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Doubly-periodic instability pattern in a smectic A liquid crystal

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We report the observation of a doubly-periodic surface defect-pattern in the liquid crystal 8CB, formed during the nematic-smectic A phase transition. The pattern results from the antagonistic alignment of the 8CB molecules, which is homeotropic at the surface and planar in the bulk of the sample cell. With the continuum Landau-deGennes theory of smectic liquid crystals, we find that the long period ($\approx 10 \mu m$) of the pattern is given by the balance between the surface anchoring and the elastic energy of curvature wall defects. The short period ($\approx 1 \mu m$) we attribute to a saddle-splay distortion, leading to a non-zero Gaussian curvature and causing the curvature walls to break up.

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thick teflon ring. The magnetic field\cite{10} is applied in the plane of the cell (along the $x$-direction). b) Polarization microscopy images of the pattern formation during cooling ($0.4 \, ^\circ\text{C/min}$) through the N–SmA phase transition of 8CB at 7 T. At $t = 0$ s ($T = 33.75 \, ^\circ\text{C}$) a line-defect is visible indicated by the dashed arrow. Upon further cooling this defect breaks up ($t = 5$ s), aligns perpendicular to the field direction ($t = 15$ s), and grows ($t = 25, 40$ s). Finally the full sample surface is covered by stripes about 10–30 µm apart ($t = 180$ s).

isotropic phase for at least 10 minutes. Then after applying the magnetic field the sample was slowly cooled ($0.4 \, ^\circ\text{C/min}$), through the magnetically aligned nematic phase, to a temperature within the smectic phase. When the LC pattern was fully developed the temperature was further decreased to room temperature and B was reduced to zero, after which the sample was taken out of the magnet to be investigated under a polarization microscope.

A typical experimental example of the pattern formation is shown in Fig. 1b. The first image ($t = 0$ s, $T = 33.75 \, ^\circ\text{C}$) corresponds to a sample (untreated glass cell) at 7 T close to the N–SmA phase transition. The overall transmitted light intensity is high because in the bulk of the sample the LC phase is uniformly aligned. An elongated defect is visible as a black line (indicated by the arrow). Upon cooling, the defect breaks up in shorter lines ($t = 5$ s), which rotate towards an orientation perpendicular to B ($t = 15$ s). Subsequently the defects rapidly grow ($t = 25, 40$ s). Finally, the entire defects is covered by a surface pattern ($t = 180$ s), consisting of many line defects oriented at 90° with respect to B. The typical distance between the stripes is 10–30 µm. Heating the system through the SmA–N transition induces the reverse phenomenon: the line-defects melt and gradually disappear in the nematic phase (not shown). Alternatively, cooling the surface pattern to room temperature leads to a stable pattern that remains after the field is switched off and that can be studied under the polarization microscope (Fig. 2a,b). The pattern is only present in the case of homeotropic boundary conditions and relatively strong surface anchoring, realized for untreated glass (Fig. 2a), and PDMS coating (Fig. 2b). When the surface anchoring is homeotropic but too weak (HMDS, Fig. 2d) or for planar surface anchoring (PVA, Fig. 2c) the pattern is absent.

Adjusting the focus of the microscope revealed that the line patterns are formed on both the top and bottom surfaces of the cell, whereas the bulk is homogeneously aligned. Most strikingly, the microscopy images at higher magnifications show that the line pattern contains an additional fine structure: between the surface-lines smaller elongated defects are visible with a periodicity of about 1–2 µm, perpendicular to the main pattern. This secondary structure develops where the primary defect structure is more dense and regular, i.e. in between straight defect lines. In contrast no secondary structure is formed in the vicinity of an interrupted line pattern. This means that the secondary structure is most clearly seen for the cell with strong surface anchoring (PDMS, (Fig. 2b)), where a regular undulation is observed in the inner structure of the line pattern, giving rise to a zig-zag pattern.

We start our theoretical description in the nematic phase (above the transition temperature $T > T_c$ ($T_c \equiv$ Figure 2: (Color) A doubly-periodic surface pattern is visible for strong homeotropic surface anchoring, realized for a) untreated glass and b) PDMS coating. c) A PVA coating (planar surface alignment) leads to an aligned monodomain and focal conics. d) A HMDS coating gives an aligned monodomain due to the small homeotropic surface anchoring.
\[ T_{N-SmA} \), which is described by a unit vector \( \mathbf{n} \) called the director, pointing along the averaged orientation of the LC molecules. In the bulk the director is aligned parallel to the magnetic field \( \mathbf{e}_z \), while close to the glass surface the molecules tend to align along the normal to the surface \( \mathbf{e}_z \) (homeotropic anchoring, see Fig. 3). The director \( \mathbf{n} \) reorients by bending in order to minimize the sum of elastic and magnetic free energy:

\[
\mathcal{F}_{\text{nem}} = \frac{1}{2} \int_V dV \left\{ K |\nabla \mathbf{n}|^2 - \chi_a B^2 (\mathbf{n} \cdot \mathbf{e}_z)^2 \right\}. \tag{1}
\]

We are far above the Freedericksz threshold \( B_{\text{cr}} = \frac{\pi}{\sqrt{2}} \frac{K}{\chi_a} \approx 10^{-2} T \) \( [1] \), where \( K \approx 7 \cdot 10^{-12} \text{ N} \) is the elastic modulus. \( \chi_a \approx 10^{-7} \) is the diamagnetic anisotropy in cgs units \( [1] \) and \( 2H = 1.6 \text{ mm} \) is the thickness of the cell. The angle \( \theta \) between the \( z \)-axis and the director \( \mathbf{n} = \sin \theta \mathbf{e}_x + \cos \theta \mathbf{e}_z \) varies along the thickness of the sample as \( [16] \):

\[
\theta(\xi) = \arcsin \left( \frac{A e^{2\xi} - 1}{A e^{2\xi} + 1} \right), \quad A = 1 + \mu \frac{1}{1 - \mu}. \tag{2}
\]

Here \( \mu = \frac{\sqrt{K e^\pi B^2}}{W_a} \), \( W_a \) is the anchoring strength associated with the free energy cost \( \frac{1}{2} W_a \sin^2 \theta |_{\xi=0} \) (Rapini–Papoular form) for the deviation of \( \mathbf{n} \) from its preferred orientation along the normal to the surface. The dimensionless coordinate \( \xi = \frac{1 - z/H}{\sqrt{\rho^2 + \frac{2}{\mu} e^\pi}} \) measures the distance from the glass surface in the \( z \)-direction in units of the coherence length \( eB H = \sqrt{\frac{K}{\chi_a}} \approx 0.45 \mu \text{m} \).

The question we pose here is what happens when the SmA order, characterized by equally spaced layers, is imposed on this bent nematic structure. The smectic order is characterized by the complex order parameter \( \rho e^{i\phi} \), where \( \rho \) is the amplitude of the smectic density modulation, \( \phi \) is the scalar function parametrizing the smectic layers, so that \( \nabla \phi \) is parallel to the layer’s normal. For SmA the director coincides with the normal to the layers, such as \( \mathbf{n} = \nabla \phi / |\nabla \phi| \). The associated Landau–deGennes free energy is \([1]\):

\[
\mathcal{F}_{\text{sm}} = \int dV \left\{ \frac{C}{2} (\mathbf{\nabla} \rho)^2 + \rho^2 |\nabla \phi - \mathbf{n}|^2 + \frac{r}{2} \rho^2 + \frac{g}{2} \rho^4 \right\}. \tag{3}
\]

For 8CB the compression modulus \( C \approx 10^6 \text{ J/m}^3 \) and the interlayer spacing \( \lambda \approx 3.2 \text{ nm} \) \([4]\). The coefficients \( r \) and \( g \) are regular Landau coefficients, where only \( r \) depends on temperature, while \( g > 0 \). We assume that the spatial variations of \( \rho \) are negligible, because they occur within the characteristic lengthscale \( \lambda \ll eB H \) and prior to the change of orientation due to the cooling. Then \( \rho = \frac{1}{\sqrt{2} \theta} \propto \sqrt{T_c - T} \) and the second term in \([3]\) is minimized if

\[
\nabla \phi = \mathbf{n} \rightarrow \frac{dx}{dz} = \cot \theta, \tag{4}
\]

yielding the formation of equidistant smectic layers \( (|\nabla \phi| = 1) \) on top of the distorted nematic phase. Substituting \( \theta \) from \([2]\) and integrating \([4]\) we find:

\[
\zeta_0(\xi) = \frac{1}{2} \log \left( \frac{\coth(\xi/2)}{A} \right), \tag{5}
\]

which parametrizes the smectic layer in terms of the dimensionless coordinate \( \xi = x/(e_B H) \) along the \( x \)-axis as shown in Fig. 3 (red curve, \( A \approx 1.1 \)). This important result for a closed form of the generating curve is derived, based on the competition between anchoring and magnetic energy, mediated by elasticity \([1]\) under the global geometric constraint of equidistant layers \([3]\), rather than assumed \( a \) \textit{priori} as in refs. \([1\, 2\, 18]\). To obtain a space-filling two dimensional structure we perform a parallel transport of the generating curve \( \zeta_0(\xi) \) along its normal \( \nu = \nu_x e_x + \nu_z e_z \) (see Fig. 3), yielding a set of equidistant layers with the \( j \)-th layer given by:

\[
x_j = (\xi + j a \nu_x) e_x + (\zeta_0(\xi) + j a \nu_z) e_z, \quad j = 0, \pm 1, \pm 2 \ldots \tag{6}
\]

Here \( a = \frac{1}{e_B H} \) is the dimensionless interlayer distance, the components of the normal \( \nu_x = \frac{\cosh(\xi)}{\sqrt{4 + \cosh^2(\xi)}} \) and \( \nu_z = \frac{2}{\sqrt{4 + \cosh^2(\xi)}} \). In the bulk \( (\zeta \rightarrow \infty) \) we obtain a set of flat parallel layers perpendicular to the \( x \)-axis. The curved smectic layers (green lines in Fig. 3) cannot fill the space without defects such as curvature walls shown by the grey and dashed-blue regions. Note that Fig. 3 illustrates only part of the whole structure, while the actual size of the grey region is determined by the energy balance between the surface and the bulk contributions (see below). Within our two dimensional model the size of the blue region is characterized by the intersection of the two red curves (at the \( z \)-axis) at \( |\xi|_{\xi=0} = \pm 2 \cosh^{-1}(A) \) and not by the energy minimizing structure of this region. This latter approach gives different space-filling structures as described previously \([2\, 18]\).

The resulting structure consists of stripes parallel to the \( y \)-axis with period \( L_x \) in the \( x \)-direction, along the magnetic field, like in the experiment. \( L_x \) is determined by two contributions (see supplemental material for derivation \([19]\)):

\[
L_x \approx 2 \frac{W_a - 2 \rho \sqrt{K C}}{\chi_a B^2} (1 - \frac{\pi}{6}) + \frac{2}{\chi_a B^2} \min |\kappa| \tag{7}
\]

The first term corresponds to the competition between the anchoring energy (green region in Fig. \(c\)) and the energy of the curvature wall (grey region), given by \( f_\omega = \rho \sqrt{K C} (\tan \omega - \omega) \cos \omega \) \([10]\) \( (\omega = \pi/4) \) per unit area. The second term originates from a non-zero curvature of the generating curve \( \zeta_0 \), given by \( \kappa = \partial_x \zeta_0/(1 + (\partial_x \zeta_0)^2)^{3/2} \). The first term in \([7]\) is non-negative only if \( W_a \geq \rho \cdot 10^{-3} \text{ J/m}^2 \), yielding \( L_x \approx (9 \mp 4) \mu \text{m} \) for a strong surface anchoring \( (W_a \approx \frac{1}{2} \frac{K}{\chi_a B^2} \approx 10^{-3} \text{ J/m}^2) \).
saddle-splay contribution \[14, 15\]: of the N–SmA phase transition, in particular an enhanced mechanism is the growth of the elastic constants in the vicinity of the N–SmA transition, which is compatible with our experimental observations. Indeed, ‘breaking’ of the smectic layers \[11\] implies that the linear correction to the level set function \(\phi + \varepsilon \hat{\phi}\) should satisfy \(\nabla \hat{\phi} = \nabla \hat{n}\), leading to:

\[
dy = -\frac{\hat{\theta}}{\varphi} \sin(q_y y) = (\cosh \zeta + \mu \sinh \zeta)^{-\sigma}.
\]

(9)

Here \(\sigma = \frac{\omega(1+\mu\omega)}{\mu^2+\mu+\omega}\), \(\omega^2 = 1 + q_y^2 \varepsilon_g^2 H^2\) and \(\tau = K_{24}l_0^6\). The correction to the level set function \(\zeta_0(\xi) + \varepsilon \zeta(y/(\varepsilon g H))\) at the onset of instability acquires a simple analytic form \(\hat{\zeta} \simeq \cosh^{-1} \left[ \sin(q_y y)^{-1/\sigma} \right]\) (\(\mu \simeq 0.06\) for strong anchoring \(W_a \simeq 3 \cdot 10^{-4} J/m^2\)).

In Fig. 4, we plot \(\zeta_0(\xi) + \varepsilon \zeta(y/(\varepsilon g H))\) for a periodically distorted generating layer (red curve in Fig. 3) with non-zero Gaussian curvature. This simplified analysis allows to identify the intrinsic periodicity of modulated stripes (4) implies that the linear correction to the level set function \(\phi + \varepsilon \hat{\phi}\) should satisfy \(\nabla \hat{\phi} = \nabla \hat{n}\), leading to:

\[
dy = -\frac{\hat{\theta}}{\varphi} \sin(q_y y) = (\cosh \zeta + \mu \sinh \zeta)^{-\sigma}.
\]

(9)

In conclusion, we have observed a new doubly-periodic defect texture in liquid crystals formed during the N–SmA phase transition. Based on a simple geometric argument and energy minimization including the saddle-splay elastic contribution, we have proposed a plausible scenario for instabilities at two different length scales. Our
findings can give insight into the organization of other lamellar microstructures, ubiquitous in nature \cite{24, 25}, and can be of general interest to analyze symmetry breaking phase transitions in condensed matter systems \cite{2, 4}.

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[19] See Supplemental Material at [URL will be inserted by publisher] for derivation of equation (13).
[23] The amplitude of the perturbation \( \varepsilon \) cannot be identified within a linear stability analysis.