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Measurements of mass, width and gauge couplings of the W boson at LEP

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Abstract

We report on measurements of mass and total decay width of the W boson and of triple-gauge-boson couplings, γWW and ZWW, with the L3 detector at LEP. W-pair events produced in e^+e^- interactions between 161GeV and 172GeV centre-of-mass energy are selected in a data sample corresponding to a total luminosity of 21.2 pb^{-1}. The mass and total decay width of the W boson are determined to be $M_W = 80.75^{+0.26}_{-0.27}$ (exp.) $\pm$ 0.03 (LEP) GeV and $\Gamma_W = 1.74^{+0.88}_{-0.78}$ (stat.) $\pm$ 0.25 (syst.) GeV, respectively. Limits on anomalous triple-gauge-boson couplings, γWW and ZWW, are determined, in particular $-1.5 < \delta_Z < 1.9$ (95% CL), excluding vanishing ZWW coupling at more than 95% confidence level.

1. Introduction

For the 1996 data taking period, the centre-of-mass energy, $\sqrt{s}$, of the e^+e^- collider LEP at CERN was increased to 161 GeV, 170 GeV and 172 GeV. This allowed for the first time the pair-production of on-shell $W^\pm$ bosons in e^+e^- interactions, $e^+e^- \to W^+W^-$. Analysis of W-pair production adds impor-

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tant knowledge to the Standard Model of electroweak interactions [1] through the measurements of mass and width of the W boson and of the triple-gauge-boson couplings γWW and ZWW [2,3]. These parameters were first measured at p$\bar{\text{p}}$ colliders [4–6].

The total W-pair production cross section as calculated within the Standard Model depends on $\sqrt{s}$ and on the mass and total width of the W boson, M_W and Γ_W. Results for M_W derived from total cross section measurements have been published by L3 [7,8] and the other LEP experiments [9,10]. In this letter a more precise determination of M_W and a first determination of Γ_W is presented based on the invariant mass of the W-boson decay products.

To lowest order within the Standard Model, three Feynman diagrams contribute to W-pair production, the s-channel γ and Z-boson exchange and the t-channel ν_e exchange. The s-channel diagrams arise as a consequence of the triple-gauge-boson vertices γWW and ZWW which are expected due to the non-Abelian gauge structure of the electroweak theory [1,3]. Results for triple-gauge-boson couplings derived from the data collected at $\sqrt{s} = 161$ GeV have been published by L3 [7,11] and the other LEP experiments [10,12]. Here a determination of triple-gauge-boson couplings is presented based on total and differential cross sections in W-pair mediated four-fermion production.

The L3 detector is described in detail in Ref. [13]. During the 1996 run the L3 detector collected total integrated luminosities of 10.9 pb^{-1} at $\sqrt{s} = 161.34$ GeV (threshold data), and 1.0 pb^{-1} and 9.3 pb^{-1} at $\sqrt{s} = 170.31$ GeV and at $\sqrt{s} = 172.32$ GeV (high-energy data). These centre-of-mass energies are known to ± 0.06 GeV [14]. The results obtained at threshold and from the high-energy data are combined to determine the mass of the W boson and triple-gauge-boson couplings.

2. Analysis of four-fermion production

The W boson decays into a quark-antiquark pair, such as W$^-$ → $\bar{\text{u}}$d or $\bar{\text{c}}$s, or a lepton-antilepton pair, $\text{W}^- \to \ell^- \bar{\nu}_\ell$ ($\ell = \text{e}, \mu, \tau$); in the following denoted as qq, $\ell\nu$ or ff in general for both W$^+$ and W$^-$ decays. All four-fermion final states expected in W-pair production are analysed:

1. $\text{e}^+\text{e}^- \to \text{qqe}\nu(\gamma)$,
2. $\text{e}^+\text{e}^- \to \text{qq}\mu\nu(\gamma)$,
3. $\text{e}^+\text{e}^- \to \text{qq}\tau\nu(\gamma)$,
4. $\text{e}^+\text{e}^- \to \ell\nu\ell\nu(\gamma)$,
5. $\text{e}^+\text{e}^- \to \text{qqqq}(\gamma)$,

where (γ) indicates the possible presence of radiative photons. The selections of these five four-fermion final states are described in detail in Ref. [7] for the threshold data and in Ref. [8] for the high-energy data.

These analyses reconstruct the visible fermions in the final state, i.e., electrons, muons, τ jets corresponding to the visible τ decay products, and hadronic jets corresponding to quarks [7,8]. Kinematic constraints as discussed below are then imposed to improve the resolution in the measured fermion energies and angles and to determine those not measured.

Parameters such as the mass or width of the W boson or triple-gauge-boson couplings are determined by comparing samples of Monte Carlo events to the data. A reweighting procedure is applied to construct Monte Carlo samples with different parameters. Selection, resolution and other detector effects are determined locally in phase space by averaging over Monte Carlo events inside a multi-dimensional box around each data event.

The following Monte Carlo event generators are used to simulate the various signal and background reactions: KORALW [15] and HERWIG [16] ($\text{e}^+\text{e}^- \to \text{WW} \to ffff(\gamma)$); EXCALIBUR [17] ($\text{e}^+\text{e}^- \to ffff(\gamma)$); PYTHIA [18] ($\text{e}^+\text{e}^- \to q\bar{q}(\gamma)$,ZZ$(\gamma)$, hadronic two-photon collisions); KORALZ [19] ($\text{e}^+\text{e}^- \to \mu^+\mu^-(\gamma)$, $\tau^+\tau^-(\gamma)$); BHAGENE3 [20] ($\text{e}^+\text{e}^- \to \text{e}^+\text{e}^-(\gamma)$). The response of the L3 detector is modelled with the GEANT [21] detector simulation program which includes the effects of energy loss, multiple scattering and showering in the detector materials and in the beam pipe.

2.1. Event reconstruction imposing kinematic constraints

The final states $qqe\nu$, $qq\mu\nu$ and $qqqq$ contain at most one unmeasured neutrino, so a kinematic fit is applicable. The kinematic fit determines energy, E_f, polar angle, θ_f, and azimuthal angle, ϕ_f, for all four fermions, f, in the final state. It adjusts the measurements of these quantities for the visible fermions according to their experimental resolutions to satisfy the constraints imposed. For hadronic jets, the velocity $\beta_f = |p_f|/E_f$ of the jet is kept at its measured value as systematic effects cancel in the ratio. Four-momentum conservation and equal mass of the two W bosons are imposed as constraints. They allow the determination of the unmeasured neutrino momentum vector. For $qqe\nu$ and $qq\mu\nu$ events, this yields a 2C kinematic fit, whereas for $qqqq$ events it is a 5C kinematic fit.

The kinematic fit mainly improves the energy resolution and less the angular resolutions. The resolutions in average invariant mass, M_{inv}, typically improve by a factor of three.

For $qq\tau\nu$ and $\ell\nu\ell\nu$ events, the event contains at least two unmeasured neutrinos in the final state. In case of $qq\tau\nu$ events, the energies of the two hadronic jets are rescaled by a common factor so that their sum equals half the centre-of-mass energy. The τ direction of flight is approximated by the direction of the visible τ jet. The τ energy and the neutrino momentum vector are then determined by overall energy-momentum conservation. This yields two equal-mass W bosons. The $\ell\nu\ell\nu$ events are used in the determination of triple-gauge-boson couplings only.

2.2. Fitting method for mass, width and gauge couplings

The maximum likelihood method is used to extract values and errors of parameters, Ψ, such as the mass and total width of the W boson or triple-gauge-boson couplings. The fit considers a set of values of reconstructed quantities Ω for each data event, which are either the average invariant mass, M_{inv}, or phase-space angles describing the four-fermion final state (see Section 2.4 below). The data are treated as unbinned; the total likelihood is the product of the normalised differential cross section, $L(\Omega,\Psi)$, for all data events. For a given four-fermion final state i, one has:

$$L_i(\Omega_i,\Psi) = \frac{1}{\sigma_i(\Psi) + \sigma_i^{\mathrm{BG}}} \times \left[\frac{\mathrm{d}\sigma_i(\Omega_i,\Psi)}{\mathrm{d}\Omega_i} + \frac{\mathrm{d}\sigma_i^{\mathrm{BG}}(\Omega_i)}{\mathrm{d}\Omega_i} \right],$$

where σ_i and σ_i^{BG} are the accepted signal and background cross sections. The total and differential cross sections of the accepted background are independent of the parameters Ψ of interest. They are taken directly from Monte Carlo simulations.

The total and differential signal cross sections depend on Ψ. For values Ψ_{fit} varied during the fitting procedure, they are determined by a reweighting procedure applied to Monte Carlo events originally generated with parameter values Ψ_{gen}. For mass and width fits, the event weights R_i are given by the ratio:

$$R_i(m_1,m_2,\Psi_{\mathrm{fit}},\Psi_{\mathrm{gen}}) = \frac{\mathrm{d}^2\sigma_i(s,m_1,m_2,\Psi_{\mathrm{fit}})/\mathrm{d}m_1\mathrm{d}m_2}{\mathrm{d}^2\sigma_i(s,m_1,m_2,\Psi_{\mathrm{gen}})/\mathrm{d}m_1\mathrm{d}m_2},$$

where m_1 and m_2 are the invariant masses of the two generated W bosons. The differential cross sections are calculated with the GENTLE [22] program. For couplings fits, the event weights R_i are calculated as the ratio:

$$R_i(p_1,p_2,p_3,p_4,k_\gamma,\Psi_{\mathrm{fit}},\Psi_{\mathrm{gen}}) = \frac{\left|\mathcal{M}_i(p_1,p_2,p_3,p_4,k_\gamma,\Psi_{\mathrm{fit}})\right|^2}{\left|\mathcal{M}_i(p_1,p_2,p_3,p_4,k_\gamma,\Psi_{\mathrm{gen}})\right|^2},$$

where $\mathcal{M}_i$ is the matrix element of the four-fermion final state i under consideration evaluated for the generated four-vectors $(p_1,p_2,p_3,p_4,k_\gamma)$ of the four fermions and any radiative photons. The matrix elements as implemented in the EXCALIBUR [17] event generator are used, which include all relevant tree-level Feynman diagrams contributing to a given four-fermion final state. The total accepted signal cross section for a given set of parameters Ψ_{fit} is then:

$$\sigma_i(\Psi_{\mathrm{fit}}) = \frac{\sigma_i^{\mathrm{gen}}}{N_i^{\mathrm{gen}}} \cdot \sum_j R_i(j,\Psi_{\mathrm{fit}},\Psi_{\mathrm{gen}}),$$

where σ_i^{gen} denotes the cross section corresponding to the total Monte Carlo sample containing N_i^{gen} events. The sum extends over all accepted Monte Carlo events j. The accepted differential signal cross section in reconstructed quantities Ω_i is determined by averaging Monte Carlo events inside a box in Ω_i around each data event [23]:

$$\frac{\mathrm{d}\sigma_i(\Omega_i,\Psi_{\mathrm{fit}})}{\mathrm{d}\Omega_i} = \frac{\sigma_i^{\mathrm{gen}}}{N_i^{\mathrm{gen}}} \cdot \frac{1}{\Delta_i^{\Omega}} \sum_{j\in\Delta_i^{\Omega}} R_i(j,\Psi_{\mathrm{fit}},\Psi_{\mathrm{gen}}),$$

where Δ_i^{Ω} is the volume of the box and the sum extends over all accepted Monte Carlo events j inside the box. This takes Ω_i-dependent detector effects and Ψ-dependent efficiencies and purities properly into account.

In addition, extended maximum likelihood fits are performed by including the overall normalisations according to the measured total W-pair cross sections. The likelihood is multiplied by the Poissonian probabilities to obtain the numbers of events observed in the data [7,8] given the integrated luminosities and the expectations for the total accepted signal and background cross sections, $\sigma_i(\Psi_{\mathrm{fit}})$ and σ_i^{BG}, at all centre-of-mass energies.

The fit method described above determines the parameters without any bias as long as the Monte Carlo describes photon radiation (ISR) and detector effects such as resolution and acceptance functions correctly. By fitting large Monte Carlo samples, typically a hundred times the data, the fitting procedure is tested to high accuracy. The fits reproduce well the values of the parameters of the large Monte Carlo samples being fitted. Also, the fit results do not depend on the values of the parameters Ψ_{gen} of the Monte Carlo sample subjected to the reweighting procedure.

2.3. Mass and width of the W boson

For mass and width fits, the weighted average of the two invariant masses in an event, M_{inv}, as determined by the kinematic fit imposing the equal-mass constraint, is fitted. The size of the box around each data event is limited by the requirement of including no more than 1000 Monte Carlo events, yielding box sizes of about ±35 MeV at the peak of the invariant mass distribution. In addition, the box size may not be larger than ±250 MeV around M_{inv}.

Based on the high-energy data, the mass of the W boson is determined for each of the final states $qqe\nu$ (19 events), $qq\mu\nu$ (9 events), $qq\tau\nu$ (12 events) and $qqqq$ (61 events) in separate maximum likelihood fits. Combined results are determined by multiplying the likelihood of the individual channels. For mass fits in the $qqqq$ channel, the pairing algorithm to assign jets to W bosons used in the event selection [8] is changed. The pairing yielding the highest likelihood in the 5C kinematic fit is chosen. The rate of correct pairings is reduced to 60% for the best combination and it is 25% for the second best combination. However, the signal-to-background ratio in the relevant signal region around $M_{\mathrm{inv}} \approx 80\,\mathrm{GeV}$ is improved. The loss of correct pairings is recovered by including the pairing with the second highest likelihood in the fits. Monte Carlo studies show that the two values for M_{W} obtained from fitting the distributions of the best and the second best combination separately have a correlation of (1.4 ± 2.2)%, which is negligible.

The observed invariant mass distributions together with the fit results are shown in Figs. 1 and 2. The results on M_{W} are summarised in Table 1. The observed statistical errors agree well with the statistical errors expected for the size of the high-energy data samples used. Systematic errors on the fitted W masses are summarised in Table 2. Hadronisation and fragmentation effects are determined by comparing different Monte Carlo programs to simulate the signal. Effects due to background are determined by varying both the total accepted background cross section and the shape of the invariant mass spectrum. Detector effects due to uncertainties in the energy scale of electrons, muons and hadronic jets and the corresponding resolutions are estimated by varying

Table 1
Results on the mass of the W boson, M_{W}, for the individual four-fermion final states in W-pair production, and their combination. The first error is statistical and the second systematic

Process	Mass of the W boson M_{W} [GeV]
$e^+e^- \to \mathrm{qqe}\nu(\gamma)$	$80.25^{+0.68}_{-0.70} \pm 0.09$
$e^+e^- \to \mathrm{qq}\mu\nu(\gamma)$	$80.94^{+1.15}_{-1.33} \pm 0.08$
$e^+e^- \to \mathrm{qq}\tau\nu(\gamma)$	$80.43^{+1.07}_{-1.06} \pm 0.09$
$e^+e^- \to qq\ell\nu(\gamma)$	$80.42^{+0.53}_{-0.55} \pm 0.07$
$e^+e^- \to \mathrm{qqqq}(\gamma)$	$80.91^{+0.41}_{-0.44} \pm 0.13$
$e^+e^- \to ffff(\gamma)$	$80.71^{+0.34}_{-0.35} \pm 0.09$