCH, and the information from the Cherenkov detector (DIRC). The electron identification efficiency is about 90%, and the hadron misidentification probability is less than 1%. Muons are identified on the basis of the energy deposited in the EMC and the penetration in the instrumented flux return (IFR) of the superconducting coil, which contains resistive plate chambers interspersed with iron. Muon candidates compatible with the kaon hypothesis in the DIRC are rejected. The muon identification efficiency is about 60%, and the hadron misidentification rate is about 2%.

**III. ANALYSIS METHOD**

A. Selection of $B^0 \rightarrow D^{\ast+} \ell^- \bar{\nu}_\ell$ decays

We select events that have more than four charged tracks. We reduce the contamination from light-quark production in continuum events by requiring the normalized Fox-Wolfram second moment [4] to be less than 0.5. We select $B^0 \rightarrow D^{\ast+} \ell^- \bar{\nu}_\ell$ events with partial reconstruction of the decay $D^{\ast+} \rightarrow \pi^+_\ell D^0$, using only the charged lepton from the $B^0$ decay and the soft pion ($\pi^+_\ell$) from the $D^{\ast+}$ decay. The $D^0$ decay is not reconstructed, resulting in high selection efficiency. BABAR has already published two measurements of $\tau_{B^0}$ [5,6] and a measurement of $\sin(2\beta + \gamma)$ [7] based on partial reconstruction of $B$ decays. This technique was originally applied to $B^0 \rightarrow D^{\ast+} \ell^- \bar{\nu}_\ell$ decays by ARGUS [8], and then used by CLEO [9], DELPHI [10], and OPAL [11].

To suppress leptons from several background sources, we use only high-momentum leptons, in the range
1.3 < p_c < 2.4 GeV/c [12]. The π_i^+ candidates have momenta \( p_{\pi_i^+} \) between 60 and 200 MeV/c. Because of the limited phase space available in the \( D^{*+} \) decay, the π_i^+ is emitted within an approximately one-radian half-opening-cone centered about the \( D^{*+} \) flight direction. We approximate the direction of the \( D^{*+} \) to be that of the \( \pi_i^+ \) and estimate the energy \( \hat{E}_{D^{*+}} \) of the \( D^{*+} \) as a function of the energy of the \( \pi_i^+ \) using a third-order polynomial, with parameters taken from the simulation. We define the square of the missing neutrino mass as

\[
\mathcal{M}_\nu^2 = \left( \frac{\sqrt{5}}{2} - \hat{E}_{D^{*+}} - E_\ell \right)^2 - (\hat{p}_{D^{*+}} + p_\ell)^2, \tag{2}
\]

where we neglect the momentum of the \( B^0 \) in the \( Y(4S) \) frame (on average, 0.34 GeV/c), and identify the \( B^0 \) energy with the beam energy \( \sqrt{5}/2 \) in the \( e^+ e^- \) center-of-mass frame. \( E_\ell \) and \( p_\ell \) are the energy and momentum vector of the lepton and \( \hat{p}_{D^{*+}} \) is the estimated momentum vector of the \( D^{*+} \). The distribution of \( \mathcal{M}_\nu^2 \) peaks at zero for signal events, while it is spread over a wide range for background events (see Fig. 1).

We determine the \( B^0 \) decay point from a vertex fit of the \( \ell^- \) and \( \pi_i^+ \) tracks, constrained to the beam-spot position in the plane perpendicular to the beam axis (the \( x-y \) plane). The beam-spot position and size are determined on a run-by-run basis using two-prong events [3]. Its size in the horizontal (\( x \)) direction is on average 120 \( \mu \)m. Although the beam-spot size in the vertical (\( y \)) direction is only 5.6 \( \mu \)m, we use a constraint of 50 \( \mu \)m in the vertex fit to account for the flight distance of the \( B^0 \) in the \( x-y \) plane.

We reject events for which the \( \chi^2 \) probability of the vertex fit, \( \mathcal{P}_V \), is less than 0.1%.

We then apply a selection criterion to a combined signal likelihood, \( \mathcal{X} \), calculated from \( p_\ell, p_{\pi_i^+} \), and \( \mathcal{P}_V \), which results in a signal-to-background ratio of about one in the signal region defined as \( \mathcal{M}_\nu^2 > -2.5 \text{ GeV}^2/c^4 \). We reject events for which \( \mathcal{X} \) is lower than 0.4 (see Fig. 2). Figure 1 shows the distribution of \( \mathcal{M}_\nu^2 \) after this selection. The distributions in the top part of the figure are obtained from events in which the \( \ell \) and the \( \pi_i \) have opposite charges ("right charge"), and the distributions in the bottom are from events in which the \( \ell \) and the \( \pi_i \) have equal charges ("wrong charge").

The points in Fig. 1 correspond to on-resonance data. The dark histograms correspond to off-resonance data, scaled by the ratio of on-resonance to off-resonance integrated luminosity. The hatched histograms correspond to \( B\bar{B} \) combinatorial background from simulation. To normalize the \( B\bar{B} \) combinatorial background, we scale the \( B\bar{B} \) Monte Carlo histogram so that, when added to the luminosity-scaled off-resonance histogram, the sum matches the on-resonance data in the region \( \mathcal{M}_\nu^2 < -4.5 \text{ GeV}^2/c^4 \). The right-charge plot is shown for illustration only. We use the wrong-charge samples as a cross check to verify that the \( B\bar{B} \) combinatorial background shape is described by the simulation. For this purpose, we compare the number of wrong-charge events in the
signal region predicted from the sum of off-resonance and $B \bar{B}$ Monte Carlo, normalized as above, to the number of wrong-charge on-resonance data events. This ratio is 0.996 ± 0.002, consistent with unity. For the rest of the analysis we consider only right-charge events.

B. Tag vertex and B flavor tagging

To measure $\Delta m_B$ we need to know the flavor of both $B$ mesons at their time of decay and their proper decay time difference $\Delta t$. The flavor of the partially reconstructed $B$ is determined from the charge of the high-momentum lepton. In order to identify the flavor of the other (“tag”) $B$ meson, we restrict the analysis to events in which another charged lepton (the “tagging lepton”) is found. To reduce contamination from fake leptons and leptons originating from charm decays, we require that the momentum of this second lepton exceed 1.0 GeV/c for electrons, and 1.1 GeV/c for muons.

The decay point of the tag $B$ is determined with the high-momentum lepton and a beam-spot constraint; the procedure is the same as that used to determine the $B \bar{B}$ vertex. We compute $\Delta t$ from the projected distance between the two vertices along the beam direction (z axis), $\Delta z = z_{\text{decay}} - z_{\text{tag}}$, with the approximation that the $B^0$ and the $\overline{B}^0$ are at rest in the $Y(4S)$ rest frame (the boost approximation): $\Delta t = \Delta z/c \beta \gamma$, where the boost factor $\beta \gamma$ is determined from the measured beam energy. To remove badly reconstructed vertices we reject all events with either $|\Delta z| > 3$ mm or $\sigma(\Delta z) > 0.5$ mm, where $\sigma(\Delta z)$ is the uncertainty on $\Delta z$, computed for each event. The simulation shows that the difference between the true and measured $\Delta t$ can be fitted with the sum of two Gaussians. The rms of the narrow Gaussian, which describes 70% of the events, is 0.64 ps; the rms of the wide one is about 1.7 ps.

We then select the best right-charge candidate in each event according to the following procedure: if there is more than one, we choose that with $M^2_\nu > -2.5$ GeV$^2/c^4$. If two or more candidates are still left, but they have different leptons, we select the one with the largest value of $\chi^2$. In a small fraction of events we select two or more candidates sharing the same lepton combined with different soft pions. We keep the candidate with the largest $\chi^2$, unless one of the $\pi^+_s$ is consistent with coming from the decay of a $D^{*+}$ from the other $B$, in which case we remove the event. For this purpose, we define the square of the missing neutrino mass in the tag-side, $M^2_{\nu,\text{tag-}B}$, by means of Eq. (2), where we replace the four-momentum of the lepton from the $B^0 \rightarrow D^{*+} \ell^- \bar{\nu}_\ell$ decay with that of the tag lepton. This variable peaks at zero for soft pions originating from the tag-$B$ decay. We require $M^2_{\nu,\text{tag-}B} < -3$ GeV$^2/c^4$. Finally we reject the events in which the signal lepton can be combined to a wrong-charge pion to produce an otherwise successful candidate, if the pion is consistent with coming from a $D^{*+}$ from the tag-$B$ according to the criterion just described. About 20% of the signal events are removed by this requirement.

For background studies, we select events in the region $M^2_\nu < -2.5$ GeV$^2/c^4$ if there is no candidate in the signal region. We find about 49,000 signal events over a background of about 28,000 events in the data sample in the region $M^2_\nu > -2.5$ GeV$^2/c^4$.

C. Sample composition

Our data sample consists of the following event types, categorized according to their origin and to whether or not they peak in the $M^2_\nu$ distribution. We consider signal to be any combination of a lepton and a charged $D^*$ produced in the decay of a single $B \bar{B}$ meson. Signal consists of mainly $B^0 \rightarrow D^{*+} \ell^- \bar{\nu}_\ell$ decays, with minor contributions from $B^0 \rightarrow D^{*+} \pi^0 \ell^- \bar{\nu}_\ell$, $B^0 \rightarrow D^{*+} \tau^- \bar{\nu}_\tau$, $B^0 \rightarrow D^{*+} D_s^-$, and $B^0 \rightarrow D^{*+} D X$ with $\tau$, $D_s^-$, or $\overline{D}$ decaying to an $\ell^-$, and from $B^0 \rightarrow D^{*+} h$, with the hadron $h$ misidentified as a muon. Peaking $B^-$ background is mainly due to the processes $B^- \rightarrow D^{*+} \pi^- \ell^- \bar{\nu}_\ell$, and $B^- \rightarrow D^{*+} \pi^- X$ with the $\pi^-$ misidentified as a muon. Other minor contributions to the peaking sample are due to decays $B^- \rightarrow D^+ \pi^- \ell^- \bar{\nu}_\ell$, and $B^- \rightarrow D^+ \pi^- X$ with the $\pi^-$ from the decay of an orbitally excited $D$ meson ($D^{*+}$). Nonpeaking contributions are due to random combinations of a charged lepton candidate and a low-momentum pion candidate, produced either in $B \bar{B}$ events ($B \bar{B}$ combinatorial) or in $e^+ e^- \rightarrow q\bar{q}$ interactions with $q = u, d, s$, or $c$ (continuum). We compute the sample composition separately for mixed and unmixed events by fitting the corresponding $M^2_\nu$ distribution to the sum of four components: continuum, $B \bar{B}$ combinatorial background, $B^0 \rightarrow D^{*+} \ell^- \bar{\nu}_\ell$ decays, and $B^- \rightarrow D^+ \pi^- \ell^- \bar{\nu}_\ell$ decays. Because of one or more additional pions in the final state, the $B^- \rightarrow D^+ \pi^- \ell^- \bar{\nu}_\ell$ events have a different $M^2_\nu$ spectrum than from the process $B^0 \rightarrow D^{*+} \ell^- \bar{\nu}_\ell$. We measure the continuum contribution from the off-resonance sample, scaled to the luminosity of the on-resonance sample. We determine the $M^2_\nu$ distributions for the other event types from the simulation, and determine their relative abundance in the selected sample from a fit to the $M^2_\nu$ distribution for the data. Assuming isospin conservation, we assign two-thirds of $B^- \rightarrow D^+ \pi^- \ell^- \bar{\nu}_\ell$ decays to peaking $B^-$ background and the rest to $B^0 \rightarrow D^+ \pi^- \ell^- \bar{\nu}_\ell$, which we add to the signal. We vary this fraction in the study of systematic uncertainties. We assume 50% uncertainty on the isospin-conservation hypothesis.

A possible distortion in the $M^2_\nu$ distribution comes from the decay chain $B \rightarrow D(X)\ell^- \bar{\nu}_\ell$, $D \rightarrow Y \pi^+$, where the state $Y$ is so heavy that the charged pion is emitted at low momentum, behaving like a $\pi^+_s$. This possibility has been extensively studied by the CLEO Collaboration [13], where the three $D^*$ decay modes most likely to cause this distortion have been identified: $K^{*0} \omega \pi^+, K^{*0} \rho^+ \pi^+$, and $K^{*0} \rho^0 \pi^+$. If we remove these events from the simulated
sample, and we repeat the fit, the number of fitted signal events increases by 0.4%. We assume therefore ±0.4% systematic error on the fraction of signal events in the sample due to this uncertainty.

Figure 3 shows the $\mathcal{M}_B^2$ fit results for unmixed (upper) and mixed (lower) events. We use the results of this study to determine the fraction of continuum ($f_{qq}^+$), $B\bar{B}$ combinatorial ($f_{BB}^+$), and peaking $B^-$ ($f_{B^-}^+$) background as a function of $\mathcal{M}_B^2$, separately for mixed ($f^+$) and unmixed ($f^-$) events. We parametrize these fractions with polynomial functions of $\mathcal{M}_B^2$ as shown in Fig. 4.

D. $\tau_{B^0}$ and $\Delta m_d$ determination

We fit data and Monte Carlo events with a binned maximum-likelihood method. We divide the events into 100 $\Delta t$ bins, spanning the range $-18 \, \text{ps} < \Delta t < 18 \, \text{ps}$, and 20 $\sigma_{\Delta t}$ bins between 0 and 3 ps. We assign to all events in each bin the values of $\Delta t$ and $\sigma_{\Delta t}$ corresponding to the center of the bin. We fit simultaneously the mixed and unmixed events. We maximize the likelihood

$$\mathcal{L} = \left( \frac{\mathcal{N}_{\text{max}}}{\prod_{k=1}^{\mathcal{N}_{\text{max}}} F_k^+} \right) \left( \frac{\mathcal{N}_{\text{max}}}{\prod_{j=1}^{\mathcal{N}_{\text{max}}} F_j} \right) \times C_{\tau^+} \times C_{\tau^-},$$

where the indices $k$ and $j$ denote the unmixed and mixed selected events. The functions $F^\pm(\Delta t, \sigma_{\Delta t}, \mathcal{M}_B^2, \Delta m_d, \rho)$ describe the normalized $\Delta t$ distribution as the sum of the decay probabilities for signal and background events:

![Graph](image-url)
where the functions \( F_i \) represent the probability density functions (PDF) for signal (\( i = B^0 \)), peaking \( B^- (i = B^-) \), \( B\bar{B} \) combinatorial (\( i = B\bar{B} \)), and continuum (\( i = q\bar{q} \)) events, modified to account for the finite resolution of the detector, and the superscript (+) applies to unmixed (mixed) events. The resolution function is expressed as

\[
\mathcal{R}(\Delta t, \sigma_{\Delta t}) = \frac{1 - f_{\text{w}} - f_{\text{o}}}{\sqrt{2\pi S_{\text{w}}\sigma_{\Delta t}}} e^{-\frac{(\Delta t-o_{\text{w}})^2}{2S_{\text{w}}^2\sigma_{\Delta t}^2}} + \frac{f_{\text{w}}}{\sqrt{2\pi S_{\text{w}}\sigma_{\Delta t}}} e^{-\frac{(\Delta t-o_{\text{w}})^2}{2S_{\text{w}}^2\sigma_{\Delta t}^2}} + \frac{f_{\text{o}}}{\sqrt{2\pi S_{\text{o}}}} e^{-\frac{(\Delta t-o_{\text{o}})^2}{2S_{\text{o}}^2}},
\]

where \( \Delta t \) is the difference between the measured and true values of \( t \), \( o_{\text{w}} \) and \( o_{\text{o}} \) are offsets, and the factors \( S_{\text{w}} \) and \( S_{\text{o}} \) account for possible misestimation of \( \sigma_{\Delta t} \). The outlier term, described by a Gaussian function of fixed width \( S_{\text{o}} \) and offset \( o_{\text{o}} \), is introduced to describe events with badly measured \( t \), and accounts for less than 1% of the events.

To account for the ±50% uncertainty on the isospin assumption (see Sec. III C), the functions \( f_{B^-} \) and \( f_{\bar{B}^-} \) are multiplied in the PDF for the peaking \( B^- \) background by a common scale factor \( S_{B^-} \). This parameter is allowed to vary in the fit, constrained to unity with variance \( \sigma_{B^-} = 0.5 \) by means of the Gaussian term

\[
C_{S_{B^-}} = e^{-\frac{(S_{B^-}-1)^2}{2\sigma_{B^-}^2}}.
\]

We constrain the expected fraction \( P_{\text{exp}} \) of mixed events to the observed one

\[
P_{\text{obs}} = \frac{N_{\text{mix}}}{N_{\text{mix}} + N_{\text{unmix}}} \]

by means of the binomial factor

\[
C_{\chi_d} = \frac{N!}{N_{\text{mix}}!N_{\text{unmix}}!} P_{\text{exp}}^{N_{\text{mix}}} (1 - P_{\text{exp}})^{N_{\text{unmix}}},
\]

For a sample of signal events with dilution \( D \), the expected fraction reads

\[
P_{\text{exp}}(\Delta m_d, \tau_{B^0}, D) = \chi_d \cdot D + \frac{1 - D}{2},
\]

where, neglecting the decay-rate difference \( \Delta \Gamma_d \) between the two mass eigenstates, the integrated mixing rate \( \chi_d \) is related to the product \( x = \Delta m_d \cdot \tau_{B^0} \) by the relation

\[
\chi_d = \frac{x^2}{2(1 + x^2)}.
\]

We divide signal events according to the origin of the tag lepton into primary (\( P\ell \)), cascade (\( C\ell \)), and decay-side (\( D\ell \)) lepton tags. A primary lepton tag is produced in the direct decay \( B^0 \rightarrow \ell^+ \nu_{\ell} X \). These events are described by Eq. (1), with \( D \) close to 1 (a small deviation from unity is expected due to hadron misidentification, leptons from \( J/\psi \), etc.). We expect small values of \( o_{\text{n}} \) and \( o_{\text{w}} \) for primary tags, because the lepton originates from the \( B^0 \) decay point.

Cascade lepton tags, produced in the process \( B^0 \rightarrow DX, D \rightarrow \ell Y \), are suppressed by the requirement on the lepton momentum but still exist at a level of 9%, which we determine by varying their relative abundance \( f_{C\ell} \) as an additional parameter in the \( \Delta m_d \) and \( \tau_{B^0} \) fit on data. The cascade lepton production point is displaced from the \( B^0 \) decay point due to the finite lifetime of charm mesons and the \( e^+e^- \) energy asymmetry. This results in a significant negative value of the offsets for this category. Compared with the primary lepton tag, the cascade lepton is more likely to have the opposite charge correlation with the \( B^0 \) flavor. The same charge correlation is obtained when the charm meson is produced from the hadronization of the virtual \( W \) from \( B^0 \) decay, which can result in the production of two opposite-flavor charm mesons. We account for these facts by applying Eq. (1) to the cascade tag events with negative dilution \( D_{C\ell} = -(1 - 2f_{C\ell}) \tau_{B^0} = -0.65 \pm 0.08 \), where we take from the PDG [14] the ratio

\[
f_{C\ell} = \frac{B(\bar{b} \rightarrow c \rightarrow \ell^+)}{B(\bar{b} \rightarrow c \rightarrow \ell^+) + B(\bar{b} \rightarrow \tau \rightarrow \ell^-)} = 0.17 \pm 0.04.
\]

The contribution to the dilution from other sources associated with the \( \pi^+ \ell^- \) candidate, such as fake hadrons, is negligible.

Decay-side tags are produced by the semileptonic decay of the unconstructed \( D^0 \). Therefore, they do not carry any information about \( \tau_{B^0} \) or \( \Delta m_d \). The PDF for both mixed and unmixed contributions is a purely exponential function, with an effective lifetime \( \tau_D \), representing the displacement of the lepton production point from the \( B^0 \) decay point due to the finite lifetime of the \( B^0 \). We determine the fraction of these events by fitting the \( \cos \theta_{\pi^+\ell^-} \) distribution [see plots in Figs. 5(a) and 6(a)], where \( \theta_{\pi^+\ell^-} \) is the angle between the soft pion and the tag lepton in the \( e^+e^- \) rest frame. We fit the data with the sum of the histograms for signal events, \( B\bar{B} \) combinatorial background, and peaking \( B^- \) background obtained from the simulation, and continuum background obtained from the off-resonance events. We fix the fraction of signal events, peaking \( B^- \) background, \( B\bar{B} \) combinatorial background and continuum.
background in the fit and we allow to vary the relative amount of decay-side tags and tag-side tags.

Using the results of the \( \cos \theta_{\tau_l^e} \) fit we parametrize the probability for each event to have a decay-side tag as a third-order polynomial function of \( \cos \theta_{\tau_l^e} \) [see plots in Figs. 5(b) and 6(b)].

The signal PDF for both mixed and unmixed events consists of the sum of PDFs for primary, cascade, and decay-side tags, each convoluted with its own resolution function. The parameters \( S_p, S_w, S_o, f_w, \) and \( f_o \) are common to the three terms, but each tag type has different offsets \( (u_p, u_o) \). All the parameters of the resolution functions, the dilution of the primary tags, the fraction of cascade tags, and the effective lifetime of the decay-side tags are free parameters in the fit. We fix the other parameters (dilution of cascade tags, fraction of decay-side tags) to the values obtained as described above, and then vary them within their uncertainties to assess the corresponding systematic error.

We adopt a similar PDF for peaking \( B^- \) background, with separate primary, cascade, and decay-side terms. Because \( B^- \) mesons do not oscillate, we use a pure exponential PDF for the primary and cascade tags with lifetime \( \tau_{B^-} = 1.671 \) ps [15]. We force the parameters of the resolution function to equal those for the corresponding signal term.

We describe continuum events with an exponential function convoluted with a three-Gaussian resolution function. The mixed and unmixed terms have a common effective lifetime \( \tau_{B^0} = 1.57 \) ps [15]. All the parameters of the continuum resolution function are set equal to those of the signal, except for the offsets, which are free in the fit.

The PDF for combinatorial \( B \bar{B} \) background accounts for oscillating and nonoscillating subsamples. It has the same functional form as the PDF for peaking events, but with
independent parameters for the oscillation frequency, the lifetimes and the fractions of $B^-$ background, primary, cascade, and decay-side tag events. The parameters $S_\text{p}^\ast$, $f_w$ and $o_o$ are set to the same values as those in the signal PDF.

IV. RESULTS

We first apply the measurement procedure on several Monte Carlo samples. We validate each term of the PDF by first fitting signal events, for primary, cascade and decay-side tags separately, and then adding them together. We then add peaking $B^-$ background, and finally add the $B\bar{B}$ combinatorial background. We observe the following features:

(i) The event selection introduces no bias on $\tau_{B^0}$ and a bias of $(-0.0029 \pm 0.0010) \text{ ps}^{-1}$ on $\Delta m_d$.

(ii) The boost approximation introduces a bias on $\tau_{B^0}$ ($+0.0054 \text{ ps}$) and an additional bias on $\Delta m_d$ ($-0.0034 \text{ ps}^{-1}$), determined by fitting the true $\Delta z$ distribution. These biases disappear however when we fit the smeared $\Delta z$ and allow for the experimental resolution in the fit function.

(iii) After the introduction of $B^-$ peaking background we observe a bias of $(-0.0079 \pm 0.0064) \text{ ps}$ on $\tau_{B^0}$ and $(-0.0034 \pm 0.0028) \text{ ps}^{-1}$ on $\Delta m_d$.

(iv) Adding combinatorial $B\bar{B}$ events, we observe a bias of $(+0.0063 \pm 0.0070) \text{ ps}$ on $\tau_{B^0}$ and $(+0.0074 \pm 0.0035) \text{ ps}^{-1}$ on $\Delta m_d$.

(v) The isospin scale factor $S_{B^-} = 0.91 \pm 0.10$ is consistent with unity.

Based on these observations, we correct the data results by subtracting 0.0063 ps from $\tau_{B^0}$, and adding 0.0074 ps$^{-1}$ to $\Delta m_d$. We include the Monte Carlo statistical errors of $\pm 0.0070$ ps for $\tau_{B^0}$ and $\pm 0.0035$ ps$^{-1}$ for $\Delta m_d$ as systematic uncertainties.

We determine the parameters for continuum events directly from the fit to on-resonance data, and we independently fit the off-resonance events to verify the consistency with the on-resonance continuum results.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Usage</th>
<th>Data</th>
<th>MC</th>
<th>Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{B^0}$ (ps)</td>
<td>Free</td>
<td>$1.510 \pm 0.013$</td>
<td>$1.554 \pm 0.007$</td>
<td>$B^0 P\ell, C\ell$</td>
</tr>
<tr>
<td>$\Delta m_d$ (ps$^{-1}$)</td>
<td>Free</td>
<td>$0.503 \pm 0.007$</td>
<td>$0.465 \pm 0.004$</td>
<td>$B^0 P\ell, C\ell$</td>
</tr>
<tr>
<td>$\tau_{B^-}$ (ps)</td>
<td>Free</td>
<td>$0.12 \pm 0.04$</td>
<td>$0.21 \pm 0.02$</td>
<td>$B^- D\ell$</td>
</tr>
<tr>
<td>$S_{B^-}$</td>
<td>Const.</td>
<td>$1.671 \pm 0.018$</td>
<td>$1.65$</td>
<td>$B^-, B\bar{B}$</td>
</tr>
<tr>
<td>$f_{\text{p}}$</td>
<td>Free</td>
<td>$0.095 \pm 0.006$</td>
<td>$0.066 \pm 0.004$</td>
<td>$B_\text{p}^0$</td>
</tr>
<tr>
<td>$D_{P\ell}$</td>
<td>Free</td>
<td>$1.00 \pm 0.02$</td>
<td>$0.970 \pm 0.006$</td>
<td>$B_\text{p}^0 P\ell$</td>
</tr>
<tr>
<td>$D_{C\ell}$</td>
<td>Free</td>
<td>$-0.65 \pm 0.08$</td>
<td>$-0.545$</td>
<td>$B_\text{p}^0 C\ell$</td>
</tr>
<tr>
<td>$o_{P\ell,s}$</td>
<td>Free</td>
<td>$-0.019 \pm 0.011$</td>
<td>$-0.012 \pm 0.007$</td>
<td>$B_\text{p}^0, B^-, P\ell$</td>
</tr>
<tr>
<td>$o_{C\ell,n}$ (Fixed)</td>
<td>$-0.18 \pm 0.07$</td>
<td>$-0.43 \pm 0.07$</td>
<td>$B_\text{p}^0 C\ell$</td>
<td></td>
</tr>
<tr>
<td>$o_{C\ell,w}$</td>
<td>Free</td>
<td>$2.8 \pm 1.1$</td>
<td>$-5.8 \pm 0.7$</td>
<td>$B_\text{p}^0 C\ell$</td>
</tr>
<tr>
<td>$o_{D\ell,n}$</td>
<td>Free</td>
<td>$-0.12 \pm 0.03$</td>
<td>$-0.14 \pm 0.02$</td>
<td>$B_\text{p}^0, B^-, D\ell$</td>
</tr>
<tr>
<td>$S_{o}$</td>
<td>Free</td>
<td>$0.952 \pm 0.015$</td>
<td>$1.007 \pm 0.006$</td>
<td>All</td>
</tr>
<tr>
<td>$S_{o}$ (ps)</td>
<td>Free</td>
<td>$12.8 \pm 5.6$</td>
<td>$17.9 \pm 8.2$</td>
<td>$B_\text{p}^0, B^-$</td>
</tr>
<tr>
<td>$f_{w}$</td>
<td>Free</td>
<td>$0.0013 \pm 0.0005$</td>
<td>$0.0008 \pm 0.0003$</td>
<td>$B_\text{p}^0, B^-$</td>
</tr>
<tr>
<td>$o_o$</td>
<td>Fixed</td>
<td>$2.57 \pm 0.13$</td>
<td>$2.63 \pm 0.15$</td>
<td>All</td>
</tr>
<tr>
<td>$f_w$</td>
<td>Free</td>
<td>$0.050 \pm 0.005$</td>
<td>$0.035 \pm 0.005$</td>
<td>All</td>
</tr>
<tr>
<td>$o_o$</td>
<td>Fixed</td>
<td>$0$</td>
<td>$0$</td>
<td>All</td>
</tr>
</tbody>
</table>
TABLE II. Parameters used in the background PDF. The upper set of parameters refers to $B\bar{B}$ combinatorial events, the central one refers to continuum parameters, and the lower set refers to those parameters of the resolution function that are common to all event types. The symbols $\alpha$ correspond to the fractions of decay-side tags in the different samples. The last line shows the statistical correlation between $\tau_{B^0}$ and $\Delta m_d$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Usage</th>
<th>Data</th>
<th>M.C.</th>
<th>Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{BKG}$ (ps)</td>
<td>Free</td>
<td>$1.22 \pm 0.07$</td>
<td>$1.37 \pm 0.07$</td>
<td>$B\bar{B}$</td>
</tr>
<tr>
<td>$\Delta m_d^{BKG}$ (ps$^{-1}$)</td>
<td>Free</td>
<td>$0.37 \pm 0.06$</td>
<td>$0.42 \pm 0.04$</td>
<td>$B\bar{B}$</td>
</tr>
<tr>
<td>$\tau_{B^0}^{BKG}$ (ps)</td>
<td>Fixed</td>
<td>$1.671 \pm 0.018$</td>
<td>1.65</td>
<td>$B^-$, $B\bar{B}$</td>
</tr>
<tr>
<td>$f_{C,a}$</td>
<td>Fixed</td>
<td>$0.030 \pm 0.006$</td>
<td>0.030</td>
<td>$B\bar{B}$ ($B^0$ only)</td>
</tr>
<tr>
<td>$\alpha_{BKG}$</td>
<td>Free</td>
<td>$0.041 \pm 0.022$</td>
<td>$0.069 \pm 0.021$</td>
<td>$B\bar{B}$ ($B^0$ only)</td>
</tr>
<tr>
<td>$f_{B^-,a}$</td>
<td>Free</td>
<td>$0.62 \pm 0.08$</td>
<td>$0.52 \pm 0.02$</td>
<td>$B\bar{B}$</td>
</tr>
<tr>
<td>$f_{B^-,m}$</td>
<td>Free</td>
<td>$0.15 \pm 0.10$</td>
<td>$0.11 \pm 0.04$</td>
<td>$B\bar{B}$</td>
</tr>
<tr>
<td>$\alpha_{B_{0,n}}$</td>
<td>Free</td>
<td>$0.11 \pm 0.19$</td>
<td>$0.25 \pm 0.03$</td>
<td>$B\bar{B}$ ($B^0$ only)</td>
</tr>
<tr>
<td>$\alpha_{B_{0,m}}$</td>
<td>Fixed</td>
<td>$0.065 \pm 0.013$</td>
<td>0.065</td>
<td>$B\bar{B}$ ($B^0$ only)</td>
</tr>
<tr>
<td>$\alpha_{B^-}$</td>
<td>Free</td>
<td>$0.21 \pm 0.10$</td>
<td>$0.20 \pm 0.02$</td>
<td>$B\bar{B}$ ($B^-$ only)</td>
</tr>
<tr>
<td>$\alpha_{B^-}$</td>
<td>Fixed</td>
<td>$0.36 \pm 0.07$</td>
<td>$0.36$</td>
<td>$B\bar{B}$ ($B^-$ only)</td>
</tr>
<tr>
<td>$f_{D\ell,2}$</td>
<td>Fixed</td>
<td>$0.60 \pm 0.12$</td>
<td>$0.60 \pm 0.12$</td>
<td>$B\bar{B}$, $D\ell$</td>
</tr>
<tr>
<td>$D_{BKG}$</td>
<td>Free</td>
<td>$0.989 \pm 0.013$</td>
<td>$0.964 \pm 0.006$</td>
<td>$B\bar{B}$ ($B^0$ only)</td>
</tr>
<tr>
<td>$D_{BKG}$</td>
<td>Fixed</td>
<td>$-0.65 \pm 0.08$</td>
<td>$-0.545$</td>
<td>$B\bar{B}$ ($B^0$ only)</td>
</tr>
<tr>
<td>$o_{BKG}$</td>
<td>Free</td>
<td>$-0.006 \pm 0.016$</td>
<td>$-0.02 \pm 0.03$</td>
<td>$B\bar{B}$, $P\ell$</td>
</tr>
<tr>
<td>$o_{BKG}$</td>
<td>Free</td>
<td>$-1.6 \pm 0.6$</td>
<td>$-0.8 \pm 0.2$</td>
<td>$B\bar{B}$, $C\ell$</td>
</tr>
<tr>
<td>$o_{D\ell}$</td>
<td>Free</td>
<td>$-0.02 \pm 0.04$</td>
<td>$-0.05 \pm 0.03$</td>
<td>$B\bar{B}$, $D\ell$</td>
</tr>
<tr>
<td>$S^{BKG}$</td>
<td>Free</td>
<td>$0.961 \pm 0.015$</td>
<td>$0.961 \pm 0.021$</td>
<td>All</td>
</tr>
<tr>
<td>$S_B^{BKG}$ (ps)</td>
<td>Free</td>
<td>$10.4 \pm 3.6$</td>
<td>$14.7 \pm 6.1$</td>
<td>$B\bar{B}$</td>
</tr>
<tr>
<td>$f_{BKG}$</td>
<td>Free</td>
<td>$0.0021 \pm 0.0009$</td>
<td>$0.0008 \pm 0.0003$</td>
<td>$B\bar{B}$</td>
</tr>
<tr>
<td>$\tau_{B^0}$ (ps)</td>
<td>Free</td>
<td>$0.27 \pm 0.05$</td>
<td>-</td>
<td>Continuum</td>
</tr>
<tr>
<td>$\alpha_{B_{0,n}}$</td>
<td>Free</td>
<td>$0.007 \pm 0.032$</td>
<td>-</td>
<td>Continuum</td>
</tr>
<tr>
<td>($= \alpha_{B_{0,n}}$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_{\nu}$</td>
<td>Free</td>
<td>$2.57 \pm 0.13$</td>
<td>$2.63 \pm 0.15$</td>
<td>All</td>
</tr>
<tr>
<td>$f_{\nu}$</td>
<td>Free</td>
<td>$0.050 \pm 0.005$</td>
<td>$0.035 \pm 0.005$</td>
<td>All</td>
</tr>
<tr>
<td>$o_{\nu}$</td>
<td>Fixed</td>
<td>0</td>
<td>0</td>
<td>All</td>
</tr>
</tbody>
</table>

$\rho(\tau_{B^0}, \Delta m_d) = 0.007 - 0.127$

We finally perform the fit to the on-resonance data. Together with $\Delta m_d$ and $\tau_{B^0}$, we allow to vary most of the parameters describing the peaking $B^-$, $B\bar{B}$ combinatorial, and continuum background events. The results of the fits to the Monte Carlo and data samples are shown in Tables I and II.

The fit results are $\tau_{B^0} = (1.510 \pm 0.013) \text{ps}$, and $\Delta m_d = (0.5035 \pm 0.0068) \text{ps}^{-1}$. We correct these values for the biases measured in the Monte Carlo simulation, obtaining the results

$\tau_{B^0} = (1.504 \pm 0.013) \text{ps}$,
$\Delta m_d = (0.5109 \pm 0.0068) \text{ps}^{-1}$.

The statistical correlation between $\Delta m_d$ and $\tau_{B^0}$ is 0.7%. $\Delta m_d$ has sizable correlations with $S_{\nu}$ (50%) and with the fraction of cascade tags (24%). $\tau_{B^0}$ is correlated with $S_{\nu}$ ($-27\%$) and the offset of the wide Gaussian for the cascade tags ($-31\%$). The complete set of fit parameters is reported in Tables I and II.

Details on the systematic error are reported in Sec. V. Figures 7 and 8 show the comparison between the data and the fit function projected on $\Delta t$, for a sample of events enriched in signal by the cut $M^2_{d} > -2.5 \text{GeV}^2/c^4$; Figs. 9 and 10 show the same comparison for events in the background region.

Figures 11 and 12 show plots of the time-dependent asymmetry.
for events in the $M^2$ signal region and events in the $M^2$ background region. For signal events, neglecting $\Delta t$ resolution, $A(\Delta t) = D \cos(\Delta m_d \Delta t)$ [see Eq. (1)].

The agreement between the fit function and the data distribution is good in both the signal and background regions. The asymmetry is quite significant for events in the background $M^2$ region because a large fraction of these events are due to combinatorial $B\bar{B}$ background.

V. SYSTEMATIC UNCERTAINTIES

The systematic errors are summarized in Table III. We consider the following sources of systematic uncertainty:

1. Sample composition.—We calculate a total uncertainty of $\pm 1.3\%$ on the number of signal events. This uncertainty is the quadratic sum of the statistical error in the $M^2$ fit ($\pm 1.2\%$), the systematic uncertainty on the shape of $B\bar{B}$ combinatorial back-
ground from the test on the “wrong-charge” sample (± 0.2%) (see Sec. III A), and the additional systematic uncertainty due to low-momentum pions from $D^+$ decays (± 0.4%) (see Sec. III C).

(2) Analysis bias (entry b).—We use the statistical error on the bias observed in the fit on the Monte Carlo sample.

(3) Signal and background PDF description.—Most of the parameters in the PDF are free in the fit and therefore do not contribute to the systematic error. We vary the parameters that are fixed in the fit by their uncertainty, repeat the fit, and use the corresponding variation in $\tau_{B^0}$ and $\Delta m_d$ as systematic errors. We take the uncertainty on $\tau_{B^-}$ (entry c), and on $D_{Cl}$ (entry d) from the PDG [14]. We find that four parameters used in the description of the combinatorial background, as determined by the fit on the Monte Carlo sample, are not in agreement with the Monte Carlo truth. They are the fraction of cascade tag-side leptons in the unmixed event sample, $f_{B^{0}\text{K}}^{B^{0}\text{K}}$, the fraction of decay-side tags in the mixed $\overline{B}^0$ and the $B^-$ event samples, $\alpha_{B_{m}^{0}}^{B_{m}^{0}}$ and $\alpha_{B^{0}_{m}B_{m}^{0}}^{B^{0}_{m}B_{m}^{0}}$, respectively, and an additional parameter used in the description of the shape of the proper time difference $\Delta t$ of the decay-side tagged mixed sample, $f_{\Delta t}^{D_{Cl}}$. Therefore we fix them to the Monte Carlo prediction. We vary the value of each of them by 20% to compute the systematic error from the comparison with the default result, and we sum the four uncertainties in quadrature (entry e).

(4) Detector alignment.—We consider effects due to the detector $z$ scale, determined by reconstructing protons scattered from the beam pipe and comparing the measured beam pipe dimensions with the optical survey data [16]. The $z$ scale indetermination corresponds to an uncertainty of ± 0.4% on $\Delta t$. We repeat the fit applying this scale correction to $\Delta t$, and use the variation with respect to the default result as the systematic error (entry f). From the measurement of the beam energies, the Y(4S) Lorentz boost factor is determined with an uncertainty which translates into a ± 0.1% indetermination on $\Delta t$. Again we repeat the fit and assume as systematic error the variation of the result (entry g). We then consider the effect of varying the beam-spot position by ± 40 $\mu$m in the y direction (entry h). We compute the uncertainty due to SVT time-dependent misalignment by comparing

---

**TABLE III.** Systematic uncertainties. See text for details.

<table>
<thead>
<tr>
<th>Source</th>
<th>Variation</th>
<th>$\delta \tau_{B^0}$ (ps)</th>
<th>$\delta \Delta m_d$ (ps$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Sample Composition</td>
<td>± 1.3%</td>
<td>±0.0003</td>
<td>±0.0002</td>
</tr>
<tr>
<td>(b) Analysis bias</td>
<td>± 0.0070</td>
<td>±0.0035</td>
<td></td>
</tr>
<tr>
<td>(c) $\tau_{B^-}$</td>
<td>1.671 ± 0.018</td>
<td>±0.0014</td>
<td>±0.0008</td>
</tr>
<tr>
<td>(d) $D_{Cl}$</td>
<td>0.65 ± 0.08</td>
<td>±0.0003</td>
<td>±0.0003</td>
</tr>
<tr>
<td>(e) Combinatorial BKG</td>
<td>±0.0007</td>
<td>±0.0002</td>
<td></td>
</tr>
<tr>
<td>(f) $z$ scale</td>
<td>±0.0070</td>
<td>±0.0020</td>
<td></td>
</tr>
<tr>
<td>(g) PEP-II boost</td>
<td>±0.0020</td>
<td>±0.0003</td>
<td></td>
</tr>
<tr>
<td>(h) Beam-spot position</td>
<td>±0.0050</td>
<td>±0.0010</td>
<td></td>
</tr>
<tr>
<td>(i) Alignment</td>
<td>+0.0132</td>
<td>-0.0038</td>
<td></td>
</tr>
<tr>
<td>(j) Decay-side tags</td>
<td>±0.0013</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(k) Binning</td>
<td>±0.0021</td>
<td>±0.0006</td>
<td></td>
</tr>
<tr>
<td>(l) Outlier parameters</td>
<td>±0.0028</td>
<td>±0.0012</td>
<td></td>
</tr>
<tr>
<td>(m) $\Delta t$ and $\sigma_{\Delta t}$ cut</td>
<td>±0.0033</td>
<td>±0.0033</td>
<td></td>
</tr>
<tr>
<td>(n) GExp model</td>
<td>-0.0016</td>
<td>+0.0011</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>+0.0182</td>
<td>+0.0068</td>
<td>-0.0131</td>
</tr>
</tbody>
</table>

---

FIG. 11. Asymmetry between unmixed and mixed events as a function of $\Delta t$, for events in the signal $M_2^z$ region. Points with error bars represent the data, and the curve is a projection of the fit result.

FIG. 12. Asymmetry between unmixed and mixed events as a function of $\Delta t$, for events in the background $M_2^z$ region. Points with error bars represent the data, and the curve is a projection of the fit result.
results obtained with different sets of alignment constants (entry i).

5. **Decay-side tags.**—We vary the parameters describing the fraction of decay-side tags by their statistical errors, repeat the fit, and take the variation with respect to the default result as the systematic error (entry j).

6. **Binned fitting.**—We vary the number of bins in \( \Delta t \) from 100 to 250 and in \( \sigma_{\Delta t} \) from 20 to 50, and we repeat the fit. We take the systematic error to be the variation with respect to the default result (entry k).

7. **Outlier description.**—We vary the value of the offset of the outlier Gaussian from \(-5 \text{ ps}\) to \(+5 \text{ ps}\). As a cross-check, we use a PDF that is uniform in \( \Delta t \) for the description of the outliers. We take the maximum variation with respect to the default result as the systematic error (entry l).

8. **Fit range.**—We vary the \( \Delta t \) fit range from \(\pm 18 \text{ ps}\) to \(\pm 10 \text{ ps}\) and the \( \sigma_{\Delta t} \) maximum value from 1.8 to 4.2 ps. Again we assume the maximum variation between the various results and the default one as the systematic error (entry m).

9. **Cascade lepton tag-side parametrization.**—For the resolution model, we use a Gaussian distribution convolved with a one-sided exponential to describe the core part of the resolution function (GExp) instead of the Gaussian resolution with a nonzero offset. We quote as systematic error the difference between the results obtained with the two different approaches (entry n).

### VI. CONSISTENCY CHECKS

We rely on the assumption that the parameters of the background PDF do not depend on \( \mathcal{M}_B^2 \). We verify this assumption for the continuum background with the fit to the off-resonance events. To check this assumption for the \( B\bar{B} \) combinatorial PDF, we perform several cross checks on the data and the Monte Carlo. We compare the simulated combinatorial \( B\bar{B} \) \( \Delta t \) distribution in several independent regions of \( \mathcal{M}_B^2 \) with Kolmogorov-Smirnov tests and always obtain a reasonable probability for agreement. We fit the \( \Delta t \) distribution of combinatorial background \( B\bar{B} \) events separately in the signal and background \( \mathcal{M}_B^2 \) region and compare the parameters of the PDF. We fit the signal plus \( B\bar{B} \) background Monte Carlo events in the signal region only, fixing all the parameters of the \( B\bar{B} \) background to the values obtained in a fit to the background region, and do not see any significant deviation from the results of the full fit. Finally, we repeat the fit on both the data and the Monte Carlo using different \( \mathcal{M}_B^2 \) ranges for the background region. Once again, we do not observe any significant difference in \( \tau_{B^0} \) and \( \Delta m_d \) relative to the default result.

We repeat the analysis with a more stringent requirement on the combined signal likelihood (a minimum \( X \) of 0.5 rather than 0.4). No significant change in the result is observed.

We validate the fit procedure with a parametrized Monte Carlo simulation. We simulate several experiments from the fitted PDF of both the Monte Carlo and the data, with parameters fixed to the values obtained from the corresponding fit. Each experiment is produced with the same number of events as the original sample. For each experiment we produce seven data sets, corresponding to \( B\bar{B} \) with primary, cascade, and decay-side lepton tags, peaking \( B^- \) background with tag-side and decay-side lepton tags, \( B\bar{B} \) combinatorial background, and continuum background. We fit every experiment with the same procedure as the corresponding original sample, and finally we compare the fitted parameters with the generated values. The result of this study is summarized in Table IV where we report the average and the root-mean-square deviation (rms) of the distribution of the difference between the fitted and the generated parameter value divided by the fit statistical error (pull). We do not find any significant statistical anomaly in the fit behavior.

We rely on the assumption that the decay-rate difference \( \Delta \Gamma_d \) between the two mass eigenstates can be neglected in the analysis. We check this assumption with a parametrized Monte Carlo simulation in which events are simulated with zero mistag probability and perfect \( \Delta t \) resolution. We produce two sets of 100 Monte Carlo experiments. In the first set, \( \Delta \Gamma_d = 0 \); in the second, \( \Delta \Gamma_d = 0.01 \). We fit every experiment with the same procedure neglecting \( \Delta \Gamma_d \) and we do not find any significant difference in the values of \( \tau_{B^0} \) and \( \Delta m_d \) in the two different sets.

We investigate a possible analysis bias due to the finite \( \tau \) and \( D_s \) lifetimes in \( B^0 \to D^{*+} \tau^- \bar{\nu}_\tau \) (\( \tau^- \to \ell^- X \)) and \( B^0 \to D^{*+} D_s^- \) (\( D_s^- \to \ell^- X \)) decays. We fit the Monte Carlo signal sample with no mistag and realistic \( \Delta t \) resolution after removing these decays and we do not find any significant variation with respect to the result obtained with the full signal sample.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Data</th>
<th>Monte Carlo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of experiments</td>
<td>54</td>
<td>124</td>
</tr>
<tr>
<td>Pull ( \tau_{B^0} ) average</td>
<td>( 0.38 \pm 0.19 )</td>
<td>( 0.38 \pm 0.12 )</td>
</tr>
<tr>
<td>Pull ( \tau_{B^0} ) rms</td>
<td>( 1.13 \pm 0.19 )</td>
<td>( 1.25 \pm 0.13 )</td>
</tr>
<tr>
<td>Pull ( \Delta m_d ) average</td>
<td>( -0.33 \pm 0.17 )</td>
<td>( 0.08 \pm 0.11 )</td>
</tr>
<tr>
<td>Pull ( \Delta m_d ) rms</td>
<td>( 1.14 \pm 0.17 )</td>
<td>( 1.09 \pm 0.08 )</td>
</tr>
</tbody>
</table>
VII. CONCLUSION

We have performed a measurement of $\Delta m_d$ and $\tau_{B^0}$ with a sample of about 50,000 partially reconstructed, lepton-tagged $B^0 \to D^{* \pm} \ell^- \bar{\nu}_\ell$ decays. We obtain the following results:

$$\tau_{B^0} = (1.504 \pm 0.013(\text{stat})^{+0.018}_{-0.013}(\text{syst})) \text{ ps},$$

$$\Delta m_d = (0.511 \pm 0.007(\text{stat})^{+0.007}_{-0.006}(\text{syst})) \text{ ps}^{-1}.$$  

The $\tau_{B^0}$ value is consistent with the published measurement performed by BABAR using $B^0 \to D^{* \pm} \ell^- \bar{\nu}_\ell$ partially reconstructed decays [5]. Our results are also consistent with published measurements of $\tau_{B^0}$ and $\Delta m_d$ performed by BABAR with different data sets [6,17–20], and with the world averages computed by the Heavy Flavor Averaging Group for the PDG 2005 web update: $\tau_{B^0} = (1.532 \pm 0.009) \text{ ps}$, and $\Delta m_d = (0.505 \pm 0.005) \text{ ps}^{-1}$.

ACKNOWLEDGMENTS

We are grateful for the extraordinary contributions of our PEP-II colleagues in achieving the excellent luminosity and machine conditions that have made this work possible. The success of this project also relies critically on the expertise and dedication of the computing organizations that support BABAR. The collaborating institutions wish to thank SLAC for its support and the kind hospitality extended to them. This work is supported by the U.S. Department of Energy and National Science Foundation, the Natural Sciences and Engineering Research Council (Canada), Institute of High Energy Physics (China), the Commissariat à l’Energie Atomique and Institut National de Physique Nucléaire et de Physique des Particules (France), the Bundesministerium für Bildung und Forschung and Deutsche Forschungsgemeinschaft (Germany), the Istituto Nazionale di Fisica Nucleare (Italy), the Foundation for Fundamental Research on Matter (The Netherlands), the Research Council of Norway, the Ministry of Science and Technology of the Russian Federation, and the Particle Physics and Astronomy Research Council (United Kingdom). Individuals have received support from CONACyT (Mexico), the A.P. Sloan Foundation, the Research Corporation, and the Alexander von Humboldt Foundation.

[2] Charge conjugate states are always implicitly assumed; $\ell$ means either electron or muon.
[12] Throughout the paper the momentum, energy and direction of all particles are computed in the $e^+e^-$ rest frame.