Measurement of the Time Dependence of $B^0_d \leftrightarrow \bar{B}^0_d$ Mixing using Leptons and $D^{\ast\pm}$ Mesons

The OPAL Collaboration

Abstract

Data collected with the OPAL detector at LEP during 1990–1993 are used to measure the time dependence of $B^0_d \leftrightarrow \bar{B}^0_d$ mixing. From a sample of $153 \pm 12$ events with a charged $D^*$ and a lepton in the opposite hemisphere, we measure the $B^0_d \leftrightarrow \bar{B}^0_d$ oscillation frequency to be

$$\Delta m_d = 0.57 \pm 0.11(\text{stat.}) \pm 0.02(\text{syst.}) \text{ ps}^{-1}.$$ 

This corresponds to an oscillation parameter of $x_d = \Delta m_d \tau_{B^0_d} = 0.82 \pm 0.16(\text{stat.}) \pm 0.03(\text{syst.}) \pm 0.09(\text{syst. } \tau_{B^0_d})$, where the second systematic uncertainty is due to the error in the measured $B^0_d$ lifetime.

(To be submitted to Physics Letters B)
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1 Introduction

Particle-antiparticle mixing in the neutral B meson system is a well established phenomenon [1, 2]. In the Standard Model, the dominant contribution is from second order box diagrams [3]. A pure $B_d^0$ (or $\bar{B}_d^0$) flavor eigenstate produced at time $t = 0$ can be described in its center of mass frame in terms of the mass eigenstates, $B_1$ and $B_2$, at a later time, $t$, as:

$$|B_d^0(t)\rangle = \frac{1}{\sqrt{2}} \left( e^{-i m_1 \frac{t}{\tau_1}} |B_1\rangle + e^{-i m_2 \frac{t}{\tau_2}} |B_2\rangle \right),$$

where $m_1$, $\tau_1$, $m_2$ and $\tau_2$ are the masses and lifetimes of the two mass eigenstates. Assuming the lifetimes are the same ($\tau_1 = \tau_2 = \tau_{B_d^0}$), the probabilities of observing a $B_d^0$ or a $\bar{B}_d^0$ at time $t$ are given as:

$$P_{B_d^0 - B_d^0}(t) = \frac{1}{2 \tau_{B_d^0}} e^{-t/\tau_{B_d^0}} \left[ 1 + \cos(\Delta m_d t) \right],$$
$$P_{B_d^0 - \bar{B}_d^0}(t) = \frac{1}{2 \tau_{B_d^0}} e^{-t/\tau_{B_d^0}} \left[ 1 - \cos(\Delta m_d t) \right],$$

where $\Delta m_d = |m_1 - m_2|$. The time integrated probability that a $B_d^0$ will decay as a $\bar{B}_d^0$ is given by

$$\chi_d = \frac{1}{2} \frac{x_d^2}{1 + x_d^2},$$

where $x_d = \Delta m_d \tau_{B_d^0}$.

This time integrated probability has been measured by the ARGUS [4] and CLEO [5] collaborations at the $\Upsilon(4S)$ resonance. Their combined result is $x_d = 0.158 \pm 0.026$ [6], corresponding to $x_d = 0.68 \pm 0.08$. Using $\tau_{B_d^0} = 1.44 \pm 0.15$ ps [7, 8], the corresponding frequency is $\Delta m_d = 0.472 \pm 0.055 \pm 0.049$ ps$^{-1}$, where the first uncertainty is the combined statistical and systematic error from the $x_d$ measurement and the second uncertainty is due to the error in the measured $B_d^0$ lifetime. Experiments at hadron colliders [1] and at LEP [2] have measured the average mixing rate integrated over time, $\chi$, which includes the contributions from both $B_d^0$ and $\bar{B}_d^0$ mixing.

The ALEPH collaboration reported the first observation of the time dependence of $B_d^0 \leftrightarrow \bar{B}_d^0$ mixing, with $\Delta m_d = 0.52^{+0.10}_{-0.11} \pm 0.04$ ps$^{-1}$ [9], using events with a reconstructed charged $D^*$ in one hemisphere and a lepton in the opposite hemisphere. There has been a subsequent measurement of $\Delta m_d = 0.50^{+0.07}_{-0.06} \pm 0.10$ ps$^{-1}$ [10] from ALEPH, using dilepton events. The OPAL collaboration has also measured the time dependence, with a result of $\Delta m_d = 0.508 \pm 0.075 \pm 0.025$ ps$^{-1}$ [11]. The decaying B mesons were reconstructed in the semileptonic channel $D^* \ell X$ and a jet charge technique was used to infer the particle-antiparticle flavor at production time.

Here we report on a measurement of the time dependence of $B_d^0 \rightarrow \bar{B}_d^0$ mixing with the OPAL detector at LEP, using partially reconstructed $B_d^0$ decays via $B_d^0 \rightarrow D^* \ell X$. The sign of

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1The convention $\bar{B} = \bar{B} = \bar{B}$ is used throughout this note.

2Throughout this note, all references to a particle or decay implicitly include the charge conjugate. The symbol $D^*$ is used to refer to charged $D^*$ mesons.
the charge of the D* tags the flavor at the time of decay. The flavor of the state at $t = 0$ is inferred by searching for a lepton from the semileptonic decay of a b-hadron in the opposite hemisphere ($b \to \ell^-X$).

2 The OPAL Detector

The OPAL detector is described in detail elsewhere [12]. Only the components relevant to this analysis are described here. A central detector system located in an axial magnetic field of 0.435 T is used for charged particle tracking. It consists of a high precision drift chamber, a large volume jet chamber and a set of $z$ chambers to measure track coordinates along the beam direction.\(^3\) For the 1991 run, two layers of silicon strip detectors were installed, with readout in the $r-\phi$ plane [13].\(^4\) Specific ionization measurement in the jet chamber, $dE/dx$, is used for charged particle identification. The $dE/dx$ resolution for tracks with the maximum number of samplings (159) is 3.5% [15]. Surrounding the magnet coil is an array of time-of-flight counters and a lead glass electromagnetic calorimeter with a presampler. The lead glass blocks of the calorimeter have a cross section of about 10 cm by 10 cm and a depth of about 25 radiation lengths. The magnet return yoke is instrumented with nine layers of streamer tubes which serve as a hadron calorimeter, and provide additional information for muon identification. Outside the hadron calorimeter are muon chambers which cover 93% of the full solid angle. A particle from the interaction point must traverse at least seven, and in most regions eight, interaction lengths of material to arrive at the muon chambers. Most muons with initial momenta larger than 3 GeV/$c$ penetrate this material.

3 Method

The goal of this analysis is to select a sample of events containing a $B^0_d$ decay in the OPAL detector and to determine the value of $\Delta m_d$ which best describes the data, assuming they follow the form of equation 2. We accomplish this by: (a) selecting a sample of events which is enriched with $B^0_d$ decays, (b) employing a method devised to ascertain the particle-antiparticle nature of the candidate $B^0_d$ at both the production point and at the time of the decay and (c) measuring some quantities related to the $B^0_d$ decay length.

Since the current data do not provide a large sample of fully reconstructed decays, we identify $B^0_d$ decays using the inclusive decay $\bar{B}^0_d \to D^{*+}X$, where only the $D^{*+}$ meson is reconstructed. The $D^{*+}$ mesons are reconstructed in the decay chain:

\[
\begin{align*}
D^{*+} & \to D^0\pi^+ \\
 & \to K^-\pi^+
\end{align*}
\]

\(^3\)The OPAL coordinate system is defined with positive $z$ along the $e^-$ beam direction and $\theta$ and $\phi$ being the polar and azimuthal angles, respectively.

\(^4\)For the 1993 run, the silicon strip detectors were upgraded to provide $z$-coordinate information, in addition to the $\phi$ readout [14]. The silicon $z$ measurement is not used for this analysis.
where the $D^*$ and $D^0$ branching ratios are 55% and 3.7% [16], respectively. In this decay, the very small $D^{**} - D^0$ mass difference provides a powerful tool for suppressing the combinatorial background. The charge of the reconstructed $D^*$ reveals whether it is from a $B_d^0$ or $\bar{B}_d^0$ decay at the time of decay. Because the pion from the $D^{**}$ decay is very soft and virtually collinear with the $D^0$ direction, it is not possible to accurately reconstruct the $\bar{B}_d^0$ decay vertex. We choose instead to use the $D^0$ decay vertex which allows for a measurement of the sum of the decay lengths of the $D^0$ and the $B$ meson.

Events are required to contain an identified lepton in the hemisphere opposite the $D^{**}$ candidate. The charge of the lepton is used to tag the flavor of the $D^*$ parent at the production point.

In addition to the $B_d^0$ decays, several other processes can produce events with the above signatures. The major sources of $D^*$ candidates are:

(a) $\bar{B}_d^0 \rightarrow D^{**} X$,  
(b) $B_d^0 \rightarrow \bar{B}_d^0 \rightarrow D^{**} X$,  
(c) $B^- \rightarrow D^{**} X$,  
(d) $c \bar{c} \rightarrow D^{**} X$ and  
(e) combinatorial background.

Hereafter, we refer to the reactions (a) and (b) as signal and the remaining processes as background.

Lepton candidates are categorized in the following way:

(i) leptons originating from the direct $b$-hadron decays $b \rightarrow \ell^-$ and $b \rightarrow \tau^- \rightarrow \ell^-$;  
(ii) cascade leptons from the reaction $b \rightarrow c(\bar{c}) \rightarrow \ell$, which includes $b \rightarrow J/\psi \rightarrow \ell^+\ell^-$;  
(iii) leptons from semileptonic decays of charm mesons in $c \bar{c}$ events, $c \rightarrow \ell^+$;  
(iv) fake leptons, including muons from the decay in flight of pions and kaons, electrons from light hadron decays and photon conversions, and misidentified hadrons.

The data sample is divided into two categories: like-sign ($D^{**} \ell^+$) and unlike-sign ($D^{**} \ell^-$). Our cuts are tuned (see section 4) so that the sample is comprised primarily of events containing $D^*$ mesons originating from $B$ meson decays and leptons from reaction (i). In this case, the like-sign sample contains most of the unmixed decays and the unlike-sign sample contains most of the mixed decays. The effect of mis-tagging, however, causes a fraction of the unmixed decays to appear in the unlike-sign sample, and vice-versa. This has the effect of reducing the amplitude of the observed $B_d^0 \leftrightarrow \bar{B}_d^0$ oscillations. The processes which lead to mis-tagging include the cascade reaction $b \rightarrow c \rightarrow \ell^+$, mixing in the hemisphere opposite the $D^*$ and fake leptons with random charge correlation.

Incorrect tagging of the flavor of the $B$ meson due to the process $B_d^0 \rightarrow D^{**} X$ is expected to be negligible and has been ignored in this analysis.

6
The oscillation frequency, $\Delta m_\alpha$, is determined by applying an unbinned maximum likelihood fit which makes use of a probability density function for each event in the two samples.

In order to construct the probability density function we must estimate the $D^\ell\ell$ sample composition using measured branching ratios and efficiencies obtained from a Monte Carlo simulation. We use the JETSET 7.3 parton shower Monte Carlo generator [17], with parameters tuned to reproduce the OPAL data [18], and a full simulation of the OPAL detector [19]. The fragmentation of b quarks is implemented according to the Peterson fragmentation function [20] with $c_b = 0.0057$, corresponding to a mean scaled energy $\langle x_E \rangle = 0.70$, in agreement with the OPAL measurement $\langle x_E \rangle = 0.697 \pm 0.012$ [21]. The modelling of semileptonic decays is the same as that used for the central results described in [21].

4 Event Selection

The data sample used in this analysis corresponds to a total of $1.9 \times 10^6$ hadronic $Z^0$ decays collected during the period 1990-1993. The selection of hadronic events, with an efficiency of $(98.4 \pm 0.4)\%$, is described elsewhere [22].

Jets are reconstructed using the JADE [23] algorithm with the E0 recombination scheme [24] and a scaled invariant mass resolution of $y_{\text{cut}} = 0.04$.

For this analysis, the following requirements are imposed on all charged tracks:

- each track must contain a minimum of 40 hits in the central detector;
- the distance of closest approach to the beam axis in the $x$-$y$ plane must be less than 0.5 cm;
- the distance to the interaction point along the beam axis, at the point of closest approach in the $x$-$y$ plane, must be less than 40 cm.

For tracks to be considered as pion candidates, we require $P_{dE/dx}^\pi > 0.01$, where $P_{dE/dx}^\pi$ is the probability that the measured rate of energy loss in the jet chamber, $dE/dx$, would be further from the expected value for a pion. For tracks to be considered as kaon candidates, we require $P_{dE/dx}^K > 0.05$.

The $D^{*+}$ candidates are identified by forming track combinations consistent with the particle composition of the decay chain described in section 3. Only tracks within the same jet are used. The transverse momentum with respect to the beam direction must exceed 0.25 GeV/$c$ for each track in the combination, except for the slow pion from the $D^{*+}$ decay. The invariant mass range of selected $D^0$ candidates is $1.79 \text{ GeV}/c^2 < M(K^-\pi^+) < 1.94 \text{ GeV}/c^2$. The mass difference of the $D^{*+}$ and $D^0$ candidate, $\Delta M = M(K^-\pi^+\pi^+) - M(K^-\pi^+)$, is required to be in the range $0.142 \text{ GeV}/c^2 < \Delta M < 0.149 \text{ GeV}/c^2$. Since $D^0$ is a pseudoscalar meson, the angular distribution of its products must be isotropic in the $D^0$ rest frame. The background, however, peaks in the forward and the backward directions with respect to the $D^0$ boost direction. We
require $|\cos \theta^*| < 0.8$, where $\theta^*$ is the angle between the $D^0$ direction and the direction of the kaon in the $D^0$ rest frame. These cuts are the same as those used in reference [25].

For each $D^0$ candidate, we perform a vertex fit in the $(r, \phi)$ plane. The decay length in the $(r, \phi)$ plane, $L_{r\phi}$, is determined from a fit to the mean event vertex and the $D^0$ decay vertex. If the fit the direction of the $B$ meson is constrained to be that of the $D^0$ momentum vector. The three-dimensional decay length, $L$, is obtained by dividing $L_{r\phi}$ by $\sin \theta$, where $\theta$ is the angle of the $D^0$ vector with respect to the $z$ axis. Monte Carlo studies show that $\theta$ is also a good approximation of the parent $B$ meson polar angle. In order to eliminate very poorly measured events, we require $|L| < 2.5$ cm and $\sigma < 0.2$ cm, where $\sigma$ is the measured uncertainty on $L$ (this eliminates less than 3% of the events satisfying all the other selection criteria).

The events are required to contain an identified electron or muon in the hemisphere opposite to the $D^{*+}$ candidate. Muons are identified by matching tracks in the central tracking system with track segments in the outer muon detectors. This is described in more detail in reference [26]. A neural-net algorithm is used for the identification of electrons (see reference [11] for a more detailed description). The network is of a feed forward type and was trained on simulated events to identify electrons on the basis of 12 measured quantities coming from the central tracking detector and the electromagnetic calorimeter.

Leptons from the cascade decay $b \to c \to \ell^+\tau$ give a large contribution to the mis-tag probability. Cascade leptons are distinguishable from prompt leptons by their soft spectrum in momentum ($p$) and transverse momentum with respect to the jet axis ($p_T$). By requiring $p > 3.0$ GeV/$c$ and $p_T > 0.75$ GeV/$c$, we reject 80% of the cascade leptons while retaining 60% of the prompt leptons.

In order to suppress the background reactions, additional cuts must be applied to the $D^*$ and lepton candidates. The mean momentum of $D^*$ mesons from $B$ decays is about 16 GeV/$c$, with less than 5% below 7 GeV/$c$ and less than 5% above 30 GeV/$c$. We require $7$ GeV/$c < p(K^-\pi^+\pi^+) < 30$ GeV/$c$, thereby suppressing the combinatorial background, which peaks at low momentum, and the $c\bar{c}$ events, which have a substantial component above 30 GeV/$c$.

The decay length ($L$), the error on the decay length ($\sigma$), the product of the $D^*$ and lepton charges ($q$) and the momentum of the candidate $D^0$ constitute the measured quantities for each event.

The $\Delta M$ distributions for like and unlike-sign signal candidates are shown in figure 1. The curves represent the predicted background shape, as determined by a hemisphere mixing technique [25]. The general idea of the hemisphere mixing technique is to form the expected $\Delta M$ distribution of combinatorial background by combining $D^0$ candidates with candidate pion tracks from the opposite hemisphere. A candidate This method was developed to ensure that the background sample is free of the correlations between the slow pion and $D^0$ tracks which form a peak in the $\Delta M$ spectrum. The background $\Delta M$ distribution is fit to a function of the form $a \cdot (\Delta M - m_{\tau})^b$, with $a$ and $b$ as free parameters. This distribution is shown in figure 2. The background levels in figure 1 are determined by fixing the exponent parameter $b$ and fitting for $a$ in the region $0.16$ GeV/$c^2 < \Delta M < 0.24$ GeV/$c^2$. The number of $D^*$ mesons is estimated by subtracting the amount of background from the total number of candidates in the signal.
region. There are 157 total combinations and 50 ± 9 background combinations in the signal region of the like-sign sample. In the unlike-sign sample there are 96 total combinations and 50 ± 8 background combinations.

5 Composition of the D*-Lepton Combinations

The relative populations of the various processes contributing to the selected sample of D*ℓ combinations are determined using their measured branching ratios and their efficiencies extracted from Monte Carlo studies.

We define \( \mathcal{N}_b \) to be the expected number of D*ℓ pairs originating from B meson decays:

\[
\mathcal{N}_b = 2 \cdot N_{\text{had}} \cdot \frac{\Gamma_{\text{b}\ell}}{\Gamma_{\text{had}}} \cdot B(b \to D^*X) \cdot B(D^* \to \pi K\pi) \cdot \epsilon_{D^*} \cdot n^b_{\ell}, \tag{4}
\]

where \( N_{\text{had}} \) is the total number of hadronic events collected, \( \frac{\Gamma_{\text{b}\ell}}{\Gamma_{\text{had}}} \) is the fraction of b\overline{b} events in hadronic Z0 decays, \( \epsilon_{D^*} \) is the efficiency for reconstructing a D*, given our set of cuts and \( n^b_{\ell} \) is the average (expected) number of leptons opposite a reconstructed D* from a b decay. We use the OPAL estimate \( \frac{\Gamma_{\text{b}\ell}}{\Gamma_{\text{had}}} \cdot B(b \to D^*X) \cdot B(D^* \to \pi K\pi) = (1.17 \pm 0.16) \cdot 10^{-3} \) [25]. The quantity \( n^b_{\ell} \) is given by:

\[
n^b_{\ell} = \sum_i B_i \epsilon_i + n^b_{\text{fake}}, \tag{5}
\]

where \( n^b_{\text{fake}} \) is the expected number of fake leptons opposite a b → D*X decay and \( B_i \) and \( \epsilon_i \) are the branching ratios and lepton reconstruction efficiencies for the processes (i) and (ii) listed in section 3. We use the OPAL measurements for \( B(b \to \ell^-) \) and \( B(b \to c \to \ell^+) \) [21]. We use \( B(b \to \ell^-) = 1.3\% \) and \( B(b \to \tau^- \to \ell^-) = (4.5 \pm 1.8)\% \) of \( B(b \to \ell^-) \), as is done in [21]. The world average value of \( B(b \to J/\psi \to \ell^+\ell^-) = (0.14 \pm 0.02)\% \) [16] is used.

Likewise, \( \mathcal{N}_c \) is defined to be the number of reconstructed D*ℓ pairs from c\overline{c} events:

\[
\mathcal{N}_c = 2 \cdot N_{\text{had}} \cdot \frac{\Gamma_{\text{c}\ell}}{\Gamma_{\text{had}}} \cdot B(c \to D^*X) \cdot B(D^* \to \pi K\pi) \cdot \epsilon_{D^*} \cdot n^c_{\ell}, \tag{6}
\]

where all the quantities are defined in a way analogous to those in equation 4, but corresponding to c\overline{c} events. The OPAL measurement of \( (1.17 \pm 0.17) \cdot 10^{-3} \) [25] is used for the quantity \( \frac{\Gamma_{\text{c}\ell}}{\Gamma_{\text{had}}} \cdot B(c \to D^*X) \cdot B(D^* \to \pi K\pi) \).

The number of combinatorial background events, \( \mathcal{N}_{\text{comb}} \), is taken directly from the \( \Delta M \) distribution. The amount of combinatorial background is found by integrating the background polynomial over the range of the signal.

The fractions \( f_b \), \( f_c \) and \( f_{\text{comb}} \) are defined as \( f_i = \mathcal{N}_i / \sum_j \mathcal{N}_j \), where the indices refer to b, c and comb. The actual values for the fractions depend on the choice of cuts.

The fraction \( f_b \) has two components: neutral and charged B mesons. The inclusive branching ratios for B0 and B+ mesons decaying into D* mesons have never been measured. There are various arguments, however, which support the claim that the inclusive charged D* rate is
dominated by $B^0_d$ mesons. A good discussion can be found in the recent OPAL publication of the $B^0_d$ and $B^+$ lifetimes [7]. The arguments presented pertain to semileptonic decays, but one does not expect large differences in hadronic decays. The expectation is that roughly 15% of $f_b$ is $B^+$ and 85% is $B^0_d$. We define the two separate fractions as $f_{B^0_d}$ and $f_{B^+}$, where $f_b = f_{B^0_d} + f_{B^+}$. The systematic uncertainty in our measurement from allowing $f_{B^+}$ to vary within the range $0 < f_{B^+}/f_b < 0.25$ is small (see section 7.1). The fraction of $D^*\ell$ pairs due to $B^0_d$ and $b$-baryon decays is expected to be negligible and is ignored.

In order to determine the fraction of each component which falls into the like-sign or unlike-sign samples, one must estimate the corresponding mis-tag probability. For $b\bar{b}$ events, we define a $D^*\ell$ combination as mis-tagged whenever the charge of the lepton candidate is opposite in sign to that expected from the prompt semileptonic decay of a $b$-hadron. The processes to consider are:

- The cascade and $J/\psi$ decay processes. The probability that a real lepton is from one of these processes is

$$P_{\text{cas}} = \frac{B(b \to c \to \ell^+)\epsilon_{\text{cas}} + B(b \to J/\psi \to \ell^+\ell^-)\epsilon_{J/\psi}}{\sum_i B_i \epsilon_i},$$

where $B_i$ and $\epsilon_i$ are the same branching ratios and reconstruction efficiencies used in equation 5.

- Leptons from the decays of neutral $B$ mesons which have oscillated. This probability, $P_{\text{mix}}$, is given by the average mixing parameter $\bar{\chi}$. We use the OPAL value $P_{\text{mix}} \equiv \bar{\chi} = 0.143 \pm 0.022 \pm 0.007$ [21].

- Fake leptons. The probability that a candidate lepton track is fake is given by $P_{\text{fake}} = n_{\text{fake}}/n_{\text{fake}} + \sum_i B_i \epsilon_i$. Due to small correlations between the fake lepton charge and the charge of the $D^*$ in the opposite hemisphere, the probability of a fake lepton giving a mis-tag is not 50%. From Monte Carlo samples we estimate that $(45 \pm 0.05)\%$ of the fake lepton tracks lead to a mis-tag.

The total lepton mis-tag probability is therefore

$$P^b_m = 0.45 \cdot P_{\text{fake}} + (1 - P_{\text{fake}})[P_{\text{mix}}(1 - P_{\text{cas}}) + (1 - P_{\text{mix}})P_{\text{cas}}].$$

We also define a mis-tag probability for $c\bar{c}$ events, $P^c_m$, which has a contribution only from fake leptons.

Our estimates for these quantities, for the cuts listed above, are given in table 1. The uncertainties originate from Monte Carlo statistics and uncertainties in measured branching fractions. The uncertainties on $f_{B^0}, f_{B^+}$ and $f_c$ include a contribution from the uncertainty on $f_{\text{comb}}$, which arises from the constraint $\sum_i f_i = 1$. 

10
Table 1: Estimated values for the fixed parameters of the fit.

6 Description of the Decay Length Distributions

In order to form the likelihood of observing each decay length, we must construct the probability density function for each sample component. This is accomplished by folding the expected theoretical decay length distributions with an experimental resolution function. The probability density is a function of the known variables, $L$, $\sigma$, $q$ and $(\beta\gamma)_D$, the Lorentz boost factor of the $D$ meson, as well as the fit parameter $\Delta m_d$.

6.1 $B_d^0$ decay length distribution

The reconstructed decay length $L$ represents the distance from the primary vertex to the $D^0$ decay vertex. We are using the approximation $L \simeq L_B + L_D$, where $L_D$ is the $D^0$ decay length and $L_B$ is the $B$ decay length. Monte Carlo studies show that this is a good approximation. The expected shape of this distribution, for the case of perfect resolution, is given by the following probability density:

$$P_0(L, (\beta\gamma)_B, (\beta\gamma)_D, q | \Delta m_d) = \int_0^L dL_B \; P_B(L_B, (\beta\gamma)_B, q | \Delta m_d) \cdot P_D(L, L_B, (\beta\gamma)_D),$$

where

$$P_B(L_B, (\beta\gamma)_B, q | \Delta m_d) = \frac{1}{2\tau_B(\beta\gamma)_B} \exp\left(\frac{-L_B}{\tau_B(\beta\gamma)_B}\right) \left[1 + q \cos\left(\Delta m_d \frac{L_B}{(\beta\gamma)_B}\right)\right],$$

$$P_D(L, L_B, (\beta\gamma)_D) = \frac{1}{\tau_D(\beta\gamma)_D} \exp\left(-\frac{(L - L_B)}{\tau_D(\beta\gamma)_D}\right).$$

$\tau_B$ and $\tau_D$ are the lifetimes of the $B_d^0$ and $D^0$, respectively, and $(\beta\gamma)_B$ and $(\beta\gamma)_D$ are the Lorentz boost factors. We use $\tau_B = 1.44 \pm 0.15$ ps [7, 8] and $\tau_D = 0.420 \pm 0.008$ ps [16]. The quantity $P_B$ describes the decay length distribution of the $B_d^0$, while $P_D$ describes the $D^0$ decay length distribution.

Since $(\beta\gamma)_B$ of each candidate is not measured, equation 9 is convoluted with the expected $(\beta\gamma)_B$ distribution, which is given by the B meson fragmentation function, $F_{frag}(x_E)$, where
\[ x_E = E(B)/E(\text{beam}). \] We use a parameterization of the Peterson form [20],

\[ F_{\text{frag}}(x_E) = \frac{\alpha}{x_E \left(1 - \frac{1}{x_E} - \frac{\epsilon'}{x_E}\right)^2}. \] (11)

The parameter \( \alpha \) is a normalization constant which is chosen so that \( \int_0^1 F_{\text{frag}}(x_E) \, dx_E = 1 \). We chose \( \epsilon' = 0.04 \), based on a fit to the \( x_E \) distribution of B meson decays in Monte Carlo in which our selection cuts have been applied. The difference between the \( x_E \) distributions before and after the selection cuts is very small. The mean value of \( x_E \) changes by less than 0.3%.

The probability density becomes

\[ P(L, (\beta \gamma)_D, q|\Delta m_d) = \int_0^1 F_{\text{frag}}(x_E) \, P_0(L, (\beta \gamma)_D, (\beta \gamma)_B, q|\Delta m_d) \, dx_E. \] (12)

Finally, the probability distribution must be smeared to take into account the effects of detector resolution. We use a resolution function of a single Gaussian with a width \( \sigma \), the estimated uncertainty on the decay length \( L \), determined on an event-by-event basis. This function provides a good description of Monte Carlo decay length distributions. The final form of the probability density is

\[ \mathcal{F}(L, \sigma, (\beta \gamma)_D, q|\Delta m_d) = \frac{1}{\sqrt{2\pi} \sigma} \int_0^\infty P(L', (\beta \gamma)_D, q|\Delta m_d) \, e^{-\frac{L^2}{2\sigma^2}} \, dL'. \] (13)

The effect of these manipulations is most evidently displayed in the asymmetry function defined as

\[ A(L|\Delta m_d) = \frac{N(L|\Delta m_d)^l - N(L|\Delta m_d)^u}{N(L|\Delta m_d)^l + N(L|\Delta m_d)^u} \] (14)

where \( N(L|\Delta m_d)^l \) (\( N(L|\Delta m_d)^u \)) is the number of like-sign (unlike-sign) events expected at a decay length \( L \), for a given \( \Delta m_d \). This quantity is simulated in figure 3 for four cases: a) using \( L_{B_0} \), the \( B_0^0 \) decay length, a fixed \( B_0^0 \) momentum and assuming perfect resolution; b) using \( L \), the distance from the event vertex to the \( D^0 \) vertex, and convoluting over \( (\beta \gamma)_D \) and the experimental resolution; c) like b) but with a 25% mis-tag rate; d) like b) but with a 25% mis-tag rate and typical background included (see section 6.2). The last case most realistically models the data.

### 6.2 Background decay length distributions

The probability density function for \( B^+ \) decays, \( \mathcal{F}_{B^+}(L, \sigma, (\beta \gamma)_D) \), is equivalent to the function in equation 13, but with \( \Delta m_d \) set to zero and the \( B^+ \) lifetime used in place of the \( B_0^0 \) lifetime. A value of \( 1.42 \pm 0.17 \) ps \([7, 8]\) is used for the \( B^+ \) lifetime.

For c\( \bar{c} \) events, the reconstructed decay length corresponds to the true \( D^0 \) decay length. Thus, the expected decay length distribution is given by an exponential, \( \exp(-L/(\beta \gamma)_D \tau_{D^0}) \), convoluted with a Gaussian resolution function. This probability function is denoted as \( \mathcal{F}_{c\bar{c}}(L, \sigma, (\beta \gamma)_D) \).

The probability density function for combinatorial background, \( \mathcal{F}_{\text{comb}} \), is determined from a fit to the decay length distribution of events in a \( \Delta M \) sideband above the signal region.
(0.18 GeV/c² < ΔM < 0.20 GeV/c²). We find that a reasonable parameterization for this distribution is given by

\[ \mathcal{F}_{\text{comb}}(L, \sigma, (\beta \gamma)_D) = a_1 G(0, \sigma) + a_2 (1 - a_1) G(0, \sqrt{\sigma^2 + \sigma_0^2}) + [1 - a_1 - a_2 (1 - a_1)] E(\tau_{bk}) \otimes G(L, \sigma), \]  

(15)

where \( G(L, \sigma) \) represents a Gaussian of width \( \sigma \) centered at \( L \) and \( E(\tau_{bk}) \otimes G(L, \sigma) \) represents the convolution

\[ E(\tau_{bk}) \otimes G(L, \sigma) = \frac{1}{\sigma \sqrt{2\pi}} \int_{0}^{\infty} e^{-(L' - L)^2/2\sigma^2} e^{-L'(\beta \gamma)_D \tau_{bk}} \, dL'. \]  

(16)

The fitted quantities are \( a_1, a_2, a_3 \) and \( \tau_{bk} \). The second Gaussian and the exponential tail arise from bottom and charm decay products being included in some random vertices. The decay length distribution of combinatorial background events is shown in figure 4.

6.3 The overall likelihood description

The decay length descriptions, fractions and mis-tag probabilities are used to form an overall probability density function for both like-sign and unlike-sign combinations. Each \( D^* \ell \) pair, with a given decay length \( L_i \), decay length error \( \sigma_i \), product of \( D^* \) and lepton charge \( q_i \), and \( D^0 \) boost factor \((\beta_i q_i)_D\), is assigned a likelihood, which is the sum of terms describing each signal and background contributing. For a like-sign pair the full likelihood is

\[ \mathcal{L}^l_i = f_{b^0} (1 - P^b_m) \mathcal{F}(L_i, \sigma_i, (\beta \gamma)_D, 1|\Delta m_d) + f_{b^0} P^b_m \mathcal{F}(L_i, \sigma_i, (\beta \gamma)_D, -1|\Delta m_d) + f_{b^+} (1 - P^b_m) \mathcal{F}^+(L_i, \sigma_i, (\beta \gamma)_D) + f_c P^c_m \mathcal{F}^c(L_i, \sigma_i, (\beta \gamma)_D) + f_{\text{comb}} C_l \mathcal{F}_{\text{comb}}(L_i, \sigma_i, (\beta \gamma)_D), \]  

(17)

and for an unlike sign pair it is

\[ \mathcal{L}^u_i = f_{b^0} (1 - P^b_m) \mathcal{F}(L_i, \sigma_i, (\beta \gamma)_D, -1|\Delta m_d) + f_{b^0} P^b_m \mathcal{F}(L_i, \sigma_i, (\beta \gamma)_D, 1|\Delta m_d) + f_{b^+} P^b_m \mathcal{F}^+(L_i, \sigma_i, (\beta \gamma)_D) + f_c (1 - P^c_m) \mathcal{F}^c(L_i, \sigma_i, (\beta \gamma)_D) + f_{\text{comb}} (1 - C_l) \mathcal{F}_{\text{comb}}(L_i, \sigma_i, (\beta \gamma)_D), \]  

(18)

The first two terms in both cases describe the contributions from correctly tagged and mis-tagged \( B_d^0 \) meson decays, respectively. The remainder of the terms describe the background contributions. The parameter \( C_l \) is the amount of combinatorial background in the like-sign sample divided by the total amount of combinatorial background. The total log-likelihood for the sample (like and unlike pairs summed) is defined as

\[ \log \mathcal{L}_{\text{tot}}(\Delta m_d) = \sum_{i=1}^{N_l} \log \mathcal{L}^l_i + \sum_{i=1}^{N_u} \log \mathcal{L}^u_i, \]  

(19)

where \( N_l \) and \( N_u \) are the number of like and unlike-sign combinations, respectively. The negative log-likelihood is minimized to find the best value of \( \Delta m_d \).
Table 2: Results of the fit when $\Delta m_d$ and one other parameter are allowed to vary.

### 7 The Results of the Fit

Performing the fit, using the estimates for the fixed parameters given in table 1, we find

$$\Delta m_d = 0.57 \pm 0.11 \text{ ps}^{-1}.$$ 

The negative log-likelihood as a function of $\Delta m_d$ is shown in figure 5. The like and unlike-sign decay length distributions are shown in figure 6. The asymmetry function, $A$, is shown in figure 7, along with the shape predicted by the fit. The $\chi^2$ is 5.1 for 4 degrees of freedom.

#### 7.1 Systematic uncertainties

The fit has been performed under a variety of different conditions to ensure that there are no systematic biases involved. These studies, as well as an estimation of the overall systematic uncertainty, are summarized in this section. been verified by $\Delta m_d = 0.61 \pm$

The fit has been performed by allowing each nominally fixed parameter to vary, one at a time, along with $\Delta m_d$. The results of these tests are summarized in table 2. Clearly, the fit is not very sensitive to any of these quantities. In all cases, the preferred values are consistent with the computed values and little variation in the fitted $\Delta m_d$ is observed.

Various sources of systematic uncertainties have been studied. They can arise from uncertainties in the $b$ quark fragmentation function, uncertainties in the fixed parameters of the fit, uncertainties in the combinatorial background size and shape, and from systematic mis-measurement of the decay length errors. The relative contributions have been studied separately and summed in quadrature to estimate the overall systematic uncertainty.

The choice of fragmentation function in the convolution over $(\beta \gamma)_B$ is the most significant source of systematic uncertainty. As mentioned previously, we use a parameterization of the Peterson form [20]. We varied the value of the parameter $\epsilon'$ in the function to cover the uncertainty in the measured $\langle x_F \rangle$ [21] in order to estimate the systematic effect on $\Delta m_d$. The variation of the measured $\Delta m_d$ is $\pm 0.012$ ps$^{-1}$. We used Monte Carlo data generated with the Lund symmetric fragmentation function [27] to investigate the systematic uncertainty associated with assuming a Peterson distribution. The estimated uncertainty in $\Delta m_d$ is $\pm 0.003$ ps$^{-1}$. The total systematic uncertainty from fragmentation assumptions is $\pm 0.0124$ ps$^{-1}$. 

<table>
<thead>
<tr>
<th>Free parameter</th>
<th>Standard value</th>
<th>Fitted value</th>
<th>Fitted value of $\Delta m_d$ (ps$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_m^m$</td>
<td>0.264 ± 0.025</td>
<td>0.21 ± 0.11</td>
<td>0.58 ± 0.11</td>
</tr>
<tr>
<td>$P_m^c$</td>
<td>0.176 ± 0.043</td>
<td>0.29 ± 0.30</td>
<td>0.60 ± 0.13</td>
</tr>
<tr>
<td>$f_{B/0}$</td>
<td>0.417 ± 0.070</td>
<td>0.458 ± 0.080</td>
<td>0.57 ± 0.11 ( f_c )</td>
</tr>
<tr>
<td>0.114 ± 0.026</td>
<td>0.100 ± 0.055</td>
<td>0.58 ± 0.12</td>
<td></td>
</tr>
<tr>
<td>$f_{B/+}$</td>
<td>0.074 ± 0.014</td>
<td>0.020 ± 0.083</td>
<td>0.57 ± 0.12 ( f_{comb} )</td>
</tr>
<tr>
<td>0.395 ± 0.048</td>
<td>0.323 ± 0.073</td>
<td>0.58 ± 0.12</td>
<td></td>
</tr>
</tbody>
</table>
The contribution from uncertainties in the fractions $f_B^0$, $f_B^+$, $f_c$ and $f_{comb}$, were investigated by varying, individually, each fraction’s value within its errors. The fraction $f_B^+$ was varied within the bounds $0 < f_{B^+}/f_h < 0.25$ to take into account the theoretical uncertainty in its value. Because we are measuring the frequency of oscillations, and not the amplitude, the fit is relatively insensitive to variations of the fractions and mis-tag probabilities.

Variations in the $B_d^0$, $B^+$ and $D^0$ lifetimes give only a small systematic uncertainty in the measurement of $\Delta m_d$. For the fit, we use $\tau_{B_d^0} = 1.44 \pm 0.15$ ps \cite{7,8}, $\tau_{B_d^0} - \tau_{B^+} = 0.02 \pm 0.23$ ps \cite{7,8} and $\tau_{D^0} = 0.420 \pm 0.008$ ps \cite{16}. The systematic uncertainty due to variations of the $B_d^0$ and $B^+$ lifetimes within their measured errors is about $\pm 0.0046$ ps$^{-1}$. The uncertainty in the $D^0$ lifetime contributes only $\pm 0.0029$ ps$^{-1}$ to the systematic uncertainty in $\Delta m_d$.

Various parameterizations of the combinatorial background decay length distribution were used to study possible systematic biases from the choice of parameterization. We find a small systematic uncertainty of $\pm 0.0017$ ps$^{-1}$ arising from the choice of parameterization and from variations of the background decay length fit parameters within their errors.

There is a possibility that the estimated decay length errors, $\sigma_i$, are systematically over- or under-estimated. To test this hypothesis we introduced a scaling factor, $s$, and performed the fit substituting $s\sigma_i$ for each $\sigma_i$ in the resolution function. The fit was performed for fixed values of $s$, ranging from 0.75 to 2.0. The range was chosen to be large so as to cover any systematic uncertainty resulting from deviation of the true resolution from a single Gaussian. The effect on the fitted value of $\Delta m_d$ is small. This is not surprising, considering that we are measuring an oscillation with a period of $\sim 2$ cm, whereas the smearing due to resolution is on the scale of 250 $\mu$m. We estimate the systematic error due to uncertainties in the resolution to be $\pm 0.0061$ ps$^{-1}$.

In order to estimate the systematic uncertainty due to calibration uncertainties and mis-alignment in the vertex detector, detailed studies have been performed on $\tau$ decays from the process $Z^0 \rightarrow \tau^+\tau^- \, \cite{28}$. We estimate a systematic uncertainty of 43 $\mu$m on the measured decay lengths of $\tau$ decays. Because the average decay length of $\tau$ leptons from $Z^0$ decay is ap-

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Systematic Uncertainty (ps$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fragmentation function</td>
<td>$\pm 0.012$</td>
</tr>
<tr>
<td>$f_B^0$</td>
<td>$\pm 0.005$</td>
</tr>
<tr>
<td>$f_B^+$</td>
<td>$\pm 0.002$</td>
</tr>
<tr>
<td>$f_c$</td>
<td>$\pm 0.003$</td>
</tr>
<tr>
<td>$f_{comb}$</td>
<td>$\pm 0.006$</td>
</tr>
<tr>
<td>Mis-tag $P_m^b$</td>
<td>$\pm 0.006$</td>
</tr>
<tr>
<td>Mis-tag $P_m^c$</td>
<td>$\pm 0.003$</td>
</tr>
<tr>
<td>$\tau_B$</td>
<td>$\pm 0.005$</td>
</tr>
<tr>
<td>$\tau_D$</td>
<td>$\pm 0.003$</td>
</tr>
<tr>
<td>Param. of comb. bkgd.</td>
<td>$\pm 0.002$</td>
</tr>
<tr>
<td>Resolution</td>
<td>$\pm 0.006$</td>
</tr>
<tr>
<td>Sys. uncertainty on $L$</td>
<td>$\pm 0.002$</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>$\pm 0.018$</td>
</tr>
</tbody>
</table>

Table 3: Contributions to systematic uncertainty on $\Delta m_d$ from various sources.
proximately the same as the average decay length in this analysis, one expects a similar decay length uncertainty. This results in an uncertainty of $\pm 0.002 \text{ ps}^{-1}$ on the measured $\Delta m_d$.

Table 3 summarizes the individual contributions to the systematic uncertainty in our measurement of $\Delta m_d$, along with the final value.

8 Summary and Conclusions

A sample of $153 \pm 12$ $D^*\ell$ pairs has been reconstructed from approximately 1.9 million hadronic $Z^0$ decays. These events have been used to measure the $B^0_d \leftrightarrow \bar{B}^0_d$ oscillation frequency:

$$\Delta m_d = 0.57 \pm 0.11(\text{statistical}) \pm 0.02(\text{systematic}) \text{ ps}^{-1},$$

which is in good agreement with other published measurements [4, 5, 9, 10, 11]. Using $\tau_{B^0_d} = 1.44 \pm 0.15 \text{ ps}$, this corresponds to

$$x_d = \Delta m_d \tau_{B^0_d} = 0.82 \pm 0.16(\text{stat.}) \pm 0.03(\text{syst.}) \pm 0.09(\text{syst.}\, \tau_{B^0_d}),$$

where the second systematic uncertainty is due to the error in the measured $B^0_d$ lifetime.

This measurement can be combined with the previous OPAL measurement of $\Delta m_d = 0.508 \pm 0.075 \pm 0.025 \text{ ps}^{-1}$ [11], which uses a jet charge technique to tag the $B$ meson flavor at production. No overlap of events is found in the two samples. The combined result is

$$\Delta m_d = 0.529 \pm 0.064(\text{stat.}) \pm 0.019(\text{syst.}) \text{ ps}^{-1},$$

where the systematic uncertainty takes into account the correlations in the systematic uncertainties of the separate measurements. Because this measurement of $\Delta m_d$ is relatively insensitive to the $B^0_d$ lifetime and the fraction of $B^+$ decays in the sample, it can also be used, along with $x_d = 0.68 \pm 0.08$ from ARGUS and CLEO [4, 5, 6], to infer the $B^0_d$ lifetime:

$$\tau_{B^0_d} = x_d/\Delta m_d = 1.29 \pm 0.16 \pm 0.15 \text{ ps}.^{6}$$

The first uncertainty is from the combined OPAL $\Delta m_d$ measurement and the second is from the ARGUS and CLEO $x_d$ measurement. This value is in good agreement with published measurements [7, 8].

9 Acknowledgements

It is a pleasure to thank the SL Division for the efficient operation of the LEP accelerator, the precise information on the absolute energy, and their continuing close cooperation with our experimental group. In addition to the support staff at our own institutions we are pleased to acknowledge the

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6In computing the uncertainty we have assumed no dependence on the $B^0_d$ lifetime in the CLEO and ARGUS $x_d$ measurements. The measured value of $x_d$ is, however, sensitive to the ratio of the product of production rates and semileptonic branching ratios for charged and neutral $B$ mesons, which has been measured by CLEO [5, 29] and ARGUS [30].
Department of Energy, USA,
National Science Foundation, USA,
Texas National Research Laboratory Commission, USA,
Science and Engineering Research Council, UK,
Natural Sciences and Engineering Research Council, Canada,
Fussefeld Foundation,
Israeli Ministry of Energy and Ministry of Science,
Minerva Gesellschaft,
Japanese Ministry of Education, Science and Culture (the Monbusho) and a grant under the
Monbusho International Science Research Program,
German Israeli Bi-national Science Foundation (GIF),
Direction des Sciences de la Matière du Commissariat à l’Energie Atomique, France,
Bundesministerium für Forschung und Technologie, Germany,
National Research Council of Canada,
A.P. Sloan Foundation and Junta Nacional de Investigação Científica e Tecnológica, Portugal.
References


Figure Captions

Figure 1. The $\Delta M$ distribution of $D^*$ candidates: a) Like-sign combinations ($D^+\ell^+$); b) Unlike-sign combinations ($D^+\ell^-$). The curves represent the predicted background. The arrows represent the region used in the $\Delta m_d$ fit.

Figure 2. The $\Delta M$ distribution of background $D^*$ candidates, as determined from the hemisphere mixing technique.

Figure 3. The expected asymmetry function $A$ for: a) Using $L_B$, the true $B_d^0$ decay length, and a fixed $B_d^0$ momentum; b) Using $L = L_B + L_D$, convoluting with the $(\beta\gamma)_B$ distribution and including resolution smearing; c) Same as b), but with 25% mistag; d) Same as b), but with 25% mistag and typical background levels.

Figure 4. The decay length distribution for combinatorial background (the solid curve is the result of the parameterization used in the final fit).

Figure 5. The value of the negative log-likelihood for the fit for a range of $\Delta m_d$ values.

Figure 6. Decay length distributions with fit results for: a) like-sign combinations; b) unlike-sign combinations.

Figure 7. The asymmetry function $A$ (the points are data, the solid curve is the expected shape using the fit result, and the dashed curve is the expected shape for no mixing—$\Delta m_d = 0$).
Figure 1

a) Like-sign (D\(^{\ast -}\)l\(^+\))

b) Unlike-sign (D\(^{\ast +}\)l\(^-\))
Figure 3
Figure 4

Decay Length (cm)

Entries/0.075 cm

-1.5  -1  -0.5  0  0.5  1  1.5  2  2.5  3
Figure 5

-log(likelihood) vs \( \Delta m_d \) (ps\(^{-1}\))

\( \pm 1\sigma \)
Figure 6

(a) $D^{*+}\pi^+$

(b) $D^{*+}\bar{\pi}^-$

![Graph showing decay lengths with entries per 0.10 cm.](image-url)
Figure 7

Asymmetry vs. Decay Length (cm)

- $\Delta m_d = 0.57 \text{ ps}^{-1}$
- $\Delta m_d = 0 \text{ ps}^{-1}$

Figure 7