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Doubly-periodic instability pattern in a smectic A liquid crystal

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We report the observation of a doubly-periodic surface defect-pattern in the liquid crystal 8CB, formed during the nematic-smectic A phase transition. The pattern results from the antagonistic alignment of the 8CB molecules, which is homeotropic at the surface and planar in the bulk of the sample cell. Within the continuum Landau-deGennes theory of smectic liquid crystals, we find that the long period (\(\approx 10 \mu \text{m}\)) of the pattern is given by the balance between the surface anchoring and the elastic energy of curvature wall defects. The short period (\(\approx 1 \mu \text{m}\)) we attribute to a saddle-splay distortion, leading to a non-zero Gaussian curvature and causing the curvature walls to break up.

The richness of thermotropic liquid crystal (LC) phases\(^1\), their susceptibility to external fields and their unique optical properties make LCs ideally suited to study symmetry breaking phase transitions. These transitions often involve the formation of isolated topological defects or complex ordered spatial structures (topological defect patterns)\(^2\), with analogues in magnetism (grain boundaries, domain walls)\(^3\), superconductivity (Abrikosov lattice, stripes)\(^4\) and cosmology (cosmic strings, monopoles)\(^5\). Contrary to cosmological or quantum systems, LC patterns can be studied at room temperature using polarization microscopy, whence the formation, organization and kinetics of the defect structures can be fully explored. Patterns in the LC nematic phase, with long range orientational order but no positional order, are mostly well understood and readily explained within a continuum elastic theory of LC\(^1\). In contrast, patterns in smectic LC phases are more difficult to describe due to the additional one-dimensional positional order. Many, sometimes rather complex, smectic patterns have been observed, such as undulations of the smectic layers in an applied magnetic field (Helfrich–Hurault instability)\(^6\),\(^7\) or other periodic structures, like stripes\(^8\),\(^9\), squares\(^10\), or hexagons\(^11\). Usually, those structures are explained by the formation of focal conic domains or curvature walls, characterized by one typical length scale\(^3\),\(^10\),\(^12\). In this Letter we report the observation of a novel doubly-periodic defect pattern, which is formed during the nematic-smectic A (N–SmA) phase transition of a liquid crystal in an applied magnetic field. The field imposes an orientation of the LC molecules in the bulk that is orthogonal to the preferred orientation at the surface of the sample cell. Most strikingly, the pattern has two distinct periods: a long one (\(\approx 10 \mu \text{m}\)) along the field direction and a short one (\(\approx 1 \mu \text{m}\)) perpendicular to the field. Interestingly, a quite similar texture develops in LC colloidal shells on cooling towards the N–SmA phase transition\(^13\). We present a model describing the pattern using a geometric construction of a space-filling, energy minimizing, structure of equidistant (smectic) layers. Within this model we identify the driving mechanism as an elastic saddle-splay contribution\(^14\),\(^15\) that breaks the symmetry in such a way that it naturally explains both distinct periodicities of the experiment and the orientation of the pattern with respect to the magnetic field direction.

For our experiments we have used the liquid crystal 8CB (4-n-octyl-4’-cyanobiphenyl) which exhibits both a N and a SmA phase (SmA \(\rightarrow 33.5^\circ \text{C}\) N \(\rightarrow 41.5^\circ \text{C}\) I). The sample is contained in a cell consisting of two 0.4 mm thick borosilicate glass plates, spaced by a teflon ring with 4.5 mm inner diameter and 1.6 mm thickness (Fig. 1a). A 7 T static magnetic field \(\textbf{B}\) was applied in the plane of the cell, along the \(x\)-direction. In-situ polarized microscopy was used to visualize the LC phase as a function of time. The sample was positioned in the \(xy\)-plane in between two crossed polarizers at \(\pm 45^\circ\) relative to \(\textbf{B}\). In this geometry the transmitted light intensity is maximal when the LC phase is aligned along \(\textbf{B}\) and minimal when the LC molecules are randomly aligned (isotropic phase), aligned along the viewing direction (\(z\)-axis) or aligned along one of the polarizer axes.

Several cell glass coatings were used to vary the orientation of the 8CB molecules at the surface and the strength of the surface anchoring. To obtain homeotropic alignment (parallel to the normal of the glass) with varying surface anchoring strength we used hexamethyldisilazane (HMDS) coatings or spin-coated polydimethylsiloxane (PDMS) layers, or the untreated glass. Alternatively, coating the cell with polyvinyl alcohol (PVA) induces a planar molecular alignment. The following standard protocol was used: the sample was heated to the
Figure 1: (Color) a) The sample cell consists of two borosilicate glass plates separated by a 4.5 mm diameter, 1.6 mm thick teflon ring. The magnetic field \( \mathbf{B} \) is applied in the plane of the cell (along the \( x \)-direction). b) Polarization microscopy images of the pattern formation during cooling (0.4 °C/min) through the N–SmA phase transition of 8CB at 7 T. At \( t = 0 \) s (\( T = 33.75 ^\circ \text{C} \)) a line-defect is visible indicated by the dashed arrow. Upon further cooling this defect breaks up (\( t = 5 \) s), aligns perpendicular to the field direction (\( t = 15 \) s), and grows (\( t = 25, 40 \) s). Finally, the full sample surface is covered by stripes about 10–30 \( \mu \text{m} \) apart (\( t = 180 \) s).

The isotropic phase for at least 10 minutes. Then after applying the magnetic field the sample was slowly cooled (0.4 °C/min), through the magnetically aligned nematic phase, to a temperature within the smectic phase. When the LC pattern was fully developed the temperature was further decreased to room temperature and \( \mathbf{B} \) was reduced to zero, after which the sample was taken out of the magnet to be investigated under a polarization microscope.

A typical experimental example of the pattern formation is shown in Fig. 1b. The first image (\( t = 0 \) s, \( T = 33.75 ^\circ \text{C} \)) corresponds to a sample (untreated glass cell) at 7 T close to the N–SmA phase transition. The overall transmitted light intensity is high because in the bulk of the sample the LC phase is uniformly aligned. An elongated defect is visible as a black line (indicated by the arrow). Upon cooling, the defect breaks up in shorter lines (\( t = 5 \) s), which rotate towards an orientation perpendicularly to \( \mathbf{B} \) (\( t = 15 \) s). Subsequently the defects rapidly grow (\( t = 25, 40 \) s). Finally, the entire defects are covered by a surface pattern (\( t = 180 \) s), consisting of many line defects oriented at 90° with respect to \( \mathbf{B} \). The typical distance between the stripes is 10–30 \( \mu \text{m} \). Heating the system through the SmA–N transition induces the reverse phenomenon: the line-defects melt and gradually disappear in the nematic phase (not shown). Alternatively, cooling the surface pattern to room temperature leads to a stable pattern that remains after the field is switched off and that can be studied under the polarization microscope (Fig. 1b). The pattern is only present in the case of homeotropic boundary conditions and relatively strong surface anchoring, realized for untreated glass (Fig. 1a) and PDMS coating (Fig. 1b). When the surface anchoring is homeotropic but too weak (HMDS, Fig. 1d) or for planar surface anchoring (PVA, Fig. 1c) the pattern is absent.

Adjusting the focus of the microscope revealed that the line patterns are formed on both the top and bottom surfaces of the cell, whereas the bulk is homogeneously aligned. Most strikingly, the microscopy images at higher magnifications show that the line pattern contains an additional fine structure: between the surface lines smaller elongated defects are visible with a periodicity of about 1–2 \( \mu \text{m} \), perpendicular to the main pattern. This secondary structure develops where the primary defect structure is more dense and regular, i.e. in between straight defect lines. In contrast no secondary structure is formed in the vicinity of an interrupted line pattern. This means that the secondary structure is most clearly seen for the cell with strong surface anchoring (PDMS, Fig. 1b), where a regular undulation is observed in the inner structure of the line pattern, giving rise to a zig-zag pattern.

We start our theoretical description in the nematic phase (above the transition temperature \( T > T_c \) (\( T_c \equiv \))...
yielding the formation of equidistant smectic layers \((\nabla \phi = 1)\) on top of the distorted nematic phase. Substituting \(\theta\) from \(2\) and integrating \(3\) we find:

\[
\zeta_0(\xi) = \frac{1}{2} \log \left( \frac{\coth(\xi/2)}{A} \right),
\]

which parametrizes the smectic layer in terms of the dimensionless coordinate \(\xi = x/(\epsilon_B H)\) along the \(x\)-axis as shown in Fig. 3 (red curve, \(A \approx 1.1\)). This important result for a closed form of the generating curve is derived, based on the competition between anchoring and magnetic energy, mediated by elasticity \(1\) under the global geometric constraint of equidistant layers \(3\), rather than assumed \(a \text{ priori}\) as in refs. \(2\) \(\text{[15]}\). To obtain a space-filling two dimensional structure we perform a parallel transport of the generating curve \(\zeta_0(\xi)\) along its normal \(\nu = \nu_x e_x + \nu_z e_z\) (see Fig. 3), yielding a set of equidistant layers with the \(j\)th layer given by:

\[
x_j = (\xi + j \alpha \nu_x) e_x + (\zeta_0(\xi) + j \alpha \nu_z) e_z, \quad j = 0, \pm 1, \pm 2 \ldots
\]

Here \(\alpha = \frac{1}{4 \epsilon_B^2}\) is the dimensionless interlayer distance, the components of the normal \(\nu_x = \frac{\cosh \xi}{\sqrt{1 + \cosh^2 \xi}}\) and \(\nu_z = \frac{2}{\sqrt{1 + \cosh^2 \xi}}\). In the bulk \((\xi \to \infty)\) we obtain a set of flat parallel layers perpendicular to the \(x\)-axis. The curved smectic layers (green lines in Fig. 3) cannot fill the space without defects such as curvature walls shown by the grey and dashed-blue regions. Note that Fig. 3 illustrates only part of the whole structure, while the actual size of the grey region is determined by the energy balance between the surface and the bulk contributions (see below). Within our two dimensional model the size of the blue region is characterized by the intersection of the two red curves (at the \(z\)-axis) at \(\xi|_{\zeta = 0} = \pm 2 \coth^{-1}(A)\) and not by the energy minimizing structure of this region. This latter approach gives different space-filling structures as described previously \(2\) \(\text{[15]}\).

The resulting structure consists of stripes parallel to the \(y\)-axis with period \(L_{x}\) in the \(x\)-direction, along the magnetic field, like in the experiment. \(L_{x}\) is determined by two contributions (see supplemental material for derivation \(13\)):

\[
L_{x} \approx 2 \frac{W_a - 2 \rho \sqrt{K C (1 - \frac{1}{\lambda^2})}}{\chi_a B^2} + 2 \frac{\epsilon_B H}{\max |\nu|}.
\]

The first term corresponds to the competition between the anchoring energy (green region in Fig. 3) and the energy of the curvature wall (grey region), given by \(f_{\omega} = \rho \sqrt{K C (\tan \omega - \omega)} \cos \omega \) \(10\) \((\omega = \pi/4)\) per unit area. The second term originates from a non-zero curvature of the generating curve \(3\), given by \(\kappa = \partial \xi \zeta_0(1 + (\partial \xi \zeta_0) \omega) \) \(3/2\). The first term in \(7\) is non-negative only if \(W_a \geq \rho \cdot 10^{-3} \text{ J/m}^2\), yielding \(L_{x} \approx (9 \pm 4) \mu\text{m}\) for a strong surface anchoring \(W_a \approx \)
The appearance of intrinsic periodicity of modulated stripes implies that the linear correction to the level set function $\phi + \varepsilon \tilde{\phi}$ should satisfy $\nabla \tilde{\phi} = \nabla \hat{n}$, leading to:

$$\frac{dy}{dz} = -\frac{\hat{\varphi}}{\varphi} \rightarrow \sin(q_y y) = (\cosh \zeta + \mu \sinh \zeta)^{-\sigma}. \quad (9)$$

Here $\sigma = \omega(1 + \mu \omega)/\mu^2(\mu + \omega)^2$, $\omega^2 = 1 + q_y^2 \varepsilon \xi H^2$, and $\tau = K_{24}/L^2$.

Figure 4: (Color online) The sum of the functions $\zeta_0(\xi) + \varepsilon \zeta(y/(\varepsilon \xi H))$ given by (9), (10), and mirror reflection with respect to $x = 0$, followed by a shift of $\pi$ along the $y$-axis. Calculated with $\varepsilon = 0.2$, $\mu = 0.06$, $\tau \approx 8$, $\omega \approx 2.4$ ($A \approx 1.1$).
findings can give insight into the organization of other lamellar microstructures, ubiquitous in nature, and can be of general interest to analyze symmetry breaking phase transitions in condensed matter systems.

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[23] The amplitude of the perturbation ε cannot be identified within a linear stability analysis.