Properties of Multihadronic Events with a 
Final State Photon at $\sqrt{s} = M_{Z^0}$

The OPAL Collaboration

Abstract

The properties of final state photons in multihadronic decays of the $Z^0$ and those of the recoiling hadronic system are discussed and compared with theoretical expectations. The yield of two and three jet events with final state photons is found to be in good agreement with the expectation from a matrix element calculation of $O(\alpha_s)$. Uncertainties in the interpretation of the theoretical calculation do not yet permit a final assessment of events with just one reconstructed jet. Comparing the rates of two jet events with a photon to those of three jet events in the inclusive multihadronic sample, the strong coupling constant in second order is determined as $\alpha_s(M_{Z^0}) = 0.122 \pm 0.010$, taking into account only the statistical and experimental systematic errors. It is found that an abelian model of the strong interaction does not describe the data. The comparison of the total yield and the jet rates with QCD shower programs shows better agreement with the ARIADNE model than with the JETSET model. Both programs are found to describe well the photon properties and the properties of the residual hadronic event.

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1 Introduction

Photons in multihadronic decays of the $Z^0$ [1, 2, 3, 4] may be used to probe fundamental properties of the Standard Model. Since the direct coupling of the $Z^0$ to the photon is forbidden, an anomalous rate of events containing photons could provide the first glimpse of a substructure of the $Z^0$ [5]. More conventionally, photons radiated from quarks (fig. 1) test the strength and structure of both the electroweak and strong sectors of the Standard Model. As suggested in [6] these photons, being messengers from the primary quarks, have been exploited to measure the weak coupling constants of up and down type quarks [2, 4]. In this paper we focus on QCD tests.

We report on the properties of final state photons in multihadronic $Z^0$ decays and on the topology of the recoiling hadronic system. We compare our measurements with a matrix element calculation of $O(\alpha\alpha_s)$ [7] and with predictions from two QCD shower models that include both photon and gluon radiation [8, 9]. Although these latter predictions are based on different concepts of the parton shower, they describe the general features of the inclusive multihadronic event sample successfully over a wide range of centre-of-mass energies. Photons from final state radiation, being sensitive to the evolution of quarks in the parton branching, offer new and specific QCD tests. They provide means of disentangling the gluon contribution. Since, apart from the coupling strength the theoretical descriptions of photon and gluon emission from quarks are alike, there is little freedom to change the photon radiation once the gluon contributions are fixed.

The photon sample in this analysis is the same as that used for the measurement of the electroweak couplings of up and down type quarks in a previous publication [2], where the selection procedure, the background estimate, and the correction procedure are discussed in detail. Since the experimental procedure and the theoretical ambiguities are important for some of the physics topics analysed here, we will summarise in sections 2 to 5 the salient features of the analysis and provide some details on our experimental checks. Section 6 gives an overview of the theoretical predictions for final state photon production. Results are given in section 7. We present measurements of the total yield of events with final state photons. From the production cross sections for events with a photon and specific jet multiplicities we derive values for the strong coupling constant $\alpha_s$ and test the existence of the gluon self coupling, one of the cornerstones of QCD. Finally, we present measurements of several properties of the photon and the general event topology and compare them to the predictions of QCD shower models. Our conclusions are summarised in section 8.

2 The OPAL Detector

This analysis is based on an integrated luminosity of approximately 6.6 pb$^{-1}$ collected with the OPAL detector [10] at LEP. The data were recorded at centre-of-mass energies $E_{cm}$ between 88.28 and 94.28 GeV around the $Z^0$ pole.

The most important components of OPAL for this study are the tracking chambers and the barrel part of the electromagnetic calorimeter. The central detector provides a measurement
of the momenta of charged particles in a magnetic field over almost the entire solid angle. The electromagnetic calorimeter covers the solid angle in $|\cos \theta| < 0.98$, where $\theta$ is the polar angle with respect to the beam direction. The barrel part ($|\cos \theta| < 0.82$) consists of 9440 lead glass blocks of 24.6 radiation lengths, pointing towards the interaction region and each subtending an angular region of approximately $40\times40$ mrad$^2$. A presampler for electromagnetic showers and a hadron calorimeter offer cross checks of the photon identification. The barrel presampler is located between the magnet coil and the lead glass calorimeter. It consists of a set of 16 double-planed chambers containing streamer-tubes with both wire and cathode-strip readout. The barrel hadron calorimeter, consisting of nine planes of streamer-tube chambers within the iron return yoke of the magnet, is located directly behind the electromagnetic calorimeter. In addition to measuring hadron energies, it provides information on longitudinal shower development.

3 Selection of Events with Final State Photons

The final state photon sample is obtained in three steps: first a sample of multihadronic events is selected. As a second step we search for isolated clusters in the lead glass calorimeter with topological properties which are unlikely for neutral hadrons. Finally the detailed structure of these clusters is used to reject background of hadronic origin. Details are described in [2].

- Multihadronic events are selected essentially by requiring a minimum number of tracks and clusters in the lead glass calorimeter together with a large energy deposition. We keep 145,095 events.

- To suppress background from neutral hadronic particles we then search for clusters of more than 7.5 GeV with a polar angle of $|\cos \theta| < 0.72$. To reject the potential background due to neutral hadrons which are overwhelmingly accompanied by additional particles, we demand the photon candidate to be isolated. No additional cluster or track is allowed within a cone of half opening angle 15 degrees around the direction of the photon candidate.

In order to be able to compare our measurements with the theoretical calculations we further require the photon to have a minimum invariant mass when combined with the jets in the hadronic system. We first group the hadrons into jets [11] such that the pair mass $y = M_{ij}^2/E_{\text{vis}}^2$ of two jets $i, j$ is always larger than a predefined $y_{\text{cut}}$. Here $M_{ij}^2 = 2 \cdot E_i E_j (1 - \cos \alpha_{ij})$, $E_i$ and $E_j$ being the energies of the jets and $\alpha_{ij}$ their opening angle. The visible energy $E_{\text{vis}}$ is the sum of the momenta of all tracks and the energies of the clusters in the event including the photon. Corrections for double counting [12] in the tracking chamber and in the lead glass calorimeter and for losses due to neutrinos and neutral hadrons have been applied.

Events are retained if the minimum mass of all combinations of the photon candidate with the jets $\min(M_{ij}^2/E_{\text{vis}}^2)$ exceeds $y_{\text{cut}}$. We choose values of $y_{\text{cut}}$ between 0.005 and 0.2. Detailed studies of the event properties are performed for $y_{\text{cut}}=0.005$ and 0.06 for which we keep 339 and 171 events, respectively.

- To further suppress hadronic background we reject clusters with properties not compatible with the expectation for genuine photons. We only accept clusters of less than 16 blocks,
a width calculated from the first moment of the cluster of less than 30 mrads, and a cluster shape, measured from the energy sharing of blocks, which is consistent with that of a photon.

After these cuts we retain 276 events for $y_{cut}=0.005$ and 149 events for $y_{cut}=0.06$.

4 Background Contributions to the Photon Sample

The background to the photon sample was determined from the data themselves and has been discussed in detail in [2]. We base our estimate on the number of isolated charged tracks which can be translated into the expected number of neutral hadrons using isospin symmetry. The dominant background is due to $\pi^0$'s. From the number of isolated charged particles folded with the probability to reject $\pi^0$'s with our cuts, we estimate their contribution to be $8.3\pm4.4$ and $1.5\pm1.4$ events for a $y_{cut}$ of 0.005 and 0.06, respectively. Additional background is due to $K^0_L$'s, neutrons or overlapping neutral particles like $K^*\rightarrow K^0\pi^0$ or $K^0\rightarrow \pi^0\pi^0$. In general, these lead to much broader clusters than photons and are mostly rejected by our cuts. We estimate their contribution as $1.2\pm1.2$ and $0.3\pm0.3$ for the two $y_{cut}$ values. Background from $\eta\rightarrow \gamma\gamma$ and $\omega\rightarrow \gamma\pi^0$, which cannot be estimated with charged particles, is expected to be negligible from simulation studies.

Cross checks were made by fitting the cluster shape distribution [2] with a sum of photon and background contributions and by using the electromagnetic presampler which is sensitive to the different shower development of $\pi^0$'s and photons in the coil and the hadron calorimeter to study the shower leakage. They lead to background estimates consistent with the above values.

Additional background to the final state photons is due to initial state photons. Taking their dependence on the centre-of-mass energy into account, we expect $13.3\pm1.3$ ($10.1\pm1.0$) initial state photons for $y_{cut} = 0.005$ ($0.06$) using the Monte Carlo generator of [13].

5 Correction Procedure

To compare our measurements with theoretical predictions, we correct for biases introduced by the detector and the selection requirements using the JETSET simulation of the final state photon radiation [8]. The simulated events are passed through a detailed simulation of the OPAL detector [14]. For any distribution $f(z)$, we obtain the bin by bin correction factors $c(z) = n_{gen}(z)/n_{det}(z)$. Here $n_{gen}$ denotes the number of generated events with a photon and $n_{det}$ the number of events after simulating the detector and applying the experimental cuts. The bin size for the determination of $c(z)$ is chosen so that there are only small bin to bin migrations from the true $z$ to the observed one. For all distributions the generated events are selected by applying $y_{cut}$ to jets formed directly from the partons produced in the QCD shower.

We distinguish two kinds of corrections. The first, $c_1$, accounts for the effects of imposing energy and isolation cuts on the photon, but neglects distortions from the detector. For low
values of $y$, $c_1$ is substantial and depends on the proper description of photon radiation in the simulation. However, since $y \propto E_\gamma \cdot (1 - \cos \alpha)$ with $\alpha$ being the angle between the photon and a jet, we expect this correction to become less important for larger values of $y$. A second correction $c_2$ then accounts for the detector performance alone.

In fig. 2 we display typical efficiencies $\epsilon_1 = 1/c_1$ for photons produced with $|\cos \theta| < 0.72$, the polar angular range of our observed photons as predicted by JETSET and ARIADNE. In fig. 2a the efficiency is displayed as a function of $y_{\text{cut}}$. At low $y_{\text{cut}}$ significant losses ($52\%$ at $y_{\text{cut}}=0.005$) are dominantly due to the energy cut. For $y_{\text{cut}} \geq 0.06$ losses are around $10\%$ independent of $y_{\text{cut}}$ and entirely due to the isolation requirement. The efficiency $\epsilon_1$ is displayed in figs. 2b and 2c as a function of the photon energy $E_\gamma$ for $y_{\text{cut}}=0.005$ and 0.06. For the low $y_{\text{cut}}$ the losses are about $25\%$ for $E_\gamma$ between 7.5 and 30 GeV. For $E_\gamma \rightarrow E_{\text{beam}}$ almost all events are accepted. The efficiency is significantly larger for $y_{\text{cut}}=0.06$, with a typical loss of $10\%$. The correction $c_2$ for the detector effects is typically $5\text{-}10\%$ independent of $y_{\text{cut}}$. Note that a further correction of $43\%$ for the restricted angular range of accepted photons has to be applied. In addition to the corrections for the topological cuts, the correction $c_2$ includes a contribution from the efficiency of the photon selection determined from a reference sample of $e^+e^- \rightarrow \gamma\gamma [15]$ and radiative lepton pair events. We lose $5.7\pm2.2\%$ of the photons due to conversions before and in the tracking chambers and $5.0\pm1.6\%$ due to the requirements on the cluster shape.

\section{The Systematic Errors of the Corrections}

We consider both the modelling of photon emission and the detector simulation as potential sources of systematic errors. As a first check of the quality of the simulation we compare the uncorrected spectra of the energy $E_\gamma$ and the polar angle $|\cos \theta_\gamma|$ of the photon for data and Monte Carlo (figs. 3a, b). The predicted yield is normalised to the observed one. Significant errors in the modelling of the event structure or in the detector simulation should lead to discrepancies between data and model. The observed good agreement gives us confidence in the correction procedure.

In the following two sections we summarise which systematic errors we will consider for the event properties discussed in section 7 and how we estimate them. They apply to all distributions. Typical values for the size of the errors will be given for the yield.

\subsection{Systematic Errors in the Modelling of Photon Emission}

Since we cannot compare the predicted photon yield directly with the data for photon energies below 7.5 GeV and for isolation cones of smaller than 15 degrees, we have to rely on the simulation to correct for these losses. We assign a systematic uncertainty $\delta c_1(y_{\text{cut}})$ of $25\%$ of the losses due to those requirements. For the total cross section this results in a systematic uncertainty of $13\%$ at $y_{\text{cut}}=0.005$ and $4.5\%$ for $y_{\text{cut}} \geq 0.04$. For some event distributions $f(x)$ this systematic error may depend on the value of $x$. For example for the efficiency in the photon energy for $y_{\text{cut}}=0.005$ as displayed in fig. 2b, we assign a systematic error of $6\%$ at $E_\gamma = 15$ GeV and of $2.5\%$ at $E_\gamma = 40$ GeV.
Various cross checks suggest that our error assignment is appropriate. As one cross check we compared the efficiencies obtained with the two Monte Carlo generators JETSET [8] and ARIADNE [9], which include final state photons. As can be seen in fig. 2 the efficiencies are similar, the largest difference being 5%. It should be noted that their predicted absolute photon yields differ by about 40%.

In addition we studied the potential bias on the observed photon yield due to the rather stringent isolation requirement. The estimate of these losses depends critically on the modelling of soft hadronisation. The energy and particle flow in multihadronic events has been analysed in much detail and found to be well described by simulations involving string fragmentation (see e.g. [16]). For our photon events we examined the particle flow just outside the isolation cone. The distribution of \((1/N)(dN/d\cos \alpha)\), \(\alpha\) being the smallest angle per event of a track or a cluster with respect to the photon candidate, is shown in figs. 4a,b for \(y_{cut} = 0.005\) and 0.06, respectively. The simulation agrees with the data. In a conservative approach we take the small (though not significant) excess of data in the first bin as an indication of a possible bias. Extrapolating the excess into the isolation cone, we estimate a potential systematic uncertainty of the yield of 2% and 3% for the two \(y_{cut}\) values. Secondly we changed the isolation requirement. By allowing a maximum energy in the isolation cone of 500 MeV instead of 0 MeV, and an isolation cone of 12.5 or 17.5 degrees we observe changes consistent with the assigned errors on the modelling of photon emission discussed above.

5.1.2 Systematic Uncertainties from the Detector Simulation

As discussed above, the photon detection efficiency has been determined with a reference sample of genuine photons. We use its statistical uncertainty as the systematic error of the photon detection efficiency for our analysis.

Another uncertainty stems from a potential misrepresentation of the measured direction and energy of the jets in the simulation. We estimate the quality of the reconstruction by using events with two jets and a photon according to the prescription of section 3. Since we assume the jets to be massless for these cross checks, we impose a small \(y_{cut} = 0.02\). To estimate the angular resolution of the jet direction we boost the hadronic part of the event into its rest system given by the photon momentum and

\[
M_{\text{recoil}} = E_{\text{cm}} \sqrt{1 - 2 \cdot E_{\gamma}/E_{\text{cm}}}. \tag{1}
\]

In this system the two jets should be approximately collinear. The acolinearity angle between the sum of momenta of the two hemispheres is displayed in fig. 4c both for the data and for the simulation. The average value of 4.0±0.4 degrees in the data agrees well with the expectation of 4.2±0.2 degrees. The quality of the energy reconstruction is estimated by using the relation

\[
E_i = E_{\text{cm}} \cdot \frac{\sin \alpha_{ji}}{\sum \sin \alpha_{mn}}, \tag{2}
\]

which is valid for three massless particles. Here \(E_i\) is the energy of the jet \(i\) and \(\alpha_{ji}\) is the angle between the other jet and the photon. The sum goes over all three angles between jets and photon. This allows us to determine the jet energies rather independently of the energy
resolution of the detector. In fig. 4d we display

\[ \frac{\delta E}{E} = \frac{E_{\text{meas}} - E_{\text{recon}}}{E_{\text{meas}}} \]

where \( E_{\text{meas}} \) is the jet energy as obtained from the sum of track and cluster energies, and \( E_{\text{recon}} \) is the energy obtained from relation (2). The data are well reproduced by the simulation. The data and Monte Carlo distributions are both centred near zero with average values of at 
\[-0.079 \pm 0.037 \text{ and } -0.019 \] 
and widths of 0.24 and 0.23, respectively.

We estimate the potential systematic uncertainty of the corrections due to the quality of the reconstruction by smearing the reconstructed energy and direction and by rescaling the jet energies. Changing the energy resolution by an additional 15%, or the angular resolution by 15 degrees has a negligible effect on the results. A rescaling of the jet energies by \( \pm 6\% \), to represent possible deviations as discussed in the previous paragraph, has small effects on the distributions (e.g. the yield changes by 2.7%) and is included in the systematic error.

In summary, several consistency checks with the data were made to estimate systematic errors. The largest one is due to uncertainties in the modelling of photon emission in phase space regions outside our selection, particularly important for low values of \( y_{\text{cut}} \). In addition, we have to account for a potential systematic distortion of the jet energies. We will consider these two error sources for all distributions discussed in section 7.

6 Theoretical Predictions for Final State Radiation

Currently there are three theoretical predictions for final state photons in multihadronic events. Whereas the matrix element calculation of \( O(\alpha_s) \) [7] is only available for the absolute yields of up to three partons and a photon, the parton shower models JETSET [8] and ARIADNE [9] predict in addition all event features and photon properties. Each is discussed below.

It should be noted that in all cases the electromagnetic coupling constant \( \alpha \) has to be taken in the Thomson limit since we are dealing with real photons [17].

6.1 The Matrix Element Calculation

Kramer and Lampe have calculated the cross sections \( \sigma_{\gamma+n \text{jets}} \) in \( O(\alpha_s) \) for \( n \leq 3 \) as a function of \( y_{\text{cut}} \) [7]. This prediction has been derived from that of the jet rates for the inclusive hadronic event sample. Their result is expressed as

\[ \sigma(Z^0 \rightarrow \gamma + \text{jets})(y_{\text{cut}}) = \sum_{n=1}^{n=3} \sigma_n(y_{\text{cut}}) \]

(3)

with the n-jet cross section \( \sigma_n \)

\[ \sigma_1(y_{\text{cut}}) \propto b_1(y_{\text{cut}}) + \frac{\alpha_s}{2\pi} \cdot b'_1(y_{\text{cut}}) \]

\[ \sigma_2(y_{\text{cut}}) \propto b_2(y_{\text{cut}}) + \frac{\alpha_s}{2\pi} \cdot b'_2(y_{\text{cut}}) \]
The coefficients $b_1$ and $b'_1$ are given for discrete values of $y_{cut}$ between 0.005 and 0.2. The primed coefficients contain the contributions from $O(\alpha_s)$ corrections. The value for $\sigma_3$ is a tree level result, $\sigma_1$ and $\sigma_2$ include complete corrections of $O(\alpha_s)$ \footnote{The coefficients $b_i$ of $[7]$ are different from those used in $[2]$. In the meantime Kramer and Lampe have calculated the $O(\alpha_s)$ correction to the one jet rate and a reevaluation of the coefficients given to us previously showed that $b_3$ was too low by a factor 2. These changes do not affect the basic conclusions and values of the weak quark couplings given in [2].}. It should be noted that since the jet rates are only given to first order in $\alpha_s$, the result only depends on the ratio of $\Lambda_{\overline{MS}}$ and the renormalisation scale $\mu^2 = f \cdot E^2_m$. Therefore no explicit variation of the scale will be considered. Both the absolute yield and the jet rates for events with final state photons depend on $\alpha_s$.

The comparison of the matrix element calculation and the data is complicated by ambiguities in the definition of $\alpha_s$ and the jet definitions. These are both discussed below.

Assuming standard model electroweak quark couplings no free parameter appears in the theoretical prediction apart from $\alpha_s$. The possible range of the predicted photon yield is therefore unambiguously given by the variation of $\alpha_s$. As the calculation of $[7]$ is first order in $\alpha_s$, we adopt the value $\alpha_s^{(1)}=0.177 \pm 0.013$, which is the first order $\alpha_s$ value as obtained from the jet rates in the multihadronic $Z^0$ decays. This value agrees well with $\alpha_s^{(1)}=0.161 \pm 0.012$, as obtained from the ratio of two and three jet fractions in our final state photon event sample (see section 7.2.1). To include the uncertainty of the theoretical prediction due to the value of $\alpha_s$ we use as a upper bound 0.19, and as a lower bound $\alpha_s=0.118$, which is the second order $\alpha_s$ value obtained from the jet rates in multihadronic $Z^0$ decays $[18]$.

The ambiguities in the jet definition arise because we have to compare the theoretical expression for massless partons with the measurement of massive jets. It was shown for the analysis of jet rates in general multihadronic events $[19]$, that the results of the calculations agree well with data if we adopt the jet finding algorithm of $[11]$ corresponding to the ‘E0’ recombination scheme.

Another discrepancy between the matrix element calculation and our analysis stems from the different procedure for the event definition.

- For this analysis the relation between photon and jets was determined in two steps. At first only the hadrons were grouped into jets according to a certain $y_{cut}$ value, and then the minimum jet-photon mass was calculated.
- Kramer and Lampe, dealing with at most three partons, immediately include the photon in the jet definition for some $y_{cut}$ value. An event is discarded if a parton and a photon are combined into a new ‘jet’.

In the absence of a matrix element for the differential distributions of the $q\bar{q}\gamma(g)$ events, we have estimated the corresponding distortions introduced by the different procedures by applying them separately to the $O(\alpha_s^3)$ matrix element for an abelian model of strong interactions as implemented in $[8]$. In this case we have selected only contributions from $q\bar{q}g$ and $q\bar{q}gg$ and
replaced one gluon by a photon. We expect very close agreement between this model and the \( q\gamma(g) \) distribution. As will be discussed in the next section, we find only small differences for the two event definitions at lower \( y_{cut} \) and large jet multiplicities but a significant difference for the one jet rate at large \( y_{cut} \) values.

6.2 Parton Shower Models

Apart from giving the absolute rates, the shower models \([8, 9]\) offer predictions on the detailed properties of the photons and of the residual hadronic event structure. On the other hand they are based on an approximate treatment of QCD. In the two models JETSET and ARIADNE the jet development is simulated respectively by a parton or colour dipole cascade with subsequent hadronisation according to the string picture. Photon emission competes at each step of the cascade with gluon emission from either a quark or a gluon or gluon splitting into a quark pair. For example the differential probability for gluon \((g)\) or photon \((\gamma)\) emission, \( q_1 \rightarrow q_2(g, \gamma) \), is approximately given by

\[
\frac{dP}{dt} \propto \int_{z_{min}}^{z_{max}} \, dz \left[ \frac{\alpha_s(Q^2)}{2\pi} P_{q_1 \rightarrow q_2, g}(z) + \frac{\alpha}{2\pi} P_{q_1 \rightarrow q_2, \gamma}(z) \right],
\]

with \( t \) being an evolution parameter (see below), \( \alpha_s, \alpha_{em} \) the strong and electromagnetic coupling strength in leading order, and \( z \) the sharing of energy and momentum between \( q_2 \) and \((\gamma, g)\). \( P_{q_1 \rightarrow q_2, g}(\gamma) \) is the Altarelli-Parisi splitting function \([20]\) (modified in ARIADNE), which is identical for gluon and photon emission apart from factors for the colour or electromagnetic charge. In fact Sudakov form factors are also included for the branching probabilities used in the parton shower models. The integration limits \( z_{min}, z_{max} \) are defined in JETSET mainly by mass cutoffs, in ARIADNE by \( p_T \) cutoffs, where \( p_T \) is the transverse momentum with respect to the initial quark direction.

Ambiguities in these prescriptions and differences between these models are due to the various possible interpretations of variables like \( z, t \), and the argument of \( \alpha_s \) (for a discussion of some of these see \([21]\)). In a given model the various parameter values are significantly constrained from the topology of multihadronic events which both programs, JETSET and ARIADNE, describe well for centre-of-mass energies between 14 and 91 GeV. The two models differ in some of these prescriptions, e.g. they require different QCD scale parameters: \( \Lambda_{JETSET} = 290 \) MeV, \( \Lambda_{ARIADNE} = 200 \) MeV to describe the inclusive multihadronic event sample and use different evolution parameters \( t_{JETSET} = \ln(m^2_{q_1}/\Lambda^2) \), \( t_{ARIADNE} = \ln(p_T^2/\Lambda^2) \). In addition there are conceptual differences of the parton evolution once a gluon has been emitted. In JETSET the gluons and quarks branch, apart from kinematical constraints, independently from the rest of the event. In ARIADNE the emission of a gluon is due to colour dipoles between gluons and quarks, and the photon emission is due to electromagnetic dipoles between quarks. As a result, the emission affects the kinematics of both quarks. This implies that the models potentially predict different properties of events with final state photon radiation although both models reproduce the event shapes and particle distributions of the inclusive hadronic event sample very well. Final state photons are therefore a new probe of the basic mechanisms of parton evolution invoked in the two models.

The ambiguities mentioned above lead to uncertainties in the predictions of the shower models. For this analysis we adopt the default parametrisations with parameter values opti-
rified to describe the event shape distributions [22] for the whole multihadronic sample. Our
analysis then tests whether photon and gluon emission can be described consistently. As the
only uncertainty we consider the variation of the QCD scale parameter \( \Lambda \) (see discussion in [6]).
Its possible range is taken from fits to the measured three jet rate [18] yielding \( \pm 30 \) MeV for
both JETSET and ARIADNE.

7 Properties of Events with Final State Photons

We now present our measurements in events with final state photons corrected for detector
effects and the event selection. We consider four classes of distributions; the photon yield, the
jet rates of the hadronic system, the properties of the photon itself, and those of the residual
event. All properties examined depend on the \( y_{\text{cut}} \) applied. The yield and the jet rates are
given for \( y_{\text{cut}} \) values between 0.005 and 0.2. For the other distributions we restrict ourselves to
a low \( y_{\text{cut}} = 0.005 \) for which the uncertainty due to the energy and isolation cut is high, and a
higher \( y_{\text{cut}} = 0.06 \) with a safer correction but limited sample size. We compare the yield to the
predictions of the matrix element calculation [7] and QCD shower models [8, 9]. Photon and
event properties are also compared to QCD shower models. In the latter case we normalise the
predictions to the observed yield. This then provides a check that is largely independent of the
yield. The uncertainties due to the energy and isolation requirement are not relevant for the
comparisons of the data with shower models; JETSET was used to determine the correction
factors and they were found to be almost identical with those from ARIADNE.

The contributions due to hadronic background and initial state photons are subtracted for
all distributions.

7.1 The Cross Section

The cross section results have already been discussed in [2], where they were used to derive the
electroweak coupling constants of up and down type quarks. For completeness we give a short
summary of these results.

In table 1 and figs. 5a, b we show the fraction of all multihadronic events which contain final
state photons as a function of \( y_{\text{cut}} \). The yield decreases by about a factor of ten for an increase
of \( y_{\text{cut}} \) from 0.005 to 0.2. It should be noted that the results for the different values of \( y_{\text{cut}} \) are
correlated. The uncertainty in \( c_1 \) leads to a large systematic error of the cross section at very
low \( y_{\text{cut}} \). This uncertainty decreases rapidly with increasing \( y_{\text{cut}} \). For \( y_{\text{cut}} \geq 0.04 \) the statistical
error dominates. Also shown are the predictions from the matrix element calculation and the
parton shower models JETSET and ARIADNE.

The ratio between data and the matrix element calculation and the parton shower models
is displayed in figs. 5c, d and e. As stated before, we neglect the error on \( c_1 \) for the comparison
of the data with the shower models. We make the following observations.

- The matrix element calculation is in good agreement with the data for \( y_{\text{cut}} \leq 0.12 \) but
  almost four standard deviations below the data at high \( y_{\text{cut}} \) (figure 5a, c).
Table 1: Photon selection for various $y_{cut}$. The corrected fraction of events with final state photons over the total number of hadronic events is listed. The first error combines the statistical and systematic uncertainty of $c_2$, added in quadrature, the second is due to the energy and isolation cut. Also shown are the predictions from the matrix element calculation [7] (ME) and the parton shower Monte Carlos JETSET [8] and ARIADNE [9]. The error on the matrix element prediction reflects the uncertainty of $\alpha_s$. The uncertainties assigned to the JETSET and ARIADNE predictions are only due to variations of the QCD scale parameter $\Lambda$. Note that the second error of the data is not relevant for the comparison of the JETSET and ARIADNE prediction to the data.
This deviation is at least partly due to the different definitions of the retained events (see section 6.1). An approximate measure of this effect may be obtained if we correct the theoretical expectation according to the results of our analysis of the abelian matrix element. In this case a much better agreement between data and theory is found. Using the abelian model to correct the theoretical calculation, we find e.g. at $y_{\text{cut}}=0.2$, where the discrepancy is largest, that the expected photon yield increases from 0.30 to 0.48. A final assessment can only be made once our event definition can be adopted in a theoretical calculation.

- JETSET (figs. 5b,d) underestimates the photon yield at very low $y_{\text{cut}}$ by 30% with a significance of about three standard deviations. For $y_{\text{cut}} \geq 0.06$ the expectation is only 10% lower than the data and agrees within one standard deviation.

- ARIADNE (figs. 5b,e) reproduces the data at low $y_{\text{cut}}$ but tends to overestimate the measured yield for $y_{\text{cut}} \geq 0.04$ by about 25%. The significance of this discrepancy is up to two standard deviations.

7.2 Jet rates

7.2.1 Results and Comparison with Theoretical Expectations

The distribution of jets reflects the underlying parton structure and can therefore be directly compared with the QCD calculations. In the case of the inclusive hadronic event sample the fraction of $n$-jet events is very well described by matrix element calculations and by the QCD shower models of JETSET and ARIADNE.

In fig. 6a we compare the relative jet rates with the parton shower models. JETSET tends to overestimate the observed two jet rate, or, equivalently, to underestimate the one jet rate for $y_{\text{cut}} > 0.06$. ARIADNE describes the observed jet rates well. The data are currently not significant enough to discriminate between the models.

In table 2 and fig. 6b we display the absolute yield of one, two and three jet events with a hard photon. The number of events with three or more jets decreases rapidly with increasing $y_{\text{cut}}$, no such events are observed for $y_{\text{cut}} \geq 0.08$. The fraction of two jet events increases slightly with $y_{\text{cut}}$ for $y_{\text{cut}} \leq 0.02$, and then decreases by almost an order of magnitude for an increase of $y_{\text{cut}}$ up to 0.2. The yield of one jet events, i.e. events where the masses of all partons satisfy $M_{\text{had}}^2/E_{\text{cm}}^2 < y_{\text{cut}}$ increases continuously with increasing $y_{\text{cut}}$. Also shown by the shaded bands are the expectations derived from the matrix element calculation; the widths indicate the uncertainty in $\alpha_s$. While the two and three jet rates are in good agreement, the one jet yield at very high $y_{\text{cut}}$ differs significantly from the theoretical prediction. For $y_{\text{cut}}=0.20$ the ratio of the expectation over the data is 0.25±0.08.

We have examined the one jet events for experimental biases by determining the recoiling hadronic mass taking advantage of the good measurement of the photon momentum and using the relation $y = 1 - z_\gamma$ with $z_\gamma = 2E_\gamma/E_{\text{cm}}$. True one jet events are to have $y < y_{\text{cut}}$. We find the assignment to be correct for typically 95% of the events. From the analysis with the abelian model of strong interactions we find that at least a large part of the discrepancy between data...
<table>
<thead>
<tr>
<th>$y_{\text{cut}}$</th>
<th>one jet</th>
<th>two jet</th>
<th>three jets</th>
<th>four jets</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>0.03±0.02</td>
<td>1.78±0.24±0.25</td>
<td>2.65±0.27±0.34</td>
<td>1.27±0.18±0.15</td>
</tr>
<tr>
<td>0.010</td>
<td>0.03±0.02</td>
<td>2.21±0.24±0.25</td>
<td>2.15±0.22±0.22</td>
<td>0.37±0.08±0.04</td>
</tr>
<tr>
<td>0.020</td>
<td>0.05±0.02</td>
<td>2.53±0.23±0.22</td>
<td>1.02±0.14±0.08</td>
<td>0.05±0.03</td>
</tr>
<tr>
<td>0.040</td>
<td>0.09±0.03</td>
<td>2.03±0.19±0.11</td>
<td>0.20±0.06±0.01</td>
<td></td>
</tr>
<tr>
<td>0.060</td>
<td>0.13±0.04</td>
<td>1.54±0.15±0.05</td>
<td>0.06±0.03</td>
<td></td>
</tr>
<tr>
<td>0.080</td>
<td>0.15±0.04</td>
<td>1.13±0.13±0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.100</td>
<td>0.20±0.05±0.01</td>
<td>0.84±0.11±0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.120</td>
<td>0.24±0.06±0.01</td>
<td>0.65±0.10±0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.140</td>
<td>0.31±0.07±0.01</td>
<td>0.49±0.09±0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.160</td>
<td>0.34±0.07±0.01</td>
<td>0.36±0.07±0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.180</td>
<td>0.35±0.07±0.02</td>
<td>0.29±0.07±0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.200</td>
<td>0.42±0.08±0.02</td>
<td>0.25±0.06±0.01</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: The number of one, two and three jet events plus photons per 1000 multihadronic events are listed. The first error combines the statistical and systematic uncertainty of $c_2$, added in quadrature, the second is due to the energy and isolation cut.

<table>
<thead>
<tr>
<th>$y_{\text{cut}}$</th>
<th>one jet</th>
<th>two jet</th>
<th>three jets</th>
<th>four jets</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>0.01±0.01</td>
<td>0.30±0.03±0.04</td>
<td>0.45±0.03±0.04</td>
<td>0.21±0.03±0.03</td>
</tr>
<tr>
<td>0.010</td>
<td>0.01±0.01</td>
<td>0.46±0.04±0.04</td>
<td>0.45±0.03±0.03</td>
<td>0.08±0.02±0.01</td>
</tr>
<tr>
<td>0.020</td>
<td>0.01±0.01</td>
<td>0.69±0.03±0.02</td>
<td>0.28±0.03±0.02</td>
<td>0.01±0.01</td>
</tr>
<tr>
<td>0.040</td>
<td>0.04±0.01</td>
<td>0.87±0.03±0.01</td>
<td>0.09±0.02±0.01</td>
<td></td>
</tr>
<tr>
<td>0.060</td>
<td>0.07±0.02</td>
<td>0.89±0.03±0.01</td>
<td>0.03±0.02</td>
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</tr>
<tr>
<td>0.080</td>
<td>0.12±0.03</td>
<td>0.88±0.03±0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.100</td>
<td>0.19±0.04</td>
<td>0.81±0.04±0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.120</td>
<td>0.27±0.06±0.01</td>
<td>0.73±0.06±0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.140</td>
<td>0.38±0.06±0.01</td>
<td>0.62±0.06±0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.160</td>
<td>0.49±0.07±0.01</td>
<td>0.51±0.07±0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.180</td>
<td>0.55±0.07±0.01</td>
<td>0.45±0.07±0.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.200</td>
<td>0.62±0.07±0.01</td>
<td>0.38±0.07±0.01</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: The fractions of one, two and three jet events plus photon are listed. The first error combines the statistical and systematic uncertainty of $c_2$, added in quadrature, the second is due to the energy and isolation cut.
and calculation can be explained by the different event definitions. Again, a final assessment can only be made once our event definition can be adopted in a theoretical calculation.

We can use the relatively strong dependence on $\alpha_s$ of the two and three jet rates at low $y_{\text{cut}}$ to determine the value of the strong coupling constant. To reduce systematic uncertainties and to be independent of unknowns in the overall production rate we use the ratio of three jet events over the sum of two and three jet events. For two different values of $y_{\text{cut}} = 0.02; 0.04$ we find

$$\frac{\sigma_{\gamma+3j}}{\sigma_{\gamma+2j} + \sigma_{\gamma+3j}} = 0.287 \pm 0.020; \quad 0.090 \pm 0.018.$$  

Translating these ratios into values of a first order $\alpha_s^{(1)}$ leads to

$$\alpha_s^{(1)} = 0.161 \pm 0.012; \quad 0.154^{+0.031}_{-0.023}$$

for the two $y_{\text{cut}}$ values respectively. These results are in good agreement with the $\alpha_s^{(1)}$ value of $0.177 \pm 0.013$ obtained from the inclusive multihadronic event sample. In the absence of differential distributions for photon plus $n$-jet events we cannot calculate the uncertainties from the hadronisation and recombination mechanism which are important for the uncertainty of the $\alpha_s^{(1)}$ value from the multihadronic jet rates.

### 7.2.2 Comparison with Jet Rates in the Inclusive Multihadronic Event Sample

It has been suggested [7, 23] that a comparison of the cross sections for the production of $q\bar{q}g$ events from the total multihadronic sample to events with two jets and a photon ($q\bar{q}\gamma$) can provide information on the strength and structure of the strong interactions. The idea is to use the well known properties of photon radiation to disentangle the specific QCD contributions.

The corresponding cross sections for $q\bar{q}g$ and $q\bar{q}\gamma$ events are given by [24]

$$\sigma(q\bar{q}g)(y_{\text{cut}}) \propto C_F \frac{\alpha_s}{2\pi} B(y_{\text{cut}}) + \left(\frac{\alpha_s}{2\pi}\right)^2 C_F \left[ C_F h_c(y_{\text{cut}}) + N_{ch} h_N(y_{\text{cut}}) + \frac{N_f}{2} h_T(y_{\text{cut}}) \right]$$  \hspace{1cm} (5)

$$\sigma(q\bar{q}\gamma)(y_{\text{cut}}) \propto A_c \left[ \frac{\alpha}{2\pi} B(y_{\text{cut}}) + \frac{\alpha}{2\pi} \frac{\alpha_s}{2\pi} C_F h_c(y_{\text{cut}}) \right]$$  \hspace{1cm} (6)

where $\alpha_s$ and $\alpha$ are the strong and electromagnetic coupling strengths, $C_F = 4/3$ is the colour factor, $N_{ch} = 3$ the number of colours and $N_f$ the number of flavours. Assuming the contributions of up and down type quarks to agree with the standard model with an electroweak mixing angle $\sin^2 \theta_w = 0.23$, the average squared quark charge $A_c$ has the value 0.2249. The functions $f(y_{\text{cut}})$ reflect the topological configurations of the parton/photon states. They account for single bremsstrahlung ($B$), double bremsstrahlung ($h_c$), the non-abelian contribution ($h_N$), and the gluon splitting into a quark pair ($h_T$). Since photons couple to quarks in the same way as gluons (apart from the strength), $B(y_{\text{cut}})$ and $h_c(y_{\text{cut}})$ are almost identical for gluon and photon emission. The terms for the gluon self coupling and for the gluon splitting are unique to the three jet distribution. The combination of (5) and (6) proposed in [7] is a convenient way to separate the common contributions from those specific to the QCD process.

$$R_{\gamma g} = \frac{\alpha A_c \sigma(q\bar{q}g)}{C_F \sigma(q\bar{q}\gamma)} = \alpha_s \left[ 1 + \frac{N_{ch} h_N(y_{\text{cut}}) + \frac{N_f}{2} h_T(y_{\text{cut}})}{2\pi B(y_{\text{cut}}) + (\alpha_s/2\pi) C_F h_c(y_{\text{cut}})} \right]$$  \hspace{1cm} (7)

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The $h_c$ term is expected to contribute about 10% to $R_{\gamma g}$ at $y_{cut}=0.06$. Those for the self coupling ($h_N$) and for $qgq$ ($h_T$) are significant with $N_c h_N \sim 4 \cdot \frac{N_c}{2} h_T$. Given that the single bremsstrahlung $B(y_{cut})$ term is well known, this implies (7) is particularly sensitive to the structure of the triple gluon vertex ($h_N$).

We will now use this ratio to measure $\alpha_s$. This measurement is strongly correlated with the determination of $\alpha_s$ from three jet event rate alone. In using jet rates the three jet cross section (5) is related to the overall event yield which is well known and depends only weakly on $\alpha_s$. In using $R_{\gamma g}$ we relate (5) to the precisely known values of $\alpha$ and $\sin^2 \theta_W$ (assuming standard model couplings). The precision of the ratio $R_{\gamma g}$ is currently limited by the statistical and systematic accuracy of the photon events. The errors on $\alpha_s$ derived from the three jet fraction are given, essentially, by the uncertainties in the recombination scheme, the hadronisation correction and the scale dependence [18]. In the absence of a differential parton distribution for $qg\gamma$ events these uncertainties cannot be estimated.

Neglecting $\alpha_s^2$ terms we can use our measurement of $R_{\gamma g}$ to determine $\alpha_s$ in first order. For $y_{cut}=0.06$, where we have the largest sensitivity, we find

$$\alpha_s^{(1)} = 0.195 \pm 0.020$$

again in agreement with the previously mentioned value determined from the three jet fraction in the inclusive hadronic event sample. The result also agrees with our measurement of $\alpha_s^{(1)}$ from the relative rates of two and three jet plus photon events, discussed in section 7.2.1. In using (7) to determine $\alpha_s$ in second order we have to be aware that the $\alpha_s$ value in the $h_c$ term stems from (6) and is of first order but all other values of $\alpha_s$ are of second order and come from the cross section for three jet events (5). In obtaining $\alpha_s^{(2)}$ from (7) we set $\alpha_s^{(1)} = \alpha_s^{(2)}$. Since $\alpha_s^{(1)}$ enters only in the $h_c$ term this leads to a small (~5%) correction. In fig. 7a we display the observed ratio $R_{\gamma g}$ as a function of $\alpha_s(M_Z)$ according to equation 7 and translate our measurement into

$$\alpha_s^{(2)}(M_Z) = 0.122 \pm 0.010$$

in agreement with the $\alpha_s$ of $0.118 \pm 0.08$ obtained from the topology of hadronic events alone. The dominant contribution to the error is from theoretical ambiguities, the pure experimental uncertainty is 0.003 [18].

Another test of the QCD structure is given by the $y$-dependence of $R_{\gamma g}$. For low $y_{cut}$ the positive contributions from the gluon self coupling tend to compensate the negative contribution of the double bremsstrahlung term, leading to an increase of the ratio at low $y_{cut}$ [7]. For $N_c=0$, i.e. the absence of the self coupling, the ratio would exhibit a slow decrease. At large $y_{cut}$, theory predicts an almost flat behaviour, over a wide range of $\alpha_s$ values, i.e.

$$\frac{N_c h_c(y_{cut}) + \frac{N_c}{2} h_T(y_{cut})}{B(y_{cut})} \sim \text{constant}.$$

This dependence is shown in fig. 7b for the data together with the expectation for various values of $\alpha_s$. The data show almost no variation with $y_{cut}$ for $y_{cut} \geq 0.04$ in agreement with QCD. However, for $y_{cut} < 0.06$ $R_{\gamma g}$ increases with decreasing $y_{cut}$ in agreement with a non-abelian theory of strong interactions. The abelian expectation (displayed for $\alpha_A=0.2$) does not describe the data.

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7.3 Properties of the Photon

We now turn to the properties of the photon itself. As yet no matrix element calculation exists for these properties and we can compare our measurement only with the parton shower models JETSET and ARIADNE. These models assume a time sequence in the parton branchings with a decreasing virtuality of the quark system in each step of the shower process. The transverse momentum of the photon with respect to the emitting quark is therefore a measure of the time scale of the emission (see also discussion in [23]): energetic photons produced at large angles stem from quarks or parton systems of high virtual masses and are, therefore, emitted at an early stage of the jet development. Within the JETSET program typically 90% of the photons retained by our cuts for $y_{cut} = 0.04-0.16$ are emitted in the first branching, i.e. before any gluon radiation. This number decreases to 75% at $y_{cut} = 0.005$ and to 81% at $y_{cut} = 0.2$. The photon properties are therefore related to the jet evolution and provide information about the masses involved in the parton branching. Because of the conceptual simplicity and the general success of this model of jet evolution we will frequently refer to it.

In the following, we restrict ourselves to $y_{cut}$ values of 0.005 and 0.06 and present distributions corrected for detector effects and our selection criteria. We have verified that these corrections are consistent for JETSET and ARIADNE. This implies in particular that the uncertainties due to the energy and isolation requirements are unimportant for the comparison of data and these models. The corrections can be large in certain kinematical regions and, as discussed in section 5, can imply substantial systematic uncertainties in the shape of the distribution. This is particularly true for $y_{cut} = 0.005$, whereas the corrections are in general slowly varying for $y_{cut} = 0.06$. Systematic uncertainties due to the energy and isolation requirement and due to detector effects are indicated by the dashed error bars in figs. 8-15. These plots are normalised to their numbers of entries. We list the observed average values of the distributions together with the expectations from the two QCD shower models in table 4.

7.3.1 The Photon Energy

Figure 8 shows the photon energy spectrum for the two values of $y_{cut}$. For $y_{cut} = 0.005$ the photon yield decreases by about a factor 10 for an increase of the energy from 7.5 to about 45 GeV. The energy distribution is much flatter for $y_{cut} = 0.06$ due to the larger photon-jet mass required.

The predictions of JETSET and ARIADNE are consistent with each other and are in agreement with the data.

7.3.2 The Transverse Momentum with Respect to the Thrust Axis

The transverse momentum distribution of the photon with respect to the thrust axis $p_T^{th}$ is displayed in fig. 9. The thrust has been calculated from all charged tracks and clusters in the electromagnetic calorimeter, including the photon candidate. From leading order calculations we expect the distribution to be proportional to $1/p_T^{th}$. Kinematically, the transverse momentum is limited to $1/\sqrt{3}E_{beam} \sim 27$ GeV. Since the thrust axis is a good measure of the direction
Table 4: Average values of photon and event properties in the data and for JETSET and ARIADNE. The first error for the data is the statistical and experimental systematic, the second is due to the energy and isolation cut. The errors in the model predictions are statistical only.

<table>
<thead>
<tr>
<th>Property</th>
<th>$y_{\text{cut}}$</th>
<th>Data</th>
<th>JETSET</th>
<th>ARIADNE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photon Energy (GeV)</td>
<td>0.005</td>
<td>19.7±0.6</td>
<td>20.3±0.1</td>
<td>20.7±0.1</td>
</tr>
<tr>
<td></td>
<td>0.060</td>
<td>23.2±0.8±0.1</td>
<td>23.2±0.1</td>
<td>24.1±0.1</td>
</tr>
<tr>
<td>$p_T$ wrt Thrust (GeV)</td>
<td>0.005</td>
<td>6.18±0.27</td>
<td>6.48±0.03</td>
<td>6.45±0.03</td>
</tr>
<tr>
<td></td>
<td>0.060</td>
<td>10.6±0.4</td>
<td>11.0±0.1</td>
<td>10.5±0.1</td>
</tr>
<tr>
<td>$p_T$ wrt Jet (GeV)</td>
<td>0.005</td>
<td>10.6±0.5±0.2</td>
<td>11.1±0.1</td>
<td>11.3±0.1</td>
</tr>
<tr>
<td></td>
<td>0.060</td>
<td>18.8±0.8±0.1</td>
<td>19.4±0.1</td>
<td>18.8±0.1</td>
</tr>
<tr>
<td>Angle wrt Jet (deg)</td>
<td>0.005</td>
<td>68.3±2.1±0.3</td>
<td>73.3±0.8</td>
<td>71.7±0.2</td>
</tr>
<tr>
<td></td>
<td>0.060</td>
<td>84.1±2.6±0.3</td>
<td>92.5±0.3</td>
<td>94.9±0.3</td>
</tr>
<tr>
<td>Jet/Photon Mass (GeV)</td>
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<td>15.2±0.5±0.1</td>
<td>15.1±0.1</td>
<td>14.9±0.1</td>
</tr>
<tr>
<td></td>
<td>0.060</td>
<td>33.3±0.9±0.4</td>
<td>36.7±0.2</td>
<td>38.2±0.2</td>
</tr>
<tr>
<td>Mass Photon Hemisphere (γ incl) (GeV)</td>
<td>0.005</td>
<td>22.2±0.5±0.1</td>
<td>21.7±0.1</td>
<td>21.2±0.1</td>
</tr>
<tr>
<td></td>
<td>0.060</td>
<td>27.5±0.9±0.1</td>
<td>27.6±0.1</td>
<td>26.6±0.1</td>
</tr>
<tr>
<td>Mass Photon Hemisphere (γ excl) (GeV)</td>
<td>0.005</td>
<td>11.9±0.4±0.1</td>
<td>10.8±0.1</td>
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<td>0.060</td>
<td>9.5±0.5±0.1</td>
<td>8.9±0.1</td>
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</tr>
<tr>
<td>Mass non-Photon Hemisphere (GeV)</td>
<td>0.005</td>
<td>16.1±0.5±0.1</td>
<td>15.5±0.1</td>
<td>15.4±0.1</td>
</tr>
<tr>
<td></td>
<td>0.060</td>
<td>16.1±0.7±0.1</td>
<td>16.6±0.1</td>
<td>16.4±0.1</td>
</tr>
</tbody>
</table>

of the primary quarks, the $p_T$ is related to the mass of the primary quark with $p_T \sim M_q/2$ assuming the photon stems from the first emission.

Also shown in figs. 9a,b are the expectations from ARIADNE and JETSET. The data and the predictions have a maximum at $p_T \sim \sqrt{y_{\text{cut}} \cdot E_{\text{cm}}} / 2 \sim M_{\gamma}^{\text{min}}/2$, i.e. 3 and 11 GeV for the two values of $y_{\text{cut}}$. For larger $p_T$ the fall off is consistent with the leading order expectation. For $y_{\text{cut}}=0.06$ there is an enhancement at low $p_T$ due to highly energetic photons that determine the thrust direction. ARIADNE and JETSET predict different fractions of these low $p_T$ events at $y_{\text{cut}}=0.06$. In JETSET there are 12.9±0.4% of events with $p_T < 2.5$ GeV, in ARIADNE 15.9±0.4%. We observe 14.3±3.6±1.8%, consistent with both predictions.

7.3.3 Transverse Momentum with Respect to the Jet Direction

A more direct relation between the mass of the emitting quark and of the photon is expected for the transverse momentum with respect to the direction of the jet with minimal jet-photon mass, $p_T^{\text{jet}}$. The corresponding distribution is shown in fig. 10.

For the two $y_{\text{cut}}$ values the measured $p_T^{\text{jet}}$ distributions peak at values of 2.5 and 12.5 GeV, similar to those for the $p_T^{\text{hr}}$. For $p_T^{\text{jet}}$ the kinematic limit is close to $E_{\text{beam}}$ which corresponds to a topology where the associated jet is soft and is emitted almost perpendicular to the thrust direction. For low values of $p_T^{\text{jet}}$ and $y_{\text{cut}}=0.06$ the two models predict different yields. JETSET predicts 4.8±0.3%, whereas ARIADNE predicts 7.6±0.4% of these events to have $p_T^{\text{jet}}$ smaller than 5 GeV. We observe the low $p_T^{\text{jet}}$ photons in 7.0±2.5±2.3% of events. This small $p_T^{\text{jet}}$ for
a $y_{\text{cut}}$ of 0.06 implies that the residual hadronic system is in the opposite hemisphere to the photon. These events are the one jet events discussed already in section 7.2.

### 7.3.4 Angle with Respect to the Jet

The angle between the photon and the jet with the minimal jet-photon mass $\alpha^{\text{jet}}$ shown in fig. 11 is strongly correlated to $p_T^{\text{jet}}$. It is strongly biased by our isolation requirement on the photon candidate. We accept essentially no events with $\alpha^{\text{jet}} \leq 20$ degrees for $y_{\text{cut}}=0.005$, or 50 degrees for $y_{\text{cut}}=0.06$.

For $y_{\text{cut}}=0.005$, we observe an almost linear decrease of the yield with increasing angle, whereas the distribution peaks at $\alpha^{\text{jet}} \sim 80$ degrees for $y_{\text{cut}}=0.06$. The bulk of the data are well described by both JETSET and ARIADNE. However, there is a significant difference between the two models for $170 < \alpha^{\text{jet}} < 180$ degrees for $y_{\text{cut}}=0.06$. This region is almost identical to the topology of $p_T \sim 0$ GeV with respect to the jet axis discussed in the previous section.

### 7.3.5 Jet Photon Mass

Fig. 12 shows the minimum mass of the photon and a jet. It was calculated using the relation to obtain the value for $y$: $M_{\gamma,\text{jet}} = \sqrt{2E_{\gamma}E_{\text{jet}}(1 - \cos\alpha^{\text{jet}})}$ with $\alpha^{\text{jet}}$ the angle between photon and jet.

JETSET and ARIADNE describe the data well, however they predict different yields for masses $M_{\gamma,\text{jet}} \sim E_{\text{cm}}$ at $y_{\text{cut}}=0.06$. Again these events are one jet events and the difference has been discussed in section 7.3.3.

### 7.4 Hemisphere Masses

We next turn to a direct investigation of the hadronic systems of these events. We compare the hemisphere masses with respect to the thrust direction in the data and the models. This analysis is also motivated by the jet development as seen in the shower models. For emissions at an early stage of the cascade, a quark $q_1$ decays either into a quark $q'_1$ and a photon with a recoiling quark $q_2$, or (in ARIADNE) an electric dipole between $q_1$ and $q_2$ emits a photon changing the quark configuration into $q'_1$ and $q'_2$. The kinematics in these models involve the masses of $q_1$, $q'_1$ and $q'_2$ on which the hemisphere masses depend. The structure of the branchings is determined by Sudakov form factors and the $\alpha_s$ value at the appropriate scale. In this section we determine the hemisphere masses from all final state particles, i.e. we do not correct back to the parton level.
7.4.1 Total Mass in Photon Hemisphere

We first consider the total mass of the hemisphere containing the photon

\[ M_{\text{in},\gamma} = \sqrt{(E_\gamma + \Sigma E_{\text{had}})^2 - (p_\gamma + \Sigma p_{\text{had}})^2} \]

where the index \( \text{had} \) refers to all particles in a hemisphere apart from the photon. In the picture given above, this corresponds to the properties of \( q_1 \). In fig. 13 \( M_{\text{in},\gamma} \) is displayed.

The distributions show a maximum at \( M_{\text{in},\gamma} \approx 20 \text{ GeV} \) and \( 30 \text{ GeV} \) for the two values of \( y_{\text{cut}} \). The larger mass for the higher \( y_{\text{cut}} \) values reflects the bias from the higher required mass in the \( y_{\text{cut}} \) selection. For \( y_{\text{cut}} = 0.005 \) the average mass is significantly larger than the minimum allowed photon-jet mass of \( \sqrt{y_{\text{cut}} \times E_{\text{cm}}} \), indicating the presence of additional jets in this hemisphere (cf. section 7.2). For \( y_{\text{cut}} = 0.06 \) the maximum is only slightly larger. The predictions of JETSET and ARIADNE agree well with the measurements for both \( y_{\text{cut}} \) values.

7.4.2 Mass in the Photon Hemisphere Excluding the Photon

In fig. 14 we show the corresponding mass \( M_{\text{ex},\gamma} \) calculated as for 7.4.1 but excluding the photon. This should be related to the mass of the quark \( q'_1 \). The observed distribution is rather independent of the \( y_{\text{cut}} \) applied apart from a tail to larger masses for \( y_{\text{cut}} = 0.005 \) due to the larger available phase space. This agreement for the two values of \( y_{\text{cut}} \) suggests that the QCD evolution is not significantly changed by the photon emission as expected by the colour blindness of the photon. The two shower models are consistent with the data although ARIADNE tends to have smaller masses.

7.4.3 Mass in the Hemisphere not Containing the Photon

The mass in the thrust hemisphere not containing the photon is also similar for the two values of \( y_{\text{cut}} \) (figs. 15a,b). They are larger than the hadronic masses in the photon hemisphere due to the larger phase space available. On the other hand, they are smaller than \( M_{\text{in},\gamma} \) since the latter is biased due to the requirement on the photon.

Both QCD shower models reproduce the data well.

8 Summary

In this paper we have discussed several tests of QCD using final state photons as a tool to isolate the gluon contributions.
From the production rate of events with two jets plus photon with that for three jet events we determine $\alpha_s$ in second order as

$$\alpha_s^{(2)} = 0.122 \pm 0.010.$$  

The error contains only statistical and experimental systematic errors, but no contributions from uncertainties in the recombination scheme, the fragmentation and the scale dependence. The value agrees with the $\alpha_s^{(2)}$ obtained from the inclusive hadronic event sample, but the measurements are correlated. The $y$-dependence of the ratio is sensitive to the structure of the triple gluon vertex. Our data agree with the QCD expectation and cannot be described by a model with an abelian strong interaction.

Although the QCD shower models JETSET and ARIADNE are in agreement with each other and with the data over a wide range of centre-of-mass energies for the inclusive multihadronic event sample, there are significant differences in the yield and in the jet rates in events with a final state photon. These differences might be caused by the different treatment of the parton evolution in the two programs. For the yield ARIADNE gives a good account of the data at low $y_{cut}$ values but tends to overestimate it at high $y_{cut}$ values ($\leq 2$ standard deviations). The jet rates are well described by ARIADNE. JETSET predicts a yield which is about three standard deviations lower than the data at low $y_{cut}$ and tends to overestimate the jet multiplicities (about 1.5 standard deviations). We have also examined the properties of the photons. Their energies and relations to jets as well as the masses of the event hemispheres including and opposite to the photon are well reproduced by both Monte Carlo models.

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References


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Figure Captions

Figure 1: Feynman diagram for final state photon emission.

Figure 2: (a) Event efficiency as a function of $y_{\text{cut}}$; (b) efficiency $\epsilon_1$ as a function of the photon energy for $y_{\text{cut}}=0.005$; (c) efficiency $\epsilon_1$ as a function of the photon energy for $y_{\text{cut}}=0.06$, circles: JETSET, squares: ARIADNE;

Figure 3: (a) Uncorrected spectrum of the photon energy for $y_{\text{cut}}=0.005$. Indicated are the data (points with error bars), the normalised prediction by JETSET (light grey) and the background contribution (dark grey); (b) Distribution of the polar angle $\cos \theta$ of the photon candidate for $y_{\text{cut}}=0.005$. Indicated are the data (points with error bars) and the normalized prediction by JETSET (light grey).

Figure 4: Distribution of the smallest angle of a particle with respect to the photon for $y_{\text{cut}}=0.005$ (a) and $y_{\text{cut}}=0.06$ (b); (c) Acollinearity angle of the two jets from two jets plus photon events, after boosting them into the hadronic rest frame. For details see text; (d) energy resolution of the jet reconstruction as defined in the text. The points with error bars represent the data, the grey area is the prediction of JETSET after a detector simulation.

Figure 5: (a) Number of events with final state photons per 1000 multihadronic events (points with error bars) compared to the matrix element calculation, the grey band represents the uncertainty due to $\alpha_s$; (b) Number of events with final state photons per 1000 multihadronic events (points with error bars) compared to JETSET (full line) and ARIADNE (dotted line); here the error of the data points does not contain contributions from the requirements on the energy and and isolation cone. Ratio of prediction/data (c) for matrix element calculation, (d) JETSET and (e) ARIADNE. Note that the results for the various $y_{\text{cut}}$ values are correlated.

Figure 6: (a) Relative jet rates compared to JETSET (full line) and ARIADNE (dotted line); (b) Absolute one, two and three jet rates compared to the matrix element calculation, the grey bands represent the uncertainties due to $\alpha_s$.

Figure 7: (a) Expectation of the ratio $R_{\gamma g}$ as a function of $\alpha_s$ for $y_{\text{cut}}=0.06$ (full line). The horizontal band represents the one standard deviation range of the measured value of $R_{\gamma g}$. Its intersection with the curve determines the range of $\alpha_s$ (vertical band); (b) $R_{\gamma q}$ as a function of $y_{\text{cut}}$, points with error bars: data, full lines: predictions for different values of $\alpha_s$, dotted line: prediction from an abelian model of QCD.

Figure 8: Corrected photon spectrum (a) $y_{\text{cut}}=0.005$, (b) $y_{\text{cut}}=0.06$. Points with error bars: data, full line: JETSET, dotted line: ARIADNE. The dashed error bars include the additional systematic error due to the energy and isolation cuts.

Figure 9: Transverse momentum $p_T^{\gamma}$ of the photon with respect to the thrust axis for (a) $y_{\text{cut}}=0.005$ and (b) $y_{\text{cut}}=0.06$. Points with error bars: data, full line: JETSET, dotted line: ARIADNE. The dashed error bars include the additional systematic error due to the energy and isolation cuts.
Figure 10: Transverse momentum $p_T^{jet}$ of the photon with respect to the jet with the minimal photon-jet mass for (a) $y_{cut}=0.005$ and (b) $y_{cut}=0.06$. Points with error bars: data, full line: JETSET, dotted line: ARIADNE. The dashed error bars include the additional systematic error due to the energy and isolation cuts.

Figure 11: Angle of the photon with respect to the jet with the minimal photon-jet mass for (a) $y_{cut}=0.005$ and (b) $y_{cut}=0.06$. Points with error bars: data, full line: JETSET, dotted line: ARIADNE. The dashed error bars include the additional systematic error due to the energy and isolation cuts.

Figure 12: Minimal photon-jet mass in GeV (a) $y_{cut}=0.005$, (b) $y_{cut}=0.06$. Points with error bars: data, full line: JETSET, dotted line: ARIADNE. The dashed error bars include the additional systematic error due to the energy and isolation cuts.

Figure 13: Invariant mass of all particles including the photon in the photon hemisphere for (a) $y_{cut}=0.005$ and (b) $y_{cut}=0.06$. Points with error bars: data, full line: JETSET, dotted line: ARIADNE. The dashed error bars include the additional systematic error due to the energy and isolation cuts.

Figure 14: Invariant mass of all particles except in the photon hemisphere for (a) $y_{cut}=0.005$ and (b) $y_{cut}=0.06$. Points with error bars: data, full line: JETSET, dotted line: ARIADNE. The dashed error bars include the additional systematic error due to the energy and isolation cuts.

Figure 15: Invariant mass of all particles in the hemisphere not containing the photon for (a) $y_{cut}=0.005$ and (b) $y_{cut}=0.06$. Points with error bars: data, full line: JETSET, dotted line: ARIADNE. The dashed error bars include the additional systematic error due to the energy and isolation cuts.
Figure 1

Figure 2a

$y_{\text{Cut}}$

Figure 2b

$y_{\text{Cut}} = 0.005$

Figure 2c

$y_{\text{Cut}} = 0.06$
Figure 3a

Figure 3b
Figure 4a

Figure 4b

Figure 4c

Figure 4d
Figure 6a

Figure 6b
Figure 7a

Figure 7b
Figure 8 a, b
$y_{\text{cut}} = 0.005$

$y_{\text{cut}} = 0.060$

Figure 9 a, b
Figure 10 a, b
Figure 11 a, b
Figure 12 a, b
Figure 13 a, b
Figure 14 a, b
Figure 15 a, b