Measurement of $B^0$-$\bar{B}^0$ Mixing in Hadronic $Z^0$ Decays

The OPAL Collaboration

Abstract

From a sample of approximately 135000 hadronic $Z^0$ decays recorded with the OPAL detector, 1536 events were selected with two lepton candidates, either electrons or muons. A signal for $B^0$-$\bar{B}^0$ mixing was observed using the sign of the lepton charge to tag the charge of the $b$ quark in decaying $b$-flavoured hadrons. A flavour discriminating variable was constructed from the lepton momentum and its component perpendicular to the jet axis. By fitting the fraction of events in which the two lepton charges are of the same sign, as a function of this variable, the average mixing parameter was measured to be

$$\chi = 0.145^{+0.041}_{-0.035} \pm 0.018,$$

where the first error is statistical and the second is systematic.

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1 Introduction

The transformation of a neutral meson into its antiparticle is made possible by flavour-changing weak interactions. If the time scale for this transformation is sufficiently short compared to the lifetime of the meson, flavour oscillations or mixing may be observed, as in the $K^0-\bar{K}^0$ system. In the Standard Model, the dominant contribution to mixing in the $B^0-\bar{B}^0$ system arises from box diagrams involving virtual top quarks [1]. The rate of mixing should therefore depend on the Cabbibo-Kobayashi-Maskawa matrix elements $V_{td}$ and $V_{ts}$ [2], and on the top quark mass. Given the rate of $B^0_d$ mixing observed by ARGUS and CLEO in decays of the $\Upsilon(4S)$ [3], where only $B_d$ and $B_s$ mesons are produced, and the relation $|V_{ts}| > |V_{td}|$ suggested by constraints on the Cabbibo-Kobayashi-Maskawa matrix [4], $B^0_s$ mixing is expected to be almost full. Full mixing would mean that 50% of produced $B^0_s$ mesons decay as $\bar{B}^0_s$ mesons and vice versa. Previous measurements [5] are consistent with full $B^0_s$ mixing.

In $Z^0$ decays, both $B^0_s$ and $B^0_d$ mesons are thought to be produced in addition to charged $B$ mesons and $b$-flavoured baryons. This paper reports a measurement of the average $B-\bar{B}$ mixing at LEP using semileptonic decays of $b$-flavoured hadrons. The mixing is expressed in terms of $\chi$, defined as

$$\chi = \frac{\text{BR}(b \to \bar{B}^0 \to B^0 \to \ell^+X)}{\text{BR}(b \to \ell^+X)},$$

where the denominator includes all $b$-flavoured hadrons produced¹, and $\ell$ is either an electron or a muon. The relationship between the average mixing parameter $\chi$ and the parameters $\chi_d$ and $\chi_s$, which respectively represent the mixing in the $B^0_d$ and $B^0_s$ systems, is

$$\chi = f_d \chi_d + f_s \chi_s,$$

where $f_d$ and $f_s$ are the respective fractions of $B^0_d$ and $B^0_s$ mesons produced relative to all $b$-flavoured hadrons. This formula assumes that the semileptonic branching ratios of different $b$-flavoured hadrons are equal.

This analysis considers hadronic decays of the $Z^0$ with two identified final state leptons, electrons or muons. If both leptons come from direct $b \to \ell$ decays, events with two leptons of the same sign indicate mixing. Backgrounds to this signature arise from

- The cascade decay $b \to c \to \ell^+$ accompanied by the decay $\bar{b} \to \ell^+$.

- Hadrons misidentified as leptons, and leptons which did not originate from heavy quarks.

Leptons from direct $b \to \ell$ decays are characterised by high momentum parallel and transverse to the $b$-flavoured hadron direction.

¹Throughout this paper, reference to a $b$ or $c$ quark decay is assumed to imply the charge conjugate process for $\bar{b}$ and $\bar{c}$, and CP violation is assumed to be negligible.
2 Event selection

The data were recorded with the OPAL detector \cite{6} at the CERN $e^+e^-$ collider LEP. Tracking of charged particles is performed by the central detector, consisting of a jet chamber, a vertex detector, and chambers measuring the $z$ coordinate\footnote{The coordinate system is defined with positive $z$ along the $e^-$ beam direction, $\theta$ and $\phi$ being the polar and azimuthal angles. The origin is taken to be the nominal interaction point.} of tracks as they leave the jet chamber. The central detector is positioned inside a solenoidal coil, which provides a uniform magnetic field of 0.435 T. The jet chamber is a large volume drift chamber, 4 m long and 3.7 m in diameter, divided into 24 azimuthal sectors with 159 layers of wires. The coil is surrounded by a time-of-flight counter array and a lead glass electromagnetic calorimeter with a presampler, divided into a cylindrical barrel and endcaps. The barrel lead glass blocks, covering the range $|\cos \theta| < 0.82$, are arranged in an approximately projective geometry on a cylinder of radius 2.45 m, and the face of each block is 10 cm by 10 cm. Outside the electromagnetic calorimeter is the instrumented return yoke of the magnet, forming the hadron calorimeter, and beyond this are muon chambers. Both the hadron calorimeter and the muon chambers are used to detect muons.

Hadronic decays of the $Z^0$ were selected according to criteria described in a previous publication \cite{7}. The jet chamber was required to be fully functioning for all events. In addition, the hadron calorimeter and muon chambers were required to be fully operational for events with muon candidates. The vertex tracking chambers, $z$-measuring tracking chambers, presampler and electromagnetic calorimeter were required to be fully functioning for events with electron candidates. To reduce background from $\tau^+\tau^-$ pairs, events were required to contain at least seven charged tracks. This gave 138 275 events available for muon identification, 135 461 events available for electron identification and 129 081 events available for both.

In this analysis, a precise knowledge of the lepton identification efficiency is not necessary, while it is important to maximise the acceptance for $b\bar{b}$ events. Lepton candidates were accepted if their measured momentum was at least 2 GeV/c and they satisfied the following track quality cuts: at least 40 jet chamber hits were required for tracks with $|\cos \theta| < 0.8$, and 20 hits for $|\cos \theta| > 0.8$. The sign of the charge of the candidate was required to be determined to at least three standard deviations. For muons, these cuts constrained the polar angle to be in the range $|\cos \theta| < 0.97$. Since electron candidate tracks were required to have matching hits in the $z$-measuring chambers, they were restricted to the range $|\cos \theta| < 0.7$. Lepton candidate tracks were also required to pass within 1 cm of the nominal beam axis at the point of closest approach in the transverse plane, with $|z| < 40$ cm at this point.

The electron selection \cite{8} is based on the ionization loss, $dE/dx$, measured in the jet chamber, the electromagnetic calorimeter energy cluster associated to the central detector track, and the amplitude of the presampler signal associated to the track. These tracks were required to have at least 40 charge samples used in the determination of $dE/dx$, and were rejected if the measured $dE/dx$ was more than 2.0 standard deviations below the value expected for electrons. Signals observed in the presampler in front of the lead glass calorimeter were required to be consistent with those expected for an electron. The lateral distribution of energy in the lead glass cluster associated to the track was measured by comparing two energies, $E_{\text{cone}}$ and
The energy $E_{\text{cone}}$ is the sum of the energies measured in the lead glass blocks, in the associated electromagnetic cluster, having centres within a cone of half-angle 30 mrad around the extrapolated track position at the front of the lead glass. The energy $E_{\text{cone2}}$ is equal to $E_{\text{cone}}$ plus the sum of the energies detected in all blocks that touch, either on the side or on the corner, any block used in $E_{\text{cone}}$. On average, electromagnetic showers are less extended laterally than hadronic showers and have $E_{\text{cone}} \approx E_{\text{cone2}}$. It was required that $E_{\text{cone}}/E_{\text{cone2}} > 0.85$ or $(E_{\text{cone2}} - E_{\text{cone}}) < 2.0$ GeV for electron candidate tracks. This latter requirement is a slight modification of the selection of reference [8] that increases efficiency at low momentum. Finally, it was demanded that $0.7 < E_{\text{cone}}/p < 1.4$ for electron candidate tracks, where $p$ is the measured momentum.

Muon candidates were identified [9] by requiring a match between an extrapolated central detector track and track segments in both muon detectors, the hadron calorimeter and the muon chambers. Candidates with a segment in only one muon subdetector were also accepted, but were subject to stricter quality requirements. Some conditions, additional to those described in reference [9], were introduced in this analysis to reduce background with only a small loss in efficiency. Fiducial cuts were placed to remove candidates passing through known holes in the calorimeter iron. Requirements on $dE/dx$ were made to reject kaons and protons. Nearly half of the remaining kaon background with reconstructed momentum larger than 2 GeV/c was rejected by this additional cut. The probability of associating the wrong central track with a muon segment (misassociation background) was reduced by asking that any additional matched track have a matching likelihood significantly less than that for the best match, otherwise the candidate was rejected. Finally, to eliminate fake pairs of close muons, muon candidates were rejected if there were a large number of muon segments reconstructed within 300 mrad in azimuth.

Requiring at least two identified leptons (e or $\mu$) per event resulted in samples of 783 $\mu\mu$, 157 ee events and 596 $\mu\mu$ events. For each event, the scaled invariant mass jet finding algorithm of JADE [10] was used to group charged tracks into jets. The E0 recombination scheme [11] was used, and the jet resolution parameter, $y_{\text{cut}}$, was chosen to be 0.02. Only those tracks which satisfied the track quality cuts described above for lepton candidates were included in the jet finding, with the exception that the 1 cm closest approach cut was relaxed to 5 cm. The transverse momentum, $p_T$, of each lepton candidate was calculated with respect to the axis of the jet to which it was associated by the jet finding algorithm. This jet axis was calculated including the lepton track, and is used as an estimate of the parent hadron direction in the case where the lepton is produced by a semileptonic decay.

### 3 Monte Carlo predictions

Monte Carlo events, together with background estimates described in the next section, were used to predict the fractions of the dilepton data due to different production mechanisms. The JETSET 7.2 Monte Carlo program [12] was used to generate 100 000 $Z^0 \rightarrow b\bar{b}$ and 100000 $Z^0 \rightarrow c\bar{c}$ events. The fragmentation was performed using the Peterson parametrization [13] with $c_3 = 0.0035$ and $c_5 = 0.06$. This value of $c_3$, measured by OPAL [14], corresponds to a mean $x_E$ of 0.72, where $x_E = 2E_{\text{hadron}}/\sqrt{s}$, $E_{\text{hadron}}$ is the energy of the first rank b-flavoured hadron.
and  is the nominal centre of mass energy. The pair of primary heavy flavour hadrons were assumed to be produced incoherently, and the flavours of quark or diquark accompanying the heavy quarks in these hadrons were assumed not to be correlated. The  branching ratio used was 11.2%, where  refers to either electrons or muons, compatible with measurements at LEP and at lower energies [4, 15, 16]. For this semileptonic decay, only the three body channels , and were included in the Monte Carlo program. The lepton momentum spectra were, however, corrected for a component constituting 15% of all semileptonic decays, close to the 11% predicted by the model of Isgur et al. [17]. The branching ratio was taken to be 11.6%, the value given by JETSET for the mix of b-flavoured hadrons produced in decays. This includes the component , which is taken to have a branching ratio of 1.7%. The branching ratio was taken to be 8.0%, consistent with an average of results from PEP and PETRA experiments [4]. The contribution from was included, but produced less than 2% of the predicted prompt leptons, where a prompt lepton is defined as a lepton coming from any of the above sources.

Monte Carlo events with at least two leptons ( or ) were processed through a detector simulation program [18]. The normalisation of the simulated events was fixed according to the number of events in the data by assuming values for the ratios and . The muon identification algorithm was applied directly to the simulated events. The muon identification efficiencies obtained in this way were corrected using the results of comparisons of events from Z0 → μ+μ− and from the two-photon process e+e− → e+e−μ+μ− with events from Monte Carlo simulations. The largest correction was 12% in the momentum range 2–3 GeV/c. The resulting efficiencies are shown as a function of momentum in Table 1 for illustration. Since the quantities used to identify electrons were not well described in the detector simulation, the electron identification efficiencies were determined [8] from several different data samples, including radiative Bhabha events and hadronic events containing identified photon conversions. A table of identification efficiencies was derived as a function of and . According to Monte Carlo predictions, the efficiency for the decay is higher than the efficiency for the decay for a given and . This difference is about 10% at high or , and was taken into account. The resulting efficiencies are shown in Table 1 for electrons from semileptonic decays. The efficiencies were applied to prompt electrons in the simulated events which fell in the polar angle range of electron candidates in the data.

The Monte Carlo events were generated without mixing, . To model non-zero values of , a fraction 2 of lepton pairs from Z0 → b̅b events was changed from one charge category to the other, either from opposite sign to like sign or vice versa, provided that the leptons did not originate from the same b quark.

4 Estimation of non-prompt backgrounds

Backgrounds to hadronic Z0 decays containing two prompt leptons include events where either one or both of the candidates are not prompt leptons. For muon candidates, backgrounds considered were decays in flight of pions and kaons, hadrons whose interaction products penetrate the material (punch-through), hadrons which do not interact strongly in the material (sail-
Muon identification efficiencies:

<table>
<thead>
<tr>
<th>Momentum</th>
<th>$2 &lt; p &lt; 3$</th>
<th>$3 &lt; p &lt; 4$</th>
<th>$4 &lt; p &lt; 6$</th>
<th>$p &gt; 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency</td>
<td>0.45</td>
<td>0.57</td>
<td>0.74</td>
<td>0.78</td>
</tr>
</tbody>
</table>

Electron identification efficiencies:

<table>
<thead>
<tr>
<th>$p_T$</th>
<th>$2 &lt; p &lt; 4$</th>
<th>$4 &lt; p &lt; 6$</th>
<th>$6 &lt; p &lt; 10$</th>
<th>$p &gt; 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_T &lt; 0.2$</td>
<td>0.31</td>
<td>0.31</td>
<td>0.33</td>
<td>0.37</td>
</tr>
<tr>
<td>$0.2 &lt; p_T &lt; 0.4$</td>
<td>0.39</td>
<td>0.32</td>
<td>0.38</td>
<td>0.39</td>
</tr>
<tr>
<td>$0.4 &lt; p_T &lt; 0.6$</td>
<td>0.47</td>
<td>0.44</td>
<td>0.41</td>
<td>0.41</td>
</tr>
<tr>
<td>$0.6 &lt; p_T &lt; 0.8$</td>
<td>0.53</td>
<td>0.57</td>
<td>0.59</td>
<td>0.50</td>
</tr>
<tr>
<td>$0.8 &lt; p_T &lt; 1.2$</td>
<td>0.64</td>
<td>0.56</td>
<td>0.56</td>
<td>0.49</td>
</tr>
<tr>
<td>$1.2 &lt; p_T &lt; 1.6$</td>
<td>0.65</td>
<td>0.62</td>
<td>0.60</td>
<td>0.58</td>
</tr>
<tr>
<td>$p_T &gt; 1.6$</td>
<td>0.65</td>
<td>0.66</td>
<td>0.64</td>
<td>0.60</td>
</tr>
</tbody>
</table>

Table 1: Efficiencies as a function of $p$ for identification of muons in the range $|\cos \theta| < 0.97$, and as a function of $p$ and $p_T$ for identification of electrons from the semileptonic decay $b \rightarrow e$ in the range $|\cos \theta| < 0.7$. The units for the values of $p$, $p_T$ quoted are GeV/c in each case. Typical statistical errors for the electron efficiencies are 5–8% of the indicated quantities.

through) and hadrons which are incorrectly associated to muon detector track segments (misassociation). For electron candidates, backgrounds arise from the misidentification of hadrons as electrons and from photon conversions.

Monte Carlo 5-flavour $Z^0 \rightarrow q\bar{q}$ events, generated by the JETSET program and passed through a full detector simulation, were used to estimate the probability for a hadron to fake a prompt muon by decay in flight, punch-through or sail-through as a function of $p$ and of $p_T$. These fake probabilities were extracted by measuring the fraction of non-prompt muon tracks that were selected by the muon identification procedure. The muon background was estimated by applying the fake probabilities to tracks in real hadronic $Z^0$ decays with a single lepton candidate. Single lepton events can be expected to have more kaons than the average hadronic $Z^0$ decay. However, this effect is estimated to lead to a negligible change in the fake probability. The calculated fake probabilities were scaled down by 30% in the region $p < 6$ GeV/c. This factor was obtained by comparing the fake probabilities of pions from identified $K^0 \rightarrow \pi^+\pi^-$ decays in simulated and real data. The resulting fake probabilities are shown as a function of momentum in Table 2. A 50% systematic error was assigned to the level of the muon background over the entire momentum range.

For electrons, separate calculations were made for the conversion and hadron misidentification backgrounds. The hadron misidentification probability per track, shown in Table 2, was estimated from the data as a function of $p$ and $p_T$, using distributions of $dE/dx$ and $E_{cone}/p$. Using information from the central detector, photon conversions were identified and rejected with an efficiency of approximately 50%. The identified conversions were then used as an estimate of the remaining conversion background. A 100% error was assigned to the level of background remaining. This accounts for 15% of the total non-prompt background in the final sample.
Fake probabilities for muon identification:

<table>
<thead>
<tr>
<th>Momentum</th>
<th>2 &lt; p &lt; 3</th>
<th>3 &lt; p &lt; 4</th>
<th>4 &lt; p &lt; 6</th>
<th>6 &lt; p &lt; 8</th>
<th>p &gt; 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.0050</td>
<td>0.0058</td>
<td>0.0055</td>
<td>0.0064</td>
<td>0.0062</td>
</tr>
</tbody>
</table>

Misidentification probabilities for electron identification:

<table>
<thead>
<tr>
<th>$p_T$</th>
<th>2 &lt; p &lt; 4</th>
<th>4 &lt; p &lt; 6</th>
<th>6 &lt; p &lt; 10</th>
<th>p &gt; 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_T$ &lt; 0.4</td>
<td>0.0006</td>
<td>0.0012</td>
<td>0.0018</td>
<td>0.0029</td>
</tr>
<tr>
<td>0.4 &lt; $p_T$ &lt; 0.8</td>
<td>0.0003</td>
<td>0.0010</td>
<td>0.0015</td>
<td>0.0018</td>
</tr>
<tr>
<td>$p_T$ &gt; 0.8</td>
<td>0.0002</td>
<td>0.0006</td>
<td>0.0012</td>
<td>0.0024</td>
</tr>
</tbody>
</table>

Table 2: Hadron fake probabilities for muon identification, and hadron misidentification probabilities for electron identification. The probabilities are shown as a function of $p$ in the range $|\cos \theta| < 0.97$ for the muon case, and as a function of $p$ and $p_T$ in the range $|\cos \theta| < 0.7$ for the electron case. The units for the values of $p$ and $p_T$ are GeV/c. Statistical errors are 2–5% of the indicated quantities for the muon case and 10–15% of the indicated quantities for the electron case.

For $\mu\mu$ events, the muon fake probability was applied to tracks in muon-inclusive hadronic $Z^0$ decays in the data. Each track was combined with the muon, and the combination weighted by the fake probability for that track, to give the absolutely normalised $\mu\mu$ background. The resulting background rate was corrected for double-counting of events with two fake muons using the single muon purity, defined as the fraction of candidates which are prompt leptons, estimated from Monte Carlo events.

For the muon background to $e\mu$ events, the fake probability was applied to all tracks except the electron candidate in electron-inclusive hadronic $Z^0$ decays selected from the data which had both electron and muon detector components fully operational. Combining each track with the electron, and weighting the combination by the fake probability for that track, gives the absolutely normalised $e\mu$ muon background. This method includes the case where both the electron and the muon are fake.

For the electron background to $e\mu$ events from misidentified hadrons, the misidentification probability was applied to each track in muon-inclusive hadronic $Z^0$ decays which had both electron and muon detector components fully operational, in the same way as described above for the muon background. In this case, the result was multiplied by the single muon purity, as estimated from Monte Carlo events, to avoid double-counting the background where both electron and muon are fake.

For the background to $ee$ events, the misidentification probability was applied to tracks in electron-inclusive hadronic $Z^0$ decays. The resulting background estimate was corrected for double-counting of events with two fake electrons, using the measured single electron purity.

The technique of muon background estimation automatically includes cases where the muon segment is caused by a decay or punch-through, but is associated to the wrong central track. However, it does not include all cases where the misassociated muon segment is caused by a
prompt muon. This additional misassociation background was estimated using Monte Carlo $Z^0 \to b\bar{b}$ and $Z^0 \to c\bar{c}$ events, which predicted that 2% of prompt muon segments are associated to the wrong central track for tracks with momentum greater than 2 GeV/c. However, in these cases, the $p$ and $p_T$ distributions for the associated track are considerably softer than that of the prompt muon. For associated tracks with $p_T > 0.8$ GeV/c, only 0.5% of Monte Carlo prompt muon segments are associated to the wrong central track. Approximately half of the misassociated tracks have the same charge sign as the prompt muon.

5 Extraction of the mixing signal

The fraction of events containing the primary decays $b \rightarrow \ell$ can be increased using the $p$ and $p_T$ of the leptons. The distribution of $p$ versus $p_T$ is shown in Figure 1 for $b \rightarrow \ell$, for $b \rightarrow c \rightarrow \ell$ (the dominant source of like sign background) and for non-prompt background. In order to simplify the analysis, a single variable was constructed from a combination of $p$ and $p_T$ to provide good separation of the $b \rightarrow \ell$ signal from the backgrounds. The variable, $p_{\text{comb}}$, is defined as

$$p_{\text{comb}} = \sqrt{\frac{p^2}{10} + p_T^2},$$

and the contour $p_{\text{comb}} = 1.2$ GeV/c is superimposed in Figure 1.

Sensitivity to mixing is obtained when both leptons come from the primary decay of b-flavoured hadrons, so that the charge sign of each lepton reflects the charge sign of the decaying parent b quark. Therefore, the sensitivity of the analysis is greater when both leptons have large $p_{\text{comb}}$. It follows that a good discriminating variable is the minimum $p_{\text{comb}}$ of the two leptons, $p_{\text{comb}}^{\text{min}}$. If the event contained more than two leptons, the two with the largest values of $p_{\text{comb}}$ were selected, and the extra lepton(s) ignored. A pair of leptons from primary $b \rightarrow \ell$ decays in an event will usually have a large opening angle. Lepton pair candidates with small opening angles may however be produced by the semileptonic decays of both the b-flavoured hadron and the c-flavoured hadron in the same decay chain, and by non-prompt background. The angle between the two leptons was required to be larger than 60° to reject events containing such candidates, reducing the sample to 510 $\mu\mu$, 388 $e\mu$ and 113 $ee$ events. The distribution of $p_{\text{comb}}^{\text{min}}$ is shown in Figure 2 separately for $\mu\mu$, $e\mu$ and $ee$ events, together with the absolutely normalised predictions. Events were classed either as like sign or as opposite sign events according to the combination of the measured charge signs of the two leptons. For events where the leptons are closer than 60°, the distribution of $p_{\text{comb}}^{\text{min}}$ provides an important check on non-prompt background predictions and cascade decays. This is shown in Figure 3 separately for opposite sign and like sign events, together with the absolutely normalised predictions. The distributions are in reasonable agreement with the predictions. The ratio, $R$, of like sign events to all events is plotted for the large-angle events as a function of $p_{\text{comb}}^{\text{min}}$ in Figure 4, together with the prediction for no mixing ($\chi = 0$) and also for $\chi = 0.15$ and 0.30. The sensitivity to $\chi$ is apparent.

Two methods were used to extract $\chi$. In the simpler method, the sensitive region of $p_{\text{comb}}^{\text{min}}$ is selected, and $R$ is calculated in this region. The measured $\chi$ is the value which, when applied to the Monte Carlo, reproduces the measurement of $R$. The second method is to fit for $\chi$ over the entire range of $p_{\text{comb}}^{\text{min}}$. This method is statistically more precise, since it includes more
For the first method, events were required to satisfy $p_{\text{comb}}^{\text{min}} > 1.2$ GeV/$c$. The cut value chosen was the one which produced the best statistical precision in Monte Carlo studies. This cut selected 182 events, of which 60 contained like sign lepton pairs. Of these 60 events, 35 contained pairs of positively charged leptons, a fraction consistent with one half. The numbers of events above the cut, the numbers of events which were like sign and the ratio $R$ are shown in Table 3 for $\mu\mu$, $e\mu$ and $ee$ events and also for the sum of the three channels.

<table>
<thead>
<tr>
<th>Number</th>
<th>Number with</th>
<th>Ratio $R$</th>
<th>Monte Carlo $R$ for $\chi = 0$ ($\chi = 0.15$)</th>
<th>Measured $\chi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>of events</td>
<td>like sign leptons</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu\mu$</td>
<td>83</td>
<td>32</td>
<td>$0.386 \pm 0.053$</td>
<td>$0.230 \pm 0.038$ (0.358)</td>
</tr>
<tr>
<td>$e\mu$</td>
<td>75</td>
<td>19</td>
<td>$0.253 \pm 0.050$</td>
<td>$0.179 \pm 0.036$ (0.336)</td>
</tr>
<tr>
<td>$ee$</td>
<td>24</td>
<td>9</td>
<td>$0.375 \pm 0.099$</td>
<td>$0.170 \pm 0.033$ (0.333)</td>
</tr>
<tr>
<td>Total</td>
<td>182</td>
<td>60</td>
<td>$0.330 \pm 0.035$</td>
<td>$0.203 \pm 0.036$ (0.346)</td>
</tr>
</tbody>
</table>

Table 3: Numbers of events with lepton pairs satisfying $p_{\text{comb}}^{\text{min}} > 1.2$ GeV/$c$. The ratio $R$ is the fraction of events which contain like sign lepton pairs. The values quoted for $\chi$ reproduce the measured $R$ values when applied to the Monte Carlo.

The predicted total number of events passing this cut was 186, with 67% containing two primary semileptonic decays of b-flavoured hadrons. The overall contribution of $Z^0 \rightarrow b\bar{b}$ events with two prompt leptons amounted to 81% of the total. Of these events, 15% were like sign for the case $\chi = 0$. Events from $Z^0 \rightarrow c\bar{c}$ with two prompt leptons contributed only 2% of the total, while non-prompt background accounted for 17%. The non-prompt background was predicted to be 46% like sign. Also included in Table 3 are the predictions for $R$ in the cases $\chi = 0$, 0.15 and the value of $\chi$ which reproduces the observed value of $R$. Adding the three channels, the result $\chi = 0.130^{+0.046}_{-0.040}$ is obtained, where the error is statistical only.

The second method used a binned maximum likelihood fit to the distributions of $R$ versus $p_{\text{comb}}^{\text{min}}$, shown in Figure 4, fitting $\chi$ to the $\mu\mu$, $e\mu$ and $ee$ data simultaneously. The fit result is $\chi = 0.145^{+0.041}_{-0.035}$, where the error is statistical, with a fit-quality chi-squared of 20.3 for 22 degrees of freedom. This result is in agreement with the first method, but has slightly smaller errors.

## 6 Systematic errors

The same sources of systematic error were considered for both methods and are listed in Table 4 together with their effects on the determined value for $\chi$. A systematic uncertainty of 20% was assigned to the overall $b \rightarrow c \rightarrow \ell$ branching ratio (including both the $b \rightarrow c \rightarrow \ell$ and the $b \rightarrow \bar{c} \rightarrow \ell$ components). Measurements of $B_u$ and $B_d$ decays [15] suggest a 15% error on this branching ratio. The error was increased to 20% because the value assumed in this paper is slightly higher than the central value in reference [15], and because additional independent
errors of 50% were allowed on this branching ratio for $B_s$ and $b$-flavoured baryon decays. In these decays, which constitute an estimated 21% of all $b$-flavoured hadron decays [20], the errors are due to the uncertainty in the mix of D states produced. The resulting systematic error on the measured value of $\chi$ is one of the largest for both methods. The branching ratio $b \to \ell$ was assigned a 10% uncertainty to reflect the range and errors of measurements both at LEP and at lower energies [4, 15, 16]. The effect of varying the fraction of $D^{**}$ in the D states from $b \to \ell$ decays was estimated. The fraction of $D^{**}$ was varied between 0, used by the JETSET program, and 32%, as favoured by a fit performed by CLEO [15]. A 10% uncertainty was assigned to the $Z^0 \to b\bar{b}$ branching ratio, but this has only a small effect on the determined values of $\chi$. The fragmentation parameter, $e_t$, was varied between 0.0015 and 0.0065 [14], corresponding to a variation in mean $x_F$ between 0.74 and 0.70. The $c \to \ell$ branching ratio for $Z^0 \to c\bar{c}$ events was varied by 13% [4], which had a negligible effect on the results. The normalisations of the muon background, the electron background from misidentified hadrons, the electron background from photon conversions and the muon misassociation background were varied by the amounts shown in Table 4. The 100% variation of the conversion background results in the largest experimental systematic uncertainty for the fitting method determination of $\chi$. The 50% variation of the muon background leads to the largest experimental systematic uncertainty for the cut method. The change in the relative importance of these errors is a consequence of the fact that the fitting method splits the sample into $\mu\mu$, $e\mu$ and $ee$ classes, each with different purities, while the cut method does not. The error due to the uncertainty on the electron efficiency was estimated by changing the efficiency to a constant value of 50%, eliminating all variation with respect to $p_t$ and the electron production process. The resulting error estimates are conservative, but are still relatively small. An overall uncertainty of 5% was assigned to the muon efficiency, and

<table>
<thead>
<tr>
<th>Source</th>
<th>Variation considered</th>
<th>Effect on $\chi$, cut method</th>
<th>Effect on $\chi$, fitting method</th>
</tr>
</thead>
<tbody>
<tr>
<td>BR($b \to c \to \ell$)</td>
<td>$\pm 20%$</td>
<td>$\pm 0.014$</td>
<td>$\pm 0.009$</td>
</tr>
<tr>
<td>BR($b \to \ell$)</td>
<td>$\pm 10%$</td>
<td>$\pm 0.012$</td>
<td>$\pm 0.006$</td>
</tr>
<tr>
<td>$D^{**}$ fraction</td>
<td>0-32%</td>
<td>$\pm 0.007$</td>
<td>$\pm 0.006$</td>
</tr>
<tr>
<td>BR($Z^0 \to b\bar{b}$)</td>
<td>$\pm 10%$</td>
<td>$\pm 0.001$</td>
<td>$\pm 0.002$</td>
</tr>
<tr>
<td>Fragmentation, $x_F$</td>
<td>0.70-0.74</td>
<td>$\pm 0.003$</td>
<td>$\pm 0.004$</td>
</tr>
<tr>
<td>BR($c \to \ell$)</td>
<td>$\pm 13%$</td>
<td>$&lt; 0.0001$</td>
<td>$&lt; 0.0001$</td>
</tr>
<tr>
<td>Muon background</td>
<td>$\pm 50%$</td>
<td>$\pm 0.009$</td>
<td>$\pm 0.005$</td>
</tr>
<tr>
<td>Electron background from misidentified hadrons</td>
<td>$\pm 25%$</td>
<td>$\pm 0.001$</td>
<td>$\pm 0.001$</td>
</tr>
<tr>
<td>Conversion background</td>
<td>$\pm 100%$</td>
<td>$\pm 0.004$</td>
<td>$\pm 0.009$</td>
</tr>
<tr>
<td>Muon misassociation background</td>
<td>$\pm 100%$</td>
<td>$\pm 0.002$</td>
<td>$\pm 0.001$</td>
</tr>
<tr>
<td>Electron efficiency</td>
<td>change to a constant 0.5</td>
<td>$\pm 0.002$</td>
<td>$\pm 0.003$</td>
</tr>
<tr>
<td>Muon efficiency</td>
<td>$\pm 5%$ overall change</td>
<td>$\pm 0.001$</td>
<td>$\pm 0.001$</td>
</tr>
<tr>
<td></td>
<td>$\pm 25%$ for $p &lt; 4$ GeV/c</td>
<td>$\pm 0.0002$</td>
<td>$\pm 0.001$</td>
</tr>
<tr>
<td></td>
<td>$\pm 25%$ for $</td>
<td>\cos \theta</td>
<td>&gt; 0.8$</td>
</tr>
<tr>
<td>Monte Carlo statistics</td>
<td></td>
<td>$\pm 0.007$</td>
<td>$\pm 0.004$</td>
</tr>
<tr>
<td>TOTAL (added in quadrature)</td>
<td></td>
<td>$\pm 0.024$</td>
<td>$\pm 0.018$</td>
</tr>
</tbody>
</table>

Table 4: Systematic errors on the determinations of $\chi$
additional uncertainties of 25% were assigned to regions at low momentum, $p < 4 \text{ GeV/c}$, and in the forward region, $|\cos \theta| > 0.8$, where the efficiency is less well known. The effect on the measured values of $\chi$ is small. Finally, the uncertainty on the value for $\chi$ due to the limited statistics of the Monte Carlo $Z^0 \rightarrow b\bar{b}$ and $Z^0 \rightarrow c\bar{c}$ events was estimated.

The combined systematic error is small compared to the statistical error for both methods. The fitting method is preferred, since both statistical and systematic errors are smaller than those of the cut method. The final result is

$$\chi = 0.145^{+0.041}_{-0.035} \pm 0.018.$$  

The $2 \text{ GeV/c}$ cut on the minimum momentum of leptons is lower than that used in other OPAL analyses [8, 9] in order to maximise the number of $b\bar{b}$ events in the sample. Such a cut also results in higher backgrounds, especially from electrons produced by photon conversions. The analysis was repeated with a momentum cut of $4 \text{ GeV/c}$ and the result $\chi = 0.140^{+0.041}_{-0.036} \pm 0.019$ was obtained. Although some contributions to the systematic errors were reduced, others increased leaving the errors essentially unchanged.

7 Discussion and conclusion

Combining the OPAL measurement of $\chi$ with the value $\chi_d = 0.17 \pm 0.04$ measured by ARGUS and CLEO [3], information can be extracted on $\chi_s$ according to equation (2). Unfortunately, the result depends critically on $f_s$, the fraction of $B^0_s$ mesons produced, which has not been measured, although it is estimated to be about 12% [20]. Assuming that equal fractions of $B_d$ and $B_u$ mesons are produced, and that the fraction of $b$-flavoured baryons is 9%, a constraint can be placed in the plane $\chi_s$ versus $f_s$. This constraint is not very sensitive to the assumed baryon fraction, and a 50% uncertainty on this fraction is included. The result is shown in Figure 5 together with the one standard deviation errors. The data are consistent with, and favour, full $B^0_s$ mixing ($\chi_s = 0.5$) for any reasonable value of $f_s$.

In conclusion, a signal for $B^0\bar{B}^0$ mixing is observed in events from hadronic $Z^0$ decays with two lepton candidates, using the sign of the lepton charge to tag the charge of the $b$ quark in decaying $b$-flavoured hadrons. Starting with approximately 135000 hadronic $Z^0$ decays, 1536 events with $\mu\mu$, $e\mu$ or $ee$ were selected. By fitting the fraction of like sign events as a function of a combination of the lepton $p$ and $p_T$ with respect to the jet axis, the mixing parameter is measured to be

$$\chi = 0.145^{+0.041}_{-0.035} \pm 0.018,$$

where the errors are statistical and systematic respectively. This is consistent with previous measurements [5]. Good agreement is obtained from a simpler analysis, where a sensitive region is selected in $p$, $p_T$ space and $\chi$ is calculated from the like sign fraction in this region.
Acknowledgements:

It is a pleasure to thank the SL Division for the efficient operation of the LEP accelerator and their continuing close cooperation with our experimental group. In addition to the support staff at our own institutions we are pleased to acknowledge the Department of Energy, USA, National Science Foundation, USA, Science and Engineering Research Council, UK, Natural Sciences and Engineering Research Council, Canada, Israeli Ministry of Science, Minerva Gesellschaft, Japanese Ministry of Education, Science and Culture (the Monbusho) and a grant under the Monbusho International Science Research Program, American Israeli Bi-national Science Foundation, Direction des Sciences de la Matière du Commissariat à l’Energie Atomique, France, Bundesministerium für Forschung und Technologie, FRG, A.P. Sloan Foundation and Junta Nacional de Investigação Científica e Tecnológica, Portugal.

References

[1] For a review see: 


[8] OPAL Collaboration, A Measurement of $\Gamma(Z^0 \rightarrow b\bar{b})$ from Electron Production in Hadronic $Z^0$ Decays, paper in preparation.


    Parameter values were tuned to describe global event shape variables:


[14] OPAL Collaboration, reported by A. Ball, proceedings of the Third Topical Seminar on
    Heavy Flavours, San Miniato, Italy, (1991), to be published in Nucl. Phys. B.


[18] D. Ward, The OPAL Monte Carlo Program - GOPAL, MC91 Workshop, Amsterdam,
    April 1991;

[19] Line shape program ZFITTER, Dubna-Zeuthen radiative correction group, (based on the
    ZBIZON package)
    R. Kleiss et al., in Z Physics at LEP1, CERN 89-08, ed. G. Altarelli et al.,
    Vol. 3 (1989) 60.
    (the Z⁰, top quark and Higgs boson masses were taken to be 91.17 GeV/c², 130 GeV/c²
    and 100 GeV/c² respectively).

[20] Assuming relative production rates during fragmentation for uū: d̅d: s̅s: diquarks of
    1.0: 1.0: 0.3: 0.23, consistent with the measured yield of hadrons of various flavours in
    e⁺e⁻ annihilation at lower energies and at LEP, gives production ratios of 39.5% B_u,
    39.5% B_d, 12% B_s and 9% b-flavoured baryon.
    For the s̅s fraction at LEP see:
    For reviews see:
    T. Sjöstrand et al., in Z physics at LEP 1, ed. G. Altarelli et al.,
    CERN 89-08, Vol 3 (1989) 143;
    D.H. Saxon, “Quark and Gluon Fragmentation in High Energy e⁺e⁻ Annihilation”,
    RAL-86-057.
Figure Captions

Figure 1: Distribution of $p$ versus $p_T$ for Monte Carlo prompt leptons with $p > 2$ GeV/c from
a) the primary decay $b \to \ell$,
b) the cascade decay $b \to c \to \ell$, and for
c) non-prompt lepton background taken from real data.
In each case the curve $p_{comb} = 1.2$ GeV/c is superimposed. The relative normalisation is
arbitrary.

Figure 2: Distributions of $p_{comb}^\text{min}$ for pairs of leptons separated by at least 60°, shown separately
for $\mu \mu$ events, $e\mu$ events and $ee$ events, with the predicted contributions indicated. These
events are used in the measurement of $\chi$.

Figure 3: Distributions of $p_{comb}^\text{min}$ for pairs of leptons closer than 60°, shown separately for oppo-
site and like sign $\mu \mu$, $e\mu$ and $ee$ events. These events provide a check on the background
estimates.

Figure 4: The fraction, $R$, of large angle dilepton events which are like sign versus $p_{comb}^\text{min}$,
shown separately for $\mu \mu$ events, $e\mu$ events, $ee$ events and the sum of the three channels.
Monte Carlo predictions for $\chi = 0$, $\chi = 0.15$ and $\chi = 0.3$ are superimposed.

Figure 5: The $B_s^0$ mixing parameter, $\chi_s$, versus the fraction of $B_s^0$ mesons produced relative to
all b-flavoured hadrons, $f_s$. The line is obtained by combining the OPAL measurement
with measurements of $B_s^0$ mixing from ARGUS and CLEO [3]. The dashed lines indicate
the one standard deviation errors.
Figure 1
Figure 2
Figure 3
Figure 4
Figure 5

![Figure 5](image-url)