Quality Measure Functions for Calibration of Speaker Recognition Systems in Various Duration Conditions

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Abstract—This paper investigates the effect of utterance duration to the calibration of a modern i-vector speaker recognition system with probabilistic linear discriminant analysis (PLDA) modeling. A calibration approach to deal with these effects using quality measure functions (QMFs) is proposed to include duration in the calibration transformation. Extensive experiments are performed in order to evaluate the robustness of the proposed calibration approach for unseen conditions in the training of calibration parameters. Using the latest NIST corpora for evaluation, results highlight the importance of considering the quality metrics like duration in calibrating the scores for automatic speaker recognition systems.

Index Terms—calibration, quality measures, duration, forensics, speaker recognition, i-vector, PLDA.

I. INTRODUCTION

The traditional challenges associated with speaker recognition system can be attributed to the within-speaker variability of the speech signals. Within-speaker or intra-speaker variability [1] refers to the changes that occur in the recorded speech produced by a single speaker. In speaker recognition, the source of within-speaker variability may originate from the language spoken by the speakers, speech register, vocal effort, emotion, background noise, duration of speech samples, recording channel and encoding, and the reverberation conditions. The within-speaker variation has been shown to reduce the performance of speaker recognition system [2]–[7].

In the real application of speaker recognition, there is a high likelihood of having different conditions between the reference (or model) and test recordings. For example, in a forensic scenario, the test recording might originate from a wire-tapped telephone conversation with the reference speech recorded in the interview session. Another example is in biometric authentication where differences may occur in the reverberation and/or background noise conditions between the enrollment of the speaker and actual authentication attempts.

There are a number of ways in dealing with the problem of within-speaker variability in speaker recognition. Since the Gaussian mixture model (GMM) was proposed for text-independent speaker recognition in the 1990s [8], there has been a strong focus on channel compensation and normalization strategies in feature, score and model domains [9]–[15]. These strategies were proposed to improve system robustness to the within-speaker variability problems.

Along with the development of speaker recognition technology, short duration cases have always been one of many problems that lead to the system performance degradation. As the speech duration is reduced, the system performance tends to follow suit. This is due to the lack of information provided by the short duration of speech samples. In [3]–[5] for example, we can find related studies to discrimination performance of speaker recognition systems in short duration conditions. Even though it is reported in [16] that the i-vector system performance is less sensitive to short utterances compared to previous techniques such as support vector machine (SVM) and joint factor analysis (JFA), performance still degrades in the presence of short duration as presented in [3], [5].

There has been numerous studies in the speaker recognition field in order to solve the short duration problem. In [17], the duration variability problem in speaker recognition is tackled using the duration pattern extracted from the automatic speech recognition prior to the modeling and scoring process. In [18], the short duration problem is addressed by doing logistic regression and fusion from several speaker recognizers.

Almost invariably, the research studying the effects of (shorter) duration in speaker recognition have concentrated on the consequences to the discrimination performance, which can be seen from the reported the performance in terms of the calibration-insensitive equal error rate or minimum decision cost function. However, for deployment of speaker recognition systems, the calibration of the scores is equally important [20]. Traditional understanding of calibration is the capability of the system to choose a threshold for detection optimally in terms of minimum expected costs. However, in the last decade the concept of calibration has been generalized to a wider range of the detection-error trade-off [21]–[23] with the introduction of the calibrated likelihood ratio and accompanying evaluation metrics such as $C_{li}$ [22] and the empirical cross entropy [24].

Presentation of recognition results in terms of calibrated log-likelihood-ratios is not only required for application in forensic evidence evaluation [25], but also presents a speaker comparison result in an application-independent way to the user [22], [23]. For the first time in the National Institute of Standards and Technology (NIST) Speaker Recognition Evaluations (SRE) in 2012 [26] it was required to submit a recognition score as a calibrated log-likelihood-ratio.

In our previous work [3], we evaluated i-vector based speaker recognition system with LDA modeling in terms of both discrimination and calibration performances on various duration conditions. However, in that study, we did not propose...
Fig. 1. Score distributions of NIST SRE’10 det-5 database for before and after conventional linear calibration [19] performed in the model/test (sec) duration conditions.

any technique to address this duration variability problem. This paper is a continuation of that work in which we propose a way to address the variability problem during calibration of the speaker recognition system.

The proposed calibration technique that is introduced in this paper is inspired by the concept of quality measures presented in [27], [28]. Here, the quality measure is defined as “knowing the quality of what you have,” which in our case are the speech segments used for speaker recognition. We treated the duration as not only the source of the within-speaker variability, but also as the quality measure or quality factor of speech samples.

Using quality measures of speech to improve the system performance is not a new thing in the speaker recognition field. It is mentioned in [28] that there are four stages in recognition process where the engagement of quality measures is potentially possible in order to improve the system performance: feature extraction, model training, score computation and score fusion. In [29], the quality measures are incorporated in a speaker recognition system in the modeling stage. Here, the authors introduced a technique for combining quality measure information in the system by classifying trials based on speaker adaptation transforms from an automatic speech recognition, and training fusion separately for each of those trial class. The incorporation of quality measures in the score fusion is quite popular in the field with most studies focusing on bi-modal person recognition1 [30]–[33].

In this paper, we use the duration of model and test segments of speech as the quality measures to improve the calibration performance of the speaker recognition system in various duration conditions. As can be seen from Figure 1, the duration variability in speaker recognition system affects the distribution of scores. By keeping the model segment duration as full and reducing the duration of the test segment, the target scores distribution approaches the non-target scores distribution (see the before calibration column). By training a typical score calibration technique [19] on full duration segments for model and test, we arrive at the right column in Figure 1. When we calibrated the scores from shorter duration conditions using the parameters trained from the longer duration, the large score shift between training and evaluation materials in calibration causes large miscalibration cost. One way of dealing with the score shift in calibration is by using discrete classes for the quality conditions, and effectively training separate calibration parameters for any of the possible combinations of quality conditions between train and test. This was, for instance, carried out for the NIST SRE 2008 by several groups [34], [35] in a calibration implementation coined bi-linear fusion of side-information. These ideas materialized later in the well-known BOSARIS toolkit [36] that can be used for calibrating speaker recognition scores with such side-information. This side-information can be used for quality measures, but inherently as discrete classes.

The new approach taken in this paper is that we model the effect of continuous quality measures to the calibration in low-parameter continuous functions. This is an attempt to capture the relation between a range of quality measure values and the calibration process in a single function, with the potential to both interpolate and extrapolate unseen quality measure values and model the interaction between quality measurements from train and test. We named this proposed calibration technique Quality Measure Function or QMF calibration. Please note that we use duration as an example quality measure, but that the approach can also be applied for other

1Person recognition based on two biometric modalities (speech, face, fingerprints, etc.).
measures, such as the signal-to-noise ratio. In this paper, we present the results from a number of linear calibration experiments in various duration conditions on a modern i-vector based speaker recognition system with probabilistic linear discriminant analysis (PLDA) modeling [37]–[39]. Besides the proposed QMF calibration, we also report the calibration performance using other linear calibration techniques such as matched, mismatched, stacked scores, and shared scaling, as comparison to QMF calibration. The proposed approach does not only show improved performance in dealing with duration variation of speech utterances, but also shows some robustness in calibration towards extrapolated durations.

This paper provides an overview of automatic speaker recognition system configuration used for the experiments in Section II. Databases explanation and calibration performance metrics are presented in Section III. In Section IV, all linear calibration approaches analyzed in this paper are explained. The experiment results discussed in Section V, and Section VI concludes the paper.

II. AUTOMATIC SPEAKER RECOGNITION SYSTEM

Text-independent speaker recognition system technologies have consistently been improving in the past decades [40]. Speaker recognition systems based on Gaussian mixture model (GMM) speaker modeling were proposed in 1995 [8], and became a fundamental approach for speaker recognition with the introduction of the universal background model (UBM) around 2000 [41]. Several milestones in the GMM-UBM based system development were achieved by the researchers subsequently. Support vector machines (SVM) [42] and joint factor analysis (JFA) [43] techniques were introduced from 2003–2007. Both of these are examples of supervector approaches [40]. Recently, the mainstream in the text-independent based speaker recognition system has moved more towards compact representations of the utterance in subspaces, known as i-vectors [16].

The text-independent speaker recognition system used in this paper is based on subspace modeling of i-vectors using probabilistic linear discriminant analysis (PLDA). This section presents a brief explanation of i-vector extraction and PLDA modeling.

A. I-vectors

The speaker recognition system used in this paper follows the i-vector framework that was proposed in [12], [16]. The i-vector is a compact representation of the speech utterance in a low-dimensional space. This space contains both speaker and channel/session variability so that our speaker- and session-dependent Gaussian mean supervector \(M\) can be modeled as:

\[
M = m + Tw
\]  

where \(m\) is the speaker- and session-independent mean supervector of the UBM, \(T\) is a low-rank matrix that defines the low-dimensional space, and \(w\) is our identity vector or so-called i-vector.

The speaker- and session-dependent mean supervector in i-vector speech representation is very similar to in the JFA speaker representation [44]. The main difference between the i-vector and JFA modeling is that JFA defines separate speaker and session subspaces, while these factors of variability are combined in a single space \(T\) in i-vector representation.

B. Probabilistic Linear Discriminant Analysis

Probabilistic linear discriminant analysis is a probabilistic approach that models the i-vectors distribution with a Gaussian assumption [37]–[39]. Computed scores from the PLDA model are directly in the form of a ratio of the likelihoods that the enrollment and test i-vectors come from the same speaker and different speakers, respectively. The PLDA method implemented in our system is similar to the approach in [45].

The PLDA models the distribution of i-vectors as the sum of Gaussians for the speaker-dependent term, \(\mu + \Phi y_k\) and an utterance dependent term \(\Gamma z_r + \epsilon_r\) with \(r = 1, \ldots, R\) utterances for a speaker \(k\) [15], [37]. The overall mean of the training vectors is denoted by \(\mu\) and the matrices \(\Phi\) and \(\Gamma\) are composed of the bases for between-speaker and within-speaker subspaces, respectively. The \(y_k\) and \(z_r\) are positioning the i-vector in between-speaker and within-speaker subspaces, respectively, and \(\epsilon_r\) is a Gaussian residual error term with covariance \(\Sigma\).

In the context of PLDA model, the hypothesis testing becomes evaluation of the probabilities if the two i-vectors \(w_1\) and \(w_2\), traditionally named enrollment/model and test, are generated by the same speaker, \(H_1\), or by different speakers, \(H_2\). This can be formulated as:

\[
s = \frac{P(w_1, w_2 | H_1)}{P(w_1, w_2 | H_2)}
\]  

It is shown in [46] and [15] that the likelihoods can be computed analytically as:

\[
s = \frac{N(w_{12} | \mu_2, \Sigma_p)}{N(w_{12} | \mu_2, \Sigma_d)},
\]  

where \(w_{12}\) is formed by stacking i-vectors \(w_1\) and \(w_2\) and \(\mu_2\) by stacking \(\mu\) twice, and the covariance matrices for the same and different speakers are obtained by using the matrix expressions:

\[
\Sigma_p = \begin{bmatrix}
\Phi \Phi^T + \Gamma \Gamma^T + \Sigma & \Phi \Phi^T \\
\Phi \Phi^T & \Phi \Phi^T + \Gamma \Gamma^T + \Sigma
\end{bmatrix}
\]

\[
\Sigma_d = \begin{bmatrix}
\Phi \Phi^T + \Gamma \Gamma^T + \Sigma & 0 \\
0 & \Phi \Phi^T + \Gamma \Gamma^T + \Sigma
\end{bmatrix}
\]

C. Experimental Setup

Features were extracted from audio samples by calculating 19 MFCC\(^2\) parameters and the log energy of speech signal using 20 ms analysis windows every 10 ms. These were augmented using delta and double delta coefficients over 9 frames forming a feature vector of 60 dimension. Speech activity detection (SAD) is performed using a two-Gaussian energy based algorithm as described in [13] and [47]. After SAD, short time Gaussianization is applied using a 5 second

\(^2\)Mel frequency cepstral coefficients.
TABLE I
GENERAL SYSTEM PERFORMANCE OF NIST SRE-2008 AND NIST SRE-2010 FOR MALE GENDER IN TERMS OF $E_m$ (%).

<table>
<thead>
<tr>
<th>Model/Test</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>40</th>
<th>full</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(%)</td>
<td>(%)</td>
<td>(%)</td>
<td>(%)</td>
<td>(%)</td>
</tr>
<tr>
<td>5</td>
<td>22.79</td>
<td>18.31</td>
<td>13.27</td>
<td>11.69</td>
<td>10.62</td>
</tr>
<tr>
<td>10</td>
<td>16.14</td>
<td>11.07</td>
<td>7.20</td>
<td>6.00</td>
<td>5.47</td>
</tr>
<tr>
<td>20</td>
<td>11.28</td>
<td>6.91</td>
<td>4.25</td>
<td>2.97</td>
<td>2.97</td>
</tr>
<tr>
<td>40</td>
<td>7.96</td>
<td>5.18</td>
<td>3.06</td>
<td>2.39</td>
<td>1.74</td>
</tr>
<tr>
<td>full</td>
<td>7.09</td>
<td>4.27</td>
<td>2.53</td>
<td>1.86</td>
<td>1.33</td>
</tr>
</tbody>
</table>

Fig. 2. $E_m$ (%) from NIST SRE-2008 det-7 (dashed lines) and NIST SRE-2010 det-5 (solid lines) databases for male trials in all 25 duration conditions.

In our experiments, we used data and protocols from NIST SRE-2008 [49] and 2010 [50]. We focus on utterances from telephone-telephone conversation in English, which are known as ‘det-7’ and ‘det-5’ conditions in SRE-2008 and 2010, respectively. Calibration performance is evaluated on the SRE-2010 trials (extended list) with the calibration parameters trained on the SRE-2008 trials. The experimental results presented in this paper concentrate on male trials only. The number of trials we used from SRE-2008 are 769 target and 10050 non-target trials, and for SRE-2010, 3601 target and 226818 non-target trials, respectively.

B. Utterances Duration and Truncation Procedure

In the NIST SRE database, the length of utterances vary in duration. In order to obtain segments for short duration conditions, all utterances from the database were truncated to $d = 5, 10, 20$, and 40 seconds. The truncation process was carried out from the beginning point of the utterances at the feature level after SAD and before short term Gaussianization, so that the duration $d$ represents the length of active speech from the utterances. Utterances that have active speech duration less than 40 seconds were excluded from the experiments in order to have the same number of trials in every duration condition.

The original segments from the NIST SRE database without any truncation form the full condition in this paper. From the full condition features and the features obtained by truncation, we have five test sets with different duration conditions in both model and test segment collections. Twenty five trial lists are formed by combining the model and test sides from every duration condition for both SRE-2008 and 2010 data sets. This set of 25 trial lists is often referred to as 25 duration conditions in this paper. We use the notation ‘(duration of model segment)/(duration of test segment)’ condition,’ in which duration is measured in seconds or ‘full’.

C. General Discrimination Performance

We have measured the discrimination performance of our PLDA based i-vector system on both the SRE-2008 and 2010 core condition, for trial sets of telephone channel, male speakers. The system’s discrimination performance in terms of equal error rate$^3$ $E_m$ is presented in Table I and depicted in Figure 2. In general, the system shows lower $E_m$ for SRE-2008 than for SRE-2010, in all duration conditions.

$^3$Equal error rate is the error rate at the operating point of a detection system where the probability of false acceptance and probability of false rejection are equal.

### III. Evaluation Databases and Metrics

#### A. NIST Speaker Recognition Evaluation Protocols

For over one decade, the National Institute of Standard and Technology (NIST) have set the standard for evaluation of text independent speaker recognition systems. The general goal of the NIST SRE is to push the technology in the field of text independent speaker recognition forward. At regular intervals, a number of research groups participate with their most advanced technology in speaker recognition, and disclose their findings in the workshop following the evaluation. [48]

We did not apply LDA prior to PLDA modeling.

The dimension of speaker and session factors are the number of components in $\Phi$ and $\Gamma$, respectively. See Section II-B for further explanations.

$^3$We did not apply LDA prior to PLDA modeling.

$^4$The dimension of speaker and session factors are the number of components in $\Phi$ and $\Gamma$, respectively. See Section II-B for further explanations.

In this paper, we used a gender-dependent 400-dimensional i-vector space which was trained on the same data as the UBM training. I-vector length normalization [14] and within class covariance normalization (WCCN) [11] were applied prior to PLDA$^4$ for optimal performance of our system [2]. We used optimal settings of 200 speaker factors and 50 session factors in applying PLDA$^4$. Subspace matrices in PLDA for both speaker and session spaces are trained using the same databases for i-vector space training, this time using the speaker labels.

In general, the system shows lower $E_m$ for SRE-2008 than for SRE-2010, in all duration conditions.
D. Evaluation Metrics for Calibration Performance

To evaluate the calibration performance of the speaker recognition system in general, we use two basic measures \( C_{llr} \) and \( C_{llr}^{\text{min}} \). The metric \( C_{llr} \) is the cost of the log-likelihood-ratio, a metric that measures calibration over the entire range of effective priors, which has both an interpretation in terms of detection cost functions \( C_{\text{det}}(O_{\text{eff}}) \), where \( O_{\text{eff}} \) are the effective prior odds, and an information theoretical interpretation [22]. The metric \( C_{llr}^{\text{min}} \) is the same metric, but after an optimal transformation of scores that minimizes \( C_{llr} \) under the condition that the order of scores stay the same, i.e., the score to likelihood function is monotonously rising. From these basic metrics, we derive the absolute and relative miscalibration costs, or calibration loss. For an introduction to \( C_{llr} \), see [23].

The metric \( C_{llr} \) can be evaluated empirically for a supervised set of evaluation log-likelihood-ratios \( x_i \) (in our case SRE-2010) using

\[
C_{llr} = \frac{1}{N_{\text{tar}}} \sum_{i \in \text{tar}} \log_2(1 + \exp(-x_i)) + \frac{1}{N_{\text{non}}} \sum_{j \in \text{non}} \log_2(1 + \exp(x_j))
\]

with \( x_i \) and \( x_j \) running over the number of target trials (\( N_{\text{tar}} \)) and non-target trials (\( N_{\text{non}} \)) respectively, i.e., trials for which either \( H_1 \) or \( H_2 \) is true.

The absolute (\( C_{mc} \)) and relative (\( R_{mc} \)) calibration loss, or miscalibration cost, are defined as:

\[
C_{mc} = C_{llr} - C_{llr}^{\text{min}},
\]

and

\[
R_{mc} = \frac{C_{mc}}{C_{llr}} = \frac{C_{llr}}{C_{llr}^{\text{min}}} - 1
\]

The value for minimum cost of the log-likelihood-ratios, \( C_{llr}^{\text{min}} \), can be obtained by isotonic regression. An efficient method for this is known as the pool adjacent violators (PAV) algorithm as explained in [22] which has relations to the receiver operating characteristic convex hull (ROC-CH) [52]. All metrics share the same property, that lower values are indicate better performance. Here, \( C_{llr} \) integrates both discrimination and calibration performance, where \( C_{llr}^{\text{min}} \) only reveals discrimination performance. The mis-calibration costs \( C_{mc} \) and \( R_{mc} \) only show calibration performance.

IV. Score Calibration

In many mathematical formulations of speaker recognition, including PLDA based systems, recognition scores are computed as likelihood ratios. However, due to a number of modeling assumptions that are probably incorrect, most notably the assumption of frame independence, these computed scores do not have a direct proper probabilistic interpretation. Using such uncalibrated scores in court as calibrated likelihood ratios will be misleading [53]. However, there is a number of ways in which we can transform the uncalibrated scores into log-likelihood-ratios, a process known as calibration and in which the field of speaker recognition has extensive experience, specifically in comparison to other biometric technologies. A remarkable property of calibrated log-likelihood-ratios \( \ell \) is relates to the probability density function of itself (for a derivation of this, see the appendix),

\[
\ell = \log \frac{P(\ell \mid H_1)}{P(\ell \mid H_2)}
\]

The metric \( C_{llr} \) can measure the validity of this property empirically for a set of evaluation trials.
In this paper we restrict ourselves to linear calibration transformations\(^6\), i.e., the function that is used to convert our uncalibrated scores \(s\) into calibrated likelihood ratios \(x\) is

\[
x = w_0 + w_1 s
\]

where \(w_0\) is the offset of the transformation and \(w_1\) is a scaling parameter. Both the offset and scaling parameters can be obtained by optimization on a development set. An effective method for this optimization is logistic regression \([56]\) training which uses an objective function quite closely related to \(C_{llr}\) \([22]\). For implementation of this linear calibration, we utilized FoCal toolkit \([19]\) and the \sretools\ analysis package \([57]\).

There are 5 linear calibration approaches discussed in this paper which we refer to as ‘mismatched,’ ‘matched,’ ‘stacked,’ ‘shared scaling’ and ‘duration quality measure function’ (QMF). All of these approaches are explained in the following subsections. The goal of this research is to describe the problems of duration for calibration (mismatched, matched and stacked scores), to understand the effects of duration to the calibration parameters (shared scaling), and to design low-parameter models to account for these effects in calibration (duration quality measures).

### A. Calibration using Mismatched and Matched Duration Conditions

Both mismatched and matched approaches employ the scores transformation defined in equation (10), which consists of two weighting parameters \(w_0\) and \(w_1\). In the mismatch approach, the two calibration parameters are trained in the full/full duration condition from the SRE-2008 calibration set. These parameters are then applied to all 25 duration conditions of the SRE-2010 evaluation set. This approach is called mismatched because the presence of many duration-mismatched conditions between the calibration and evaluation data, i.e., the calibration parameters remain trained on the full/full condition even if they are applied to, e.g., the 40/20 seconds condition for evaluation. This is a 2-parameter calibration, and we expect it to be the worst performing based on previous work on an LDA i-vector system \([3]\).

In the matched approach, calibration parameters are trained on each of 25 duration conditions in the SRE-2008 calibration set. This approach uses 50 calibration parameters, 25 pairs of weighting parameters \(w_0\) and \(w_1\). Each of these weighting pairs are then applied to the corresponding matched condition in the SRE-2010 evaluation data. One may consider this as a “poor-man’s” solution, because it does not rely on understanding the effect of the quality measures on calibration, but requires to match the quality of the calibration data with the conditions under evaluation. For (shorter) duration, this may actually be feasible (although perhaps not very practical), but for other quality factors such as language, reverberation or background noise level and type, this may be less than trivial.

In the more realistic forensic cases, it may not always be possible to find training calibration material that match the condition of evaluation data. The matched approach needs 50 parameters, and we expect this to outperform most other methods, simply because the only reason for miscalibration is the data set shift, which is hard to deal with anyway.

### B. Calibration using Stacked Scores

In the case of mismatched duration, the training calibration condition used (full/full) is quite peripheral to the conditions used in evaluation. The purpose of the stacked scores approach is to understand the potential of two-parameter linear calibration if calibration training data shows the same variability as the evaluation. The stacked scores approach uses all available data for calibration in all 25 conditions in order to train a single set of transformation parameters according to (10). Similarly to the mismatched approach, this single set of 2 calibration parameters is applied to all duration conditions of the evaluation data. We expect that this approach performs better, on average, than the mismatched approach.

### C. Calibration using Shared Scaling

Keeping in mind that we would like to design a calibration function that takes duration of model and test segments into account, we want to study the effect of duration on the shift parameter \(w_0\) while keeping the scaling \(w_1\) constant. This approach is somewhat between stacked scores and matched approaches. We stack all calibration trials, but use a single scaling parameters \(w_1\) while allowing for a duration-dependent offset parameter \(w_0\) for each of the 25 duration conditions. This approach is similar to the ‘side information’ calibration technique mentioned earlier that was employed in the NIST SRE-2008 in order to deal with varying language and

\(^6\)It is presented in \([54]\) that the scores calibration can be conducted by doing simple normalization (like Z- or T-norm), isotonic regression, etc. besides the linear logistic regression. One of the authors is also contributed in the proposing of line-up calibration method for speaker recognition system which is not a form of linear calibration \([55]\).
transducer [34]. We expect this approach to be almost as good as the matched approach, because of its many parameters. However, this is not an approach we would like to propose as a viable method of dealing with continuous quality factors in general, but rather as an inspiration for designing quality measure functions. To this effect, the 25 offset parameters are presented in Figure 4. The behavior study of the calibration parameters presented in the figure becomes the foundation of the duration quality measures approach that we propose in the next section.

D. Linear Calibration with Duration Quality Measure Functions

Finally, we propose a calibration approach for calibration in various duration conditions that models the effect of calibration in a low-parameter model. The general score transformation model is:

\[ x = w_0 + w_1 s + Q(d_m, d_t, w_2, \ldots) \quad (11) \]

where \( Q(d_m, d_t, w_2, \ldots) \) is the quality measure function (QMF) that is related to duration of model segment \( d_m \) and duration of test segment \( d_t \). We propose four QMFs for improving the global calibration performance in various duration conditions in Table II.

The results on calibration using shared scaling (cf. Figure 4 (further explained in Section V) shows that the larger magnitude of offset parameters \( (w_0) \) occurs where the difference between model and test segments duration are larger. The first two QMFs, \( Q_1 \) and \( Q_2 \), model this behavior. These two functions, however, do not model any difference in offset where model and test segments have the same duration. Observing Figure 4 there clearly is a dependency on the duration even for \( d_m = d_t \). Therefore, the QMFs \( Q_3 \) and \( Q_4 \) were proposed in order to better model the offset parameters behavior from the shared scaling approach.

The mathematical form of \( Q_3 \) and \( Q_4 \) are modeled after the saddle-like shape of the surface in Figure 4. In general, a two-dimensional saddle function can be described by

\[ f(y, z) = \alpha \cdot y^2 - \beta \cdot z^2, \quad (12) \]

with \( \alpha \beta > 0 \). In our case, we use a rotated version of the axes \( y \) and \( z \) which work with log-duration, and placing the origin at \( d_c = 20 \) s for both model and test durations, so that

\[ y = \log \frac{d_m}{d_c} + \log \frac{d_t}{d_c}, \quad (13) \]

\[ z = \log \frac{d_m}{d_c} - \log \frac{d_t}{d_c}. \quad (14) \]

The rotation is because the distribution of offset parameters has the saddle plane shape which lies along the diagonal axis of \( d_m \) and \( d_t \) as depicted in Figure 4. By using \( y \) and \( z \) defined in (13), the QMFs that model the saddle-plane \( Q_3 \) and \( Q_4 \) from Table II can be found with

\[ w_2 = 2(\alpha + \beta), \quad (15) \]

\[ w_3 = \alpha - \beta. \quad (16) \]

The third QMF \( Q_3 \) is the case where \( \alpha = \beta \), i.e., forcing the ‘tails’ at the extremes full/5 and 5/full to go as much up as the ones at 5/5 and full/full go down. The parameter \( d_c \) is fixed in our experiments to 20 seconds as it is the center of our saddle-shaped parameter distribution (as seen in Figure 3).

The proposed QMFs are not designed to handled the condition where duration goes to zero. In this extreme condition, the speaker recognition system should output \( \ell = 0 \) as there is no speaker information, and hence both hypotheses are equally likely. The log-duration dependence of the QMFs in Table II is inspired by Figure 4, which has logarithmic axes, and may find some further motivation in the observation that the number of unique phones found in a random speech sample scales logarithmically with duration over a fairly wide range [58].

V. Experiment Results

The calibration results for all linear calibrations mentioned in previous section are presented in Table IV, and analyzed in the paragraphs V-B until V-E. Table V is the summary of Table IV in which we take the average \( \mu \) and standard deviation \( \sigma \) across all 25 duration conditions for each calibration technique.

A. Discrimination performance based on \( C_{\text{llr}}^{\text{min}} \)

The minimum achievable values \( C_{\text{llr}}^{\text{min}} \) were measured and showed in Table III for all 25 duration conditions in the SRE-2010 evaluation set. The numbers presented in Table III are the \( C_{\text{llr}}^{\text{min}} \) values from all calibration techniques but the QMFs approach. The \( C_{\text{llr}}^{\text{min}} \) values on the full/full QMFs calibration

\[ \text{Table III: Minimum cost of log-likelihood-ratio calibration} \quad C_{\text{llr}}^{\text{min}} \]

\begin{tabular}{|c|c|c|c|c|c|}
\hline
\( d_m/d_t \) & 5 & 10 & 20 & 40 & full \\
\hline
5 & 0.695 & 0.581 & 0.476 & 0.416 & 0.372 \\
10 & 0.572 & 0.428 & 0.321 & 0.264 & 0.223 \\
20 & 0.470 & 0.319 & 0.219 & 0.171 & 0.138 \\
40 & 0.401 & 0.248 & 0.156 & 0.119 & 0.094 \\
full & 0.351 & 0.210 & 0.122 & 0.090 & 0.071 \\
\hline
\end{tabular}

\[ ^{7}\text{We took the averaging approach to summarize the results instead of pooling the scores then computing the calibration metrics. This is due to the pooling method that causing} \quad C_{\text{llr}}^{\text{min}} \text{less sensitive because of the pooled}
\]

\( C_{\text{llr}}^{\text{min}} \) is increasing.
are slightly different to what is presented in the table due to the effect of variable duration in the full/full condition. As explained in Section III-D, \( C_{\text{mis}} \) is a representation of discrimination loss [23]. Thus, it has similar information as \( E_{\text{mix}} \) that is presented in Section III-C. We can observe that \( C_{\text{mir}} \) is increasing as the duration of model/test segments are decreasing. Note that \( C_{\text{mir}} \) stays the same after linear calibration applied unless \( C_{\text{mis}} \) is computed on pooled scores over all 25 duration conditions, because the pooling makes \( C_{\text{mis}} \) sensitive to “relative calibration” between the duration conditions. Further, linear calibration will have an effective objective to minimize \( C_{\text{mir}} \), but we know beforehand that \( C_{\text{mir}} \) is a lower bound to this.

### B. Calibration using Mismatched and Matched approaches

In this section, we compare the calibration performance from the mismatched and matched approaches. The calibration results for these two approaches are presented in the top sections of Table IV. In the mismatched condition, where we trained calibration parameters on the full/full duration condition only, the miscalibration values are higher, as the durations of model or test segments are shorter, and deviate more from the calibration condition. The highest miscalibration values are present when there is a large difference between the duration in model and test segments. These results confirm that by training the calibration parameters using a data set which is not representative of evaluation data in the sense of speech segments duration, the calibration performance is not very good.

The matched calibration approach is an easy solution to come up with the deficiency of mismatched approach by simply matching the duration condition of training calibration data to the evaluation data. As presented in Table V, the calibration system performance using matched approach totally surpass the mismatched approach based on the miscalibration values. The average miscalibration values from all 25 duration conditions drops from 0.084 for mismatched to 0.006 for

### TABLE IV

<table>
<thead>
<tr>
<th>Calibration: ( d_{\text{cal}}/d_{\text{tr}} )</th>
<th>( C_{\text{mir}} )</th>
<th>( C_{\text{mis}} )</th>
<th>( R_{\text{mis}}(%) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.71 .684 .603 .591 .600</td>
<td>0.76 .103 .127 .174 .228</td>
<td>10.96 17.73 26.68 41.84 61.19</td>
</tr>
<tr>
<td>10</td>
<td>0.68 .652 .398 .346 .308</td>
<td>0.96 .093 .078 .081 .086</td>
<td>16.81 21.63 24.26 30.72 38.64</td>
</tr>
<tr>
<td>20</td>
<td>0.60 .469 .267 .205 .163</td>
<td>0.57 .064 .049 .034 .025</td>
<td>27.89 34.26 23.33 19.77 18.02</td>
</tr>
<tr>
<td>40</td>
<td>0.57 .328 .187 .134 .101</td>
<td>0.70 .081 .031 .015 .007</td>
<td>42.34 32.49 20.15 12.59 7.67</td>
</tr>
<tr>
<td>full</td>
<td>0.57 .328 .187 .134 .101</td>
<td>0.70 .081 .031 .015 .007</td>
<td>42.34 32.49 20.15 12.59 7.67</td>
</tr>
</tbody>
</table>

### TABLE V

<table>
<thead>
<tr>
<th>Approach</th>
<th>( C_{\text{mir}} )</th>
<th>( C_{\text{mis}} )</th>
<th>( R_{\text{mis}}(%) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{\text{mir}} )</td>
<td>( \mu )</td>
<td>( \sigma )</td>
<td>( \mu )</td>
</tr>
<tr>
<td>Mismatched</td>
<td>0.385</td>
<td>0.216</td>
<td>0.084</td>
</tr>
<tr>
<td>Matched</td>
<td>0.307</td>
<td>0.173</td>
<td>0.006</td>
</tr>
<tr>
<td>Stacked Scores</td>
<td>0.317</td>
<td>0.177</td>
<td>0.016</td>
</tr>
<tr>
<td>Shared Scaling</td>
<td>0.313</td>
<td>0.173</td>
<td>0.012</td>
</tr>
<tr>
<td>Duration ( Q_1 )</td>
<td>0.312</td>
<td>0.175</td>
<td>0.011</td>
</tr>
<tr>
<td>Duration ( Q_2 )</td>
<td>0.312</td>
<td>0.175</td>
<td>0.011</td>
</tr>
<tr>
<td>Duration ( Q_3 )</td>
<td>0.311</td>
<td>0.177</td>
<td>0.010</td>
</tr>
<tr>
<td>Duration ( Q_4 )</td>
<td>0.311</td>
<td>0.177</td>
<td>0.010</td>
</tr>
</tbody>
</table>

\( n_p = \) number of calibration parameters.
matched. This is of course an expected effect, but it is interesting to note here that the effect on miscalibration cost in the PLDA system is much smaller than in the i-vector LDA system reported on our earlier work [3]. For the LDA system, we had observed average miscalibration values $C_{mc} = 0.494$ and $R_{mc} = 141\%$ over all 25 duration conditions following the mismatched calibration approach. This shows that with the classifier in i-vector systems becoming better at discrimination (going from LDA with cosine distance scoring via normalized cosine to PLDA with probabilistic scoring), the calibration behavior improves as well.

Even though the matched calibration approach offers a very good calibration performance, the training process within this approach can be tough because we have to match the condition of our training data to the evaluation data. This condition matching might not possible in an extreme condition such as when we have only limited amount of training data in which we do not have long enough duration to match the evaluation data condition.

C. Calibration using Stacked Scores

The experiment results of the stacked scores approach is presented in the third section of Table IV. Compared to the mismatched approach, the stacked calibration results is generally better in terms of miscalibration results over all 25 duration conditions. The miscalibration metrics in the condition where the duration of model and test segments have large difference (i.e., 5/full and full/5 conditions) are lower in the stacked scores approach than in the mismatched approach. These results show that the stacked scores calibration works generally better than the mismatched approach even though we have to sacrifice the calibration performance in the longer duration condition.

The average values of miscalibration using the stacked scores approach is 0.016, still a notable increase over the optimal value of 0.006 found using the matched approach. This is due to a fact that in the stacking calibration, we only use 2 calibration parameters even though we had more training data available compared to the matched approach, while in the matched approach, we use 50 calibration parameters in total. Hence, there is room for improvement for the system calibration performance in various duration conditions, where we can have a good performance with using only a small number of calibration parameters. In order to find out about the pattern of calibration parameters with respect to duration of model/test segments, we will now present the results for shared scaling calibration which are discussed in the next section.

D. Calibration using the shared scaling approach

This section presents the experimental results on calibration using the shared scaling approach. This experiment was performed to demonstrate the relation between the bias term (offset parameter) of linear calibration and the speech segment duration. This formed the inspiration of the proposed duration quality measure function which explained in Section V-E. The 25 offset parameters trained from this calibration approach is presented in Table VI and have been shown before in Figure 4.

The standard deviation of this 25 offset parameters is 0.511. This means that there is a large variability of offset parameters due to the variation of duration condition in the calibration.

From Table V, the average of miscalibration values of the shared scaling\(^8\) approach is 0.010 which is better than the stacked scores calibration results. Despite its low average of miscalibration costs, the shared scaling approach requires a large number of calibration parameters (26) which need to be trained. As indicated earlier, the motivation was to use the offset parameters (cf. Figure 4) to find a simpler calibration technique with few calibration parameters and comparable calibration performance. In the next subsection, we present the duration quality measures calibration results.

E. Calibration using the Duration Quality Measures Approach

This section presents the results of duration quality measure calibration which is based on the QMFs proposed in Section IV-D. As can be seen from the average of miscalibration values of all 25 duration conditions in Table V, all of the proposed duration QMFs offer better calibration performance than the stacked scores approach. It has also comparable performance to the shared scaling approach, even though the duration QMF approaches use only 3 or 4 calibration parameters. By adding 1 or 2 extra parameters in the duration QMF approach, it provides much better calibration performance than the 2-parameter mismatched or stacked scores approaches. The differences in $R_{mc}$ are statistically significant at $p < 0.05$, as tested in a one-sided paired t-test\(^9\) [59]. The results for this t-test are presented in Table VII.

Presented in Table V, there are differences in performance which are offered by each of the proposed duration QMFs. The first two functions, $Q_1$ and $Q_2$, have similar calibration

---

\(^8\)The calibration experiment results for shared scaling are not shown in Table IV because this calibration method is carried out only as an inspiration for QMF design, and not as a calibration method per se.

\(^9\)via R programming language.
performance trend in terms of miscalibration values, while the last two functions, $Q_3$ and $Q_4$, have almost identical calibration performance trend. In terms of the average of miscalibration values of all 25 duration conditions, $Q_3$ and $Q_4$ offer slightly better calibration performance than the $Q_1$ and $Q_2$ functions, but it has bigger variance. A one-sided paired t-test shows that $Q_3$ and $Q_4$ have statistically significantly lower $R_{mc}$ than $Q_1$ at the $p < 0.05$ level as presented in Table VII.

In the miscalibration metrics of the proposed QMFs shown in Table IV, $Q_1$ and $Q_2$ functions have better calibration performance at the 5/5 duration condition, while it has worse calibration performance at the full/full condition compared to the $Q_3$ and $Q_4$ functions. In general, $Q_3$ and $Q_4$ functions have lower miscalibration value across all duration conditions except for 3 conditions, which are 5/5, 5/10 and 10/5. From this observation, we summarize that the $Q_3$ and $Q_4$ functions have better performance than the $Q_1$ and $Q_2$ functions except in the conditions in which the model and/or test segments contain 5 second of duration. Even though there are slight differences between the calibration performance among the proposed duration QMFs, all of them bring improved results in the calibration performance of the system compared to the stacked calibration technique, as revealed by one-sided paired t-tests we carried out.

To better analyzed how the QMFs can give better performance in calibration with duration variability problem, we present the score distributions of calibrated scores from the mismatched and $Q_4$ calibrations in Figure 5. As can be seen from the figure, the calibrated scores from mismatched technique are shifted further to the left when the test segment duration is decreased. In the calibrated scores using the $Q_4$ function, however, the QMF is able to normalized the duration effect in the scores distribution. Therefore, the $Q_4$ calibrated scores is pushed back to the center of the log likelihood ratio (LLR) axis (LLR = 0).

We further evaluate the robustness of proposed QMF calibration to the mismatched channel problem. We applied the same calibration parameters used to calibrate NIST SRE’10 det-5 condition to the det-3 condition. Since the calibration parameters were trained in the NIST SRE’08 det-7 (telephone-telephone), the evaluation on NIST SRE’10 det-3 (interview-telephone) is therefore incurring ‘mismatched channel’ challenge in calibration. The average of $C_{llr}$ values from all 25 duration conditions in det-3 trial set are 0.550 (mismatched); 0.376 (matched); 0.380 (stacked scores); 0.379 ($Q_2$ and $Q_2$); and 0.375 ($Q_3$ and $Q_4$). The $C_{llr}$ values from det-3 evaluation deviate by similar amount for each of the calibration techniques compared to the det-5 results. In addition, similar trends in performance are observed across all calibration techniques. These results show that the QMF calibration is robust in dealing with duration variability, even though the mismatched channel problem occurs in the calibration process.

**F. Extrapolation Experiment**

In this section, we will present the results from our extrapolation experiments on the duration QMF calibration approach. The extrapolation experiments were performed in order to test the robustness of the calibration approach to unseen values of the quality measures. There are two experiments in which we tested extrapolation performance of our proposed duration QMF calibration:

- **Short missing**: Calibration parameters were trained by using only 16 duration conditions by excluding durations of 5 s present in the model and/or test segments.
- **Long missing**: Calibration parameters were trained by using only 16 duration conditions by excluding conditions in which full condition present in the model and/or test segments.

These trained calibration parameters were then applied in evaluation to the nine duration conditions which were not seen in the calibration training. These extrapolation experiments
were conducted for all four duration QMFs, and the results of these experiments are presented in Table VIII for both short missing and long missing extrapolations. The calibration metrics presented in Table VIII are averaged over the nine extrapolation conditions.

In order to be able to compare the performance of all calibration approaches in the extrapolation experiments, Figure 6 depicts the miscalibration rate values for every observed duration conditions in the short and long missing extrapolations. In both extrapolations, the system calibration performance drops compared to the experiments with calibration training on all 25 duration conditions. This may be expected because there is still an obvious duration mismatch. However, the extrapolated QMFs show a lot better performance than the stacked scores approach that did have access to the missing duration conditions.

In the short missing extrapolation, the $Q_2$ function has the best extrapolation performance compared to the other QMFs based on the average of miscalibration values which are presented in Table VIII. The $Q_1$ function has the next best performance, and both $Q_1$ and $Q_2$ perform statistically significantly better than the stacked calibration technique based on one-sided paired $t$-test similar to what we did for the results in Section V-E. As presented in Table VIII, the $Q_3$ and $Q_4$ functions do not perform as well as the other two in this extrapolation experiment, perhaps because they utilize a too complex duration QMFs. Specifically in the short duration condition such as $5/5$, $Q_3$ and $Q_4$ functions give limited performance improvement, similar to what we observed in the all duration training condition from the last paragraph of Section V-E.

In the long missing extrapolation, in contrary, the $Q_3$ and $Q_4$ functions offer statistically significantly better extrapolation performance compared to the $Q_2$ function, as we determined by a one-sided paired $t$-test. As it is mentioned in Section V-E, the $Q_3$ and $Q_4$ functions are, in fact, performing well in the presence of longer duration in both model and test segments. However, inspecting the difference of miscalibration average between the duration QMF approach for the full training and extrapolation experiment, the $Q_1$ and $Q_2$ functions show only a 0.001 and 0.003 absolute miscalibration increase, respectively. This is not very much and comparable to the increase found in $Q_3$ and $Q_4$ which shows the robustness of all proposed QMFs to unseen duration condition in calibration.

In general, the trend of the extrapolation results for both short missing and long missing for all duration QMFs indicate good extrapolation performance: in almost every evaluated condition in the extrapolation experiment, the calibration performance is better than the calibration performance of the stacked scores approach. We conclude that the proposed duration QMF approaches have successfully maintained a good calibration performance in the various duration conditions at the cost of adding at most two extra parameters in scores transformation for calibration, even if the durations have to be extrapolated beyond the range of durations seen in calibration training.

### Table VIII

<table>
<thead>
<tr>
<th>Calibration approach</th>
<th>Short missing</th>
<th>Long missing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_{llr}$</td>
<td>$C_{llc}$</td>
</tr>
<tr>
<td>Mismatch*</td>
<td>.628</td>
<td>.147</td>
</tr>
<tr>
<td>Match*</td>
<td>.489</td>
<td>.008</td>
</tr>
<tr>
<td>Stacked scores</td>
<td>.519</td>
<td>.038</td>
</tr>
<tr>
<td>$Q_1$ Extrapolation</td>
<td>.495</td>
<td>.022</td>
</tr>
<tr>
<td>$Q_2$ Extrapolation</td>
<td>.495</td>
<td>.013</td>
</tr>
<tr>
<td>$Q_3$ Extrapolation</td>
<td>.496</td>
<td>.015</td>
</tr>
<tr>
<td>$Q_4$ Extrapolation</td>
<td>.497</td>
<td>.015</td>
</tr>
</tbody>
</table>

* No extrapolation experiments applied in this approach.
G. Comparison of Duration QMFs and k-means approaches

This section contrasts our proposed duration QMFs to other calibration techniques that also include quality measures. We use two contrasting approaches: calibration through k-means clustering of duration conditions and calibration via side-information using the BOSARIS toolkit [36] . Calibration through k-means clustering uses discrete calibration classes (clusters of duration) based on the duration in model and test segments. In applying calibration on the evaluation set, the parameters from the cluster closest to the evaluation train and test durations are used. The BOSARIS approach is the current common practice performing calibration for speaker recognition systems, and allows duration to be ‘fused in’ as side-information in the calibration step. In other parts of this paper, the calibration experiments were carried out using the FoCal toolkit, which is the predecessor of BOSARIS toolkit.

In order to evaluate the calibration approaches compared in this section, we generated a set of test segments with uniformly random durations in both model and test, ranging from 5 seconds to full length durations from the NIST SRE-2010 det-5 condition. The scores were then calibrated and evaluated using stacked scores, k-means clustering, BOSARIS, and QMFs approaches. In the k-means clustering approach, we use 25 clusters corresponding to the duration combinations used earlier, i.e., the calibration parameters were taken from the ‘Matched Duration Condition’. In evaluation, the calibration parameters were used corresponding to the closest duration combination, e.g., a trial between a 12.5 s train segment and a 23.2 s test segment was calibrated with the \( (12.5, 23.2) \) combination, e.g., a trial between a 12.5 s train segment and a 23.2 s test segment was calibrated with the \( (10.0, 20.0) \) calibration parameters from the Matched Duration Condition.

In the BOSARIS side-information, we used discrete classes for duration in both train and test as indicator vectors, e.g., for the example above the indicator vectors for train and test are \( v_m = (0, 1, 0, 0, 0)^T \) and \( v_t = (0, 0, 1, 0, 0)^T \) respectively. BOSARIS uses these in a bilinear fashion to train a symmetric offset matrix \( V \) using a term \( v_m^T V v_t^T \). As such, it is a symmetricized version of the k-means approach.

The calibrations results of these comparisons are presented in Table IX. As can be seen from the \( C_{\text{mc}} \) values, all calibration approaches produce very low miscalibration cost with k-means clustering has the highest cost. However, based on the \( E_\ell \) and \( C_{\text{llr}} \) values, the QMFs have generally better discrimination performance than other approaches. The \( Q_1 \) and \( Q_2 \) the best performance in both calibration and discrimination shown by all performance metrics presented in Table IX. With relatively less calibration parameters employed in the calibration process, the proposed duration QMF approach can outperform other techniques that also incur information of duration quality measures in calibration, e.g., with k-means clustering approach.

VI. CONCLUSION

Using a simple modification in the linear scores transformation for calibration by adding a quality measure function of duration is an easy and straightforward idea to improve the calibration performance of speaker recognition system. This is observed from the calibration performance of the proposed duration quality measures approach and its comparison with other linear calibration approaches. Four duration quality measure functions are proposed and evaluated in this paper. All of them have their own advantages in counteracting the duration variability problem in calibration. Based on the one-sided paired t-test, all proposed QMFs perform statistically significantly better than the stacked scores calibration, and the saddle-shaped \( Q_3 \) and \( Q_4 \) functions offer better performance compared to the wedge-shaped \( Q_1 \) in terms of \( R_{\text{mc}} \). We have also shown from the extrapolation experiments that the duration quality measures approach is fairly robust against the calibration problem of unseen duration condition in the calibration.

Future work in the topic of calibration with QMF technique includes using other quality measures such as background noise level which can be quantified as signal to noise ratio (SNR). Evaluation of the proposed duration QMFs are planned using different databases with more variation in duration conditions. With encouraging results achieved from the good calibration performance offered by QMF technique, further research on this topic is highly encouraged.

### APPENDIX

In this appendix we will derive (9) following an argument put forward by Niko Brümmer. The relation is well-known by forensic statisticians, but we are not aware of any published derivation. The basic premise is that the likelihood ratio \( \ell(x, y) \) for a speaker recognition system comparing speech samples \( x \) and \( y \)

\[
\ell = \frac{P(x, y | H_1)}{P(x, y | H_2)}
\]

is well-calibrated if it results in the same posterior distribution over \( H \), whether \( \ell \) or the speech input \( (x, y) \) is given. This means that all speaker comparison information is encoded in \( \ell \):

\[
P(H|\ell) = P(H|x, y).
\]

Applying Bayes’ rule, and converting to the log odds domain this becomes

\[
\log P(\ell|H_1)P(H_1) = \log P(x, y|H_1)P(H_1)/P(x, y|H_2)P(H_2)
\]

where in the odds domain the factors with \( P(\ell) \) and \( P(x, y) \) cancel. In (19) the prior odds \( P(H_1)/P(H_2) \) cancel as well, so that with the definition of the log likelihood ratio (17) we have

\[
\log P(\ell|H_1) = \log P(x, y|H_1)/P(x, y|H_2) = \ell,
\]

which proves (9).

### TABLE IX

<table>
<thead>
<tr>
<th>Calibration approach</th>
<th>( C_{\text{llr}} )</th>
<th>( C_{\text{min}} )</th>
<th>( C_{\text{mc}} )</th>
<th>( R_{\text{mc}}(%) )</th>
<th>( E_\ell(%) )</th>
<th>( n_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stacked scores*</td>
<td>0.318</td>
<td>0.317</td>
<td>0.001</td>
<td>0.34</td>
<td>9.28</td>
<td>2</td>
</tr>
<tr>
<td>k-means clustering</td>
<td>0.312</td>
<td>0.311</td>
<td>0.002</td>
<td>0.48</td>
<td>9.19</td>
<td>50</td>
</tr>
<tr>
<td>BOSARIS</td>
<td>0.315</td>
<td>0.313</td>
<td>0.001</td>
<td>0.43</td>
<td>9.23</td>
<td>7</td>
</tr>
<tr>
<td>QMF 1</td>
<td>0.313</td>
<td>0.311</td>
<td>0.001</td>
<td>0.39</td>
<td>9.26</td>
<td>3</td>
</tr>
<tr>
<td>QMF 2</td>
<td>0.312</td>
<td>0.311</td>
<td>0.001</td>
<td>0.41</td>
<td>9.23</td>
<td>3</td>
</tr>
<tr>
<td>QMF 3</td>
<td>0.311</td>
<td>0.310</td>
<td>0.001</td>
<td>0.30</td>
<td>9.18</td>
<td>3</td>
</tr>
<tr>
<td>QMF 4</td>
<td>0.311</td>
<td>0.310</td>
<td>0.001</td>
<td>0.29</td>
<td>9.18</td>
<td>4</td>
</tr>
</tbody>
</table>

* Stacked scores does not include quality measures in calibration.